APPENDIX E

Implementation of WIM Error Filtration Algorithm on Load Effect Histogram

The procedure is presented through an example based on the WIM data collected at the I-81 site in NY. To verify the validity of the proposed approach for filtering out the WIM errors, the WIM histogram for the moment effect of a simply support 60-ft bridge span is used. The histogram of the moment effect on a 60-ft simple span for the I-81 WIM data as collected on site is shown in Figure E-1. The moments are normalized to the moment of the HL-93 design truck. The frequencies are also normalized and presented in percent.

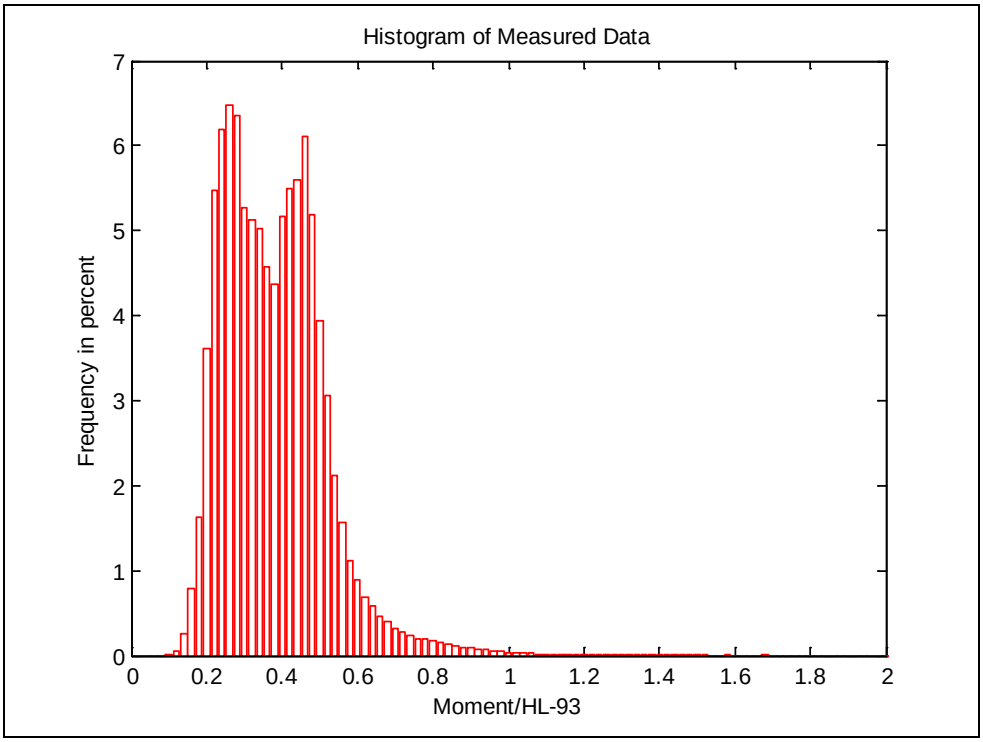

Figure E-1 Histogram of load effect of I-81 data on a simple 60-ft span.

In this example, because of the lack of site-specific calibration data, we are assuming that the WIM calibration error data of the I-87 site as provided in Chapter 3, Tables 16 and 17 are valid for this I-81 site also. This assumption is only used to illustrate the procedure and should not be made for actual calculations of maximum load effect L_{max} because the WIM calibration errors are highly site dependent.

A simulated histogram of a random sample of 10,000 values of the error, ε, with mean=1.04 and standard deviation =0.078 is obtained as shown in Figure E-2 and plotted against the histogram obtained from the Normal distribution function. This comparison serves to illustrate that the random sample of the error is consistent with the assumption of a Normal probability distribution.

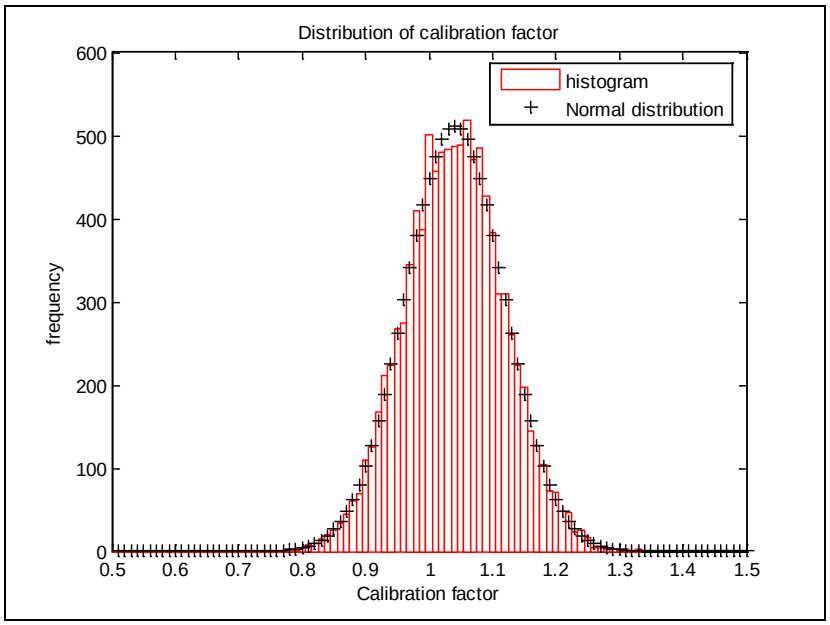

Figure E-2 Histogram of simulated WIM error distribution.

The application of the WIM error algorithm produces the results that are shown in black + signs in Figure E-3. These + signs give the frequency in percent for each of the bins for the histogram of the actual 60-ft simple span moment effects of the trucks that crossed this bridge after filtering out the WIM errors. The results of the filtration are compared to those obtained through a Monte Carlo simulation assembled into the red histogram of Figure E-3. The Monte Carlo simulation consisted of randomly generating 10,000 samples of truck moment effects, x_m, from the measured WIM data histogram of Figure E-1. Also, a random sample of 10,000 errors, ε, was generated from the calibration error histogram of Figure E-2. By taking a value of x_m and dividing by one value of ε, we obtain a single value of x_r. Repeating the process 10,000 times, a sample of 10,000 x_r values is obtained and assembled into the red histogram of Figure E-3.

A visual comparison between the values indicated by the symbol + and the red histogram show overall good agreement. In order to obtain an objective measure of the differences, a normalized error function is used that has been defined as:

$$E = \frac{\sum_{i=1}^{N} (e_i - n_i)^2}{e_i^2} \qquad \text{(E-1)}$$

Where N is the total number of bins, n_i is the number of samples that fall in bin i from the simulation where the total number of simulated samples is 10,000, e_i is the total number of samples in bin i obtained from the filtered histogram re-normalized for a total number of 10,000 samples. The error E was found to be equal to E=46.55 when comparing the simulated no-error histogram to the filtered histogram. However, when n_i represents the number of samples of measured WIM data that fall in bin i, the error was calculated to be E=91.70 confirming the improved accuracy of the filtered histogram in representing the actual load effect of the trucks when compared to the measured WIM data histogram. In all cases, the number of samples in each bin is re-normalized to provide an overall total number of 10,000 samples in all the bins.

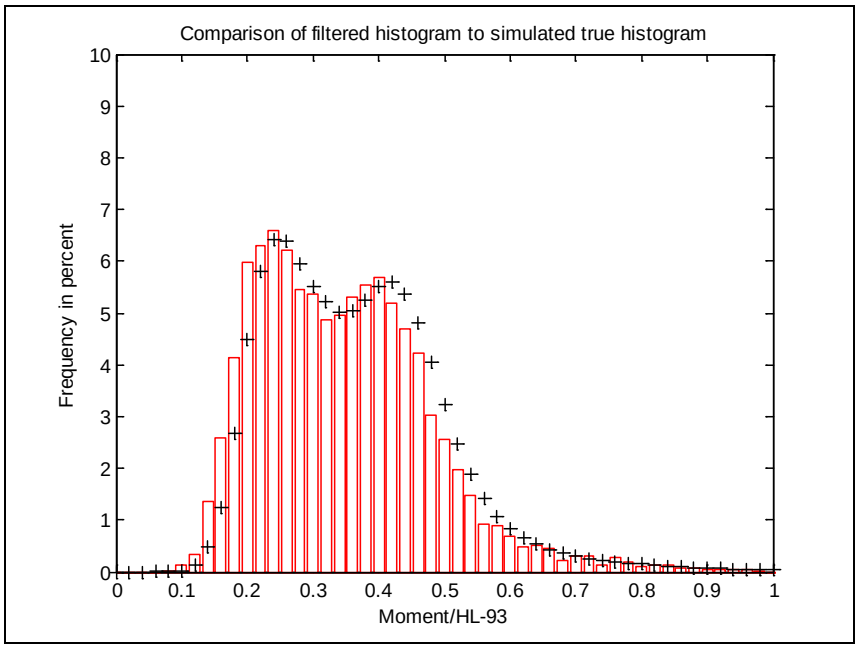

Figure E-3 Histogram of simulated WIM error distribution.

The results of the filtration and those of the originally measured data are plotted on Normal probability curves in Figure E-4. For the regression fit of the tail end, the highest values are not considered to reduce the effect of the numerical errors introduced when the bins are very lightly populated. The plots show that the tail end of the original histogram representing the upper 5% of the data as collected from the original WIM data may be represented on the Normal probability plot by a straight line represented by the equation:

$$y = 3.0159x - 0.0784 \qquad \text{(E-2)}$$

This indicates that the tail end of the WIM data matches that of normal distribution with a mean μ_{event} =0.026 and a standard deviation σ_{event}=0.332. Similarly, the filtered histogram's upper tail can be modeled by a Normal distribution with mean μ_{event} =0.089 and a standard deviation σ_{event}=0.326. The tail end of the histogram obtained from simulation matches that of Normal distribution with mean μ_{event} =0.020 and a standard deviation σ_{event}=0.326.

The above calculated means and standard deviations are then implemented into the equations for calculating the maximum load effect in a 75-year return period, L_{max} which are given by applying the following steps:

- Set the number of events in 75 years given n_{day}=2000 truck loading events/day to be:

$$N = n_{day} * 365 * 75 \qquad \text{(E-3)}$$

- Find the most probable value, u, for the Gumbel distribution that models the maximum value in 75 years L_{max} given by:

$$u_N = \mu_{event} + \sigma_{event} \times \left[\sqrt{2\,ln(N)} - \frac{ln(ln(N)) + ln(4\pi)}{2\sqrt{2\,ln(N)}} \right] \quad \text{(E-4)}$$

- Find the dispersion coefficient for the Gumbel distribution that models the maximum load effect L_{max} as:

$$\alpha_N = \frac{\sqrt{2\,ln(N)}}{\sigma_{event}} \quad \text{(E-5)}$$

- The mean value of L_{max} is given as:

$$L_{max} = u_N + \frac{0.577216}{\alpha_N} \quad \text{(E-6)}$$

- The standard deviation of the Gumbel distribution that best models the maximum 75-year load effect is given as:

$$\sigma_{max} = \frac{\pi}{\sqrt{6}\alpha_N} \quad \text{(E-7)}$$

The application of the above equations lead to L_{max}=1.887 and σ_{max}=0.071 for the original WIM data, L_{max}=1.920 and σ_{max}=0.001 for the filtered data, and L_{max}=1.853 and σ_{max}=0.070 for the simulated data, where the differences in L_{max} are less than 2%. This small difference in the estimated L_{max} values is due to the small standard deviation of 0.078 in the WIM error as observed during the calibration of the WIM system. A sensitivity analysis is performed in the next section to study the effect of the WIM errors on the estimated L_{max} values.

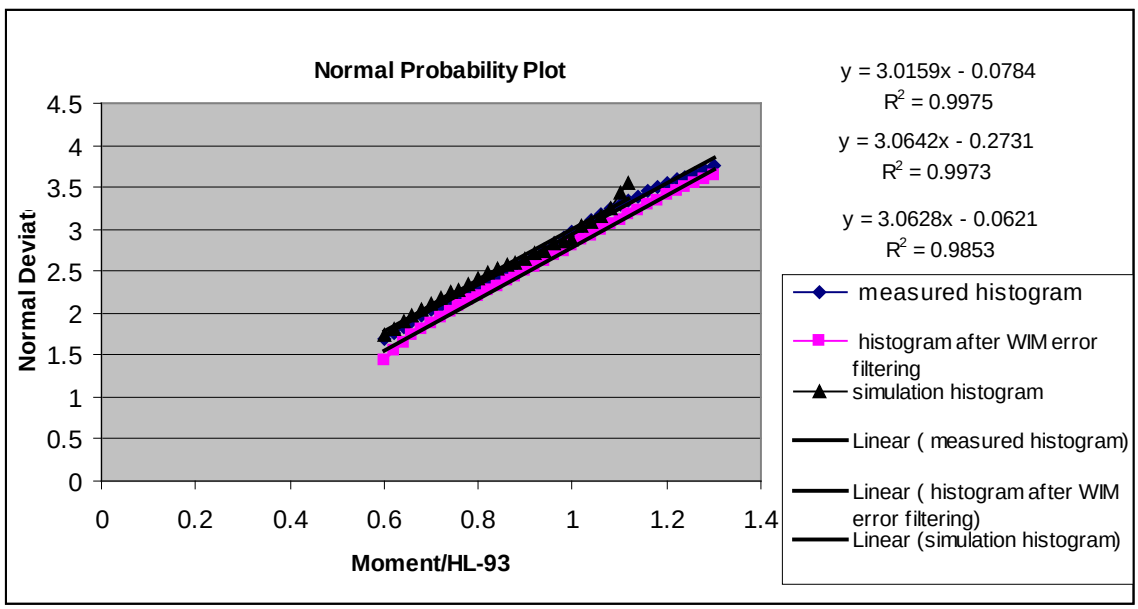

Figure E-4 Normal Probability plots of upper 5% of original and filtered histograms

Sensitivity Analysis

The analysis performed above uses a mean value for the calibration error ε, with mean=1.04 and a standard deviation =0.078. A comparison between the filtered histogram and the original measured histogram is provided in Figure E-5a and Figure E-5b where the latter shows the tail end of the histogram. The comparison between the filtered histogram and the one obtained from a Monte Carlo simulation of a histogram without measurement errors is provided in Figure E-5c and Figure E-5d where the latter gives a comparison of the tail ends of the histograms. The visual comparison shows little difference between all three histograms (measured, filtered and simulated with no WIM errors). However, the application of the error function of Equation E-2 confirms that the simulated histogram with no error and the filtered histogram are compatible as evidenced by a reduction of the error obtained when comparing the measured histogram to the simulated no-error histogram.

When the standard deviation of the WIM calibration error is assumed to increase from 0.078 to 0.10, 0.15 and 0.20, the application of the filtration algorithm will result in a large improvement in the match between the filtered histogram and the simulated no-error histogram. This comparison is presented in Figures E-6, E-7 and E-8 for the assumed higher standard deviations. The figures show a large improvement in the match between the filtered histogram and the simulated histogram as opposed to the comparison between filtered histogram and the measured histogram. The fourth and fifth columns of Table E-1 also provide the error function confirming that the filtered histogram and the simulated no-error histogram are compatible.

Table E-1 Sensitivity Analysis Cases for 60-ft moment.

	Mean of WIM calib. error $\bar{\varepsilon}$	Standard deviation σ_ε	E (filtered vs simulated)	E (filtered vs measured)	L_{max} (% difference from Measured data L_{max} =1.887)
Case 1	1.04	0.078	46.55	91.68	1.920 (1.75%)
Case 2	1.00	0.10	62.21	84.54	2.015 (6.78%)
Case 3	1.00	0.15	80.39	95.79	2.134 (13.1%
Case 4	1.00	0.20	73.39	105.9	2.340 (24.0%)

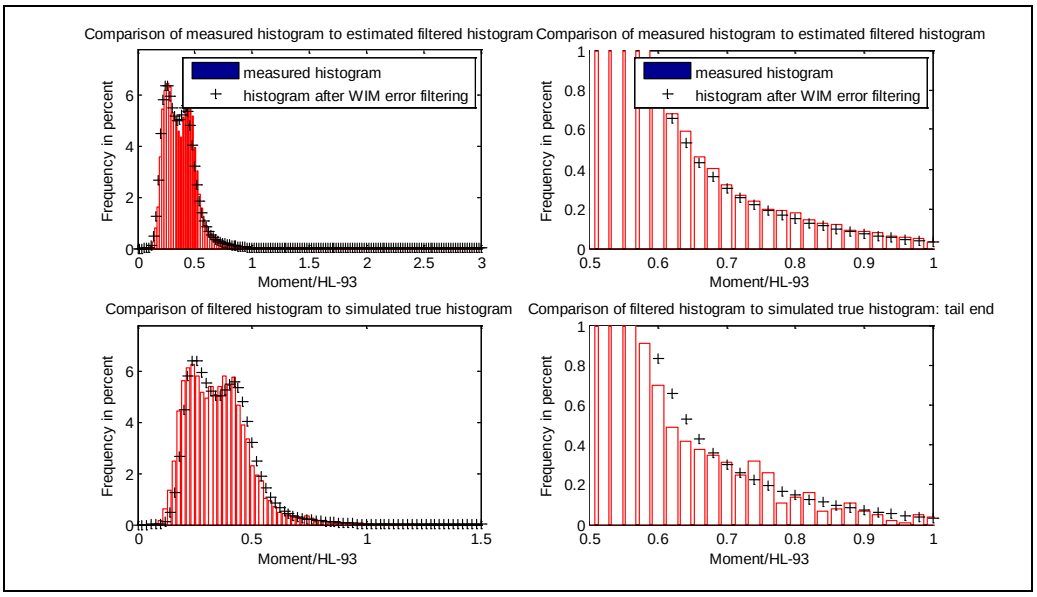

Figure E-5 Comparison of Filtered histogram to measured and simulated no-error histogram

for $\bar{\varepsilon}$ *=1.04 and* σ_{ε}*=0.078*

a) Comparison between measured histogram and filtered histogram.
b) Zoom on tail ends of filtered and measured histograms.
c) Comparison between simulated true histogram and filtered histogram.
d) Zoom on tail ends of filtered and simulated true histograms

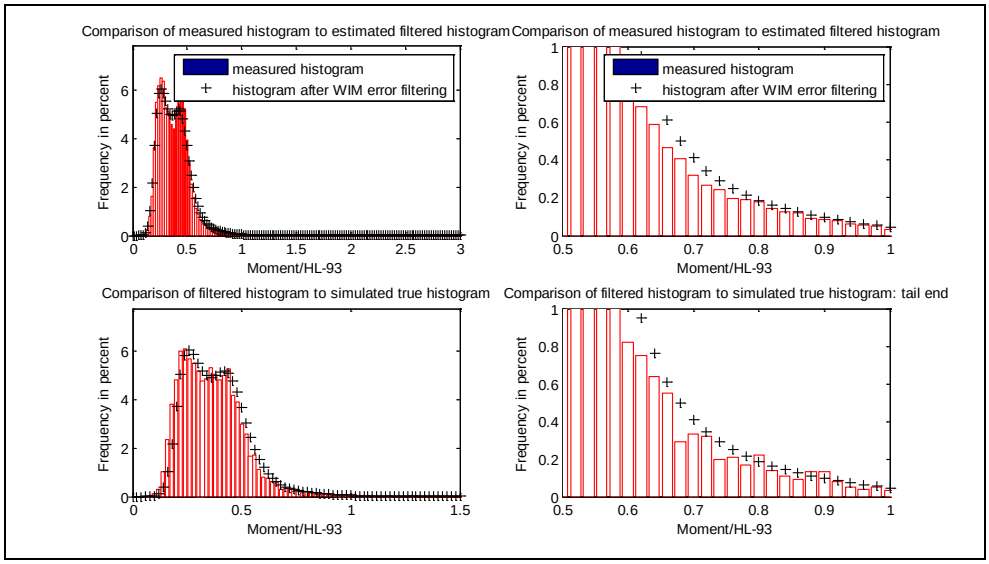

Figure E-6 Comparison of Filtered histogram to measured and simulated no-error histogram

for $\bar{\varepsilon}$ *=1.0 and* σ_{ε}*=0.10*

a) Comparison between measured histogram and filtered histogram.
b) Zoom on tail ends of filtered and measured histograms.
c) Comparison between simulated true histogram and filtered histogram.
d) Zoom on tail ends of filtered and simulated true histograms

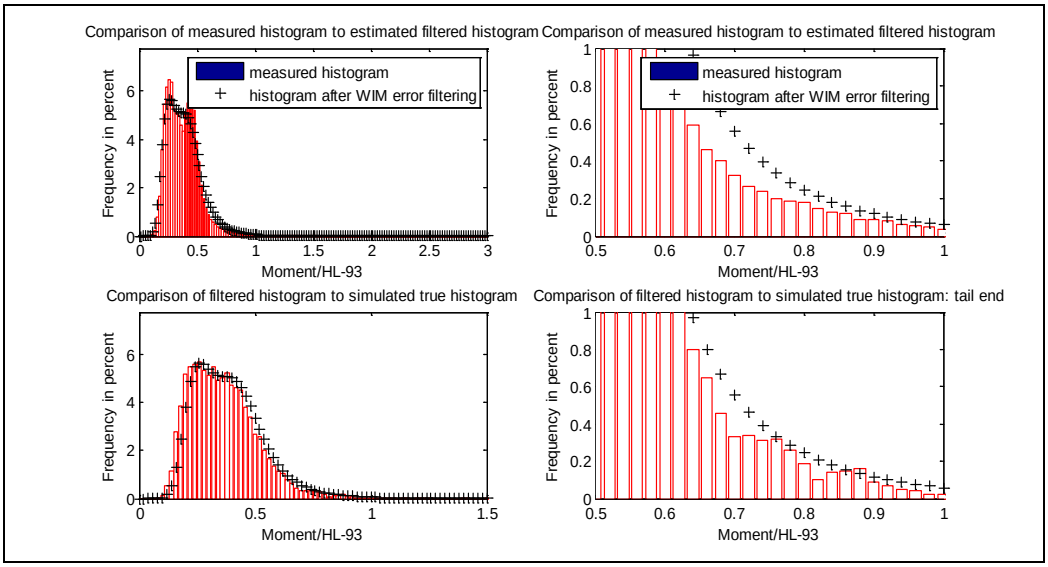

Figure E-7 Comparison of Filtered histogram to measured and simulated no-error histogram

for $\bar{\varepsilon}$ *=1.0 and* σ_ε*=0.15*

a) Comparison between measured histogram and filtered histogram.
b) Zoom on tail ends of filtered and measured histograms.
c) Comparison between simulated true histogram and filtered histogram.
d) Zoom on tail ends of filtered and simulated true histograms

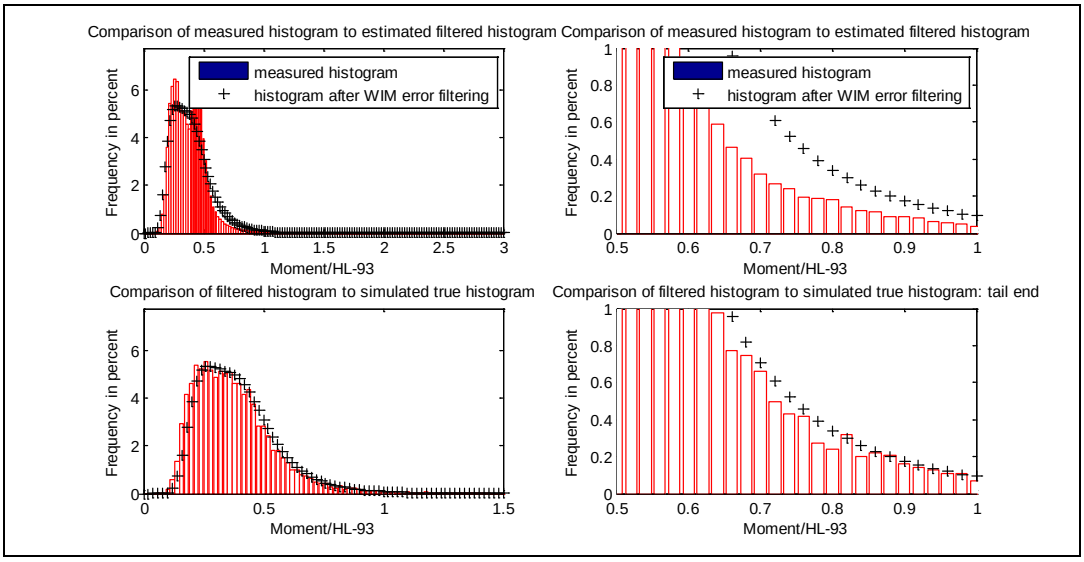

Figure E-8 Comparison of Filtered histogram to measured and simulated no-error histogram

for $\bar{\varepsilon}$ *=1.0 and* σ_ε*=0.20*

a) Comparison between measured histogram and filtered histogram.
b) Zoom on tail ends of filtered and measured histograms.
c) Comparison between simulated true histogram and filtered histogram.
d) Zoom on tail ends of filtered and simulated true histograms

Implementation of WIM Error Filtration Algorithm on Axle Weight Histogram

The Coefficient of Variation (COV) in the WIM errors for the moment load effect may sometimes be reduced due to the random nature of the errors that may be higher on some of the axles and lower on some other axles. However, this compensating effect would not be triggered when dealing with deck design for which the axle weight is the controlling loading parameter. For this reason, it may be more important to filter out the WIM errors when analyzing the axle weight histograms as compared to load effect histograms. In this section, the filtration algorithm is further tested by analyzing the axle weight histograms collected at three sites in New York State. These sites are site 0580 (over I-495), 2680 (over Rt. 12) and 9121 (over I-81). Each site is analyzed for four difference cases:

1. WIM calibration error with mean = $\bar{\varepsilon}$ =0.97 and standard deviation σ_ε=0.10 corresponding to the WIM error for axle weights as listed in Chapter 3 Table 17.
2. An assumed WIM calibration error with mean = $\bar{\varepsilon}$ =1.0 and standard deviation σ_ε=0.15.
3. An assumed WIM calibration error with mean = $\bar{\varepsilon}$ =1.0 and standard deviation σ_ε=0.20.
4. An assumed WIM calibration error with mean = $\bar{\varepsilon}$ =1.0 and standard deviation σ_ε=0.40.

Cases 2–4 are used as a sensitivity analysis to determine at which level of the WIM calibration error the filtration become necessary. The filtered histogram for each of the cases studied is first compared to the measured histogram, which includes the WIM errors and a simulated histogram where the WIM errors are filtered out using a Monte Carlo simulation procedure. In all the cases studied, the filtered histogram shows good agreement with the simulated histogram. For low standard deviations of the WIM errors, the filtered histogram is reasonably close to the measured histogram. When the L_{max} value calculated from the measured data is compared to the L_{max} value calculated from the filtered data, the difference is on the order of 7% to 13% when the standard deviation of the WIM calibration error is 10% as shown in the summary of results given in Tables E-2 thru E-4. The difference increases dramatically as the standard deviation of the WIM error increases beyond 10%. These results illustrate the importance of filtering out the WIM errors if the standard deviation of the WIM error is high on the order of 0.10 or higher. For low standard deviations, there is little difference between the filtered histogram and the originally measured histogram.

Conclusion

This report presented a procedure to filter out the WIM calibration errors from the measured WIM histograms of gross weights or load effects. A comparison between the L_{max} values obtained from the filtered histograms to those obtained from the measured histograms show the importance of filtering out the errors when the WIM calibration error has a standard deviation higher than 0.10.

Table E-2. Sensitivity Analysis Cases for axle weights of I-495 site

Case	Mean of WIM calib. error $\bar{\varepsilon}$	Standard deviation σ_ε	E (filtered vs simulated)	E (filtered vs measured)	L_{max} (% difference from Measured data L_{max} =97.88)
Case 1	0.97	0.10	14.08	69.77	104.73 (7.0%)
Case 2	1.00	0.15	36.24	71.20	108.16 (10.5%)
Case 3	1.00	0.20	30.08	76.69	119.67 (22.3%)
Case 4	1.00	0.40	77.34	85.00	213.87 (118%)

Table E-3 Sensitivity Analysis Cases for axle weights of Route 12 site

Case	Mean of WIM calib. error $\bar{\varepsilon}$	Standard deviation σ_ε	E (filtered vs simulated)	E (filtered vs measured)	L_{max} (% difference from Measured data L_{max}=61.52)
Case 1	0.97	0.10	3.79	12.96	69.98 (13.8%)
Case 2	1.00	0.15	12.96	79.26	75.99 (23.5%)
Case 3	1.00	0.20	5.11	85.03	89.07 (44.8%)
Case 4	1.00	0.40	52.45	87.63	168.15 (173%)

Table E-4 Sensitivity Analysis Cases for axle weights of I-81 site

Case	Mean of WIM calib. error $\bar{\varepsilon}$	Standard deviation σ_ε	E (filtered vs simulated)	E (filtered vs measured)	L_{max} (% difference from Measured data L_{max}=72.37)
Case 1	0.97	0.10	7.98	76.76	81.76 (13.0%)
Case 2	1.00	0.15	8.72	80.06	89.06 (23.1%)
Case 3	1.00	0.20	19.77	84.94	106.26 (46.8%)
Case 4	1.00	0.40	65.54	90.08	209.49 (189%)