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# Distribution of Wheel Loads on Highway Bridges

*An NCHRP digest of the findings from the final report on NCHRP Project 12-26,* "Distribution of Wheel Loads on Highway Bridges," *conducted by Imbsen & Associates, Inc., Dr. Toorak Zokaie, Principal Investigator.*

## INTRODUCTION

NCHRP Project 12-26 was initiated in the mid-1980s in order to develop comprehensive specification provisions for distribution of wheel loads in highway bridges. The study was performed in two phases: Phase I concentrated on beam-and-slab and box girder bridges; Phase II concentrated on slab, multibox beam, and spread box beam bridges.

Three levels of analysis were considered for each bridge type. The most accurate level, level 3, involves detailed modeling of the bridge deck. Level 2 includes either graphical methods, nomographs and influence surfaces, or simplified computer programs. Level 1 methods provide simple formulas to predict lateral load distribution, using a wheel load distribution factor applied to a truck wheel line to obtain the longitudinal response of a single girder.

The formulas developed in this study for the level 1 analysis were based on the standard AASHTO "HS" trucks. Levels 2 and 3 analyses may be applied for trucks outside the AASHTO family of trucks. A limited parametric study conducted as part of this research showed that variations in the truck axle configuration or truck weight do not significantly affect the wheel load distribution factors. However, it is anticipated that smaller gage widths would result in larger distribution factors and larger gage widths would result in smaller distribution factors. Figure 1 shows a schematic drawing of various truck types and their axle configurations used in the parametric study mentioned above. Table 1 gives the variation of wheel load distribution factors with different axle configurations applied to a number of beam-and-slab bridges. The differences were below 1% in most cases, and in all cases the formulas resulted in good predictions. Therefore, with some caution, these formulas may be applied to other truck types.

The major part of this research was devoted to the level 1 analysis methods because of its ease of application, established use, and the surprisingly good correlation with the higher levels of analysis in their application to a majority of bridges. The formulas presented in the current AASHTO specifications were evaluated, and alternate formulas were developed that offer improved accuracy, wider range of applicability, and in some cases, easier application than the current AASHTO formulas. These formulas were developed for interior and exterior girder moment and shear load distribution for single or multiple lane loadings. In addition, correction factors for continuous superstructures and skewed bridges were developed.

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These formulas present a complete, consistent, accurate set for simplified analysis of new and existing bridges and have been submitted to the AASHTO Highway Subcommittee for Bridges and Structures for consideration for adoption. If adopted by the Subcommittee, the formulas will replace the existing wheel load distribution provisions in the AASHTO *Standard Specifications for Highway Bridges*.

In addition, the study resulted in recommendations for use of computer programs to achieve more accurate results. The recommendations focus on the use of plane grid analysis as well as detailed finite element analysis, where different truck types and their combinations may be considered.

## THE PROBLEM AND ITS SOLUTION

Wheel load distribution on highway bridges is a key response quantity in determining member size and, consequently, strength and serviceability. It is of critical importance both in the design of new bridges and in the evaluation of the load-carrying capacity of existing bridges.

Using wheel load distribution factors, engineers can predict bridge response by treating the longitudinal and transverse effects of wheel loads as uncoupled phenomena. Empirical wheel load distribution factors for stringers and longitudinal beams have appeared in the AASHTO *Standard Specifications for Highway Bridges* with only minor changes since 1931. Findings of recent studies suggest a need to update these specifications in order to provide improved predictions of wheel load distribution.

Wheel load distribution is a function of the magnitude and location of truck wheel loads and the response of the bridge to these loads. This study focused on the second factor mentioned above: the response of the bridge to a predefined set of wheel loads, namely, the HS family of trucks.

The current AASHTO specifications allow for simplified analysis of bridge superstructures using the concept of a wheel load distribution factor for bending moment in interior girders of most types of bridges, i.e., beam-and-slab, box girder, slab, multi-box beam, and spread box beam. This distribution factor is given by:

$$g = S/D \tag{1}$$

where $g$ = a factor used to multiply the total longitudinal response of the bridge due to a single longitudinal line of wheel loads in order to determine the maximum response of a single girder; $S$ = the center-to-center girder spacing; and $D$ = a constant that varies with bridge type and geometry.

A major shortcoming of the current specifications is that the piecemeal changes that have taken place over the last 55 years have led to inconsistencies in the load distribution criteria including: inconsistent consideration of a reduction in load intensity for multiple lane loading; inconsistent changes in distribution factors to reflect the changes in design lane width; and, inconsistent approaches for verification of wheel

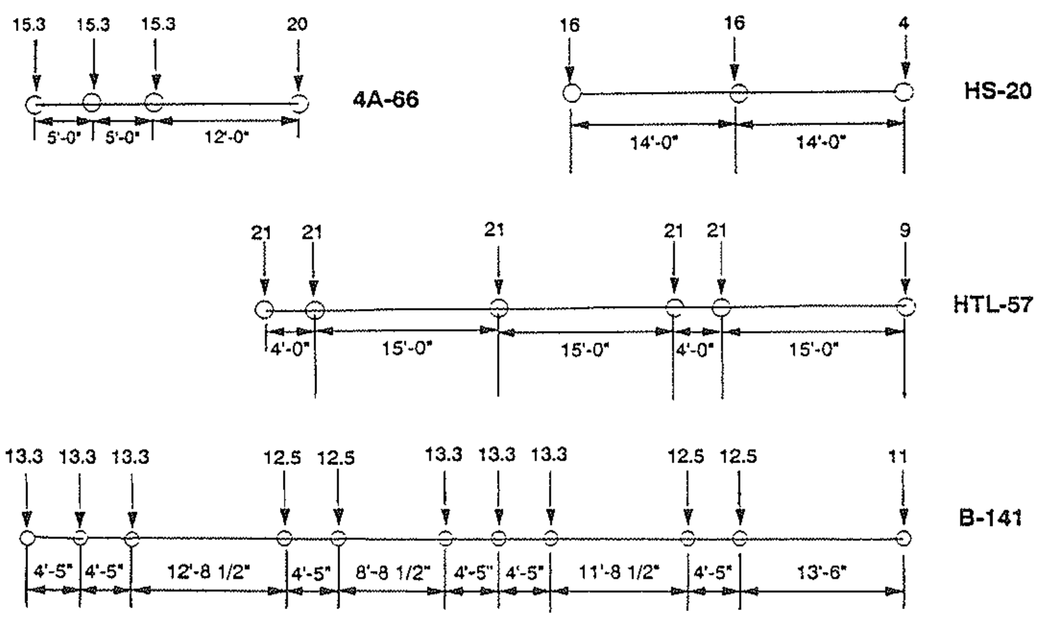


Figure 1. *Axle configurations for truck types considered in study*

Table 1. Distribution factors for girder moment for various truck types and two-lane loading

| | DISTRIBUTION FACTOR (g) | | | | | PERCENT DIFFERENCE WITH HS-20 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | HS-20 | HTL-57 | 4A-66 | B-141 | NCHRP 12-26 | HTL-57 | 4A-66 | B-141 | NCHRP 12-26 |
| Average* | 1.293 | 1.261 | 1.285 | 1.268 | 1.304 | -2.4 | -0.6 | -1.9 | +0.9 |
| Max. S (16') | 2.220 | 2.162 | 2.205 | 2.178 | 2.308 | -2.6 | -0.7 | -1.9 | +4.0 |
| Min. S (3.5') | 0.713 | 0.717 | 0.713 | 0.715 | 0.755 | +0.6 | 0.0 | +0.3 | +5.9 |
| Max. L (200') | 0.982 | 0.958 | 0.983 | 0.952 | 1.033 | -2.4 | +0.1 | -3.1 | +5.2 |
| Min. L (20') | 1.630 | 1.625 | 1.624 | 1.623 | 1.807 | -0.3 | -0.3 | -0.4 | +10.9 |

*S = 7.5'; L = 64'; $t_s$ = 7.25"; $K_g$ = 560,000 in$^4$

load distribution factors for various bridge types.

The current AASHTO simplified procedures were developed for nonskewed, simply-supported bridges. Although the current specifications state that these procedures apply to the design of normal (i.e., supports oriented perpendicular to the longitudinal girders) highway bridges, there are no other guidelines for determining when the procedures are applicable. Because modern highway and bridge design practice requires a large number of bridges to be constructed with skewed supports, on curved alignments, or continuous over interior supports, it is increasingly important that the limitations of wheel load distribution criteria be fully understood by designers.

Advanced computer technology has become available in recent years which allows detailed finite element analysis of bridge decks. However, many computer programs exist which employ different formulations and techniques. It is important that the computer methodology and formulation that produces the most accurate results be used to predict the behavior of bridge decks. In order to identify the most accurate computer programs, data from full scale and prototype bridge load tests were compiled. The bridge tests were then modeled by different computer programs and the experimental and computer results were compared. The programs that produced the most accurate results were then considered as the basis for evaluation of the other method levels, i.e., levels 2 and 1 methods.

An important part of the development or evaluation of simplified methods is range of applicability. In order to ensure that common values of various bridge parameters were considered, a database of actual bridges was compiled. Bridges from various states were randomly selected in order to achieve national representation. This resulted in a database of 365 beam-and-slab bridges, 112 prestressed concrete and 121 reinforced concrete box girder bridges, 130 slab bridges, 67 multi-box beam bridges, and 55 spread box beam bridges. This bridge database was studied to identify the common values of various parameters such as beam spacing, span length, slab thickness, and so on. The range of variation of each parameter was also identified. A hypothetical bridge that has all the average properties obtained from the database, referred to as the "average bridge," was created for each of the beam-and-slab, box girder, slab, multibox beam, and spread box beam bridge types. For the study of moment responses in box girder bridges, separate reinforced concrete and prestressed concrete box girder average bridges were also prepared.

In evaluating simplified formulas, it is important to understand the effect of various bridge parameters on wheel load distribution. Bridge parameters were varied one at a time in the average bridge for the bridge type under consideration. Wheel load distribution factors for both shear and moment were obtained for all such bridges. Variation of wheel load distribution factors with each parameter shows how important that parameter is. Simplified formulas can then be developed to capture the variation of wheel load distribution factors with each of the important parameters. A brief description of the method used to develop such formulas is as follows.

In order to derive a formula in a systematic manner certain assumptions must be made. First, it is assumed that the effect of each parameter can be modeled by an exponential function of the form $ax^b$, where $x$ is the value of the given parameter, and $a$ and $b$ are coefficients to be determined based on the variation of $x$. Second, it is assumed that the effects of different parameters are independent of each other, which allows each parameter to be considered separately. The final distribution factor will be modeled by an exponential formula of the form: $g = (a)(S^{b1})(L^{b2})(t^{b3})(\ldots)$ where $g$ is the wheel load distribution factor; $S$, $L$, and $t$ are parameters included in the formula; $a$ is the scale factor; and $b1$, $b2$, and $b3$ are determined from the variation of $S$, $L$, and $t$, respectively. Assuming that for two cases all bridge parameters are the same except for $S$, then:

$$g^1 = (a)(S_1^{\,b1})(L^{b2})(t^{b3})(\ldots) \tag{2a}$$

$$g^2 = (a)(S_2^{\,b1})(L^{b2})(t^{b3})(\ldots) \tag{2b}$$

therefore:

$$g_1 / g_2 = (S_1 / S_2)^{b1} \tag{3}$$

or:

$$b1 = [\ln(g_1 / g_2)] / [\ln(S_1 / S_2)]. \tag{4}$$

If $n$ different values of $S$ are examined and successive pairs are used to determine the value of $b1$, $n-1$ different values for $b1$ can be obtained. If these $b1$ values are close to each other, an exponential curve may be used to accurately model the variation of the distribution factor with $S$. In that case the average of $n-1$ values of $b1$ is used to achieve the best match. Once all the power factors (i.e., $b1$, $b2$, and so on) are determined, the value of $a$ can be obtained from the average bridge, i.e.,

$$a = g_0 / [(S_0^{\,b1})(L_0^{\,b2})(t_0^{\,b3})(\ldots)]. \tag{5}$$

This procedure was followed during the entire course of the study to develop new formulas as needed. In certain cases where an exponential function was not suitable to model the effect of a parameter, slight variation from this procedure was used to achieve the required accuracy. However, this procedure worked quite well in most cases and the developed formulas demonstrate high accuracy.

Because certain assumptions were made in the derivation of simplified formulas and some bridge parameters were ignored altogether, it is important to verify the accuracy of these formulas when applied to real bridges. The database of actual bridges was used for this purpose. Bridges to which the formula can be applied were identified and analyzed by an accurate method. The distribution factors obtained from the accurate analysis were compared to the results of the simplified methods. The ratio of the approximate to accurate distribution factors was calculated and examined to assess the accuracy of the approximate method. Average, standard deviation, and minimum and maximum ratio values were obtained for each formula or simplified method. The method or formula that has the smallest standard deviation is considered to be the most accurate. However, it is important that the average be just slightly greater than unity to assure slightly conservative results. The minimum and maximum values show the extreme predictions that each method or formula produced when a specific database was used. Although these values may change slightly if a different set of bridges is used for evaluation, the minimum and maximum values allow identification of where shortcomings in the formula may exist that are not readily identified by the average or standard deviation values.

It was previously mentioned that different subsets of the database of bridges were used to evaluate different formulas. When a subset included a large number of bridges (100 or more) a level 2 method was used as the basis of comparison. When it included a smaller number of bridges (less than 100) a level 3 method was used. As a result, LANELL (an influence surface method) was used for verification of formulas for moment distribution in box girder bridges, and a Multidimensional Space Interpolation (MSI) method was used for verification of formulas for straight beam-and-slab and slab bridges.

## FINDINGS

### Level 3 Methods: Detailed Bridge Deck Analysis

Recent advances in computer technology and numerical analysis have led to the development of a number of computer programs for structural analysis. Programs that can be applicable to bridge deck analysis can be divided into two categories. One includes general purpose structural analysis programs such as SAP, STRUDL, and FINITE. The other category is specialized programs for analysis of specific bridge types, such as GENDEK, CURVBRG, and MUPDI. (Note: detailed references for all computer programs listed in this digest are included in the agency final report.)

In the search for the best available computer program for analysis of each bridge type, all suitable computer programs (general and specific) that were available were evaluated. In order to achieve meaningful comparisons and assess the level of accuracy of the programs, a number of field and laboratory tests were modeled by each program. The results were then compared in three ways: (1) by visual comparison of the results plotted on the same figure; (2) by comparison of the averages and standard deviations of the ratios of analytical to experimental results; and (3) by comparison of statistical differences of analytical and experimental results. Five bridge types were considered: beam-and-slab, box girder, slab, multibox beam, and spread box beam. Detailed results of the computer program evaluations are presented in Appendix D of the agency final report.

For analysis of beam-and-slab bridges, the following computer programs and models were evaluated: GENDEK-PLATE, GENDEK-3, GENDEK-5, CURVBRG, SAP, and MUPDI. It was found that, in general, GENDEK-5 analysis using plate elements for the deck slab and eccentric beam elements for the girders is very accurate. This program is also general enough to cover all typical cases, i.e., straight, skew, moment, and shear. However, for analysis of curved open girder steel bridges, CURVBRG was the most accurate program. MUPDI was also found to be a very accurate and fast program; however, skewed bridges cannot be analyzed with this program and shear values near the point of application of load, or near supports, lack accuracy. GENDEK-5 was therefore selected to evaluate level 2 and level 1 methods.

For analysis of box girder bridges, computer programs MUPDI, CELL-4, and FINITE were evaluated. MUPDI was the fastest and most practical program for analysis of straight bridges for moment, but FINITE was found to be the most practical program for skewed bridges and for obtaining accurate shear results. Therefore, MUPDI was selected for the evaluation of LANELL (a level 2 method for moment in straight bridges which was in turn used for evaluation of level 1 methods) and FINITE was selected for other cases.

For the analysis of slab bridges, computer programs MUPDI, FINITE, SAP, and GENDEK were evaluated. Shear results cannot be obtained accurately in slab bridges and therefore were not considered. The GENDEK-5 program, without beam elements, proved to be very accurate. However, MUPDI was found to be the most accurate and practical method for nonskewed prismatic bridges and was selected to evaluate level 2 and level 1 methods.

For the analysis of multibeam bridges, the following computer programs were evaluated: SAP, FINITE, and a specialized program developed by Professor Powell at the University of California, Berkeley, for analysis of multibeam bridges (referred to as the POWELL program in this digest). Various modeling techniques were studied using different grillage models and different plate elements. The program that is capable of producing the most accurate results in all cases (i.e., straight and skewed for shear and moment) is the FINITE program. This program was later used in evaluation of more simplified methods. POWELL is also very accurate in reporting moments in straight bridges, but it uses a finite strip formulation, similar to MUPDI, and, therefore, is incapable of modeling skewed supports, and shear results near supports and load locations cannot be accurately obtained. This program was used to evaluate simplified methods for straight bridges.

For analysis of spread box beam bridges, computer programs SAP, MUPDI, FINITE, and NIKE-3D were evaluated. FINITE produced the most accurate results, especially when shear was considered. MUPDI was selected to evaluate simplified methods for calculation of moments in straight bridges, and FINITE was selected for all other cases.

### Level 2 Methods: Graphical And Simple Computer-Based Analysis

Nomographs and influence surface methods have traditionally been used when computer methods have been unavailable. The *Ontario Highway Bridge Design Code* uses one such method based on orthotropic plate theory. Other graphical methods have also been developed and reported. A good example of the influence surface method is the computer program SALOD developed by the University of Florida for the Florida Department of Transportation. This program uses influence surfaces, obtained by detailed finite element analysis, which are stored in a database accessed by SALOD. One advantage of influence surface methods is that the response of the bridge deck to different truck types can be readily computed.

A grillage analysis using plane grid models can also be used with minimal computer resources to calculate the response of bridge decks in most bridge types. However, the properties for grid members must be calculated with care to assure accuracy. Level 2 methods used to analyze the five bridge types (beam-and-slab, box girder, slab, multibeam, and spread box beam bridges) are discussed below.

The following methods were evaluated for analysis of beam-and-slab bridges: plane grid analysis, the nomograph-based method included in the *Ontario Highway Bridge Design Code* (OHBDC), SALOD, and Multidimensional Space Interpolation (MSI). All of these methods are applicable for single and multilane loading for moment. The OHBDC curves were developed for a truck other than HS-20, and using the HS-20 truck in the evaluation process may have introduced some inaccuracy. The method presented in OHBDC was also found to be time consuming, and inaccurate interpolation between curves was probably a common source of error. SALOD can be used with any truck and, therefore, the "HS" truck was used in its evaluation. The MSI method was developed based on HS-20 truck loading for single and multiple lane loading. MSI was found to be the fastest and most accurate method and was therefore selected for the evaluation of level 1 methods. This method produces results that are generally within 5 percent of the finite element (GENDEK) results.

In the analysis of box girder bridges, OHBDC curves and the LANELL program were evaluated. The comments made about OHBDC for beam-and-slab bridges are valid for box girder bridges as well. As LANELL produced results that were very close to those produced by MUPDI, it was selected for evaluation of level 1 methods for moment.

OHBDC, SALOD, and MSI were evaluated for the analysis of slab bridges. MSI was found to be the most accurate method and, thus, was used in the evaluation of level 1 methods. SALOD also produced results that were in very good agreement with the finite element (MUPDI) analysis. Results of OHBDC were based on a different truck and therefore do not present an accurate evaluation.

In the analysis of multibeam bridges, a method presented by R. A. Jones ["A Simple Algorithm for Computing Load Distribution in Multibeam Bridge Decks," Proceedings, 8th ARRB Conference, 1976] was evaluated. The method is capable of calculating distribution factors due to a single concentrated load and was modified for this study to allow wheel line loadings. The results were found to be in very good agreement with POWELL. However, because this method was only applicable for moment distribution in straight single span bridges, it was not used for verification of level 1 methods.

In the analysis of spread box beam bridges, only plane grid analysis was considered as a level 2 method.

In general, level 2 graphical and influence surface methods were found to produce accurate and dependable results. These methods are at times difficult to apply, but a major advantage of some of them is that different trucks, lane widths, and multiple presence live load reduction factors may be considered. Therefore, if a level 2 procedure does not provide that flexibility, and the accuracy of it is on the same order as a simplified formula, its use is not warranted. MSI is an example of such a method for calculation of wheel load distribution factors in beam-and-slab bridges.

A plane grid analysis would require computer resources similar to those needed for some of the methods mentioned above. In addition, a general purpose plane grid analysis program is available to most bridge designers. Therefore this method of analysis is considered a level 2 method. However, the user has the burden of producing a grid model that will produce sufficiently accurate results. As part of NCHRP Project 12-26, various modeling techniques were evaluated and it was found that a plane grid model may be used to accurately produce wheel load distribution factors for each of the bridge types studied.

**Level 1 Methods: Simplified Formulas**

The current AASHTO specifications recommend use of simplified formulas for determining wheel load distribution factors. Many of these formulas have not been updated in years and do not provide optimum accuracy. A number of other formulas have been developed by researchers in recent years. Most of these formulas are for moment distribution for beam-and-slab bridges subjected to multilane truck loading.

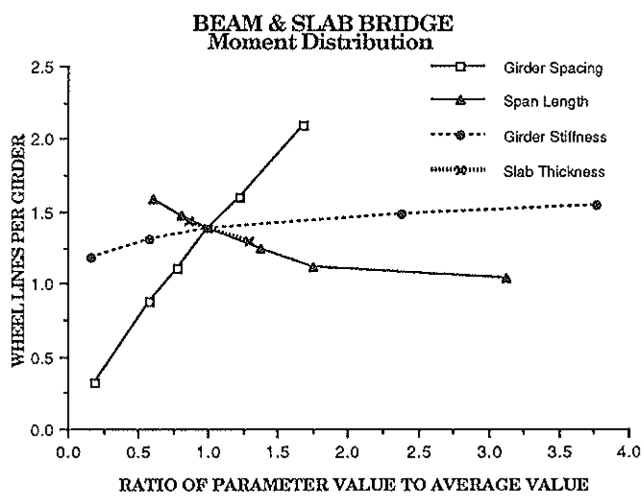


Figure 2. *Effect of parameter variation on beam-and-slab bridges*

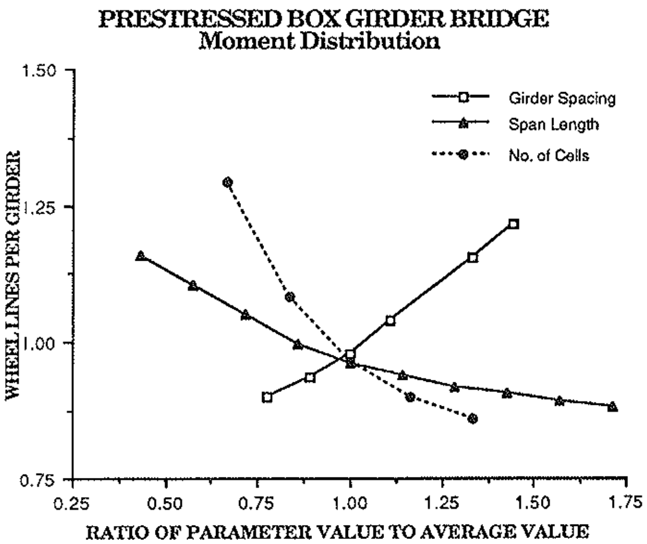


Figure 3. *Effect of parameter variation on box girder bridges*

While some have considered correction factors for edge girders and skewed supports, very little has been reported on shear distribution factors or distribution factors for bridges other than beam-and-slab.

The sensitivity of wheel load distribution factors to various bridge parameters was also determined as part of this study. In general, beam spacing is the most significant parameter. However, span length, longitudinal stiffness, and transverse stiffness also affect the wheel load distribution factors. Figures 2 through 6 show the variation of wheel load distribution factors with various bridge parameters for each bridge type. Ignoring the effect of bridge parameters other than beam spacing can result in highly inaccurate (either conservative or unconservative) results.

A major objective of this research was to evaluate current AASHTO specifications and other researchers' published work to assess their accuracy and develop alternate formulas whenever a more accurate method could be obtained. The formulas that were evaluated and developed are briefly described below, according to bridge type; i.e., beam-and-slab, box girder, slab, multibeam, and spread box beam. Details on each of these formulas are provided in the agency final report.

*Simplified Formulas for Beam-and-Slab Bridges*

This type of bridge has been the subject of many previous studies, and many simplified methods and formulas were developed by previous researchers for multilane loading moment distribution factors. The AASHTO formula, the formulas presented by other researchers, and the formulas developed in this study are discussed in the following according to their application.

***Moment distribution to interior girders, multi-lane loading.*** The AASHTO formula for moment distribution for multilane loading is given as $S/6$ for reinforced concrete T-beam bridges with girder spacing up to 10 ft, and as $S/5.5$ for steel girder bridges and prestressed concrete girder bridges with girder spacing up to 14 ft, where $S$ is the girder spacing. When the girder spacing is larger than the specified limit, simple beam distribution is to be used to calculate the wheel load distribution factors.

Marx et al., at the University of Illinois, developed a formula for wheel load distribution for moment which included multiple lane reduction factors and is applicable to all beam-and-slab bridges. The formula is based on girder spacing, span length, slab thickness, and bridge girder stiffness.

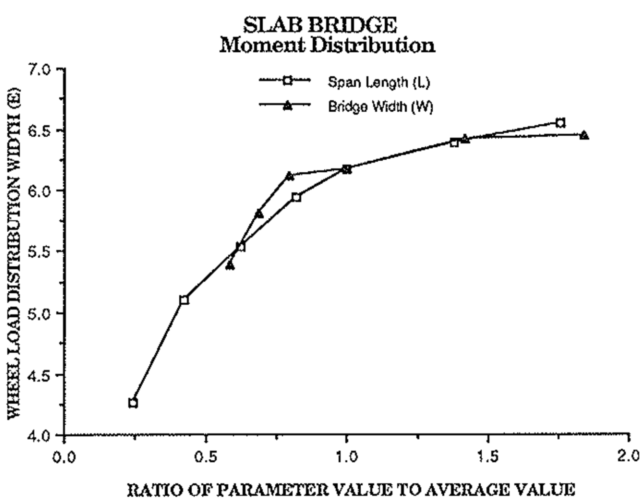


Figure 4. *Effect of parameter variation on slab bridges*

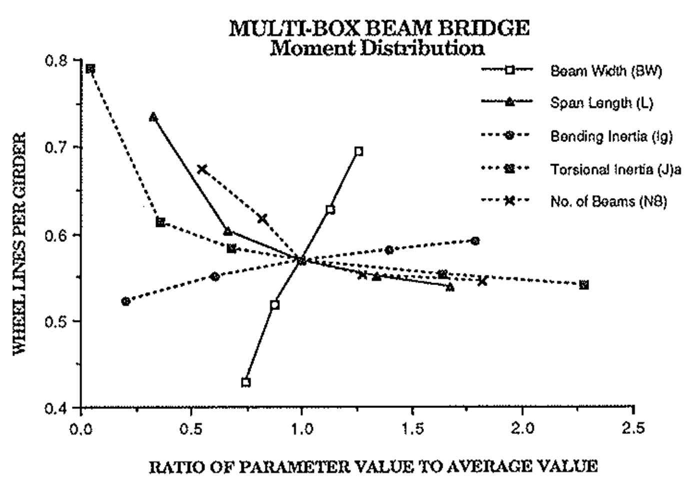


Figure 5. *Effect of parameter variation on multi-box beam bridges*

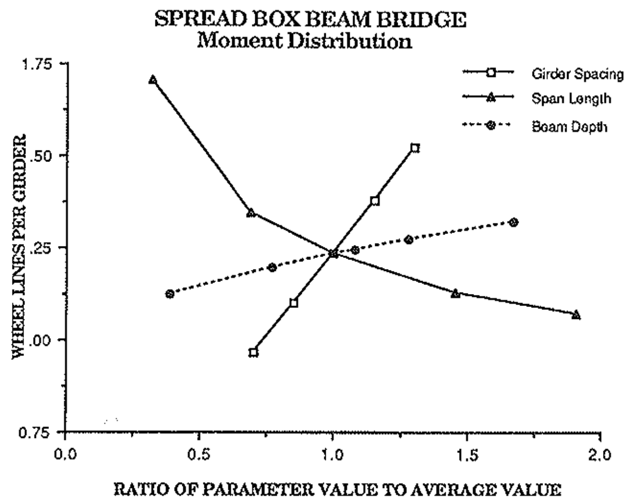


Figure 6. *Effect of parameter variation on spread box beam bridges*

A formula which does not consider a reduction for multilane loading was developed at Lehigh University. The Lehigh formula includes terms for the number of traffic lanes, number of girders, girder spacing, span length, and total curb-to-curb deck width.

Sanders and Elleby [NCHRP Report 83] developed a simple formula based on orthotropic plate theory for moment distribution on beam-and-slab bridges. Their formula includes terms for girder spacing, number of traffic lanes, and a stiffness parameter based on bridge type, bridge and beam geometry, and material properties.

A full width design approach, known as Henry's Method, is used by the State of Tennessee. Henry's Method includes factors for number of girders, total curb-to-curb bridge deck width, and a reduction factor based on number of lanes.

A formula developed as part of NCHRP Project 12-26 includes the effect of girder spacing, span length, girder inertia, and slab thickness. The multiple lane reduction factor is built into the formula. This formula is given by:

$$g = 0.15 + (S/3')^{0.6}\,(S/L)^{0.2}\,(K_g/Lt_s^3)^{0.1} \quad (6)$$

where:

$S$ = girder spacing ($3.5' \leq S \leq 16'$)

$L$ = span length ($20' \leq L \leq 200'$)

$K_g$ = $n(I+Ae^2)$ ($10{,}000 \leq K_g \leq 7 \times 10^6$ in$^4$)

$n$ = modular ratio of girder material to slab material

$I$ = girder moment of inertia

$e$ = eccentricity of the girder (i.e., distance from centroid of girder to midpoint of slab)

$t_s$ = slab thickness ($4.4'' \leq t_s \leq 12''$).

This formula is dependent on the inertia of the girder and, thus, a value for $K_g$ must be assumed for initial design. For this purpose, $K_g/Lt_S^3$ may be taken as unity.

All of the above formulas were evaluated using direct finite element analysis with the GENDEK-5 program and a database of 30 bridges; subsequently, they were evaluated using the MSI method and database of more than 300 bridges. It was found that Eq. 6 and the Illinois formulas are accurate and produce results that are as accurate as the level 2 methods.

***Moment distribution to exterior girders, multilane loading.*** The AASHTO specifications recommend a simple beam distribution of wheel loads in the transverse direction for calculating wheel load distribution factors in edge girders. Any load that falls outside the edge girder is assumed to be acting on the edge girder and any load that is between the edge girder and the first interior girder is distributed to these girders by assuming that the slab acts as a simple beam in that region. Any wheel load that falls inside of the first interior girder is assumed to have no effect on the edge girder.

Marx et al., at the University of Illinois, developed a formula for the exterior girder based on certain assumptions in the placement of wheel loads and may not be applicable to all bridges. This formula includes terms similar to those used in their formula for moment distribution to interior girders.

A formula depending on wheel position alone was developed as part of this study which results in a correction factor for the edge girder. The factor must be applied to the distribution factor for the interior girder to obtain a distribution factor for the edge girder. This formula is given by:

$$e = (7' + d_e)/9.1' \geq 1.0 \quad (7)$$

where $d_e$ is the distance from center of the exterior girder to the edge of the exterior lane. If the edge of lane is outside of the exterior girder, the distance is positive; if the edge of lane is to the interior side of the girder, the distance is negative.

It was found that the formula developed here resulted in accurate correction factors and was simpler than the current AASHTO procedure.

***Moment distribution to interior girders, single-***

***lane loading***. The literature search performed in this study did not reveal any simplified formula for single lane loading of beam-and-slab bridges. The formula developed as part of this study is as follows:

$$g = 0.1 + (S/4')^{0.4} (S/L)^{0.3} (K_g/Lt_s^3)^{0.1} \qquad (8)$$

where the parameters are the same as those given for Eq. 6.

This formula is applicable to interior girders only. Simple beam distribution in the transverse direction should be used for single lane loading of edge girders.

***Shear distribution***. No formula was found from previous research for the calculation of wheel load distribution factors for shear. Therefore, the formulas developed as part of this study are reported for different cases as follows.

The formula for *multilane loading* of interior girders is:

$$g = 0.4 + S/6' - (S/25')^2. \qquad (9)$$

The correction formula for multilane loading edge girder shear is:

$$e = (6' + d_e)/10'. \qquad (10)$$

The formula for shear distribution factor due to *single-lane loading* is:

$$g = 0.6 + S/15'. \qquad (11)$$

Equation 11 is applicable to interior girders only. Simple beam distribution in the transverse direction should be used for single-lane loading of edge girders.

***Correction for skew effects***. Current AASHTO specifications do not include approximate formulas to account for the effect of skewed supports. However, some researchers have developed correction factors for such effects on moments in interior girders.

Marx et al., at the University of Illinois, developed four correction formulas for skew, one each for skew angles of 0, 30, 45, and 60 degrees. Corrections for other values of skew are obtained by straight-line interpolation between the two enveloping skew values. These correction formulas are based on girder spacing, span length, slab thickness, and bridge girder stiffness.

A formula for a correction factor for prestressed concrete I-girders was developed as part of the research performed at Lehigh University. This formula is based on the number of traffic lanes, number of girders, girder spacing, span length, and total curb-to-curb deck width, and includes a variable term for skew angle.

A correction factor for *moment* in skewed supports was also developed as part of this study. This formula is:

$$r = 1 - c_1 (\tan\theta)^{1.5} \qquad (12)$$

where:

$$c_1 = 0.25 (K_g/Lt_s^3)^{0.25} (S/L)^{0.5}$$

and the other parameters are defined in Eq. 6.

From the literature review, no correction formulas were obtained for shear effects due to skewed supports. In the current study it was found that shear in interior girders need not be corrected for skew effects; that is, the shear distribution to interior girders is similar to that of the straight bridge. A correction formula for *shear* at the obtuse corner of the exterior girder was developed as part of this study and is given as:

$$r = 1 + c_1 (\tan\theta) \qquad (13)$$

where:

$$c_1 = 1/[5 (K_g/Lt_s^3)^{0.3}]$$

and the other parameters are defined in Eq. 6.

Equation 13 is to be applied to the shear distribution factor in the exterior girder of nonskewed bridges. Therefore, the product of factors $g$, $e$, and $r$ must be obtained to find the obtuse corner shear distribution factor in a beam-and-slab bridge.

*Simplified Formulas for Box Girder Bridges*

Research on box girder bridges has been performed by various researchers in the past. Bridge deck behavior has been well studied and many recommendations have been made for detailed analysis of these bridges. However, there is a limited amount of information on simplified wheel load distribution formulas in the literature.

***Moment distribution to interior girders***. Scordelis, at the University of California, Berkeley, presented a formula for prediction of wheel load distribution for moment distribution in prestressed and reinforced concrete box girder bridges. The formula is based on modification of distribution factors obtained for a rigid cross section and is too complex to be presented here. The formula predicts wheel load distribution factors in reinforced concrete box girders with high accuracy, and for prestressed concrete box girders with acceptable accuracy.

Sanders and Elleby also presented a simple formula for moment distribution factors which is similar to their formula for beam-and-slab bridges.

The following formulas, developed as part of the current study, may be used to predict the moment wheel load distribution factors in the interior girders of concrete box girder bridges due to single-lane and multilane loadings. These formulas are applicable to both reinforced and prestressed concrete bridges, and the multiple presence factor is accounted for.

For *single-lane loading*:

$$g = (3 + S/2.2')(1'/L)^{0.35}\,(1/N_c)^{0.45}. \quad (14)$$

For *multilane loading*:

$$g = 2.5/N_c - 1/N_L + L/800' + (S/9')(90'/L)^{0.25} \quad (15)$$

where $S$ = girder spacing, in feet; $L$ = span length, in feet; $N_c$ = number of cells; $N_L$ = number of lanes = $W_c/12'$, rounded down to a whole number; and $W_c$ = bridge deck width, curb to curb. Eqs. 14 and 15 are to be used for interior girders only.

***Moment distribution to exterior girders.*** The factor for wheel load distribution for exterior girders shall be $W_e/7'$, where $W_e$ is the width of the exterior girder, taken as the top slab width measured from the midpoint between girders to the edge of the slab.

***Shear distribution.*** No formula for shear wheel load distribution was obtained from previous research for box girder bridges, but the following were developed as part of this study.

The shear distribution factor for interior girder *multilane loading* of reinforced and prestressed concrete box girder bridges is:

$$g = (S/3.4')^{0.9}\,(d/L)^{0.1} \quad (16)$$

where $S$ = girder spacing, in feet; $d$ = girder depth; and $L$ = span length.

The distribution factor for shear in the interior girders due to *single-lane loading* may be obtained from:

$$g = (S/4')^{0.6}\,(d/L)^{0.1} \quad (17)$$

where the parameters are as defined in Eq. 16.

A correction formula for shear in the exterior girder for *multilane loading* is:

$$e = (8' + d_e)/12.5' \quad (18)$$

where $d_e$ = distance from edge of lane to the center of exterior girder, in feet.

***Correction for skew effects.*** The following formula was developed for correction of *moment* due to skewed supports:

$$r = 1.05 - 0.25\,(\tan\theta) \leq 1.0. \quad (19)$$

Another formula was developed for correction of *shear* at the obtuse corner of an edge girder. It must be applied to the shear distribution factor for the edge girder of a nonskewed bridge and must therefore be used in conjunction with the edge girder correction factor of Eq. 18. This formula is:

$$r = 1 + c_1\,(\tan\theta) \quad (20)$$

where $c_1 = 0.25 + (L/70d)$; $d$ = bridge depth; and $L$ = span length.

*Simplified Formulas for Slab Bridges*

The literature search did not reveal any simplified formulas for wheel load distribution in slab bridges other than those recommended by AASHTO. Therefore, the following are formulas that were developed as part of this study.

***Moment distribution, multilane loading.*** Equation 21 was developed to predict wheel load distribution (distribution design width) for moment in slab bridges due to multilane loading. Multiple presence factors are accounted for in the formula:

$$E = 3.5' + 0.06\,(L_1\,W_1)^{0.5} \quad (21)$$

where $E$ = the transverse distance over which a wheel line is distributed; $L_1 = L \leq 60$ ft; $W_1 = W \leq 60$ ft; $L$ = span length, in feet; and $W$ = bridge width, in feet, edge to edge.

***Moment distribution, single-lane loading.*** This formula predicts wheel load distribution for moment due to single-lane loading:

$$E = [2' + (L_1\,W_1)^{0.5}]/4 \quad (22)$$

where the parameters are as defined in Eq. 21.

***Correction for skew effects.*** A formula to account for the reduction of moment in skewed bridges was also developed:

$$r = 1.05 - 0.25\,(\tan\theta) \leq 1.0. \quad (23)$$

According to the AASHTO specifications, slab bridges are adequate for shear if they are designed for moment. A quick check of this assumption was made and it was concluded that it is a valid assumption. Therefore, no formula or method is presented for calculation of shear in slab bridges.

*Simplified Formulas for Multibeam Bridges*

Only one formula other than those presented in the AASHTO specifications was obtained for wheel load distribution in multibeam bridges. This formula, developed by Arya at the University of Illinois, is applicable to both box and open section multibeam bridges and predicts interior beam moment responses due to single-lane and multilane loading. However, a number of simplified formulas developed in the study are valid only for multi-box beam bridges and do not apply to open sections. Therefore, the response of multibeam bridges made of open members, such as channels, may or may not be accurately predicted by the formulas developed in that study.

***Moment distribution to interior girders, multilane loading.*** The formula developed by Arya for interior girder load distribution in multibeam bridges includes terms for the maximum number of wheels that can be placed on a transverse section of the bridge, number of beams, beam width, and span length. A variation of the formula was also proposed for multibeam bridges made of channels, which includes consideration of the overall depth of the channel section and its average thickness, defined as its area divided by its length along the centerline of the thickness.

The following formula was developed in Project 12-26 to predict wheel load distribution factors for interior beam moment due to multilane loading. The multiple presence reduction factor is accounted for in the formula.

$$g = (2b/3')^{0.6}\,[(b/L)(1/N_b)]^{0.2}\,(I/J)^{0.06} \quad (24)$$

where $b$ = beam width, in feet; $L$ = span length; $N_b$ = number of beams; $I$ = moment of inertia of a beam; and $J$ = torsional constant of a beam. This formula is dependant on the inertia and torsional constant of a beam; an estimated value for these properties must therefore be used in preliminary design. The term $I/J$ may be taken as unity for this case.

***Moment distribution to interior girders, single-lane loading.*** Arya also presented a load distribution formula for multibeam bridges designed for one traffic lane. The formulation and parameters were similar to those presented for multilane loading. A variation of that equation was also presented for calculation of the interior beam moment distribution factor for a single-lane, channel section multibeam bridge. It should be noted that Arya's equations are not applicable to cases of only one lane loading with more than one traffic lane.

A formula for wheel load distribution for moment in the interior girders due to single-lane loading was also developed in this study. This formula is as follows:

$$g = k(b/L)^{0.5}\,(I/J)^{0.25} \quad (25)$$

where $k = 2.5(N_b)^{-0.2} \geq 1.5$; $N_b$ = total number of beams; and the other parameters are defined in Eq. 24. Equation 25 is also dependent on inertia and torsional constants and a value of 1.0 may again be used as an approximation for the term $I/J$ during preliminary design.

***Moment distribution to exterior girders.*** The moment in the edge girder due to *multilane loading* is obtained by using a correction factor applied to the interior girder distribution factors for multilane loading. This correction factor may be found from the following formula:

$$e = (26' + d_e)/25' \quad (26)$$

where $d_e$ = distance from edge of lane to the center of exterior web of the exterior girder, in feet.

***Shear distribution.*** Distribution factors for shear in interior girders due to *multilane loading* may be calculated from the following formula:

$$g = (b/3.2')^{0.4}\,(b/L)^{0.1}\,(I/J)^{0.05} \quad (27)$$

where the parameters are as defined in Eq. 24.

Distribution factors for shear in the interior girders due to *single-lane loading* is obtained from the following formula:

$$g = 1.15(b/L)^{0.15}\,(I/J)^{0.05} \quad (28)$$

where the parameters are again as defined in Eq. 24.

Note that Eqs. 27 and 28 are dependent on inertia and torsional constants, and a value of 1.0 may be used as an approximation for the term $I/J$ during preliminary design.

The shear in the edge girder due to *multilane loading* can be found using a correction factor applied to interior girder distribution factors. This correction factor is obtained from the formula:

$$e = (51' + d_e)/50' \qquad (29)$$

where $d_e$ = distance from edge of lane to the center of exterior web of the exterior girder, in feet.

The wheel load distribution factor for moment or shear in the edge girder due to *single-lane loading* may be obtained by simple beam distribution in the same manner as described for beam-and-slab bridges.

***Correction for skew effects.*** The *moment* in any beam in a skewed bridge may be obtained by using a skew reduction factor given by:

$$r = 1.05 - 0.25\,(\tan\theta) \leq 1.0. \qquad (30)$$

The *shear* in the interior beams of a skewed multibeam bridge in most cases is of the same order as that of the shear in the obtuse corner and must be obtained by applying a correction factor to the response of the edge girder in a straight bridge. This correction factor may be calculated from the formula:

$$r = 1 + c_1\,(\tan\theta)^{0.5} \qquad (31)$$

where $c_1 = L/90d$.

*Simplified Formulas for Spread Box Beam Bridges*

Only one formula other than those recommended by AASHTO was obtained from previous research for determining wheel load distribution factors in spread box beam bridges. This formula was developed at Lehigh University for predicting the response of interior beams due to multilane loading and was later adopted by AASHTO. A correction factor for skewed bridges was also presented. In addition, a number of simple formulas were developed as part NCHRP Project 12-26.

***Moment distribution to interior beams, multi-lane loading.*** This formula was developed at Lehigh University for wheel load distribution to interior beams of spread box beam bridges due to multilane loading. It was adopted by AASHTO in special provisions for this type of bridge. The formula is:

$$g = (2N_l)/N_b + k(S/L) \qquad (32)$$

where $N_l$ = number of design traffic lanes; $N_b$ = number of beams; $S$ = average beam spacing; $L$ = span length; and $k = 0.07W_c - N_l(0.1L - 0.26) - 0.2N_b - 0.12$, in which $W_c$ = roadway width between curbs.

A formula developed in Project 12-26 for moment in interior beams due to multilane loading is as follows:

$$g = (S/2')^{0.6}\,[(S/L)(d/L)]^{0.125} \qquad (33)$$

where $S$ = girder spacing; $L$ = span length; and $d$ = beam depth.

***Moment distribution to interior beams, single-lane loading.*** A similar formula was developed for distribution to interior beams due to single-lane loading:

$$g = 2\,(S/5')^{0.35}\,[(S/L)(d/L)]^{0.25} \qquad (34)$$

where the parameters are as defined in Eq. 33.

***Moment distribution to exterior girders.*** The moment in edge girders due to *multilane loading* may be calculated by applying a correction factor to the interior girder distribution factor:

$$e = (27.7' + d_e)/28.5' \geq 1.0 \qquad (35)$$

where $d_e$ = distance from edge of lane to the center of exterior web of the exterior girder, in feet.

The distribution factor for moment in the edge girder due to *single-lane loading* may be obtained by simple beam distribution in the same manner as was described for beam-and-slab bridges.

***Shear distribution.*** The distribution factor for shear in the interior girders due to *multilane loading* may be calculated from the following:

$$g = (S/3.1')^{0.8}\,(d/L)^{0.1} \qquad (36)$$

where the parameters are as defined in Eq. 33.

The distribution factor for shear in the interior girders due to *single-lane* loading may be obtained from:

$$g = (S/4.4')^{0.6}\,(d/L)^{0.1} \qquad (37)$$

where the parameters are again as defined in Eq. 33.

The shear in the edge girder due to *multilane loading* can be found by applying a correction factor to the interior girder equation. This correction factor is:

$$e = (8' + d_e)/10' \qquad (38)$$

where $d_e$ = distance from edge of lane to the center of exterior web of the exterior girder, in feet.

The wheel load distribution factor for shear in the edge girder due to *single-lane loading* may be obtained by simple beam distribution in the same manner as was described for beam-and-slab bridges.

***Correction for skew effects.*** Research at Lehigh University also resulted in a formula for correction of wheel load distribution for *moment* in interior girders due to *multilane loading* in skewed bridges. A formula was also developed in Project 12-26 which is given by the following:

$$r = 1.05 - 0.25 (\tan \theta) \leq 1.0 \qquad (39)$$

The shear in the interior beams of a skewed bridge is the same as that of a straight bridge. However, the *shear* in the obtuse corner must be obtained by applying a correction factor to the distribution factor for the edge girder in a straight bridge:

$$r = 1 + c_1 (\tan \theta) \qquad (40)$$

where $c_1 = (Ld)^{0.5} / 6S$.

*Response of Continuous Bridges*

The response of continuous bridges was studied by modeling a number of two-span continuous bridges where each span is similar to the average bridge. Each abutment is supported by a simple support and the shear responses at the abutment and bent, and maximum positive and negative moments in the span and at the bent, were obtained. The wheel load distribution factor for each case was compared to that of a simple bridge and correction factors for continuity were obtained. In the case of beam-and-slab bridges, a complete parameter study was performed, and it was found that the correction factor is generally independent of bridge geometry. These factors are given as follows:

<u>Beam-and-slab bridges</u>

| | |
|---|---|
| Positive moment: | $c = 1.05$ |
| Negative moment: | $c = 1.10$ |
| Shear at simply supported end: | $c = 1.00$ |
| Shear at continuous bent: | $c = 1.05$ |

<u>Box girder bridges</u>

| | |
|---|---|
| Positive moment: | $c = 1.00$ |
| Negative moment: | $c = 1.10$ |
| Shear at simply supported end: | $c = 1.00$ |
| Shear at continuous bent: | $c = 1.00$ |

<u>Slab bridges</u>

| | |
|---|---|
| Positive moment: | $c = 1.00$ |
| Negative moment: | $c = 1.10$ |

<u>Multibeam bridges</u>

| | |
|---|---|
| Positive moment: | $c = 1.00$ |
| Negative moment: | $c = 1.10$ |
| Shear at simply supported end: | $c = 1.00$ |
| Shear at continuous bent: | $c = 1.05$ |

<u>Spread box beam bridges</u>

| | |
|---|---|
| Positive moment: | $c = 1.00$ |
| Negative moment: | $c = 1.10$ |
| Shear at simply supported end: | $c = 1.00$ |
| Shear at continuous bent: | $c = 1.05$ |

*Summary of Specification Recommendations*

A draft of the proposed revised specifications is presented in Appendix A of this Research Results Digest. The Appendix illustrates how the results of NCHRP Project 12-26 can be incorporated into the 14th edition of the AASHTO *Standard Specifications for Highway Bridges*. A side-bar distinguishes the recommended revisions to the present specifications.

## RECOMMENDATIONS FOR THE THREE LEVELS OF ANALYSIS

### Level 3: Finite Element Computer Programs

Detailed bridge deck analysis using a finite element computer program may be used to produce accurate results. However, extreme care must be taken in preparation of the model, or inaccurate results will be obtained. Important points to consider are selection of a program capable of accurately modeling responses being investigated, calculation of element properties, mesh density, and support conditions. Every model should be thoroughly checked to ensure that nodes and elements are generated correctly.

Another important point is the loading. Truck loads should be placed at positions that produce the maximum response in the components being investigated. In many cases, the truck location is not known before preliminary analysis is performed and therefore many loadings should be investigated. This problem is more pronounced in skewed bridges.

Many computer programs have algorithms that allow loads to be placed at any point on the elements. If this feature is not present, equivalent nodal loads must be calculated. Distribution of wheel loads to various nodes must also be performed with care, and the mesh should be fine enough to minimize errors that can arise because of load approximations.

Many computer programs, especially the general purpose finite element analysis programs, report stresses and strains, not shear and moment values. Calculation of shear and moment values from the stresses must be carefully performed, usually requiring an integration over the beam cross section. Some programs report stresses at node points rather than Gaussian integration points. Integration of stresses at node points is normally less accurate and may lead to inaccurate results.

Detailed analysis of bridge decks can produce incorrect and inaccurate results if not carefully performed. The additional accuracy gained by such an analysis is usually not enough to warrant its use for everyday design practice. However, in some cases, unusual geometry or complex configurations may not allow the use of a simplified procedure, and detailed finite element analysis is only recommended in these cases.

**Level 2: Graphical And Simplified Computer Programs**

Many graphical and computer-based methods are available for calculating wheel load distribution. One popular method consists of design charts based on the orthotropic plate analogy, similar to that presented in the Ontario *Highway Bridge Design Code.* As computers become readily available to designers, simple computer-based methods such as SALOD become more attractive than nomographs and design charts. Also grillage analysis presents a good alternative to other simplified bridge deck analysis methods, and will generally produce more accurate results.

The grillage analogy may be used to model any one of the five bridge types studied in this research. Each bridge type requires special modeling techniques. A major advantage of plane grid analysis is that shear and moment values for girders are directly obtained and integration of stresses is not needed. Loads normally need to be applied at nodal points, and it is recommended that simple beam distribution be used to distribute wheel loads to individual nodes. If the loads are placed in their correct locations, the results will be close to those of detailed finite element analysis.

**Level 1: Simplified Formulas**

The formulas developed in this study may be used to determine the wheel load distribution factors for moment and shear in interior and exterior girders of straight or skewed, simply supported or continuous bridges. These formulas are generally more complex than those currently included in the AASHTO specifications, but they also present a greater degree of accuracy.

Some of the formulas developed in this study are dependent on stiffness parameters that are not known before preliminary design. In these cases an approximation of the stiffness parameter may be used.

The formulas currently presented in the AASHTO specifications, although simpler, do not present the degree of accuracy demanded by today's bridge engineers. In some cases these formulas can result in highly unconservative results (more than 40 percent); in other cases they may be highly conservative (more than 50 percent). In general, the formulas developed in this study are within 5 percent of the results of an accurate analysis. Histogram plots comparing the accuracy of current AASHTO formulas and those developed in this study are provided in the agency final report. Table 2 shows comparisons with moment distribution factors obtained from AASHTO, level 1, level 2, and level 3 methods for simple span bridges.

Bridge design engineers use the simplified methods and formulas whenever possible because of the efficiency gained by the simplicity of these methods. However, in general, simplified formulas have limitations which should be understood. These limitations apply to the current AASHTO formulas, those presented by other researchers, the ones developed in this study, and any other simplified formulas. These limitations are briefly described below.

The formulas are normally developed for single-lane loading and multilane loading. The formulas for multilane loading predict the maximum distribution factor for each of two-lane, three-lane, and four-lane loadings and include multiple presence reduction factors. Therefore, if other reduction factors are to be considered, these formulas should be reevaluated to assess their accuracy.

The formulas are developed for a specific truck type, normally the AASHTO HS family of trucks, and the effect of other truck configurations should be considered. Limited investigation of this matter revealed that if the gauge width is the same and the longitudinal axis positions or loads change, the distribution factors are not greatly affected. However, if two different truck types are considered simultaneously, e.g., one permit truck along with an HS-20 truck, the formulas are not applicable.

The formulas are developed to predict wheel load distribution factors for bridges of common types and dimensions. Therefore, their validity has been veri-

Table 2. Comparison of interior girder moment distribution factors by varying levels of accuracy using the "average bridge" for each bridge type

| Bridge Type | AASHTO | NCHRP 12-26 (Level 1) | Grillage (Level 2) | Finite Element (Level 3) |
|---|---|---|---|---|
| Beam-and-slab[a] | 1.413 (S/5.5) | 1.458 | 1.368 | 1.378 |
| Box girder[a] | 1.144 | 1.143 | 0.970 | 1.005 |
| Slab[b] | 5.980 | 5.625 | 6.242 | 6.204 |
| Multi-box beam[a] | 0.646 | 0.597 | 0.540 | 0.552 |
| Spread box beam[a] | 1.564 | 1.282 | 1.248 | 1.241 |

[a]Number of wheel lines per girder.
[b]Wheel line distribution width, in feet.

fied for parameter variations within specific ranges; if bridge parameters fall outside of those ranges, the accuracy is reduced or the formula may not be applicable.

The simplified formulas have many advantages that should not be overlooked. The most obvious advantage is their simplicity. They are quick to use and do not require any special tools other than a calculator; no special knowledge of finite element modeling techniques is required. If the simplified formulas are applied in their applicable range and the bridge has a regular geometry, accurate answers can be obtained. Therefore, for bridges of regular geometry and properties, simplified formulas present the best alternative.

## CONCLUSIONS AND SUGGESTED RESEARCH

### Conclusions

Three levels of analysis were considered and evaluated in this study and five bridge types were investigated: beam-and-slab bridges, box girder bridges, slab bridges, multibeam bridges, and spread box beam bridges.

Level 3 analysis involves detailed bridge deck analysis. The following computer programs were found useful for detailed finite element analysis of different bridge types: GENDEK-5 for general beam-and-slab bridges; CURVBRG for open girder steel bridges; FINITE for all box girder bridges; MUPDI for nonskewed simply-supported box girder bridges; GENDEK-5 for slab bridges; FINITE for multibeam bridges; POWELL for simply-supported, single span, nonskewed, multibeam bridges; FINITE for spread box beam bridges; and MUPDI for nonskewed spread box beam bridges.

Level 2 analysis involves use of nomographs, design charts, or simple computer methods. When these methods are employed, it is desirable to have the flexibility to analyze different truck types and multiple presence factors. Therefore, methods such as SALOD and LANELL, where influence surfaces are used to calculate wheel load distribution factors, or plane grid analysis, are the most useful methods for this level of analysis.

Level 1 analysis involves the use of simplified formulas for the calculation of wheel load distribution factors. This method has some limitations, but is very simple and effective. Simplified formulas presented in AASHTO specifications were found to be inaccurate in some cases. A set of simple formulas were developed for each one of the bridge types under study. These formulas were evaluated using detailed level 3 and level 2 analyses and were found to have the same order of accuracy in their ranges of applicability. The formulas developed in this study allow calculation of wheel load distribution factors for moment and shear in the interior girders for single-lane and multilane loading. Additional formulas are presented to calculate correction factors for the response of edge girders and to account for the effects of skewed supports. Correction factors are also presented to calculate distribution factors in continuous bridges.

### Suggested Research

The present research covered most common bridge types; however, some special cases need more attention than this project could offer. For example, the effects of curvature on load distribution were not investigated. These effects are specially important in steel girder and box girder bridges, mainly because these bridges are curved more often than other types. More work in this area could produce results that may allow use of simplified formulas for the calculation of wheel load distribution in curved bridges.

It was found that plane grid analysis can produce accurate results if the model is prepared in accordance with the recommendation previously presented. A simple computer program that would employ plane grid analysis and produce the correct model for each bridge type is needed to simplify the designer's work. Such a computer program could be easily developed and could also be enhanced to produce either wheel load distribution factors or direct response of the girders (i.e., shear and moment values). With this program, the designer would only need to specify a few bridge parameters such as those used for simplified formulas, bridge type, and truck type. A model could be generated internally and responses would calculated and reported.

## FINAL REPORT NCHRP PROJECT 12-26

The overall objective, research approach, findings, and recommendations are presented in the main body of the agency final report on NCHRP Project 12-26 titled, "Distribution of Wheel Loads on Highway Bridges (Volume I)." However, much of the analytical details of the study are documented in the appendixes of that report (Volume 2). Appendix A includes the recommended specifications and commentary. Appendix B presents the responses received to the questionnaire which was sent to state DOTs. The database of bridges used for evaluating various analytical methods is listed in Appendix C. Evaluation of detailed analysis methods is explained in Appendix D. Sensitivity studies are presented in Appendix E. Appendix F includes verification details of simplified methods. Guidelines and sample problems for grillage analysis are presented in Appendix G. Guidelines and sample problems for detailed finite element analysis are discussed in Appendix H. A complete bibliography of related literature is presented in Appendix I.

The agency final report will not be published in the regular NCHRP report series. However, loan copies of the agency report are available by contacting: Transportation Research Board, National Cooperative Highway Research Program, 2101 Constitution Avenue, N.W., Washington, D.C., 20418.

## Acknowledgment

The research reported herein was performed under NCHRP Project 12-26 in two phases by Imbsen & Associates, Inc. (IAI). This digest and the agency final report, which focuses on the second phase, include the findings of both phases of the project. Dr. Toorak Zokaie and Mr. Robert Schamber, of the IAI research team, were the co-principal investigators on this phase. Dr. Roy Imbsen, President of IAI, was the overall project manager. The report was written by Drs. Zokaie and Imbsen, and Mr. Timothy Osterkamp, another member of the IAI research team. Another participating member was Mr. Michael Wong, an assistant engineer at IAI.

Many state departments of transportation assisted in the preparation of the bridge database files by providing bridge plans and by completing a questionnaire. Their assistance is greatly appreciated. The guidance and constructive comments received from the NCHRP project panel are also very much appreciated.

# APPENDIX A: Recommended Specifications

## Part C
## DISTRIBUTION OF LOADS

### 3.23 DISTRIBUTION OF LOADS TO STRINGERS, LONGITUDINAL BEAMS, AND FLOOR BEAMS*

#### 3.23.1 Notation

The following notation is used throughout Section 3.23 and in the tables included in this section. If the units of any measurement are not specified, units of feet must be assumed.

$A$ = Area of a stringer
$b$ = Width of a beam
$d$ = Depth of a beam or stringer
$d_e$ = Edge distance of traffic lanes—to be calculated as the distance between the center of the outside roadway stringer web to the edge of the exterior lane. (If the web is out side of the lane, then $d_e$ is negative).
$e_g$ = Eccentricity of a stringer with respect to the slab—to be calculated as the distance between the geometric centroid of the stringer and mid-depth of the slab.
$g$ = Distribution factor—i.e., the fraction of wheel loads (front and rear) to be applied to the stringers.
$I$ = Moment of inertia of a stringer
$J$ = Torsional inertia of a stringer
$k$ = $2.5\,(N_b)^{-0.2}$ but not less than 1.5—used in calculation of distribution factor for multi-beam bridges.
$K_g$ = Longitudinal stiffness parameter = $n\left(I+Ae_g^2\right)$
$L$ = Span length—to be calculated as the center-to-center spacing between abutments or bents but need not be larger than the clear spacing plus one girder depth.
$n$ = Modular ratio—to be calculated as the ratio of the elastic modulus of stringer to that of the slab.
$N_b$ = Number of beams or stringers
$N_c$ = Number of cells in a box girder bridge
$N_L$ = Total number of traffic lanes from Article 3.6.
$S$ = Average stringer spacing
$t_s$ = Slab thickness
$W$ = Bridge width, edge-to-edge
$w_e$ = Top slab width—to be measured from the midpoint between girders to the outside edge of the slab. The cantilever dimension of any slab extending beyond the exterior girder shall preferably not exceed half the spacing of interior girders.
$\theta$ = Skew angle—to be calculated as the lesser of the skew angles of the two supports for moment, and as the skew angle of the support where shear or reaction is calculated. The skew angle is the angle between the centerline of a support and a line normal to the roadway centerline.
$\mu$ = Poisson's ratio of girders

#### 3.23.2 Bending Moments in Stringers and Longitudinal Beams**

##### 3.23.2.1 General

In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel loads shall be assumed. The lateral distribution shall be determined in accordance with the following sections. However, the value of $\left(\frac{K_g}{Lt_s^3}\right)$ and $\left(\frac{I}{J}\right)$ may be taken as unity in any of the given formulae. This results in slight loss of accuracy which may be conservative or unconservative. Therefore, it is only recommended for preliminary design use.

##### 3.23.2.2 Interior Stringers and Beams

The live load bending moment for each interior stringer shall be determined by applying to the stringer the fraction (g) of a wheel load (both front and rear) determined in Table 3.23.1. The range of applicability of each formula is given in the table.

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* Provisions in this Article shall not apply to orthotropic deck bridges.

** In view of the complexity of the theoretical analysis involved in the distribution of wheel loads to stringers, the empirical method herein described is authorized for the design of normal highway bridges.

**TABLE 3.23.1 Distribution of Wheel Loads in Longitudinal Beams for Calculation of Bending Moments in Interior Longitudinal Stringers**

| Kind of Floor | Bridge Designed for One Traffic Lane | Bridge Designed for Two or More Traffic Lanes | Range of Applicability |
|---|---|---|---|
| Timber:[a] | | | |
| Plank[b] | S/4.0 | S/3.75 | N/A |
| Nail laminated[c] 4" thick or multiple layer[d] floors over 5" thick | S/4.5 | S/4.0 | N/A |
| Nail laminated[c] 6" or more thick | S/5.0<br>If S exceeds 5' use footnote f. | S/4.25<br>If S exceeds 6.5' use footnote f. | N/A |
| Glued Laminated[e] Panels on Glued Laminated Stringers | | | |
| 4" thick | S/4.5 | S/4.0 | N/A |
| 6" or more thick | S/6.0<br>If S exceeds 6' use footnote f. | S/5.0<br>If S exceeds 7.5' use footnote f. | N/A |
| On Steel Stringers | | | |
| 4" thick | S/4.5 | S/4.0 | N/A |
| 6" or more thick | S/5.25<br>If S exceeds 5.5' use footnote f. | S/4.5<br>If S exceeds 7' use footnote f. | N/A |

S = average stringer spacing in feet.

[a] Timber dimensions shown are for nominal thickness.

[b] Plank floors consist of pieces of lumber laid edge to edge with the wide faces bearing on the supports (see Article 20.17—Division II).

[c] Nail laminated floors consist of pieces of lumber laid edge to edge with the narrow edges bearing on the supports, each piece being nailed to the preceding piece (see Article 20.18—Division II).

[d] Multiple layer floors consist of two or more layers of planks, each layer being laid at an angle to the other (see Article 20.17—Division II).

[e] Glued laminated panel floors consist of vertically flued laminated members with the narrow edges of the laminations bearing on the supports (see Article 20.1.1—Division II).

[f] In this case the load on each stringer shall be the reaction of the wheel loads, assuming the flooring between the stringers to act as a simple beam.

**TABLE 3.23.1 (Cont.) Distribution of Wheel Loads in Longitudinal Beams for Calculation of Bending Moments in Interior Longitudinal Stringers**

| Kind of Floor | Bridge Designed for One Traffic Lane | Bridge Designed for Two or More Traffic Lanes | Range of Applicability |
|---|---|---|---|
| Concrete: On Timber Stringers | S/6.0' If S exceeds 6' use footnote f. | S/5.0' If S exceeds 5' use footnote f. | N/A |
| On Steel I-Beam Stringers and Prestressed Concrete Girders; Concrete T-Beams[g] | $0.1 + \left(\frac{S}{4'}\right)^{0.4} \left(\frac{S}{L}\right)^{0.3} \left(\frac{K_g}{Lt_S^3}\right)^{0.1}$ or: $0.1 + \left(\frac{S}{4'}\right)^{0.4} \left(\frac{S}{L}\right)^{0.3}$ If S exceeds 16' use footnote f. | $0.15 + \left(\frac{S}{3'}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left(\frac{K_g}{Lt_S^3}\right)^{0.1}$ or: $0.15 + \left(\frac{S}{3'}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2}$ If S exceeds 16' use footnote f. | $3'\text{-}6'' \le S \le 16'\text{-}0''$ $20' \le L \le 200'$ $4.5'' \le t_s \le 12.0''$ $10{,}000 \le K_g \le 7{,}000{,}000 in^4$ $N_b \ge 4$ |
| Prestressed and Reinforced Concrete Box Girders[g,h] | $\left(3 + \frac{S}{2.2'}\right)\left(\frac{1'}{L}\right)^{0.35} \left(\frac{1}{Nc}\right)^{0.45}$ If S exceeds 13' | $\frac{2.5}{N_c} - \frac{1}{N_L} + \frac{L}{800'} + \left(\frac{S}{9'}\right)\left(\frac{90'}{L}\right)^{0.25}$ use footnote F | $7' \le S \le 13'$ $60' \le L \le 240'$ $3 \le N_c$ |
| | If S ≤ 7' use 7' to | be conservative | |
| | If L ≤ 60' use L, but distribution | factor will be more conservative | |
| On Steel Box Girders | See Article 10.39.2. | | N/A |
| On Prestressed Concrete Spread Box Beams[g] | $2\left(\frac{S}{5'}\right)^{0.35} \left[\left(\frac{S}{L}\right)\left(\frac{d}{L}\right)\right]^{0.25}$ If S exceeds 11'-6" | $\left(\frac{S}{2'}\right)^{0.6} \left[\left(\frac{S}{L}\right)\left(\frac{d}{L}\right)\right]^{0.125}$ use footnote f. | $6' \le S \le 11'\text{-}6''$ $20' \le L \le 135'$ $1'\text{-}6'' \le d \le 5'\text{-}6''$ $N_b \ge 3$ |
| Precast Box Beams Used in Multi-Beam Decks[g] | $k\left(\frac{b}{L}\right)^{0.5} \left(\frac{I}{J}\right)^{0.25}$ or $k\left(\frac{b}{L}\right)^{0.5}$ | $\left(\frac{2b}{3'}\right)^{0.6} \left[\left(\frac{b}{L}\right)\left(\frac{1}{N_b}\right)\right]^{0.2} \left(\frac{I}{J}\right)^{0.06}$ or $\left(\frac{2b}{3'}\right)^{0.6} \left[\left(\frac{b}{L}\right)\left(\frac{1}{N_b}\right)\right]^{0.2}$ | $3' \le b \le 5'$ $20' \le L \le 105'$ $5 \le N_b \le 20$ $25{,}000 \le J \le 610{,}000 in^4$ $40{,}000 \le I \le 610{,}000 in^4$ |
| Precast Beam Other than Box Beams Used in Multi-Beam Decks | See Article | 3.23.2.7 | N/A |
| Steel Grid: (Less than 4" thick) (4" or more) | S/4.5 S/6.0 If S exceeds 6' use footnote f. | S/4.0 S/5.0 If S exceeds 10.5' use footnote f. | |
| | | | |
| Steel Bridge Corrugated Plank[i] (2" min. depth) | S/5.5 | S/4.5 | |

g From Imbsen & Associates, Inc. (NCHRP Project 12-26).

h The sidewalk live load (see Article 3.15) shall be omitted for interior and exterior box girders designed in accordance with the wheel load distribution indicated herein.

i Distribution factors for Steel Bridge Corrugated Plank set forth above are based substantially on the following reference: *Journal of Washington Academy of Sciences*, Vol. 67, No. 2, 1977 "Wheel Load Distribution of Steel Bridge Plank," by Conrad P. Heins, Professor of Civil Engineering, University of Maryland. These distribution factors were developed based on studies using 6" x 2" steel corrugated plank. The factors should yield safe results for other corrugation configurations provided primary bending stiffnesses is the same as or greater than the 6" x 2" corrugated plank used in the studies.

### 3.23.2.3 Outside Roadway Stringers and Beams

*3.23.2.3.1 Steel, Timber, Concrete T-Beams, and Concrete Multi-Beams*

*3.23.2.3.1.1* The dead load supported by the outside roadway stringer or beam shall be that portion of the floor slab carried by the stringer or beam. Curbs, railings, and wearing surface, if placed after the slab has cured, may be distributed equally to all roadway stringers or beams.

*3.23.2.3.1.2* The live load bending moment for outside roadway stringers or beams shall be determined by applying to the stringer or beam the fraction (g) of a wheel load (both front and rear) as determined in Table 3.23.2. The range of applicability of each formula is given in the table.

*3.23.2.3.1.3* When the outside roadway beam or stringer supports the sidewalk live load as well as traffic live load and impact and the structure is to be designed by the service load method, the allowable stress in the beam or stringer may be increased by 25 percent for the combination of dead load, sidewalk live load, traffic live load, and impact, providing the beam is of no less carrying capacity than would be required if there were no sidewalks. When the combination of sidewalk live load and traffic live load plus impact governs the design and the structure is to be designed by the load factor method, 1.25 may be used as the beta factor in place of 1.67.

*3.23.2.3.1.4* In no case shall an exterior stringer have less carrying capacity than an interior stringer.

*3.23.2.3.2 Concrete Box Girders*

*3.23.2.3.2.1* The dead load supported by the exterior girder shall be determined in the same manner as for steel, timber, concrete T-beams, as given in Article 3.23.2.3.1.

*3.23.2.3.2.2* The live load bending moment for outside roadway girders shall be determined by applying to the girders the fraction (g) of a wheel load

**TABLE 3.23.2: Distribution of Wheel Loads in Longitudinal Beams for Calculation of Bending Moments in Outside Roadway Stringers**

| Kind of Floor | Bridge Designed for One Traffic Lane | Bridge Designed for Two or More Traffic Lanes | Range of Applicability |
|---|---|---|---|
| Timber: | Use footnote f | Use footnote f | N/A |
| Concrete: | | | |
| On Timber Stringers | Use footnote f. | Use footnote f. | N/A |
| On Steel I-Beam Stringers[g] and Prestressed Concrete Girders; Concrete T-Beams[g] | Use footnote f. | $e * g_{interior}$<br>$e = \frac{7' + d_e}{9.1'} \geq 1.0$ | $-1' \leq d_e \leq 5'\text{-}6''$ |
| Concrete Box Girders[h] | $\frac{W_e}{7'}$ | $\frac{W_e}{7'}$ | $W_e \leq S$ |
| On Steel Box Girders | See Article | 10.39.2. | |
| On Prestressed Concrete Spread Box Beams | Use footnote f. | $e * g_{interior}$<br>$e = \frac{27.7' + de}{28.5'}$ | $0' \leq d_e \leq 4'\text{-}6''$ |
| Precast Box Beams Used in Multi-Beam Decks | Use footnote f. | $e * g_{interior}$<br>$e = \frac{26' + d_e}{25'}$ | $-1' \leq d_e \leq 2'$ |
| Precast Beams other than Box Beams Used in Multi-Beam Decks | Use footnote f. | Use footnote f. | N/A |
| Steel Grid: | Use footnote f. | Use footnote f. | N/A |

For footnotes see Table 3.23.1.

(both front and rear) determined in Table 3.23.2. The range of applicability of each formula is also given in the same table.

#### *3.23.2.3.3 Total Capacity of Stringers and Beams*

The combined design load capacity of all the beams and stringers in a span shall not be less than required to support the total live and dead load in the span.

### 3.23.2.4 Skewed Supports

When the supports are skewed the bending moment in the stringers may be reduced. In order to reduce the bending moment a reduction factor may be applied to the distribution factors obtained from Table 3.23.1 or 3.23.2. If the two supports are nearly parallel, the value of the reduction factor (r) is obtained from Table 3.23.3; otherwise a higher level analysis is recommended.

### 3.23.2.5 Continuous Superstructures

When the superstructure is continuous over the bent, the bending moments must be increased. The correction factors (c) to be applied to the bending moments are given in Table 3.23.4. These correction factors must be applied to the moments obtained from a continuous frame analysis.

**TABLE 3.23.3: Reduction of Wheel Load Distribution Factors for Calculation of Moment in Longitudinal Beam Supported on Skewed Supports**

| Kind of Floor | Bridge Designed for Any Number of Traffic Lanes | Range of Applicability |
|---|---|---|
| Timber: | 1.0 | N/A |
| Concrete: | | |
| On Timber Stringers | 1.0 | N/A |
| On Steel I-Beam Stringers and Prestressed Concrete Girders; Concrete T-Beams[g] | $1-c_1 (\tan\theta)^{1.5}$<br>$c_1 = 0.25 \left(\frac{K_g}{Lt_s^3}\right)^{0.25} \left(\frac{S}{L}\right)^{0.5}$<br>If $\theta$ is less than 30°, $c_1 = 0.0$<br>If $\theta$ is larger than 60° use $\theta$ as 60° | $30° \le \theta \le 60°$<br>$3'\text{-}6" \le S \le 16'\text{-}0"$<br>$20' \le L \le 200'$<br>$4.5" \le t_s \le 12.0"$<br>$10{,}000 \le K_g \le 7{,}000{,}000 in^4$<br>$N_b \ge 4$ |
| On Steel Box Girders | 1.0 | N/A |
| On Prestressed Concrete Spread Box Beams; Concrete Box Girders; and Precast Box Beams Used in Multi-Beam Decks[g] | $1.05 - 0.25 \tan(\theta) \le 1.0$<br>If $\theta$ is larger than 60°, use $\theta$ as 60° | $0 \le \theta \le 60°$ |
| Precast Concrete Beams Other Than Box Beams Used in Multi-Beam Decks[g] | 1.0 | N/A |
| Steel Grid: | 1.0 | N/A |

For footnotes see Table 3.23 1.

**TABLE 3.23.4: Correction Factors for Calculation of Bending Moments in Continuous Longitudinal Beams**

| Kind of Floor | Correction Factor for Positive Moments | Correction Factor for Negative Moments |
|---|---|---|
| Timber: | 1.00 | 1.00 |
| Concrete: | | |
| On Timber Stringers | 1.00 | 1.00 |
| On Steel I-Beam Stringers and Prestressed Concrete Girders; Concrete T-Beams[g] | 1.05 | 1.10 |
| On Steel Box Girders | 1.00 | 1.00 |
| On Prestressed Concrete Spread Box Beams; Concrete Box Girders; and Precast Box Beams Used in Multi-Beam Decks[g] | 1.00 | 1.10 |
| Precast Concrete Beams Other Than Box Beams Used in Multi-Beam Decks | 1.00 | 1.00 |
| Steel Grid: | 1.00 | 1.00 |

For footnotes see Table 3.23.1.

### 3.23.2.6 Bending Moments in Floor Beams (Transverse)

*3.23.2.6.1* In calculating bending moments in floor beams, no transverse distribution of the wheel loads shall be assumed.

*3.23.2.6.2* If longitudinal stringers are omitted and the floor is supported directly on floor beams, the beams shall be designed for loads determined in accordance with Table 3.23.5.

### 3.23.2.7 Precast Concrete Beams Other Than Box Beams Used in Multi-Beam Decks

*3.23.2.7.1* A multi-beam bridge is constructed with precast reinforced or prestressed concrete beams that are placed side by side on the supports. The interaction between the beams is developed by continuous longitudinal shear keys and lateral bolts that may, or may not, be prestressed. Deep, rigid end diaphragms are needed to ensure proper load distribution for precast stemmed members.

**TABLE 3.23.5 Distribution of Wheel Loads in Transverse Beams**

| Kind of Floor | Fraction of Wheel Load to Each Floor Beam |
|---|---|
| Plank[a,b] | $\frac{S}{4}$ |
| Nail laminated[c] or glued laminated, 4 inches in thickness, or multiple layered[d] floors more than 5 inches thick | $\frac{S}{4.5}$ |
| Nail laminated[c] or glued laminated, 6 inches or more in thickness | $\frac{S^f}{5}$ |
| Concrete | $\frac{S^f}{6}$ |
| Steel grid (less than 4 inches thick) | $\frac{S}{4.5}$ |
| Steel grid (4 inches or more) | $\frac{S^f}{6}$ |
| Steel bridge corrugated plank (2 inches minimum depth) | $\frac{S}{5.5}$ |

Note:
S = spacing of floor beams in feet.
For footnotes a through e, see Table 3.23.1.

[f] If S exceeds denominator, the load on the beam shall be the reaction of the wheel loads assuming the flooring between beams to act as a simple beam.

*3.23.2.7.2* In calculating bending moments in multibeam precast concrete bridges, conventional or prestressed, no longitudinal distribution of wheel load shall be assumed.

*3.23.2.7.3* The live load bending moment for each section shall be determined by applying to the beam the fraction of a wheel load (both front and rear) determined by the following relations:

$$\text{Load Fraction} = \frac{S}{D} \qquad (3\text{-}11)$$

where

S = Width of precast member; when S is less than 4 feet or more than 10 feet for precast stemmed members, a special analytical investigation may be necessary;

$$D = (5.75 - 0.5N_L) + 0.7N_L\ (1\text{-}0.2C)^2 \quad \text{when } C \leq 5 \qquad (3\text{-}12)$$

$$D = (5.75 - 0.5N_L) \text{ when } C > 5 \qquad (3\text{-}13)$$

$$C = K(W/L) \qquad (3\text{-}14)$$

where

$K = \{(1+\mu)\ I/J\}^{1/2}$

for preliminary design, the following volumes of K may be used

| Bridge Type | Beam Type | K |
|---|---|---|
| Multi-Beam | Nonvoided rectangular beams | 0.7 |
| | Rectangular beams with circular voids | 0.8 |
| | Channel Beams | 2.2 |

### 3.23.3 Shear in Stringers and Longitudinal Beams

#### 3.23.3.1 General

In calculating shear forces in longitudinal beams or stringers, no longitudinal distribution of the wheel loads shall be assumed.

#### 3.23.3.2 Interior Stringers and Beams

The live load shear for each interior stringer shall be determined by applying to the stringer the fraction (g) of a wheel load (both front and rear) determined in Table 3.23.6. The range of applicability for each formula is given in the table.

*3.23.3.2.1* For stringer and beam types not listed in Table 3.23.6, lateral distribution of the wheel or axle load adjacent to the end of span shall be that produced by assuming the flooring to act as a simple span between stringers or beams.

For loads in other positions on the span, the distribution for shear shall be determined by the method prescribed for moment, except that the calculations of horizontal shear in rectangular timber beams shall be in accordance with Article 13.3.

#### 3.23.3.3 Outside Roadway Stringers and Beams

*3.23.3.3.1* The live load shear for outside roadway stringers or beams shall be determined by applying to the stringer or beam the fraction (g) of a wheel line load (both front and rear) as determined in Table 3.23.7. The range of applicability for each formula is given in the table.

#### 3.23.3.4 Skewed Supports

Shear in the exterior obtuse corner girder must be corrected when the support is skewed. The value of the correction factor is obtained from Table 3.23.8. These factors are applied to the distribution factors obtained from Table 3.23.7.

In calculation of shear in multi-beam bridges, all beams must be treated like the beam at obtuse corner, i.e., the correction is applicable to all beams and the correction factor shall be applied to the distribution factors obtained from Table 3.23.6 for interior girders.

#### 3.23.3.5 Continuous Superstructures

When the superstructure is continuous over the bent, the support shears must be increased. The correction factors (c) to be applied to the support shears are given in Table 3.23.9. These correction factors are applied to the support shears given in Tables 3.23.6 and 3.23.7.

#### 3.23.3.6 Shear in Floor Beams (Transverse)

Lateral distributtion of the wheel load shall be that produced by assuming the flooring to act as a simple span between floor beams.

**TABLE 3.23.6: Distribution of Wheel Loads in Longitudinal Beams for Calculation of Shears in Interior Longitudinal Stringers**

| Kind of Floor | Bridge Designed for One Traffic Lane | Bridge Designed for Two or More Traffic Lanes | Range of Applicability |
|---|---|---|---|
| Timber: | Use footnote f | Use footnote f | N/A |
| Concrete: On Steel I-Beam Stringers, Prestressed Concrete Girders, and Concrete T-Beams[g] | $0.6 + \frac{S}{15'}$ | $0.4 + \frac{S}{6'} - \left(\frac{S}{25'}\right)^2$ | 3'-6" $\leq S \leq$ 16'-0"<br>20' $\leq L \leq$ 200'<br>4.5" $\leq t_s \leq$ 12.0"<br>$10{,}000 \leq K_g \leq 7{,}000{,}000 in^4$<br>$N_b \geq 4$ |
| On Concrete Box Girder[g] | $\left(\frac{S}{4'}\right)^{0.6}\left(\frac{d}{L}\right)^{0.1}$ | $\left(\frac{S}{3.4'}\right)^{0.9}\left(\frac{d}{L}\right)^{0.1}$ | 6' $\leq S \leq$ 13'<br>20' $\leq L \leq$ 240'<br>3' $\leq d \leq$ 9'<br>$N_c \geq 3$ |
| On Prestressed Concrete Spread Box Beams[g] | $\left(\frac{S}{4.4'}\right)^{0.6}\left(\frac{d}{L}\right)^{0.1}$ | $\left(\frac{S}{3.1'}\right)^{0.8}\left(\frac{d}{L}\right)^{0.1}$ | 6' $\leq S \leq$ 11'-6"<br>20' $\leq L \leq$ 135'<br>1'-6" $\leq d \leq$ 5'-6"<br>$N_b \geq 3$ |
| Precast Box Beams Used in Multi-Beam Decks[g] | $1.15\left(\frac{b}{L}\right)^{0.15}\left(\frac{I}{J}\right)^{0.05}$<br>or<br>$1.15\left(\frac{b}{L}\right)^{0.15}$ | $\left(\frac{b}{3.2'}\right)^{0.4}\left(\frac{b}{L}\right)^{0.1}\left(\frac{I}{J}\right)^{0.05}$<br>or<br>$\left(\frac{b}{3.2}\right)^{0.4}\left(\frac{b}{L}\right)^{0.1}$ | 3' $\leq b \leq$ 5'<br>20' $\leq L \leq$ 105'<br>$5 \leq N_b \leq 20$<br>$25{,}000 \leq J \leq 610{,}000 in^4$<br>$40{,}000 \leq I \leq 610{,}000 in^4$ |
| Precast Beams Other Than Box Beams Used in Multi-Beam Decks | Use footnote f. | Use footnote f. | N/A |
| Steel Grid: | Use footnote f. | Use footnote f. | N/A |

For footnotes see Table 3.23.1.

**TABLE 3.23.7: Distribution of Wheel Loads in Longitudinal Beams for Calculation of Shears in Outside Roadway Stringers**

| Kind of Floor | Bridge Designed for One Traffic Lane | Bridge Designed for Two or More Traffic Lanes | Range of Applicability |
|---|---|---|---|
| Timber:[a] | Use footnote f | Use footnote f | N/A |
| Concrete: | | | |
| On Timber Stringers | Use footnote f. | Use footnote f. | N/A |
| On Steel I-Beam Stringers[g] and Prestressed Concrete Girders; Concrete T-Beams[g] | Use footnote f. | $g = e * g_{interior}$ $e = \frac{6' + d_e}{10'}$ | $-1' \le d_e \le 5'\text{-}6''$ |
| Concrete Box Girders[g,h] | Use footnote f. | $g = e * g_{interior}$ $e = \frac{8' + d_e}{12.5'}$ | $-2' \le d_e \le 5.0'$ |
| On Steel Box Girders | See Article 10.39.2 | | |
| On Prestressed Concrete Spread Box Beams[g] | Use footnote f. | $g = e * g_{interior}$ $e = \frac{8' + d_e}{10'}$ | $0' \le d_e \le 4'\text{-}6''$ |
| Precast Box Beams Used in Multi-Beam Decks[g] | Use footnote f. | $g = e * g_{interior}$ $e = \frac{51' + d_e}{50'}$ | $-1' \le d_e \le 2'$ |
| Precast Beams other than Box Beams Used in Multi-Beam Decks | Use footnote f. | Use footnote f. | N/A |
| Steel Grid: | Use footnote f. | Use footnote f. | N/A |

For footnotes see Table 3.23.1.

**TABLE 3.23.8: Correction of Wheel Load Distribution Factors for Support Shear at the Obtuse Corner**

| Kind of Floor | Correction | Range of Applicability |
|---|---|---|
| Timber: | Use footnote j. | N/A |
| Concrete: | | |
| On Timber Stringers | Use footnote j. | N/A |
| On Steel I-Beam Stringers, Prestressed Concrete Girders, and Concrete T-Beams[g] | $1.0 + c_1 \tan\theta$ <br> $c_1 = \dfrac{1}{5\left(\dfrac{K_g}{Lt_s^3}\right)^{0.3}}$ | $0° \le \theta \le 60°$ <br> 3'-6" $\le S \le$ 16'-0" <br> 20' $\le L \le$ 200' <br> 4.5" $\le t_s \le$ 12.0" <br> $10{,}000 \le K_g \le 7{,}000{,}000 in^4$ <br> $N_b \ge 4$ |
| On Steel Box Beams | Use footnote j. | N/A |
| On Concrete Box Girders[g] | $1.0 + c_1 \tan(\theta)$ <br> $c_1 = 0.25 + \dfrac{L}{70d}$ | $0° < \theta \le 60°$ <br> 6' $\le S \le$ 13' <br> 20' $\le L \le$ 240' <br> 3' $\le d \le$ 9' <br> $N_c \ge 3$ |
| On Prestressed Concrete Spread Box Beams[g] | $1.0 + c_1 \tan(\theta)$ <br> $c_1 = \dfrac{\sqrt{Ld}}{6S}$ | $0° < \theta \le 60°$ <br> 6' $\le S \le$ 11'-6" <br> 20' $\le L \le$ 135' <br> 1'-6" $\le d \le$ 5'-6" <br> $N_b \ge 3$ |
| On Precast Box Beams Used in Multi-Beam Decks[g] | $1.0 + c_1 \sqrt{\tan(\theta)}$ <br> $c_1 = \dfrac{L}{90d}$ | $0° < \theta \le 60°$ <br> 20' $\le L \le$ 105' <br> 1'-4" $\le d \le$ 5' <br> 3' $\le b \le$ 5' <br> $5 \le N_b \le 20$ |
| Precast Concrete Beams Other Than Box Beams Used in Multi-Beam Decks | Use footnote j. | N/A |
| Steel Grid: | Use footnote j. | N/A |

For footnotes a thru i, see Table 3.23.1.

[j] For this bridge type no approximate method is available. Use a more detailed analysis.

**TABLE 3.23.9: Correction Factors for Calculation of Support Shear in Continuous Longitudinal Beams**

| Kind of Floor | Correction for Simply Supported End | Correction for Continuous Bent |
|---|---|---|
| Timber: | 1.00 | 1.00 |
| Concrete: | | |
| On Timber Stringers | 1.00 | 1.00 |
| On Steel I-Beam Stringers, Prestressed Concrete Girders, and Concrete T-Beams[g] | 1.00 | 1.05 |
| On Steel Box Girders[g] | 1.00 | 1.00 |
| On Prestressed Concrete Spread Box Beams[g] | 1.00 | 1.05 |
| On Concrete Box Girders[g] | 1.00 | 1.00 |
| On Precast Box Beams Used in Multi-Beam Decks | 1.00 | 1.05 |
| Precast Concrete Beams Other Than Box Beams Used in Multi-Beam Decks | 1.00 | 1.00 |
| Steel Grid: | 1.00 | 1.00 |

For footnotes see Table 3.23.1.

## 3.24 DISTRIBUTION OF LOADS AND DESIGN OF CONCRETE SLABS*

### 3.24.1 Span Lengths (See Article 8.8)

**3.24.1.1** For simple spans the span length shall be the distance center to center of supports but need not exceed clear span plus thickness of slab.

* The slab distribution set forth herein is based substantially on the "Westergaard" theory. The following references are furnished concerning the subject of slab design.

*Public Roads*, March 1930, "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard.

University of Illinois Bulletin No. 303, "Solutions for Certain Rectangular Slabs Continuous over Flexible Supports," by Vernon P. Jensen; Bulletin 304, "A Distribution Procedure for the Analysis of Slabs Continuous over Flexible Beams," by Nathan M. Newmark; Bulletin 315, "Moments in Simple Span Bridge Slabs with Stiffened Edges," by Vernon P. Jensen; and Bulletin 346, "Highway Slab Bridges with Curbs: Laboratory Tests and Proposed Design Method."

**3.24.1.2** The following effective span lengths shall be used in calculating the distribution of loads and bending moments for slabs continuous over more than two supports:

(a) Slabs monolithic with beams or slabs monolithic with walls without haunches and rigid top flange prestressed beams with top flange width to minimum thickness ratio less than 4.0. "S" shall be the clear span

(b) Slabs supported on steel stringers, or slabs supported on thin top flange prestressed beams with top flange width to minimum thickness ratio equal to or greater than 4.0. "S" shall be the distance between edges of top flange plus one-half of stringer top flange width.

(c) Slabs supported on timber stringers, S shall be the clear span plus one-half thickness of stringer.

### 3.24.2 Edge Distance of Wheel Loads

**3.24.2.1** In designing slabs, the centerline of the wheel load shall be 1 foot from the face of the curb. If curbs or sidewalks are not used, the wheel load shall be 1 foot from the face of the rail.

**3.24.2.2** In designing sidewalks, slabs and supporting members, a wheel load located on the sidewalk shall be 1 foot from the face of the rail. In service load design, the combined dead, live, and impact stresses for this loading shall be not greater than 150 percent of the allowable stresses. In load factor design, 1.0 may be used as the beta factor in place of 1.67 for the design of deck slabs. Wheel loads shall not be applied on sidewalks protected by a traffic barrier.

### 3.24.3 Bending Moment

The bending moment per foot width of slab shall be calculated according to methods given under Cases A and B, unless more exact methods are used considering tire contact area. The tire contact area needed for exact methods is given in Article 3.30.

In Cases A and B:

$S$ = effective span length, in feet, as defined under "Span Lengths" Articles 3.24.1 and 8.8;
$L_1$ = $S$ if $S \leq 60'$
= $60'$ if $S > 60'$;
$W$ = bridge width, edge to edge, in feet;
$W_1$ = $W$ if $W \leq 60'$;
= $60'$ if $W > 60'$;
$E$ = width of slab in feet over which a wheel load is distributed;
$P$ = load on one rear wheel of truck ($P_{15}$ or $P_{20}$);
$P_{15}$ = 12,000 pounds for H15 loading;
$P_{20}$ = 16,000 pounds for H20 loading.

#### 3.24.3.1 Case A—Main Reinforcement Perpendicular to Traffic (Spans 2 to 24 Feet Inclusive)

The live load moment for simple spans shall be determined by the following formulas (impact not included):

HS20 Loading:

$$\frac{S+2}{32}P_{20} = \text{Moment in foot-pounds per foot-width of slab} \quad (3\text{-}15)$$

HS15 Loading:

$$\frac{S+2}{32}P_{15} = \text{Moment in foot-pounds per foot width of slab} \quad (3\text{-}16)$$

In slabs continuous over three or more supports, a continuity factor of 0.8 shall be applied to the above formulas for both positive and negative moment.

#### 3.24.3.2 Case B—Main Reinforcement Parallel to Traffic

For wheel loads, the distribution width, E, shall be:

$$\frac{2+\sqrt{L_1W_1}}{4} \quad \text{for a bridge designed for one traffic lane}$$

or

$$3.5 + 0.06\sqrt{L_1W_1} \quad \text{for a bridge designed for two or more traffic lanes.}$$

Lane loads are distributed over a width of 2E. Longitudinally reinforced slabs shall be designed for the appropriate HS loading.

For simple spans, the maximum live load moment per foot width of slab, without impact, is closely approximated by the following formulas:

HS20 Loading:

Spans up to and including 50 feet: LLM = 900S foot-pounds
Spans 50 feet to 100 feet: LLM = 1000 (1.30S-20.0) foot-pounds

HS15 Loading:

Use 3/4 of the values obtained from the formulas for HS20 loading.

Moments in continuous spans shall be determined by suitable analysis using the truck or appropriate lane loading.

##### *3.24.3.2.1 Skewed Supports*

If the slab in supported on skewed supports, the distribution width (E) may be increased by dividing it by the following factor:

$$C = 1.05 - 0.25 \tan(\theta) \leq 1.0$$

where $\theta$ = average angle of skew

##### *3.24.3.2.2 Continuity of Supports*

If the slab is continuous over the bents, the distribution width (E) shall be reduced by 10% in the negative moment regions.

No changes will be made in the following sections or subsections:

**3.24.5 Cantilever Slabs**

**3.24.6 Slabs Supported on Fours Sides**

**3.24.7 Median Slabs**

**3.24.8 Longitudinal Edge Beams**

**3.24.9 Unsupported Transverse Edge Beams**

**3.24.10 Distribution Reinforcement**

## 3.25 DISTRIBUTION OF WHEEL LOADS ON TIMBER FLOORING

## 3.26 DISTRIBUTION OF WHEEL LOADS AND DESIGN OF COMPOSITE WOOD-CONCRETE MEMBERS

## 3.27 DISTRIBUTION OF WHEEL LOADS ON STEEL GRID FLOORS

## 3.28 MOMENTS, SHEARS, AND REACTIONS

Maximum moments, shears, and reactions are given in tables, Appendix A, for H15, H20, HS15, and HS20 loadings. They are calculated for the standard truck or the lane loading applied to a single lane on freely-supported spans. It is indicated in the table whether the standard truck or the lane loadings produces the maximum stress.

## 3.29 THE CONTACT AREA

The tire contact area shall be assumed as a rectangle with an area in square inches of 0.01P, and a Length in Direction of Traffic/Width of Tire ratio of 1/2.5, in which P = wheel load, in pounds.

## A.2 PARTIAL COMMENTARY

In this section, the "previous specifications" refers to the "AASHTO Standard Specifications for Highway Bridges, 14th edition, with interim 1990 revisions; and the current specification refers to the proposed draft specifications given in Section A.1.

### A.2.1 GENERAL

The formulas presented for calculation of wheel load distribution factors are based on power curves. Some of such formulas include stiffness and inertia terms. In order to predict most accurate factors using these formulas, an iteration approach is needed. However, the value of stiffness terms $\left(\frac{k_g}{Lt_s^3}\right)$ and $\left(\frac{I}{J}\right)$ are close to unity for most common bridges. Therefore, these factors may be taken as unity for initial design or when greater accuracy is not desired.

### A.2.2 RECOMMENDATIONS FOR MORE ACCURATE ANALYSIS

**Recommendations For Detailed Bridge Deck Analysis**—Detailed bridge deck analysis using a finite element computer program may be used to produce accurate results. However, extreme care must be taken in preparation of the model, or inaccurate results will be obtained. Important points to consider are selection of a program capable of accurately modeling responses being investigated, calculation of element properties, mesh density, and support conditions. Every model should be thoroughly checked to make sure that nodes and elements are generated correctly.

Another important point is the loading. Truck loads should be placed at positions that produce the maximum response in the components being investigated. In many cases, the truck location is not known before preliminary analysis is performed and therefore many loadings should be investigated. This problem is more pronounced in skewed bridges.

Many programs have algorithms which allow loads to be placed at any point on the elements. If this feature is not present, equivalent nodal loads must be calculated. Distribution of wheel loads to various nodes must also be performed with care, and the mesh should be fine enough to minimize errors which can arise due to load approximations.

Many computer programs, especially the general purpose finite element analysis programs, report stresses and strains, not shear and moment values. Calculation of shear and moment values from the stresses must be carefully performed. Some kind of integration over the beam cross section is usually required. Some computer programs report stresses at node points rather than Gaussian integration points. Integration of stresses reported at nodal points is normally less accurate and may lead to inaccurate results.

Detailed analysis of bridge decks can produce incorrect and inaccurate results if not carefully performed. The additional accuracy gained by such an analysis is usually not enough to warrant its use for everyday design practice. However, in some cases, unusual geometry or complex configurations may not allow the use of a simplified procedure, and detailed finite element analysis is only recommended in these cases. Guidelines for detailed analysis of bridge decks and sample problems to illustrate their application are given in Appendix H of the NCHRP Project 12-26/1 final report.

**Recommendations For Graphical and Simplified Computer Analysis**—Many graphical and computer based methods are available for calculating wheel load distribution. One popular method for such analysis is design charts based on orthotropic plate analogy, similar to those presented in the Ontario Highway Bridge Design Code. As computers become more and more available to designers, simple computer based methods such as SALOD (developed by University of Florida) become more attractive than nomographs and design charts. Also grillage analysis presents a good alternative to other simplified bridge deck analysis methods, and will generally produce more accurate results.

Grillage analogy may be used to model most common bridge types. Each bridge type requires special modeling techniques. Guidelines for modeling these bridge types and sample problems to illustrate their application are given in Appendix G of the NCHRP Project 12-26/1 final report. A major advantage of plane grid analysis is that shear and moment values for girders are directly obtained and integration of stresses is not needed. Loads normally need to be applied at nodal points, and it is recommended that simple beam distribution be used to distribute wheel loads to individual nodes. If the model is generated according to Appendix G recommendations and the loads are placed in their correct locations, the results will be close to those of detailed finite element analysis.

**Recommendations For Simplified Formulas**—The wheel load distribution formulas may be used to determine the wheel load distribution factors for moment and shear, in interior and exterior girders of straight or skewed, simply supported or continuous bridges. These formulas are generally more complex than those previously recommended by AASHTO specifications, but they also present a greater degree of accuracy.

The formulas previously presented in the AASHTO specifications—although simpler—do not present the degree of accuracy demanded by today's bridge engineers. In many cases these formulas can result in highly unconservative results (more than 40%); and in other cases they may be highly conservative (more than 50%). In general, the formulas presented here are within 5% of the results of an accurate analysis. The following figures present histogram plots comparing the accuracy of previous and current AASHTO formulas. These figures show the accuracy of the distribution factors for moments in five bridge types. More detailed evaluation of these formulas and further comparisons are presented in Appendix F of the NCHRP Project 12-26/1 final report.

Bridge design engineers use the simplified methods and formulas whenever possible because of the efficiency gained by the simplicity of these methods. However, in general, simplified formulas have limitations which should be understood. These limitations are briefly described below.

The formulas are normally developed for single-lane loading and multi-lane loading. The formulas for multi-lane loading predict the maximum distribution factor for each of two-lane, three-lane, and four-lane loadings and include the multiple presence reduction factors. Therefore, if other reduction factors are to be considered, the formulas developed to date should be reevaluated to assess their accuracy.

The formulas are developed for a specific truck type, namely AASHTO HS family of trucks, and the effect of other truck configurations should be kept in mind. Limited investigation of this matter has revealed that if the gauge width is the same and the longitudinal axis positions or loads change, the distribution factors are not effected greatly. However, if two different truck types are considered simultaneously, e.g. one permit truck along with a HS-20 truck, the formulas are not applicable.

The formulas are developed to predict wheel load distribution factors for bridges of common types and dimensions. Therefore, their validity has been verified for parameter variations within specific ranges; and if bridge parameters fall outside of those ranges, the accuracy is reduced or the formula may not be applicable.

The simplified formulas have many advantages which should not be overlooked. The most obvious advantage is their simplicity. They are very quick to use, and do not require any special tools other than a calculator. No special computers or computer programs are needed, and no special knowledge of finite element modeling techniques is required. If the simplified formulas are applied in their applicable range and the bridge has a regular geometry, accurate answers will be obtained. Therefore for bridges of regular geometry and properties, simplified formulas present the best alternative.

Special Notes:

1. Whenever girder inertia is used in calculating the distribution factors and the girder has variable cross sections, the cross section properties at the key locations shall be used. For instance, the cross section at the bent is used for the negative moment and the one near midspan is used for positive moment.

2. A concrete girder is a T-beam or an I-girder; A concrete box girder is a multi-cell box girder; and a concrete box beam is a precast single cell box or stemmed beam.

3. For more detail on the background of the new formulas, refer to the NCHRP 12-26 final report.

4. The NCHRP Report 287, "Load Distribution and Connection Design for Precast Multibeam Bridge Superstructures", page 71, recommends end diaphragms be used "to ensure proper load distribution". The report also says that interior diaphragms may cause a reduction in the live load distribution to the exterior girders predicted under Article 3.24.4.3.

   Article 3.23.4 currently does not limit the range of girder widths that the formulas cover. The NCHRP Report 287, page 71, recommends that the girder width be limited to a range of four to ten feet beyond which special investigation is required.