SHRP-P-689

Evaluation of a Time Domain Reflectometry Technique for Seasonal Monitoring of Soil Moisture Content Under Road Pavement Test Sections

Strategic Highway Research Program
National Research Council

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Strategic Highway Research Program
National Research Council
Washington, DC 1994

SHRP-P-689
Product no. 5030

Program Manager: *Neil F. Hawks*
Program Area Secretaries: *Cynthia Baker, Francine A. Burgess*
Production Editor: *Carrie Kent*

June 1994

key words:
pavement performance
soil moisture content
time domain reflectometry

Strategic Highway Research Program
National Research Council
2101 Constitution Avenue N.W.
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(202) 334-3774

425/NAP/694

Acknowledgments

The research described herein was supported by the Strategic Highway Research Program (SHRP). SHRP is a unit of the National Research Council that was authorized by section 128 of the Surface Transportation and Uniform Relocation Assistance Act of 1987.

Contents

1. Introduction

Time domain reflectometry (TDR) is a relatively new method for determining soil moisture content under road pavements from the measurement of the soil's dielectric permitivity.[1]

TDR equipment consists of a pulse generator, a sampler that produces a low frequency output, and an oscilloscope for the display of the sampler output.[2] Electromagnetic pulses in the range from 1 Mhz to 1 Ghz are applied through a transmission line ending in a waveguide which is embedded in the soil. When a pulse enters the soil, the change of impedance at the air/soil interface causes part of the pulse to be reflected. The travel time of the pulse can be measured on an oscilloscope and is a function of the relative permitivity of the soil surrounding the sensors. This permitivity is in turn strongly dependent on soil moisture content since the relative values of the dielectric constant for water, dry soil, and air are 80, 4, and 1, respectively.

2. The Purpose of the Study

The purpose of this study is to evaluate time domain reflectometry method and equipment used for seasonal monitoring of soil moisture content. The study is based on data collected before and during the installation of instrumentation[4], as well as seasonal data collected over a period of three months in 1992 at a Strategic Highway Research Program (SHRP) test section in Boise, Idaho.[3]

More specifically, the study has the following objectives:

a) To evaluate existing methods for reading TDR traces and recommend a method specific to the type of TDR waveguide used.

b) To compare soil moisture values obtained using various empirical equations.[5,6,8]

c) To calculate and represent graphically the effective seasonal moisture content for the test section studied.

3. Methods for Locating the Initial and Final Points of a TDR Trace

Three methods have been suggested for determining the locations of the initial and final points of TDR traces: the Method of Peaks, the Method of Diverging Lines, and the Method of Tangents. A typical TDR trace rises smoothly to a local maximum and then falls smoothly to a local minimum. Figures 1, 2, and 3 depict typical TDR traces and illustrate these methods, which require the construction of various tangents and determination of locations where tangents intersect or where trace and tangent diverge.

a) The Method of Peaks

In Figure 1, the initial point ("A") is located on the TDR trace at the intersection of tangents drawn on either side of the local maximum. The final point ("B") is located on the trace at the intersection of tangents drawn on either side of the local minimum.

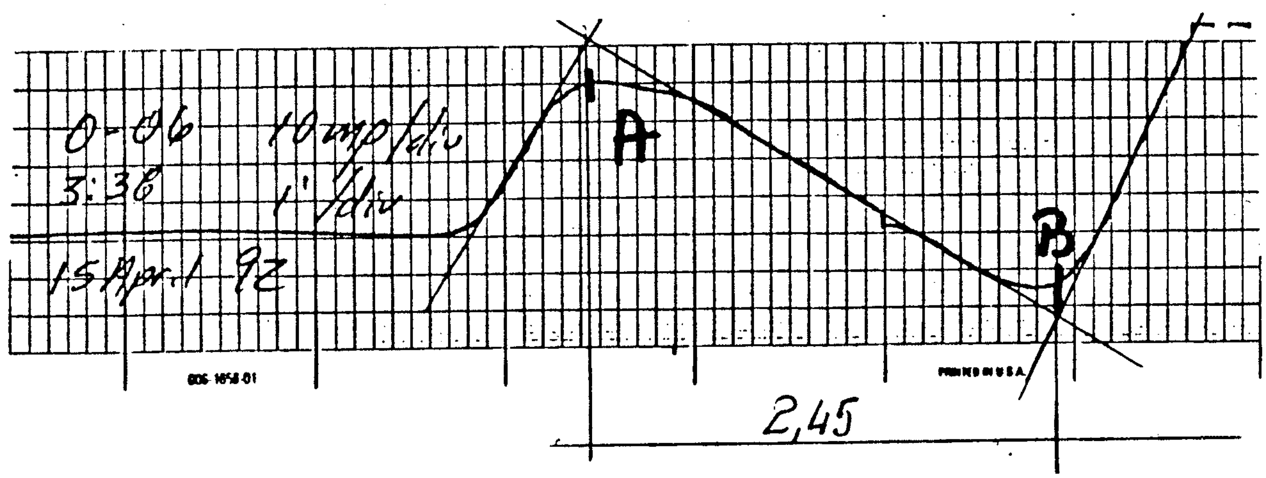

Fig.1. Method of Peaks, suggested for locating the initial and final points of a typical TDR trace.

b) The Method of Diverging Lines

In Figure 2, "A" is located where the trace diverges from the local maximum's positively sloped tangent. "B" is located where the trace diverges from the local minimum's negatively sloped tangent.

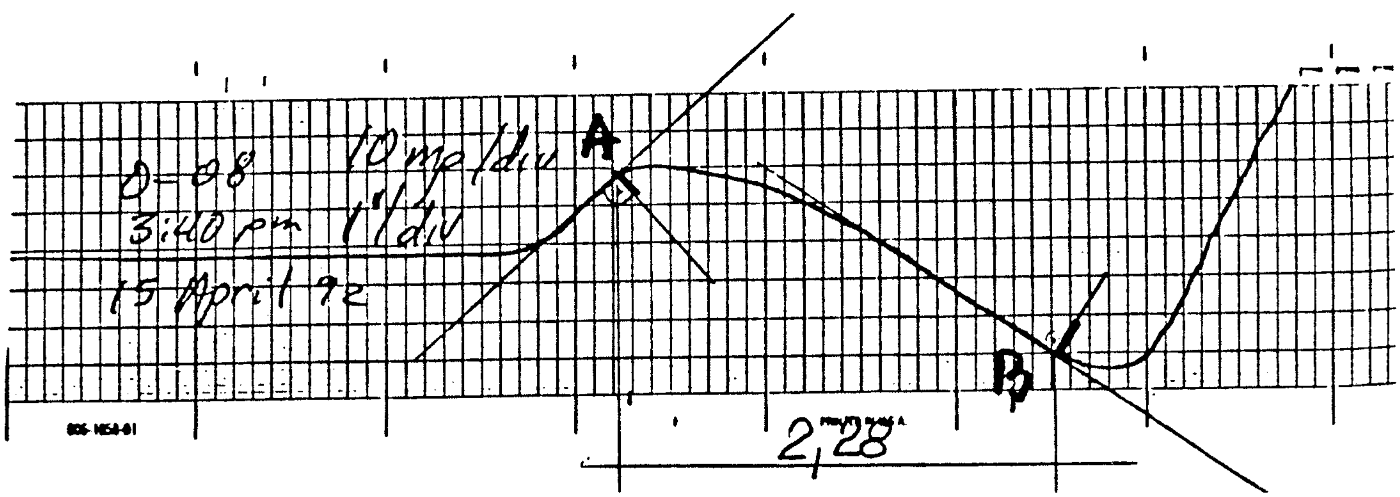

Fig.2. The Method of Diverging lines, for locating the initial and final points of a typical TDR curve.

c) The Method of Tangents

In Figure 3, "A" is located at the intersection of the horizontal and negatively sloped tangents to the trace's local maximum value. "B" is located at the intersection of the horizontal and positively sloped tangents to the trace's local minimum value.

d) The Alternative Method of Tangents (AMT)

In Figure 4, "A" is located at the intersection of the horizontal and positively sloped tangents to the trace's maximal value. "B" is located at the intersection of the horizontal and negatively sloped tangents to the trace's local minimum value.

4. Comparative Study of Different Methods of Reading TDR Traces and Selection of a Recommended Method

Lengths (i.e., time) from "A" to "B" have been determined graphically from TDR traces and compared statistically to select a recommended method of reading field measurement traces.

These traces were obtained from measurements made in February through April 1992 at SHRP General Pavement Study (GPS) Test Section 163203 in Boise, Idaho for Boreholes #2 and #3.

Taking into consideration the small quantity of data available, the non-parametric Wilcox's signed-rank test was used.[9] This test considers both the direction and magnitude of differences between pairs of observations.

Appendix 1 contains the TDR traces taken at Borehole #2 using 7.5" flat-pronged waveguides. The initial and final points of the traces have been determined using all three methods. Table 1 lists the lengths between "A" and "B".

Appendix 2 contains the TDR traces taken at Borehole #3 using 12.0" curved-pronged waveguides. Again, the initial and final points have been determined using the three methods. Table 2 lists the lengths between "A" and "B".

Appendix 3 contains the detailed statistical approach implied by the non-parametric Wilcox's signed-rank test. Table 3 lists the statistical comparisons made between pairs of TDR readings and suggests that the Method of Peaks leads to higher results than the Method of Diverging Lines.

The "t-Student test" has been used to investigate the meaning of these differences.[9] This test assumes normality of the data and requires that all pairs of data be observed under the same conditions. It yields more information than is achievable using Wilcox's signed-rank test.

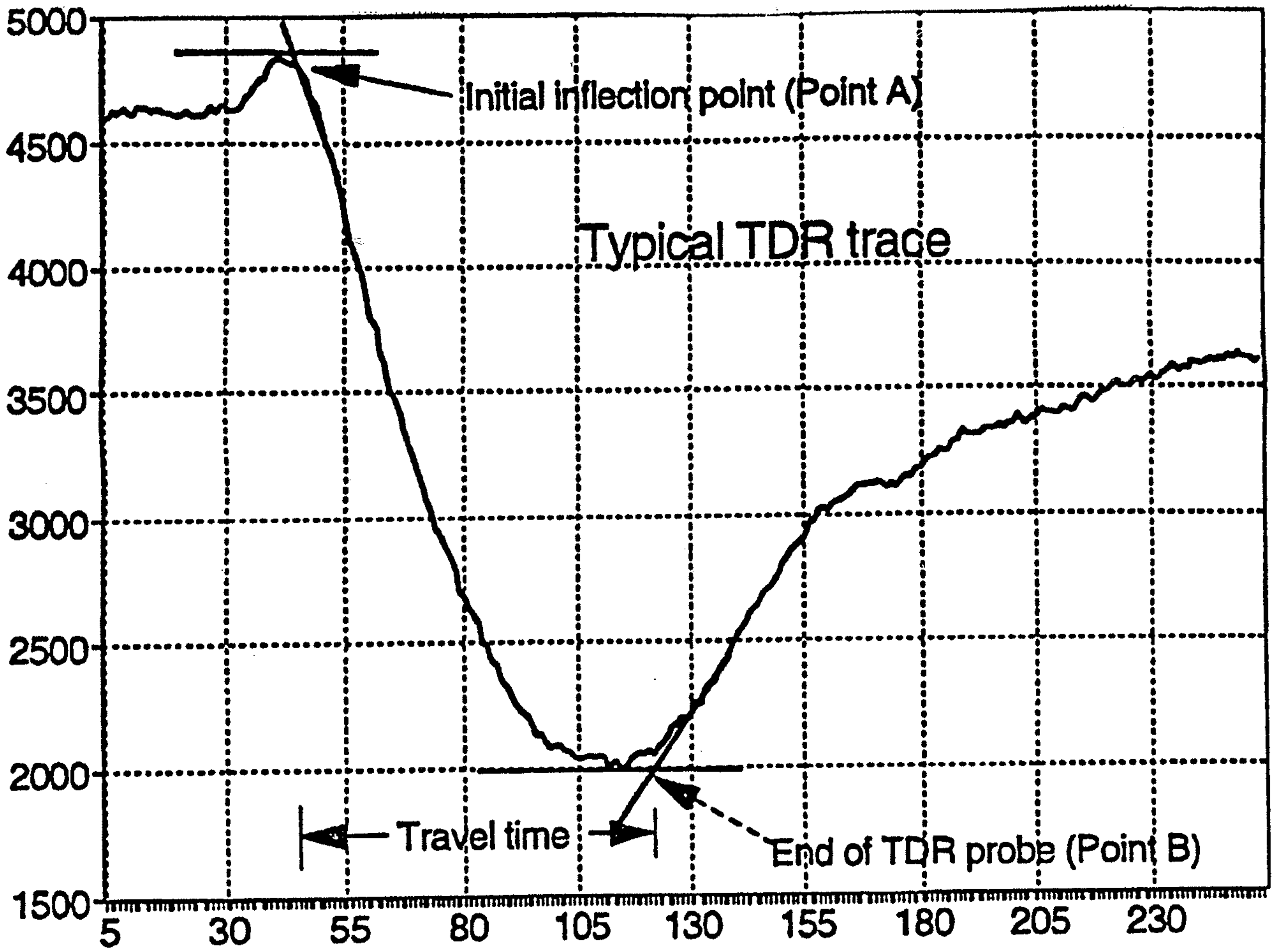

Fig.3. The Method of Tangents, for locating the initial and final points of a typical TDR trace.

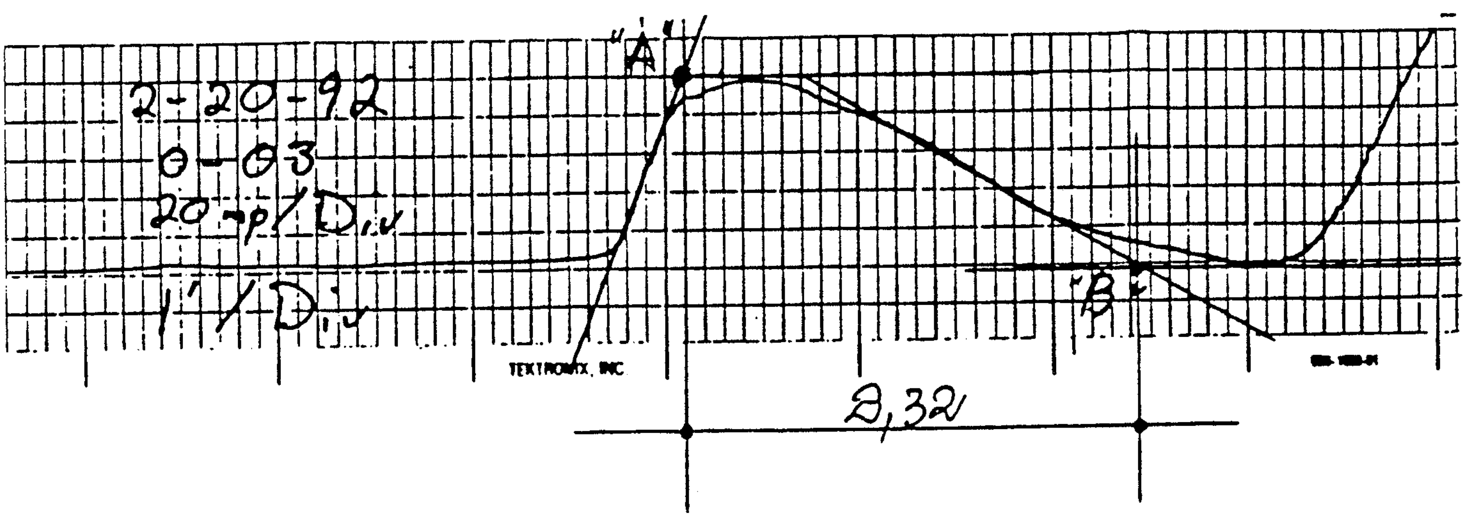

Fig.4. The Alternative Method of Tangents for locating the initial and final points of a typical TDR curve.

Calculating the means and standard deviations of the resulting differences for each pair of data and analyzing their significance, one may establish confidence limits for these differences (see Appendix 5). These confidence limits are given in Table 4.

The influence of the average confidence limit d,avg = 0.18 ft. on soil moisture content determined with flat and curved sensors was studied (see Appendix 8) and found to be less then 1% for curved and 2% for flat sensors. Thus, the influence of the method of reading TDR curves is negligible in the case of curved sensors. When flat-pronged sensors are used, the Method of Diverging Lines is recommended.

The Method of Tangents has not been studied in detail, but a rough comparative examination (see Appendices 1 and 2) shows that this method produces lengths somewhat smaller then those obtained by the Method of Diverging Lines.

From this statistical study, it appears that the data obtained with the Method of Diverging Lines fits well between the data obtained with the other two methods and is the most suitable one for future use.

5. Calculation of Soil Moisture Content Using Empirical Equations and Recommendation for Monitoring of a Specific Road Test Section

Considering soil as composed of three constituent materials, air, dry solids, and water, for which the dielectric constants are as follows:

$$K(air) = 1$$
$$K(dry\ soil) = 3 \text{ to } 6$$
$$K(water) = 79 \text{ to } 82$$

the dielectric constant of a moist soil (Ka) will be somewhere in the range from 6 to 82 and can be calculated from the formula:

$$Ka = (L/VP)^2$$

where L is the trace length, V is the phase velocity, and P is the probe length.

Topp has developed an empirical relationship between a soil's volumetric moisture content (X) and its dielectric constant (Ka):

$$X = -0.053 + (0.0292)Ka - (0.00055)Ka^2 + (0.0000043)Ka^3$$

where $0 < X < 1$. This equation is known as Topp's Universal Equation.[10] Despite its minor limitations for clay materials, in which the soil and water interact chemically and physically, this equation has been proved to be valid in most cases.[6,7,8,10]

The explicit Universal Equation has been compared with two implicit equations:

$$X^3 - (198.8/417.3)X^2 - (30.08/417.3)X - (3.91\text{-}Ka)/417.3 = 0$$

$$X^3 - (146/76.7)X^2 - (9.3/76.7)X - (3.03\text{-}Ka)/76.7 = 0$$

for gravel and soil, respectively,[11] and has produced more pertinent results (see Appendix 9, Borehole #3, February 1992).

The use of existing tabulated values for the solutions of these two implicit equations was found to be slow and burdonsome. To remove these deficiencies, a Lotus-based spreadsheet routine was developed and used. This routine, described in Appendix 10, permits the user to select with equal ease any of the three equations to calculate the soil moisture content.

Except for the case of Borehole #3, February 1992, for which all three equations were used, the Universal Equation was used to calculate soil mositure content for all seasonal tests performed on the Idaho test section. The volumetric moisture contents thus obtained were then transformed into moisture contents by weight using the dry density values of the soil determined during the installation of the TDR sensors.[3] These calculations are contained in Appendix 9.

The final worksheet of Appendix 9 presents the seasonal average soil moisture content under the road pavement and related statistical values characterizing its variability (standard deviation and coefficient of variation). These figures illustrate the change in moisture content with depth and time.

6. Conclusions

1. Both the Method of Diverging Lines and the Alternative Method of Tangents are recommended for the interpretation of TDR traces obtained using 7.5" flat sensors for determining soil moisture content (MC) under road pavements.

2. Use of the other two methods studied requires corrections as follows:

   a) Method of Peaks: MC = MC(Peaks) - 1.8

   b) Method of Tangents: MC = MC(Tangents) + 1.8

3. Any of the three methods for interpreting traces obtained using 12.0" curved sensors could be used without correction.

4. This statistical study could be repeated to evaluate more fully the Method of Tangents using the data in Appendices 1 and 2.

5. Curved TDR sensors produce more reliable results than shorter flat probes.

6. Topp's Universal Equation is recommended for calculating moisture content from experimental data.

7. The Lotus-based spreadsheet routine developed for determining soil moisture content from any of the three equations studied is recommended.

Table 1. The lengths of TDR traces, graphically determined through the use of different methods on Borehole No. 2, from measurements taken in February, March, and April, 1992, at Boise Test Section 163023.

Table 1, Part 1. Measurements Taken During February, 1992.

Wire (No.)	Depth (in.)	Prong (No.)	Vrt. mp/d	Hrz. ft/d	"Peaks" (ft.)	"Div. Lines" (ft.)	"Tangents" (ft.)
1-01	12	3	10	1	1.40	1.40	1.30
			10	1	1.37	1.48	1.30
1-02	17	2	10	1	1.72	1.60	1.53
1-03	22	3	50	1	1.75	1.79	1.69
			20	1	1.65	1.67	1.66
1-04	27	2	20	1	2.16	1.88	2.10
1-05	33	3	50	1	2.25	2.00	2.20
1-06	39	2	10	1	1.68	1.60	1.52
			5	1	1.65	1.67	1.55
1-07	45	3	50	1	1.72	1.81	1.49
			20	1	1.65	1.60	1.52
1-08	57	2	20	1	2.12	1.84	2.09
1-09	69	3	50	1	2.77	2.45	2.85
1-10	81	2	50	1	2.60	2.39	2.70
					3.10	2.22	3.79

Table 1, Part 2. Measurements Taken During March, 1992.

Wire (No.)	Depth (Ins.)	Prong (No.)	Vrt mp/d	Hrz ft/d	"Peaks" (ft.)	"Div. Lns." (ft.)	"Tang." (ft.)
1-01	12	3	10	1	1.42	1.38	1.30
			10	1	1.47	1.40	1.10
1-02	17	2	5	1	1.60	1.60	1.10
1-03	22	3	20	1	1.89	1.82	1.72
			20	1	1.90	1.90	1.80
1-04	27	2	20	1	2.08	1.78	2.00
1-05	33	3	50	1	2.29	2.51	2.22
			50	1	2.18	2.02	2.20
1-06	39	2	20	1	1.73	1.60	1.60
1-07	45	3	20	1	1.72	1.49	1.63
			20	1	1.75	1.45	—
1-08	57	2	20	1	2.02	1.77	2.05
			20	1	2.07	1.98	2.05
1-09	69	3	50	1	2.88	2.40	3.07
			50	1	2.80	2.26	3.90
1-10	81	2	50	1	2.80	2.40	3.40
			50	1	2.50	2.1	3.70

Table 1, Part 3. Measurements Taken During April, 1992.

Wire (No.)	Depth (Ins.)	Prong (No.)	Vrt. mp/d	Hrz. ft/d	"Peaks" (ft.)	"Div. Lns" (ft.)	"Tang." (ft.)
1-01	12	3	50	1	1.60	1.67	1.40
1-02	17	2	5	1	1.68	1.62	1.57
1-03	22	3	50	1	1.90	1.88	1.82
1-04	27	2	20	1	2.04	1.78	2.12
1-05	33	3	50	1	2.21	2.10	2.23
1-06	39	2	5	1	1.72	1.65	1.58
1-07	45	3	50	1	1.77	1.59	1.65
1-08	57	2	20	1	2.10	2.00	2.20
1-09	69	3	50	1	2.80	2.15	3.16
1-10	81	2	50	1	2.60	2.54	3.79

Table 2. The lengths of TDR traces, graphically determined using three different methods on Borehole No. 3 (12-inch curved-prong sensors) from measurements taken in February, March, and April, 1992, at Boise, Idaho Test Section 163023.

Table 2, Part 1. Measurements Taken During February, 1992.

Wire (No.)	Depth (Ins.)	Prong (No.)	Vrt. mp/d	Hrz. ft/d	"Peaks" (ft.)	"Div. Ins." (ft.)	"Tang." (ft.)
0-01	12	3	10	1	1.69	1.66	1.03
			5	1	1.47	1.69	1.05
0-02	17	2	20	1	2.44	2.43	2.03
			10	1	2.44	2.43	2.03
			5	1	2.44	2.34	2.05
0-03	22	3	20	1	4.12	3.05	2.78
0-04	27	2	20	1	3.00	2.90	2.90
0-05	33	3	10	1	2.49	2.19	1.90
			5	1	2.45	2.04	2.03
0-06	39	2	20	1	2.50	2.39	2.11
0-07	45	3	100	1	2.40	2.74	-
0-08	57	2	10	1	2.38	2.50	2.11
0-09	69	3	50	1	3.55	3.45	3.65
0-10	81	2	20	1	3.30	2.66	3.15

Table 2, Part 2. Measurements Taken During March, 1992.

Wire (No.)	Depth (Ins.)	Prong (No.)	Vrt. mp/d	Hrz. ft/d	"Peaks" (ft.)	"Div. Lns" (ft.)	"Tang." (ft.)
0-01	12	3	10	1	2.02	1.90	1.68
0-02	17	2	10	1	2.40	2.40	2.08
0-03	22	3	20	1	2.98	2.29	2.90
			50	1	3.05	2.58	2.90
0-04	27	2	20	1	2.95	2.97	2.95
0-05	33	3	10	1	2.60	2.48	2.48
			10	1	2.63	2.40	2.50
0-06	39	2	10	1	2.46	2.40	2.05
0-07	45	3	50	1	2.79	2.53	2.60
			50	1	2.76	2.58	–
0-08	57	2	10	1	2.36	2.35	2.35
0-09	69	3	50	1	3.63	3.37	3.85
0-10	81	2	20	1	3.05	2.35	3.15

Table 2, Part 3. Measurements Taken During April, 1992.

Wire (No.)	Depth (ins.)	Prong (No.)	Vrt. mp/d	Hrz. ft/d	"Peaks" (ft.)	"Div. lns" (ft.)	"Tang." (ft.)
0-01	12	3	50	1	2.17	2.10	2.60
0-02	17	2	10	1	2.29	2.38	2.29
0-03	22	3	50	1	3.00	2.40	3.16
0-04	27	2	20	1	2.93	2.85	2.48
0-05	33	3	20	1	2.70	2.58	2.30
0-06	39	2	10	1	2.45	2.38	2.30
0-07	45	3	50	1	2.72	2.63	2.64
0-08	57	2	10	1	2.33	2.28	2.72
0-09	69	3	50	1	3.62	3.00	4.05
0-10	81	2	20	1	3.15	2.63	3.30

Table 3. Statistical Comparison Between Pairs of TDR Readings Obtained by Using the "Method of Peaks" and the "Method of Diverted Lines."

Month	Annex	W	Wcr	Relation	Conclusion
A) Borehole No. 2, 7½-inch flat prongs:					
February	31	24	25	W<Wcr	Different Results
March	32	7	35	W<Wcr	Different Results
April	32	3.5	8	W<Wcr	Different Results
B) Borehole No. 3, 12-inch curved prongs:					
February	41	32.5	25	W>Wcr	Similar Results
March	42	2	7	W<Wcr	Different Results
April	43	5.5	8	W<Wcr	Different Results

Table 4. Confidence limits for the differences (di) between the reading of typical TDR curves, utilizing the method of "Peaks" and the method of "Divided Lines" in the hypothesis:

TDR(Peaks) - TDR(Div.Lns) + di

Confidence limits for di (ft.)			
Probability (P%)	**d,min**	**d,max**	**d,avg**
95	0.06	0.29	0.18
90	0.09	0.26	0.18
80	0.12	0.23	0.18
60	0.15	0.22	0.18

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List of Appendices

1. Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

2. Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

3. Application of the Wilcox's Signed-Rank Test, for pairs of TDR readings, obtained in Borehole #2, by using different graphical methods of reading.

4. Application of the Wilcox's Signed-Rank Test, for pairs of TDR readings, obtained on Borehole #3, by using different graphical methods of reading.

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9. Evaluation of soil moisture contents from TDR readings, graphically determined, by Method of Diverging Lines. Worksheets and graphical representation.

10. Instructions for use of Lotus-based spreadsheet routine for calculation of volumetric moisture content (VMC) from TDR experimental data using various empirical equations.

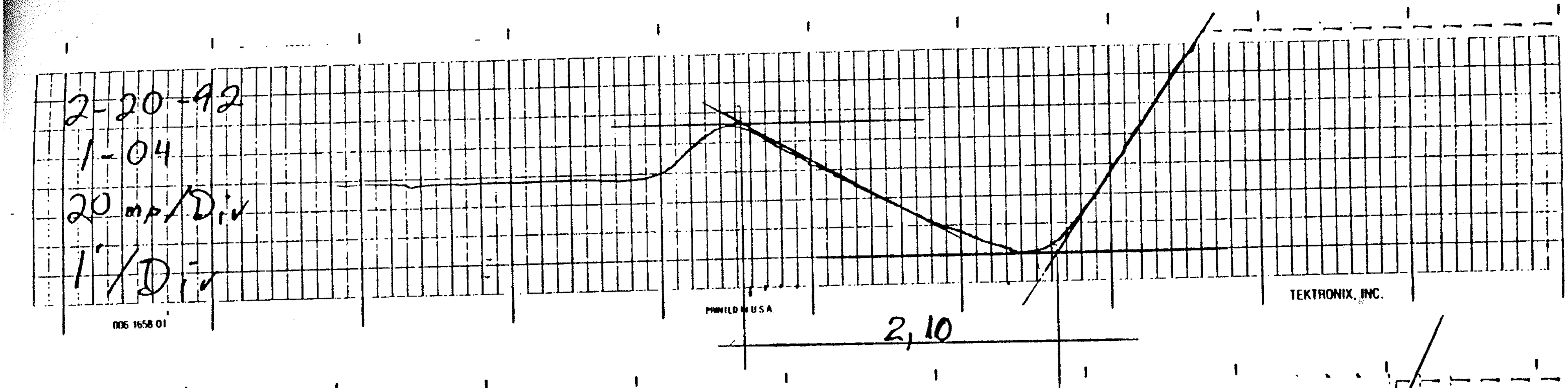
Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

aaa) "Method of Tangents", Borehole #2/February 1992



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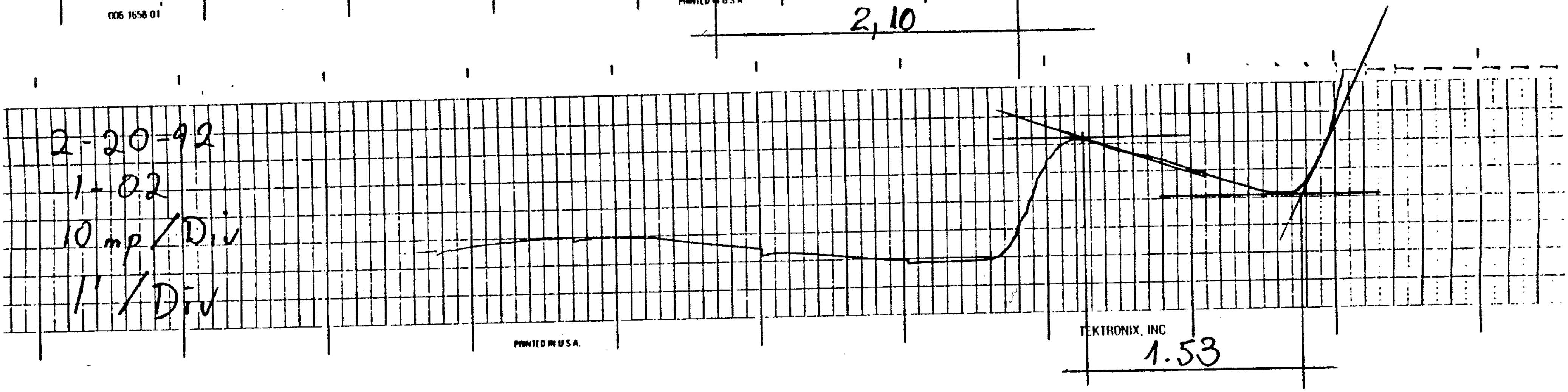

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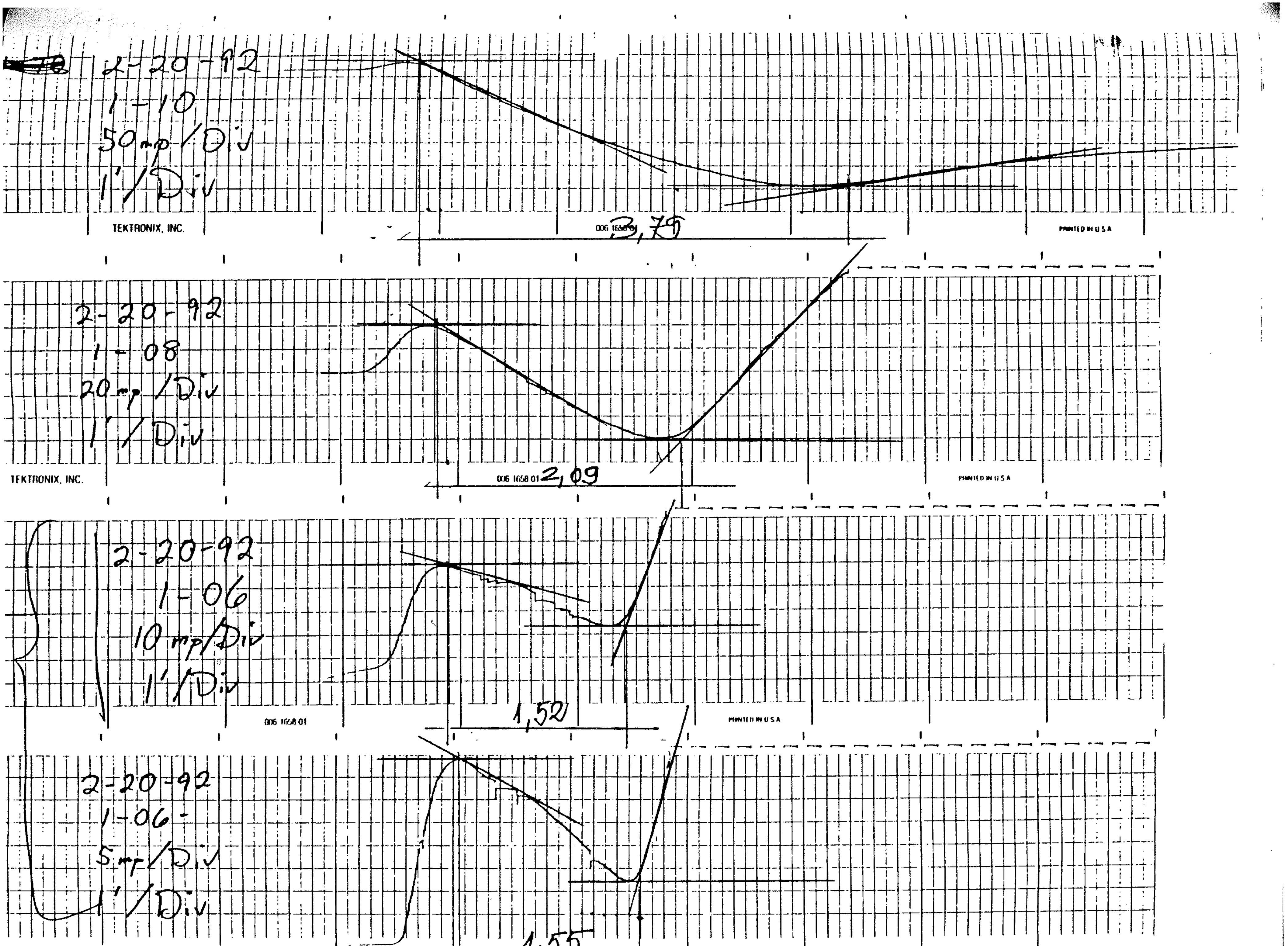
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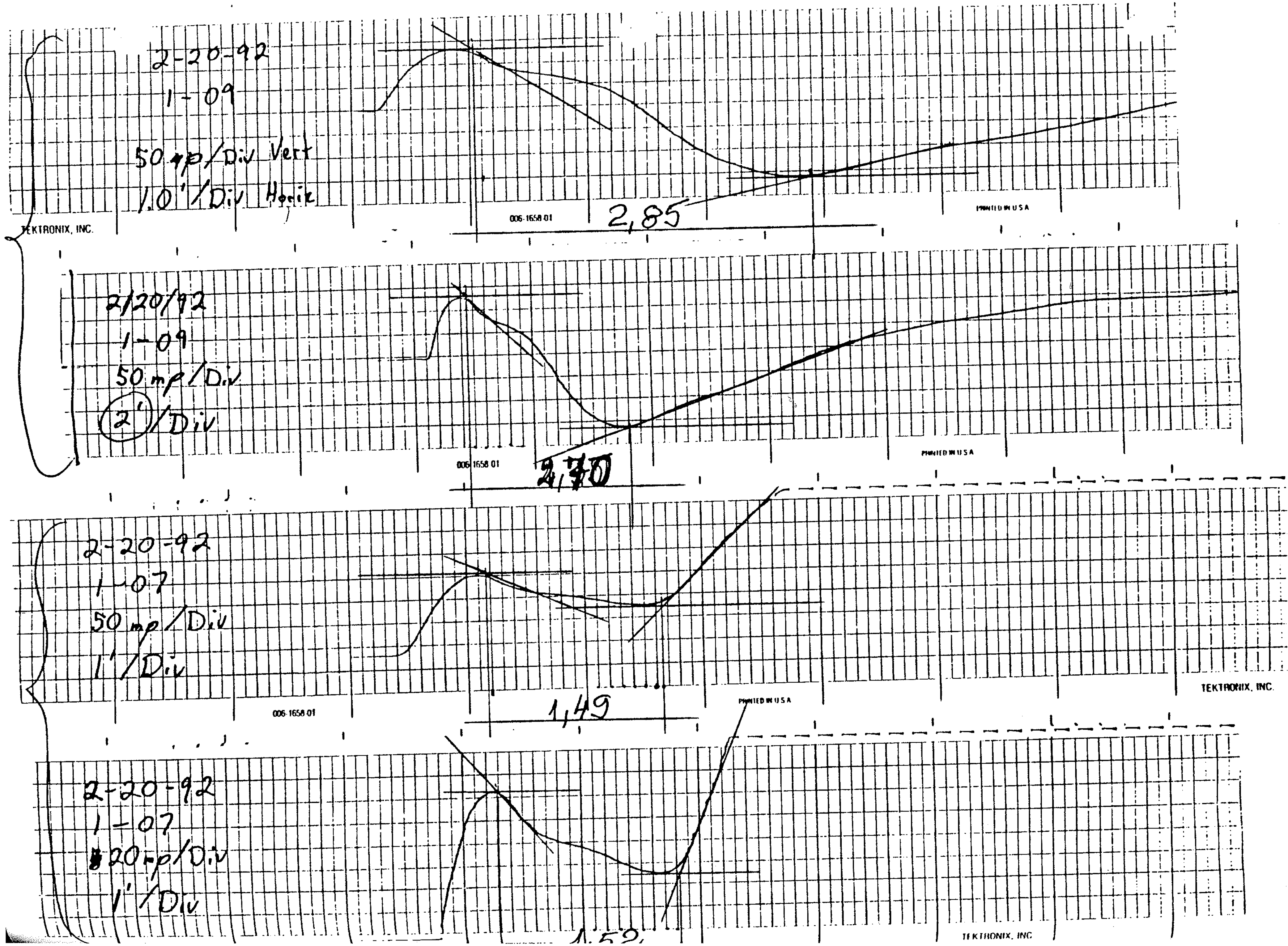

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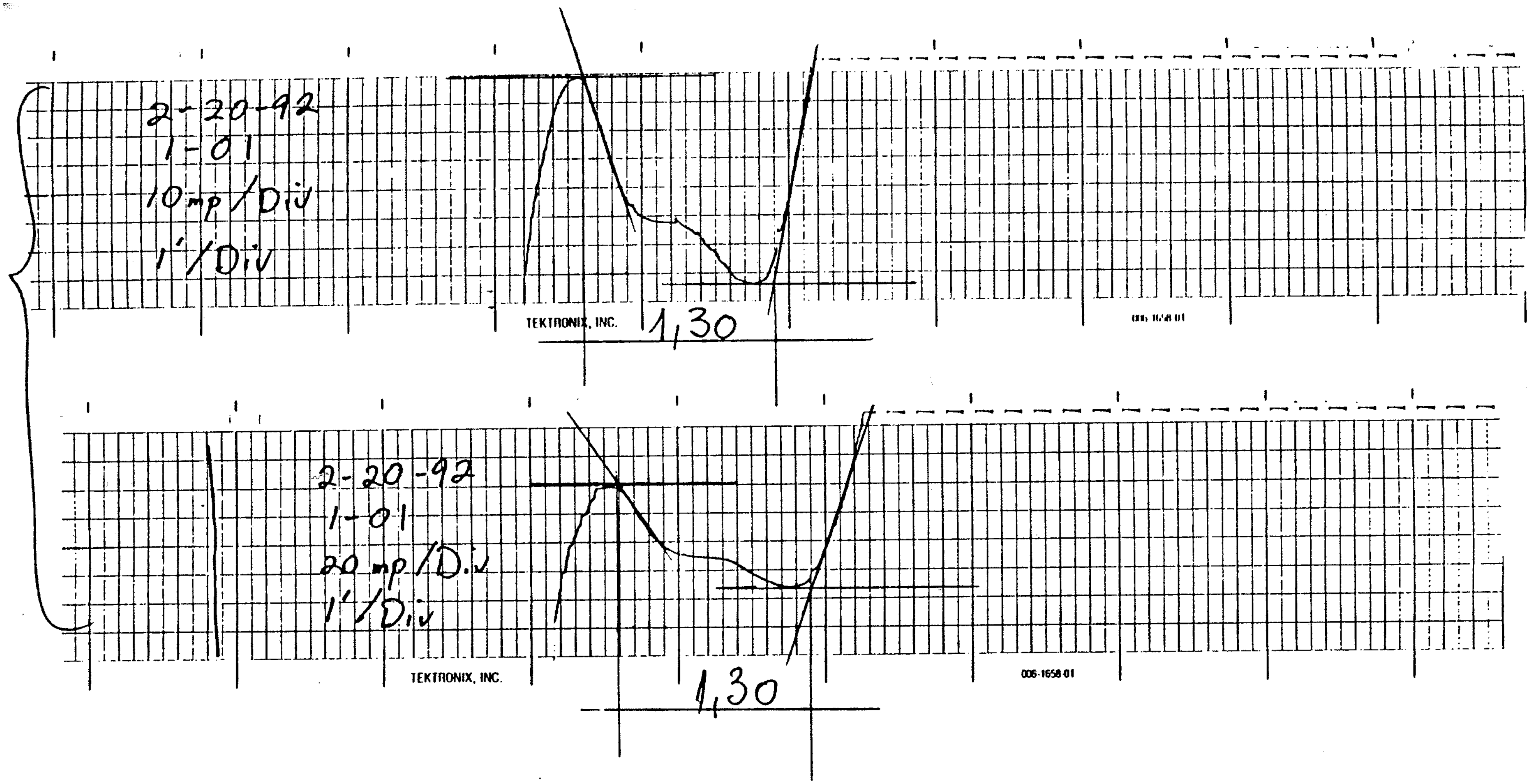
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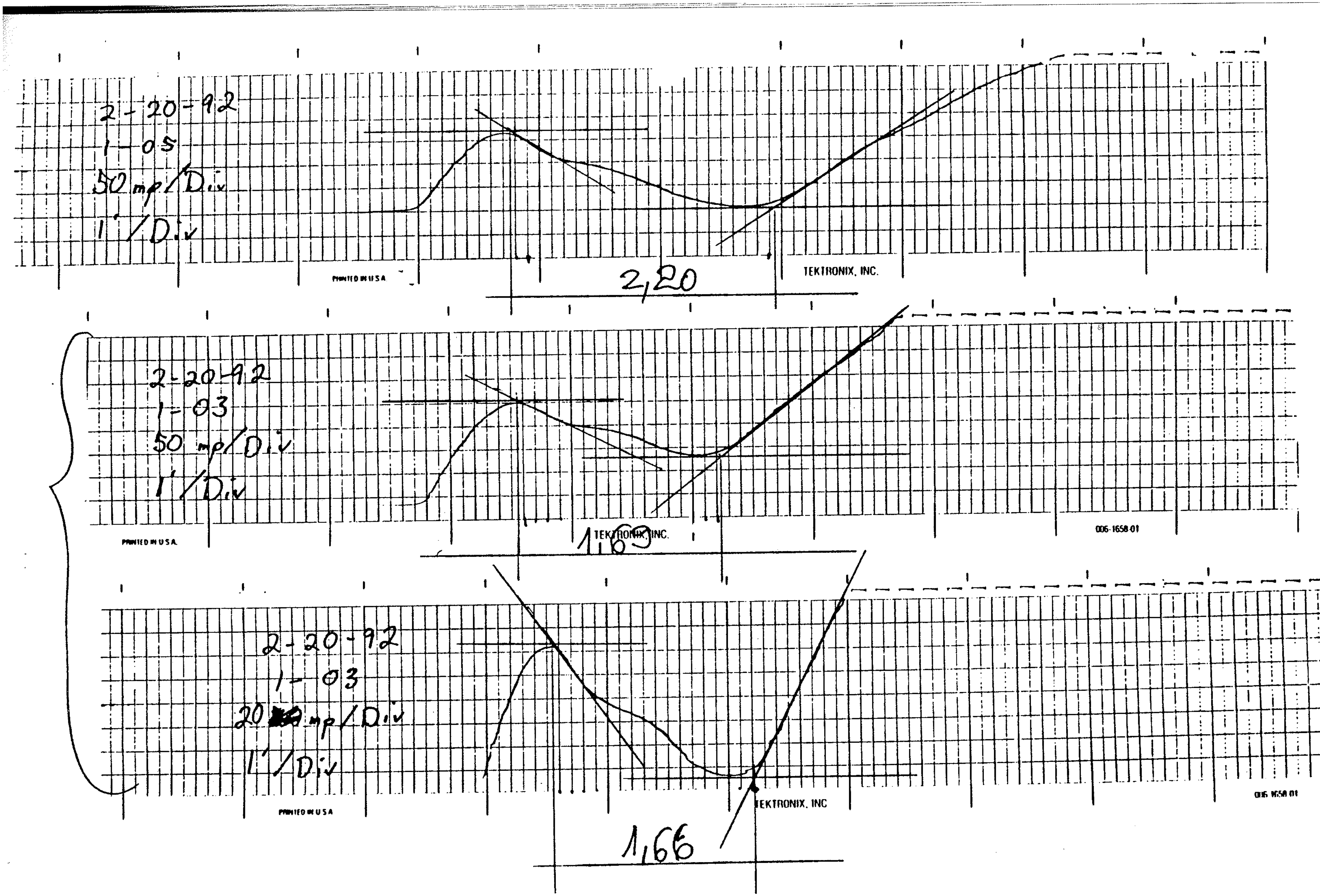
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1,66
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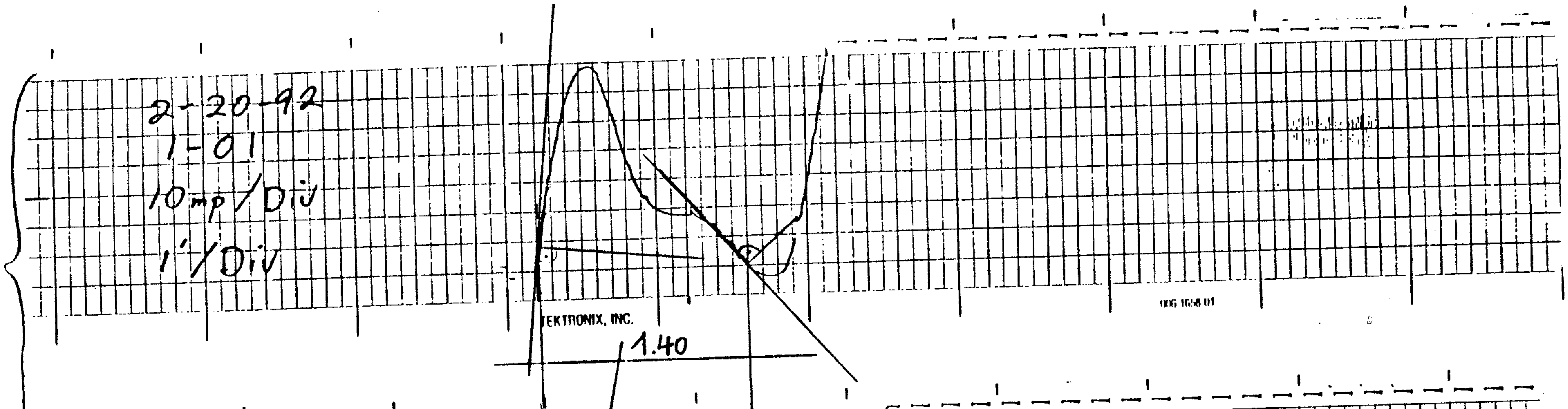
TEKTRONIX, INC.
006-1658-01

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

bbb) "Method of Diverging Lines", Borehole #2/February 1992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1'/Div
TEKTRONIX, INC.
006-1658 01
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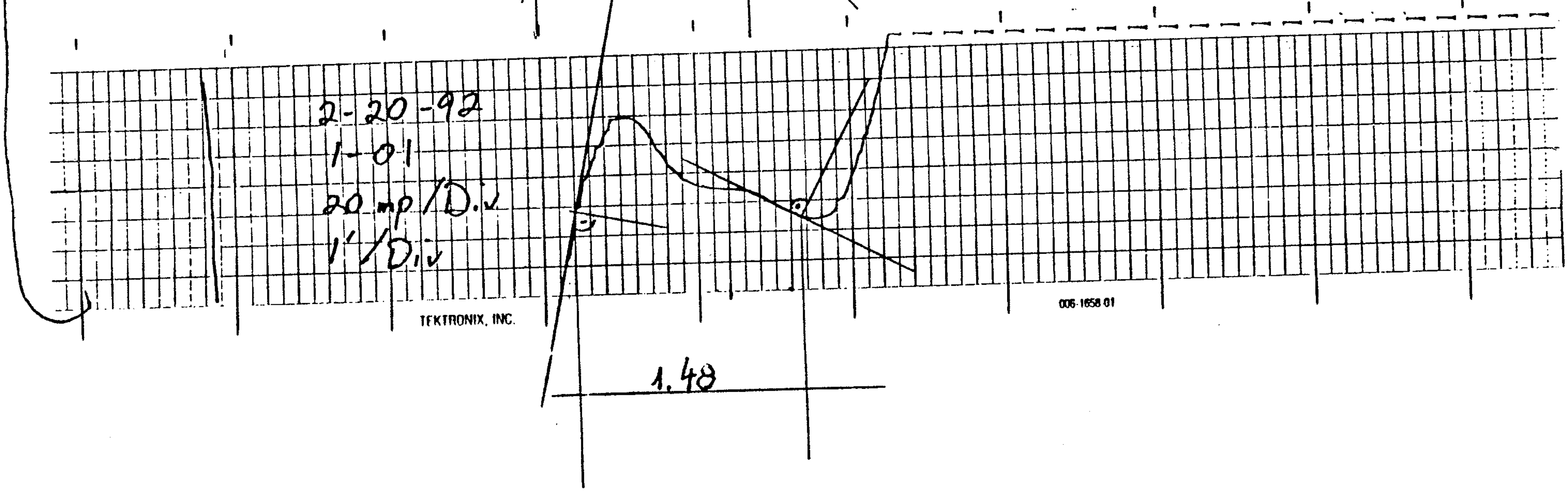
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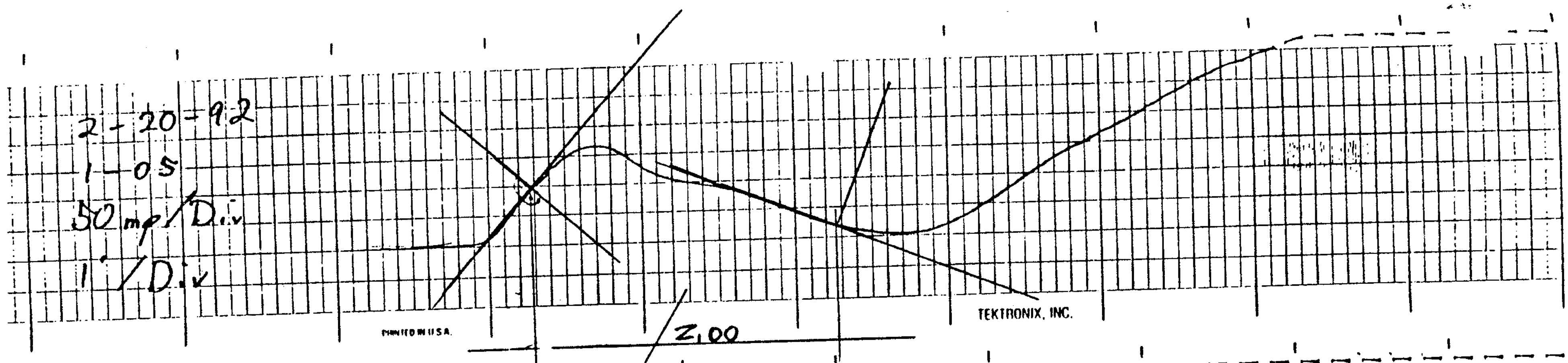
006-1658 01
1.48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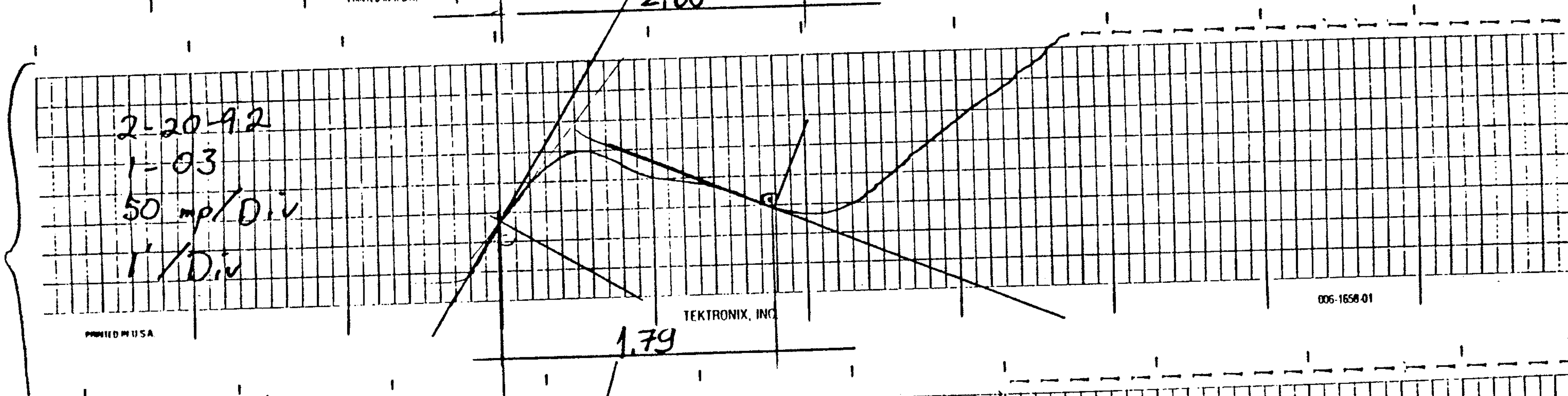
TEKTRONIX, INC.
2.00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TEKTRONIX, INC.
006-1658-01
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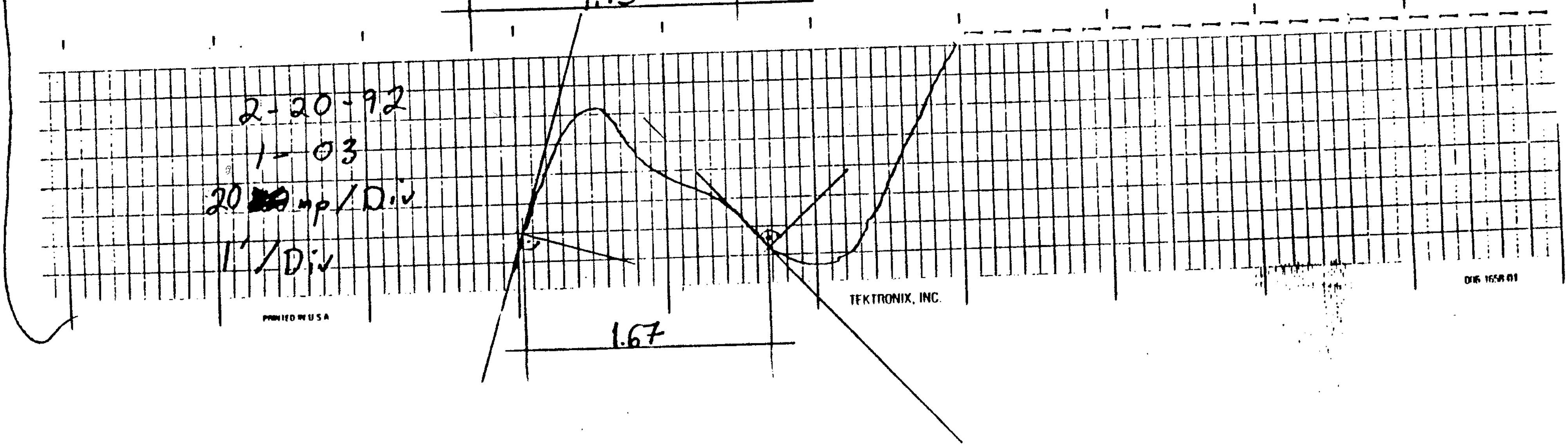
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TEKTRONIX, INC.
006-1658-01
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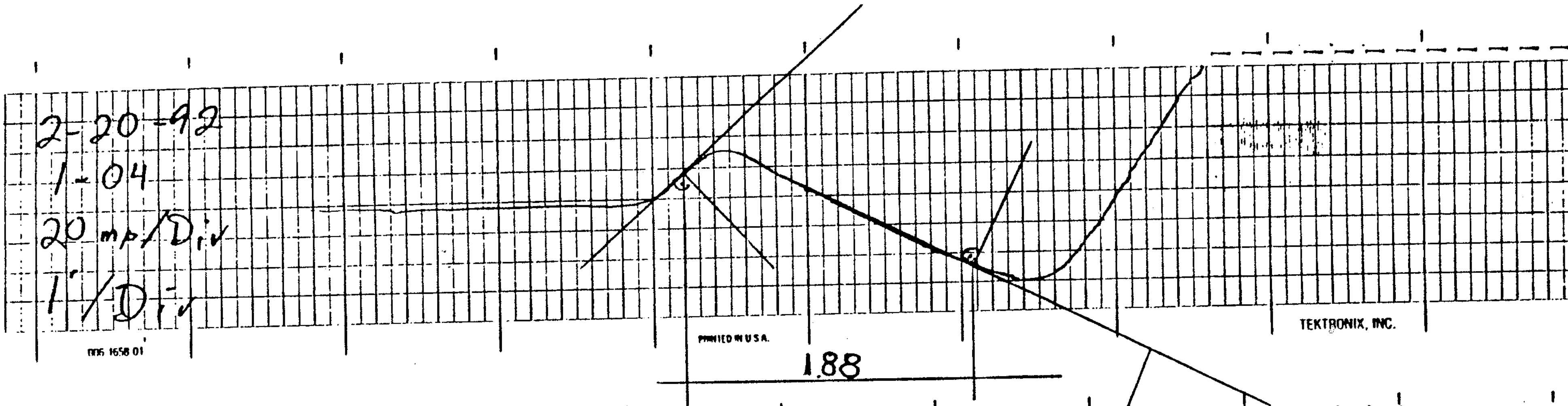
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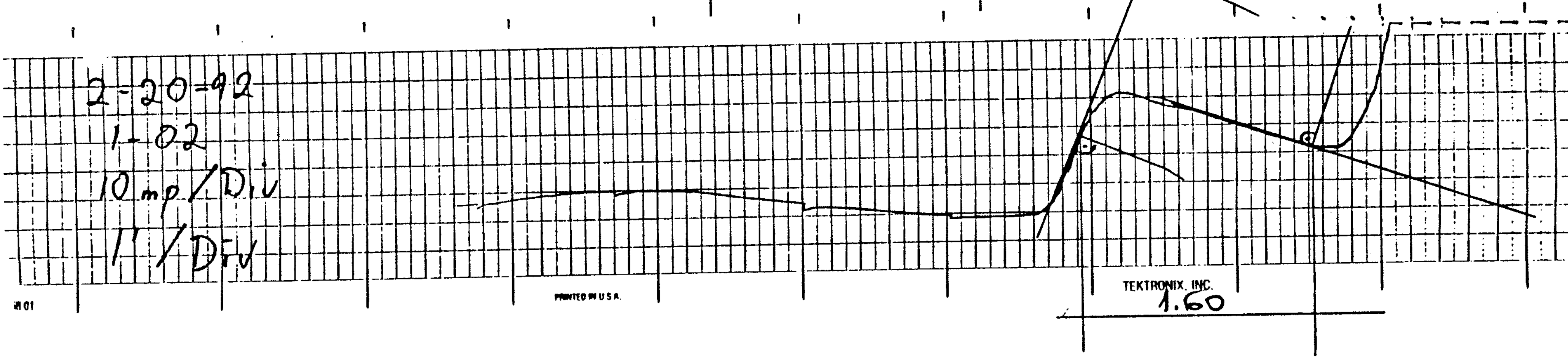
TEKTRONIX, INC.
006 1658 01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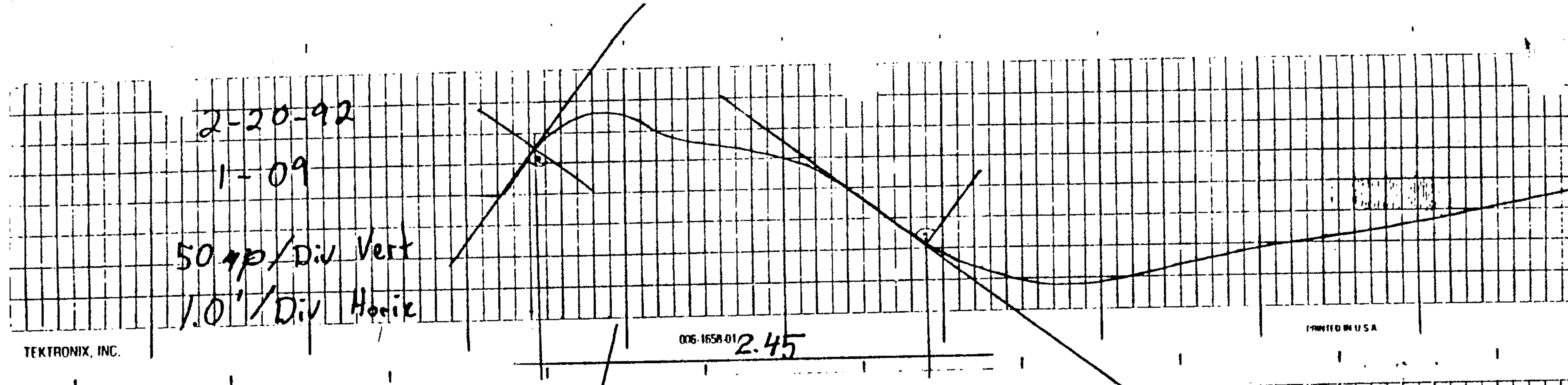
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1.0'/Div Horiz
TEKTRONIX, INC.
006-1658-01
2.45
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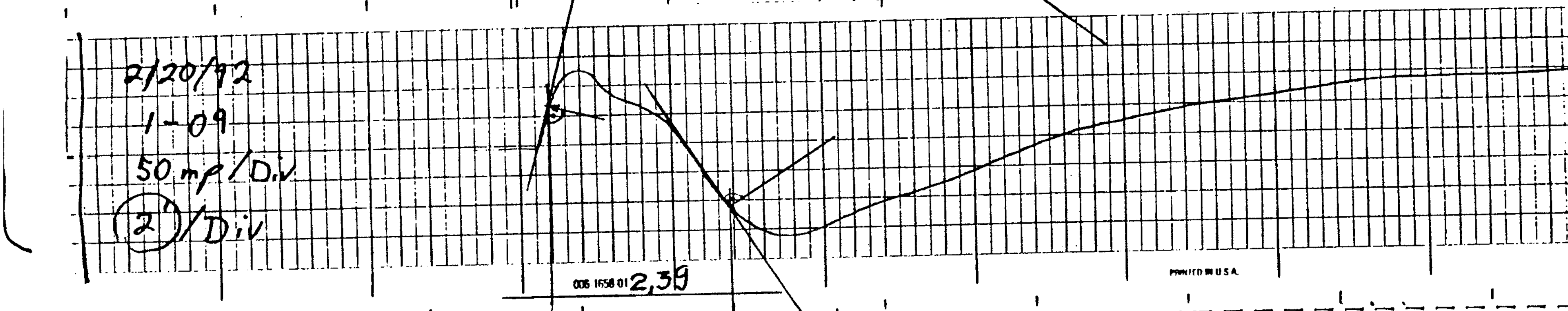
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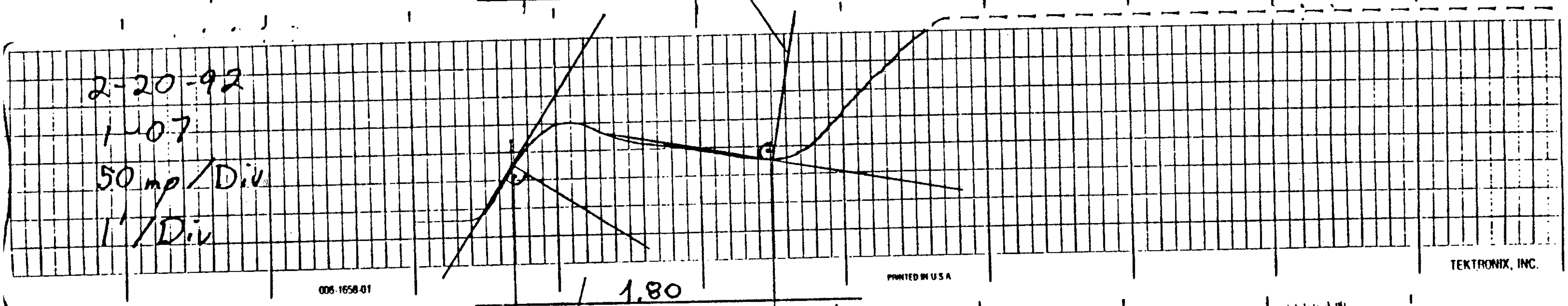
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006-1658-01
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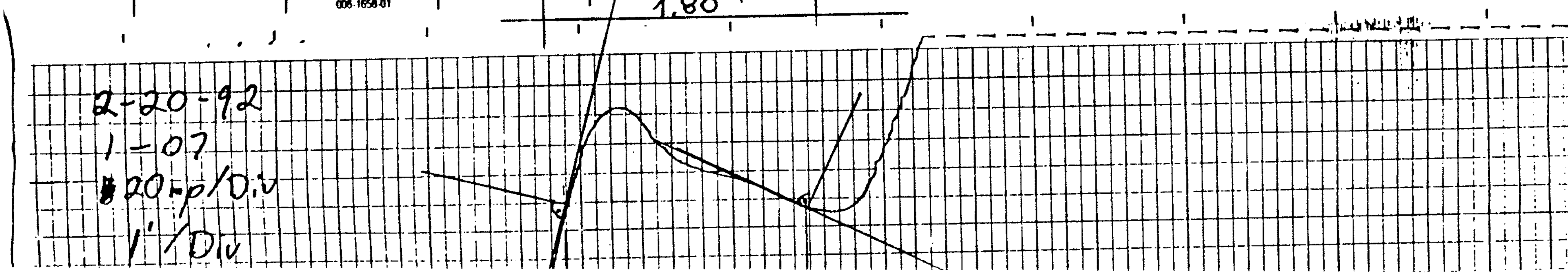
TEKTRONIX, INC.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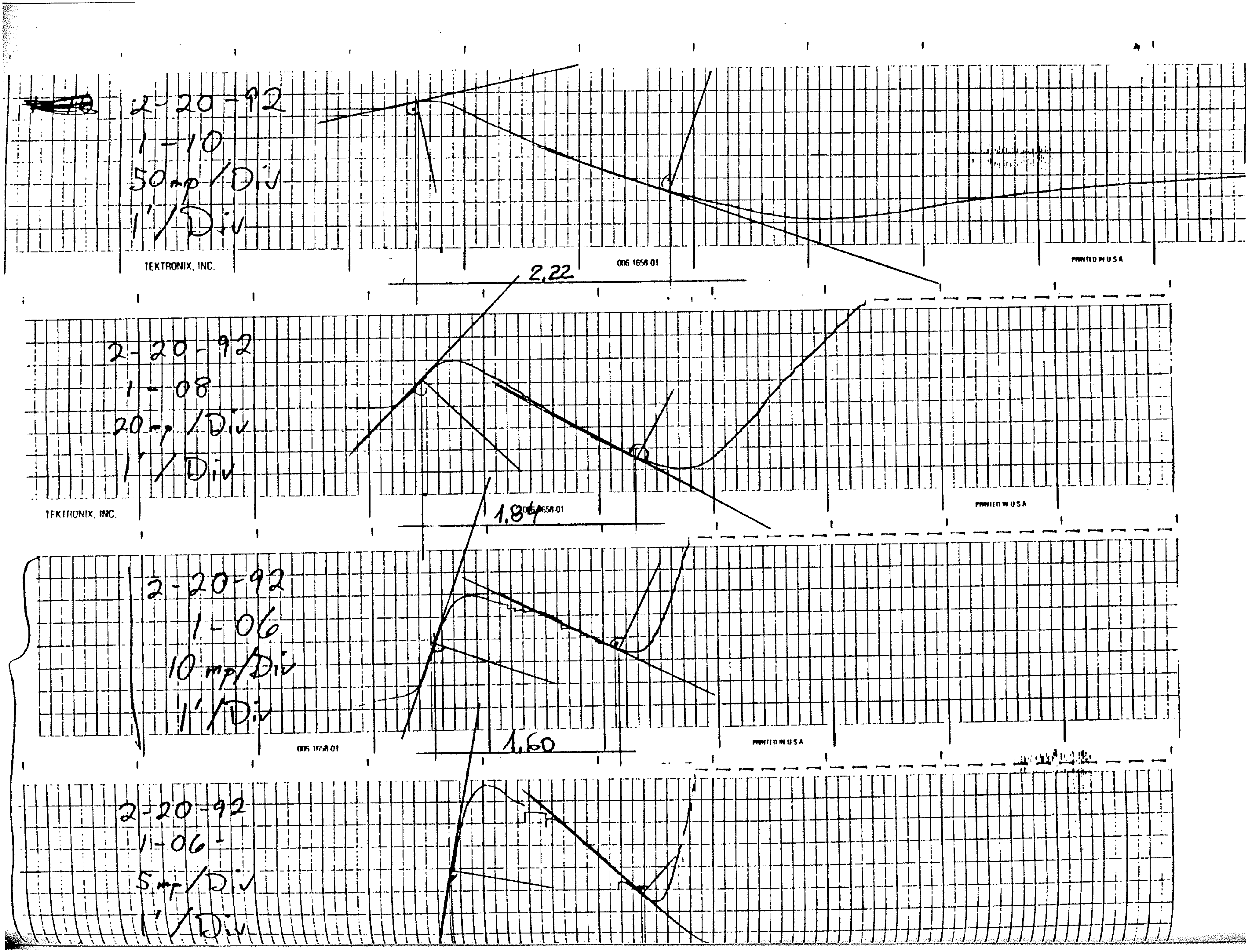
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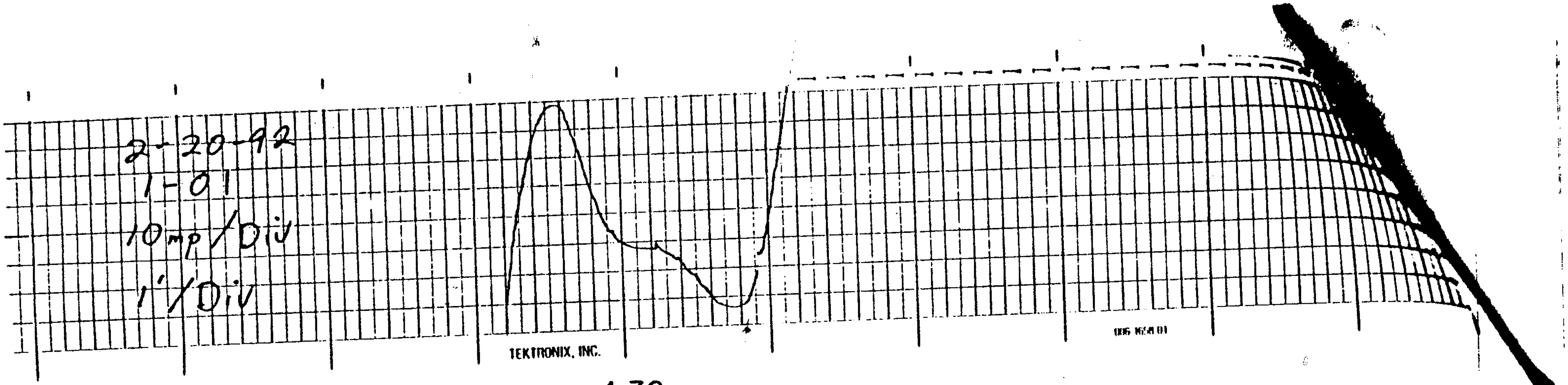
1-10
50mp/Div
1'/Div
TEKTRONIX, INC.
006 1658 01
PRINTED IN U.S.A.
2.22
2-20-92
1-08
20mp/Div
1'/Div
TEKTRONIX, INC.
006 1658 01
PRINTED IN U.S.A.
1.84
2-20-92
1-06
10mp/Div
1'/Div
006 1658 01
PRINTED IN U.S.A.
1.60
2-20-92
1-06
5mp/Div
1'/Div

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

ccc) "Method of Peaks", Borehole #2/February 1992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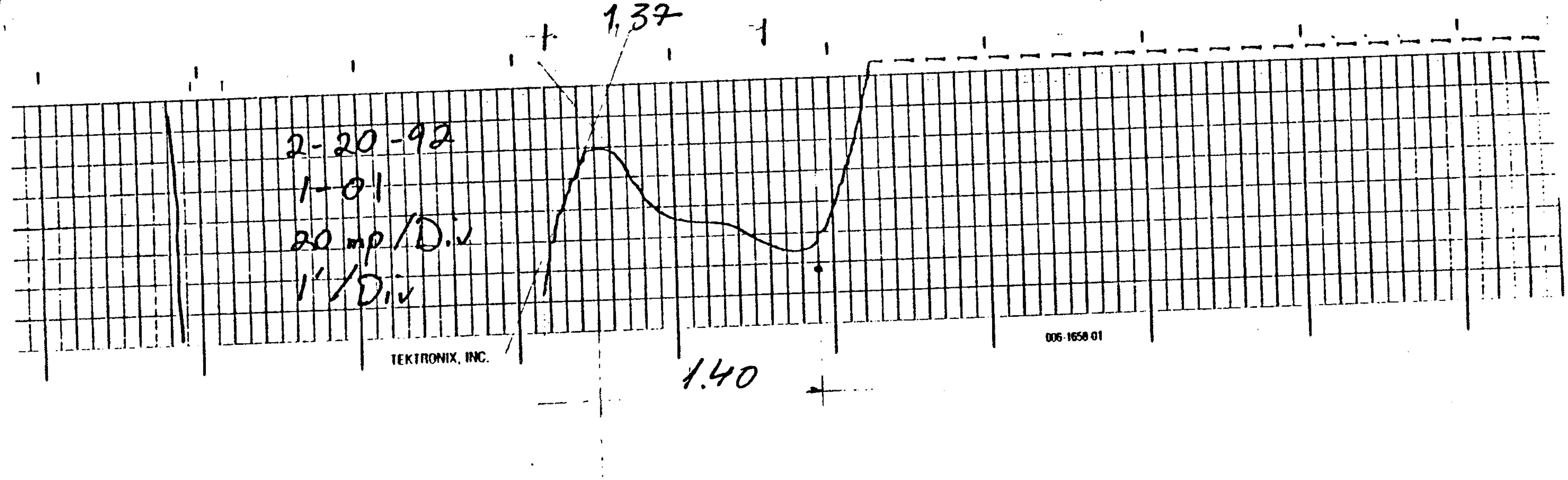
TEKTRONIX, INC.
006-1658-01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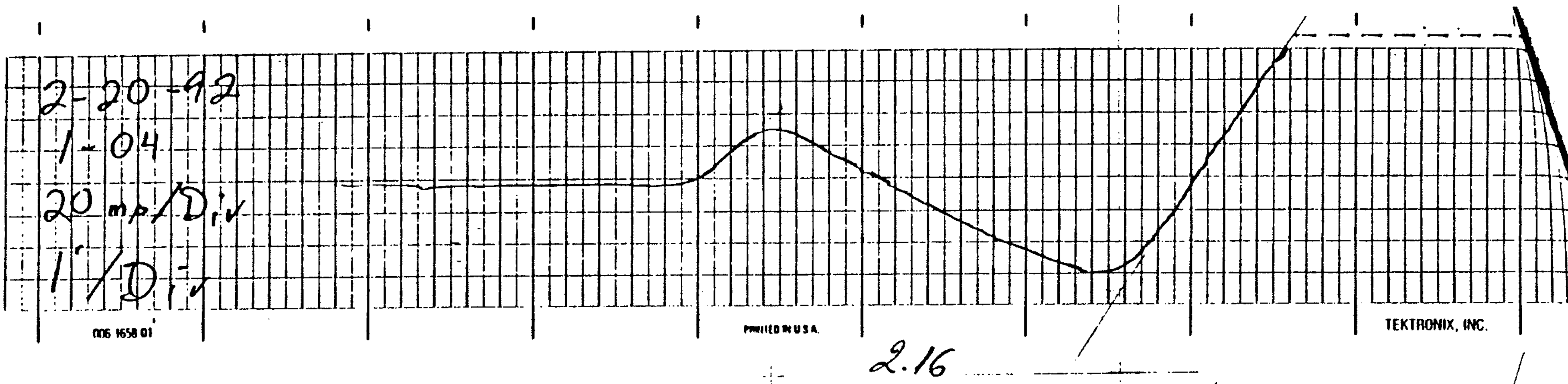
TEKTRONIX, INC.
006-1658-01
1.40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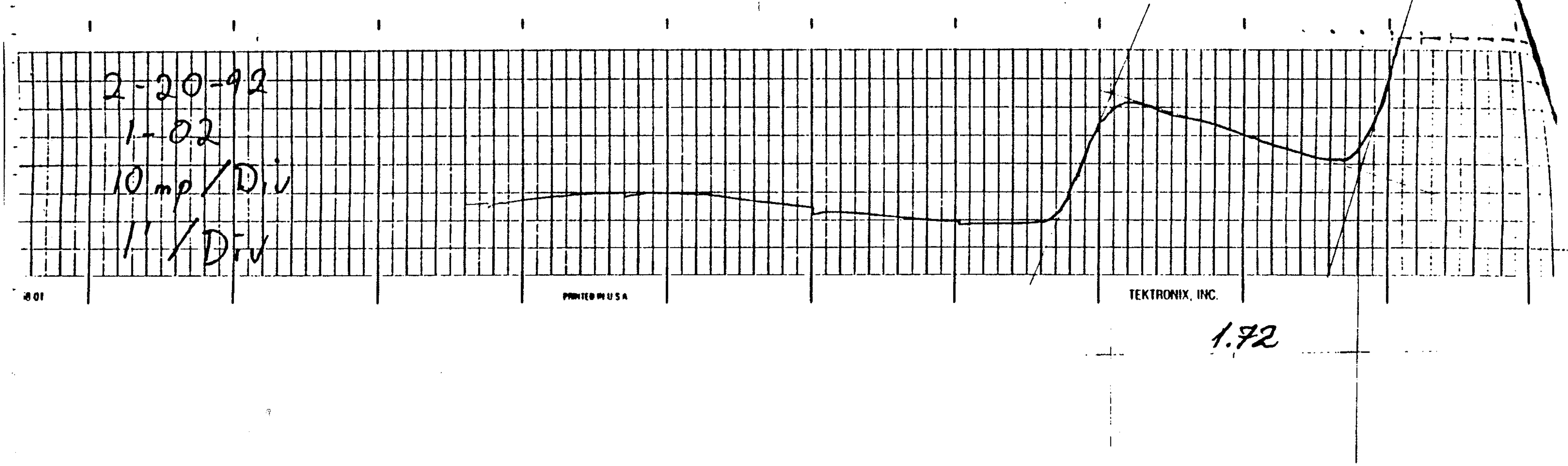
TEKTRONIX, INC.
2.16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10 mp/Div
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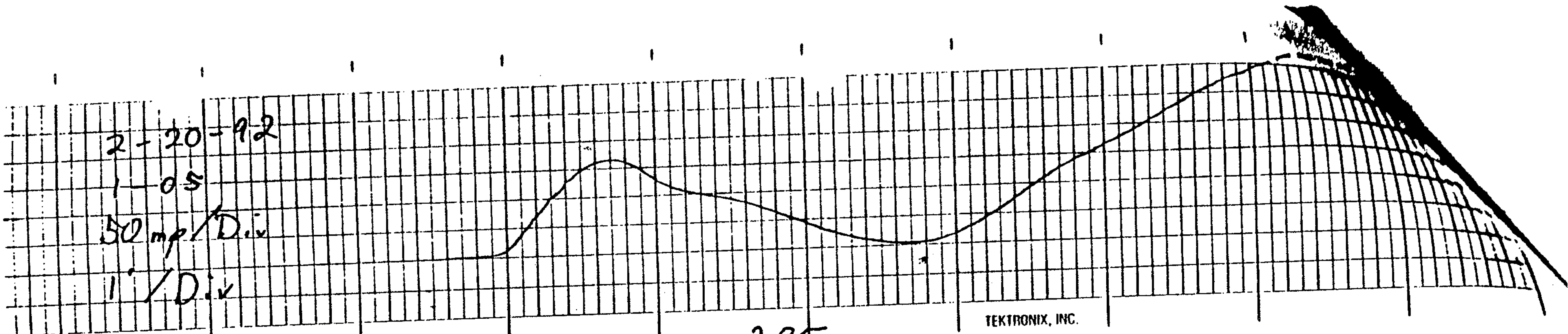
PRINTED IN U.S.A.
TEKTRONIX, INC.
1.72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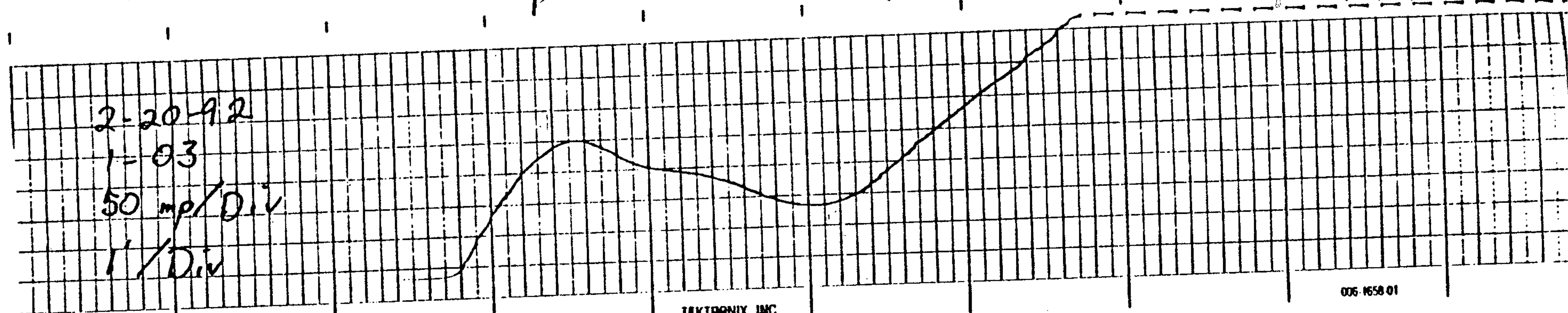
TEKTRONIX, INC.

2,25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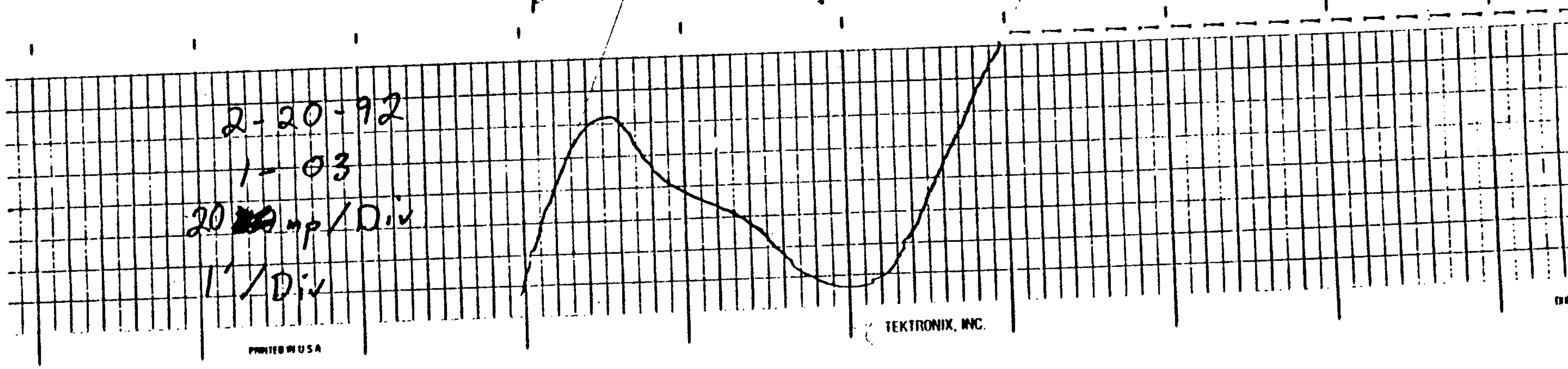
TEKTRONIX, INC.
006-1658-01

1,75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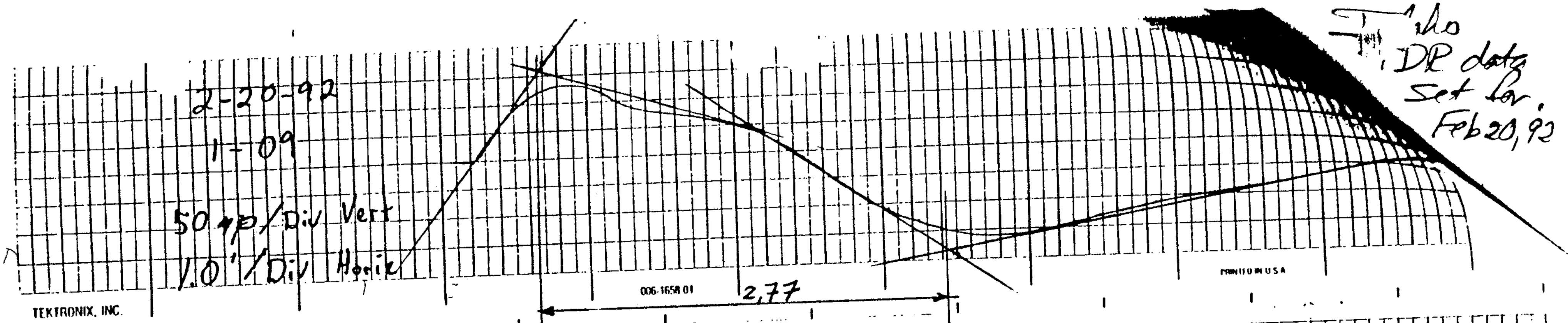
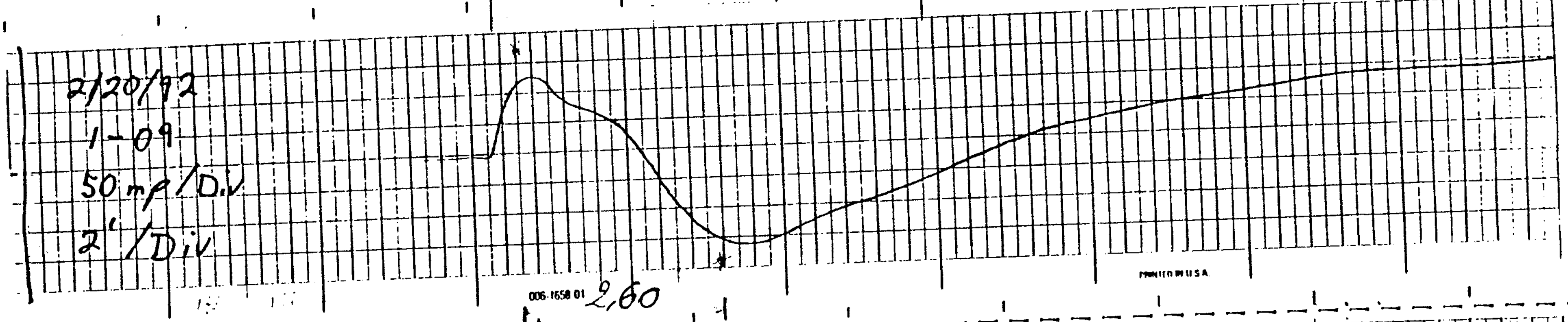
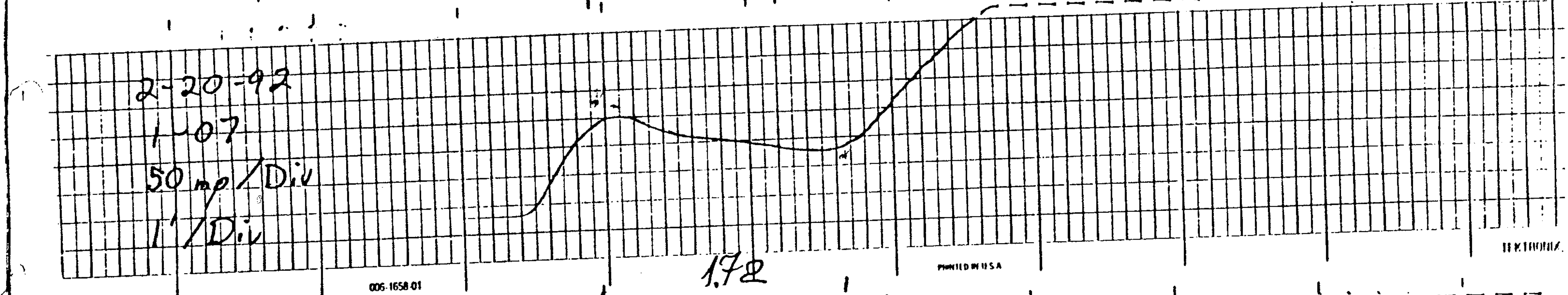
PRINTED IN U.S.A.
TEKTRONIX, INC.

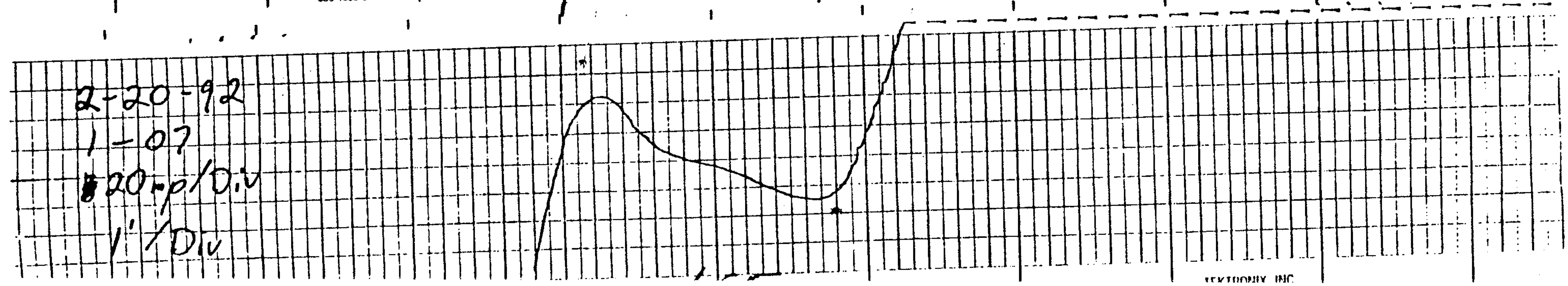
1.65

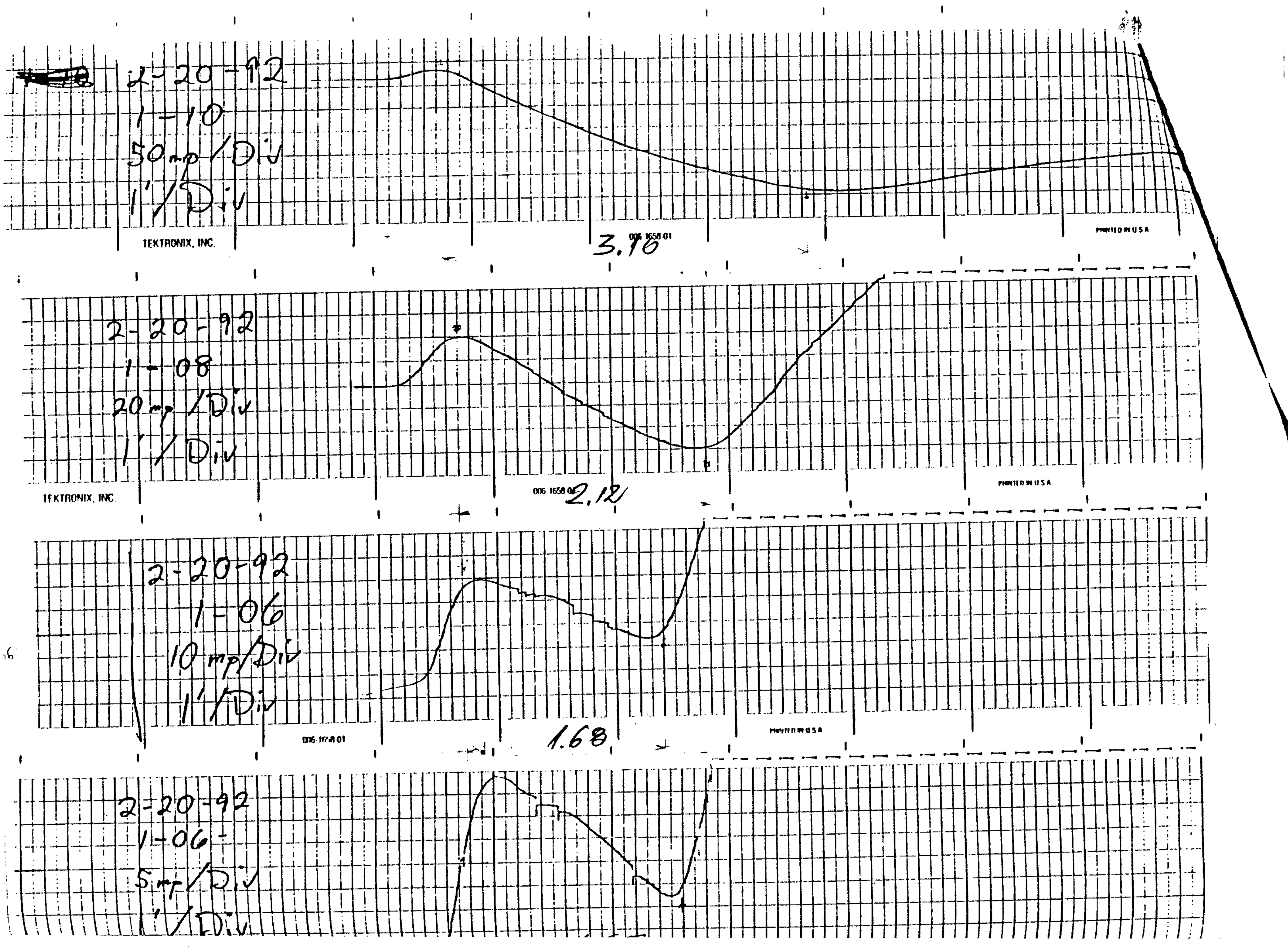
File, DP data Set for, Feb 20, 92







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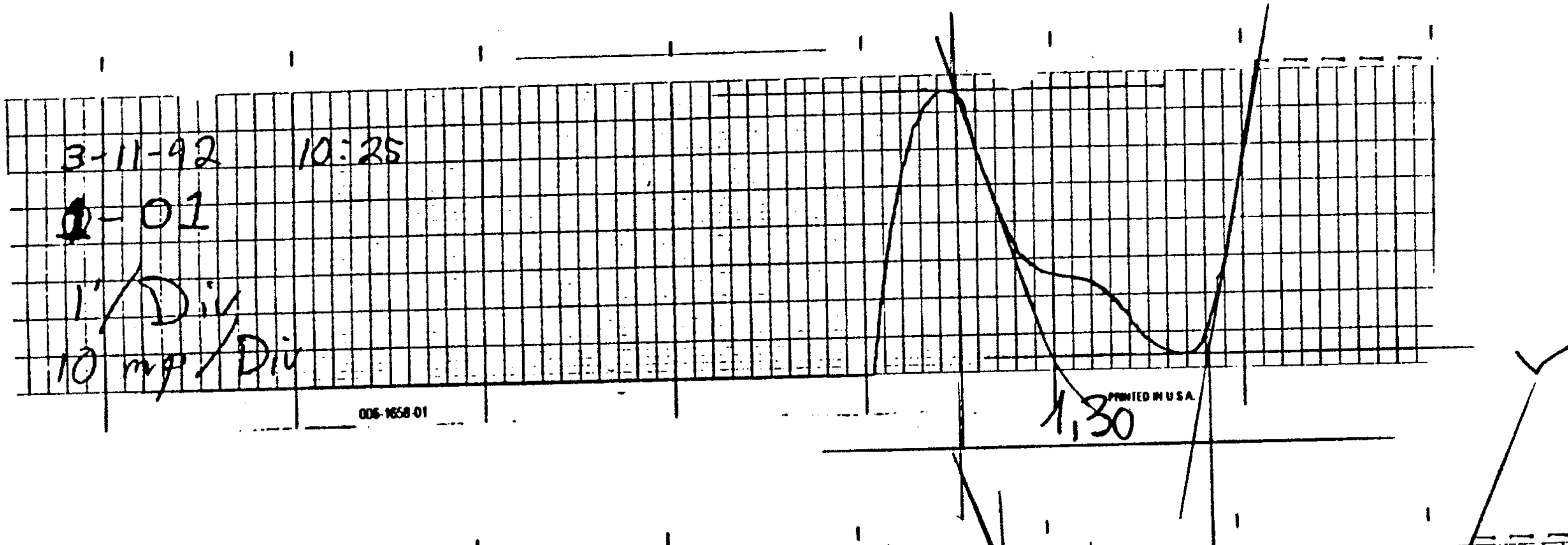
TEKTRONIX, INC.
006 1658-01
3.16
PRINTED IN U.S.A
2-20-92
1-08
20mp/Div
1'/Div
TEKTRONIX, INC.
006 1658-01
2.12
PRINTED IN U.S.A
2-20-92
1-06
10 mp/Div
1'/Div
006 1658-01
1.68
PRINTED IN U.S.A
2-20-92
1-06
5mp/Div
1'/Div

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

aaa) "Method of Tangents", Borehole #2, March 1992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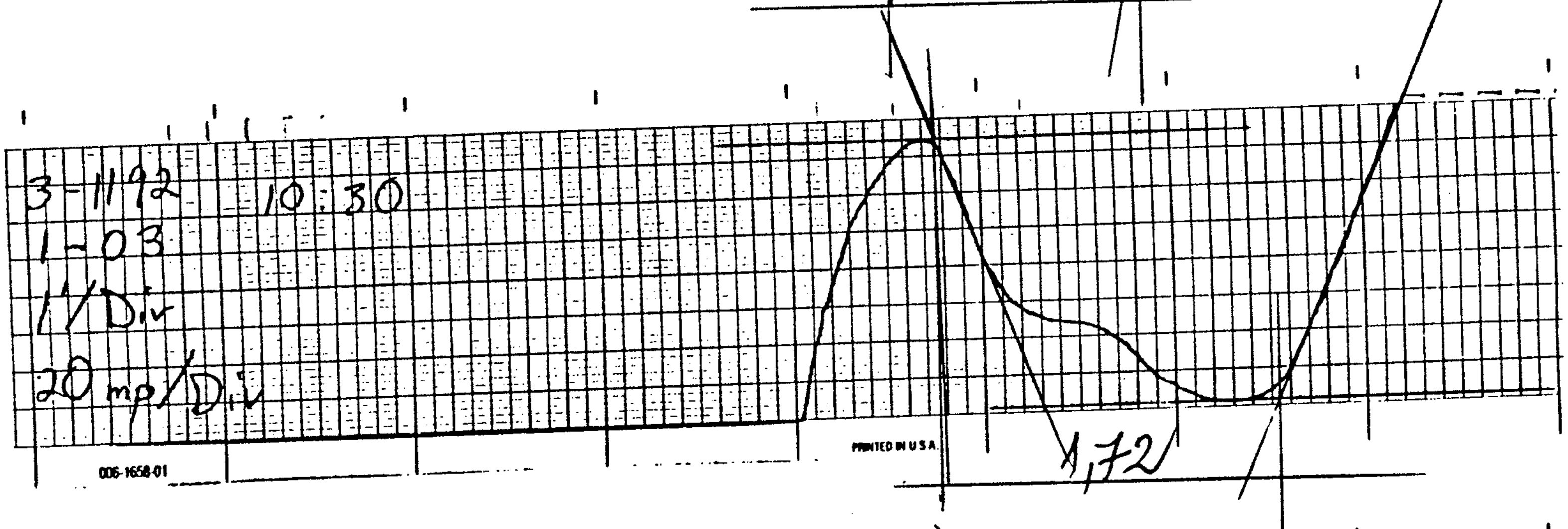
006-1658-01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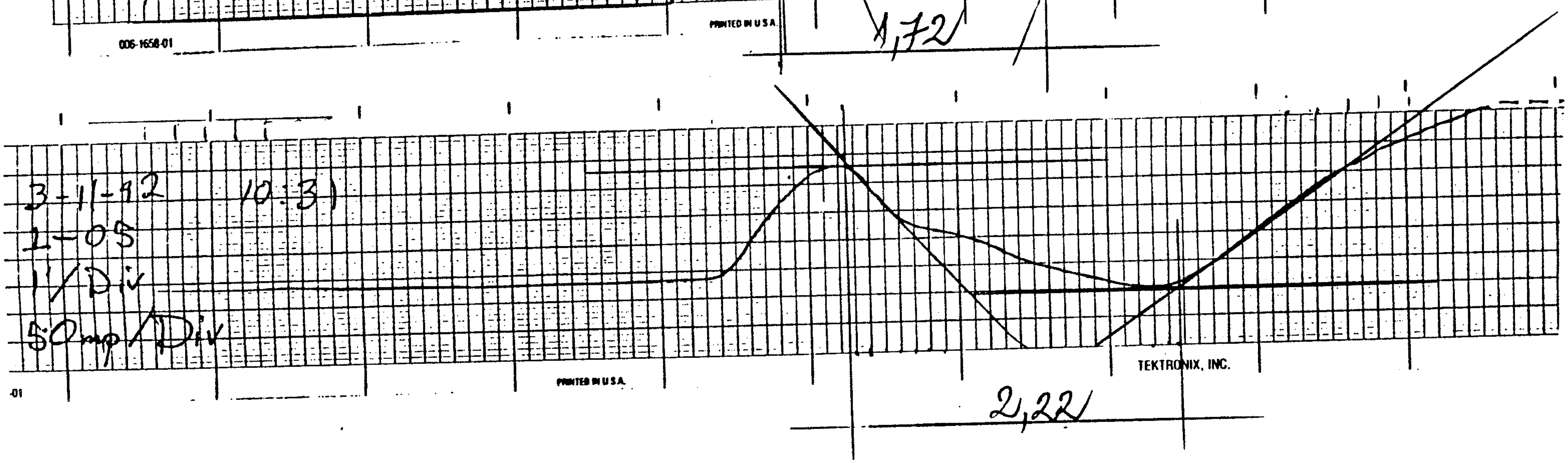
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TEKTRONIX, INC.
2,22
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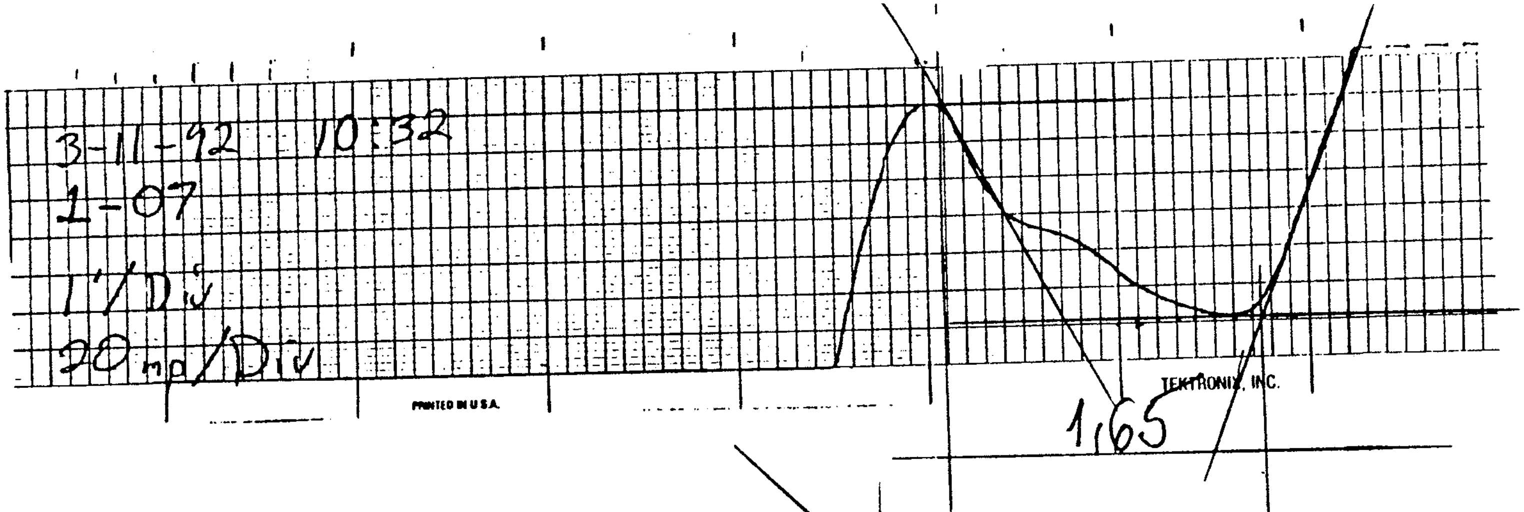
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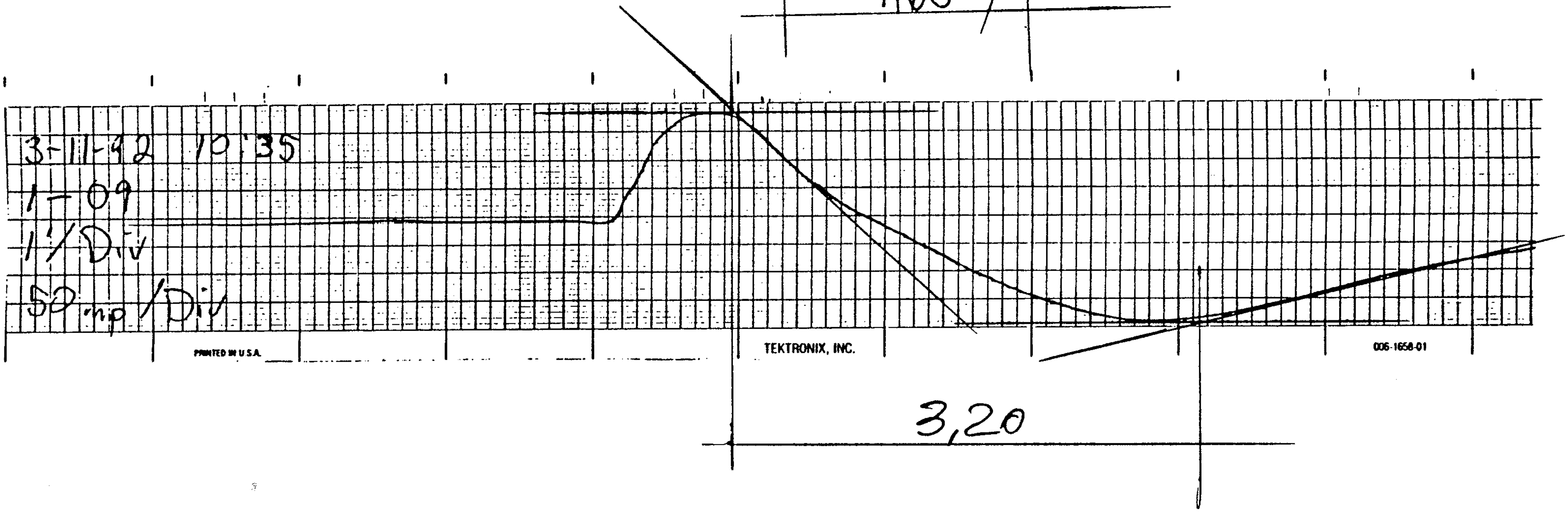
TEKTRONIX, INC.
1,65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TEKTRONIX, INC.
006-1658-01
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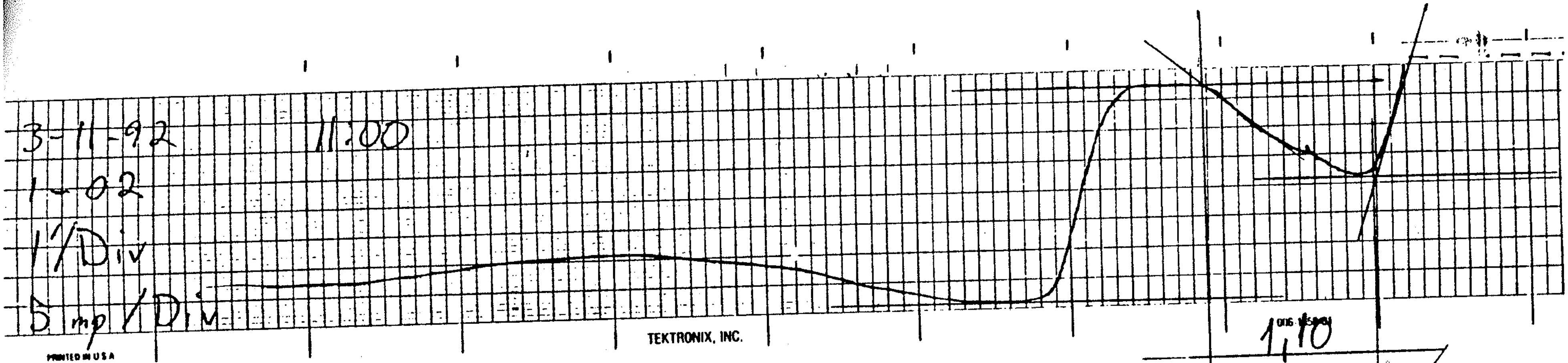
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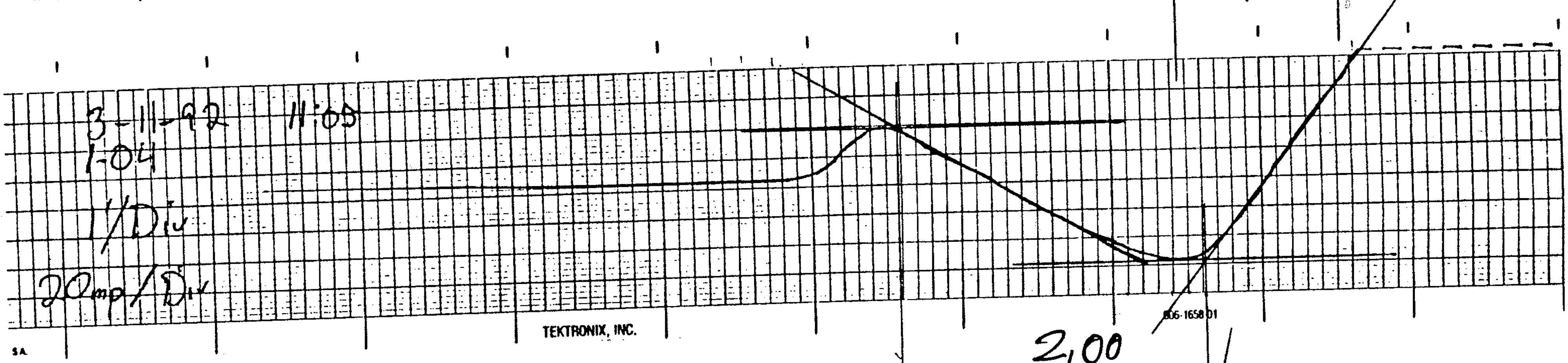
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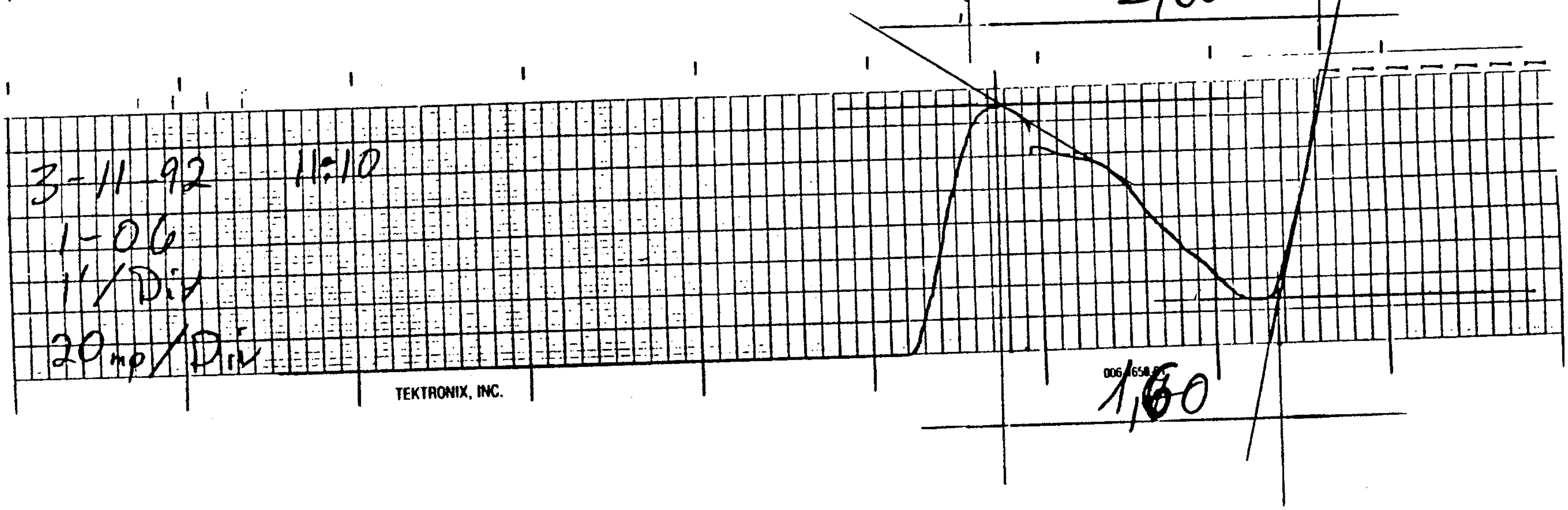
006-1658-01
2,00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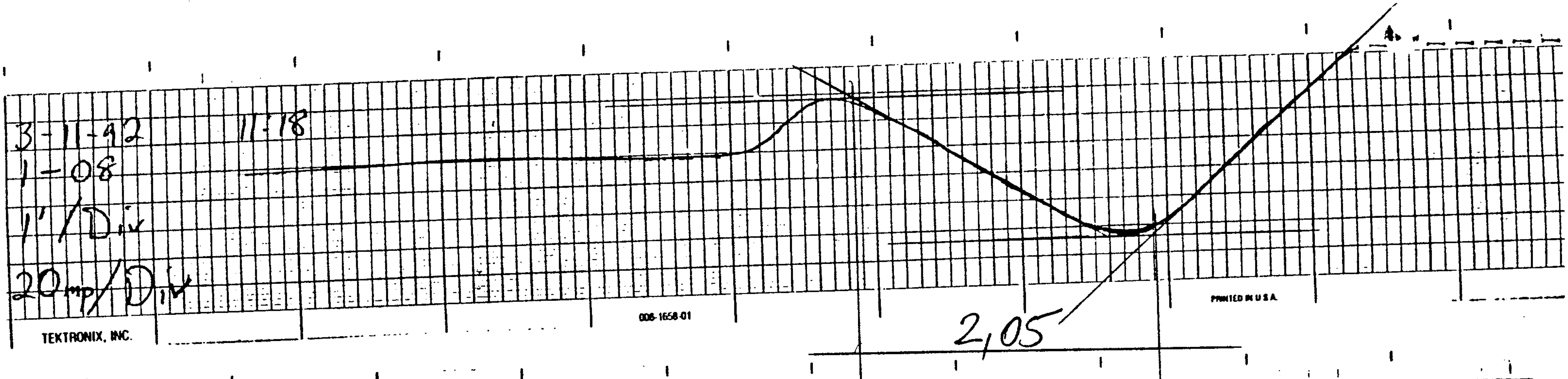
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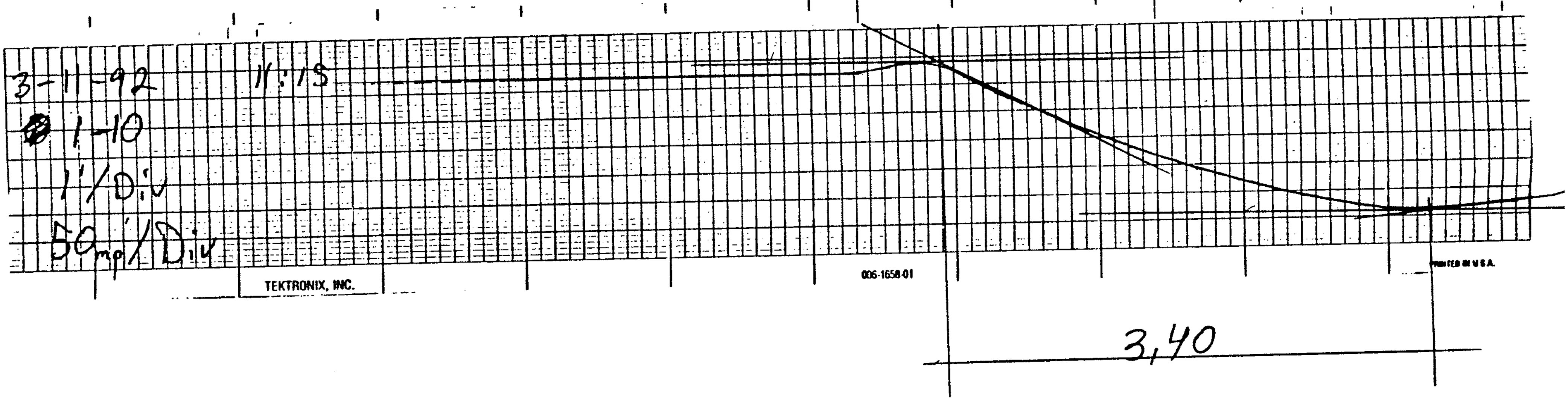
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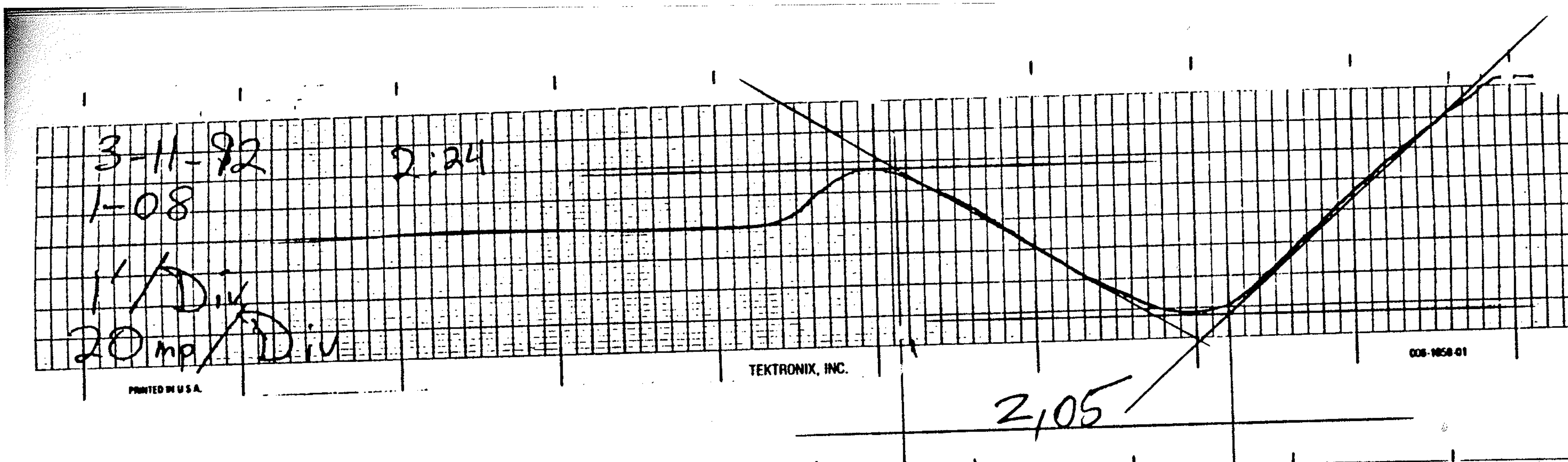

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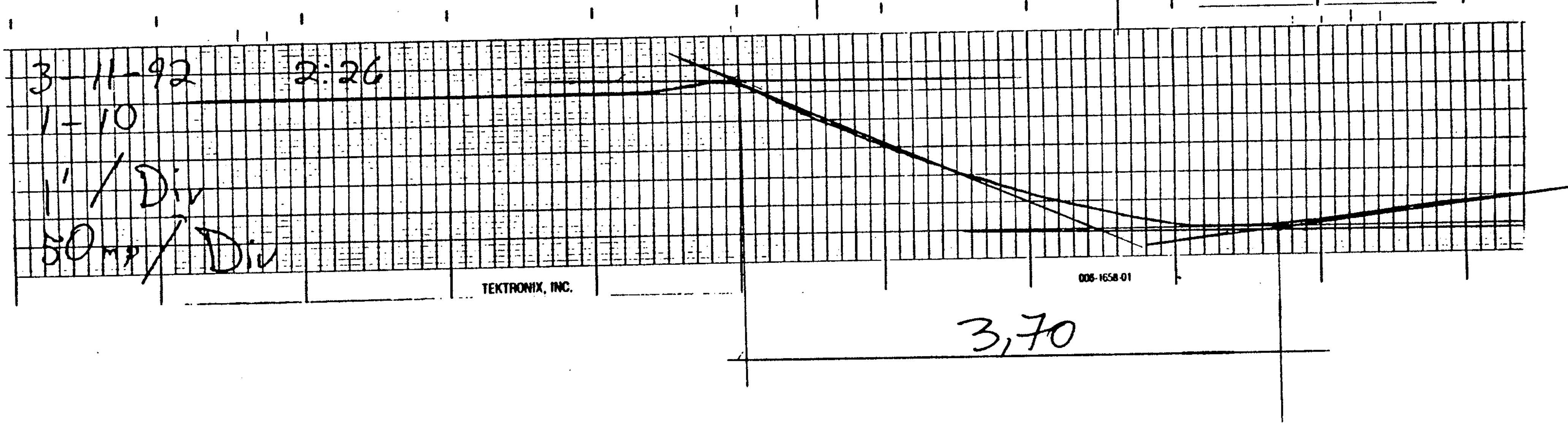

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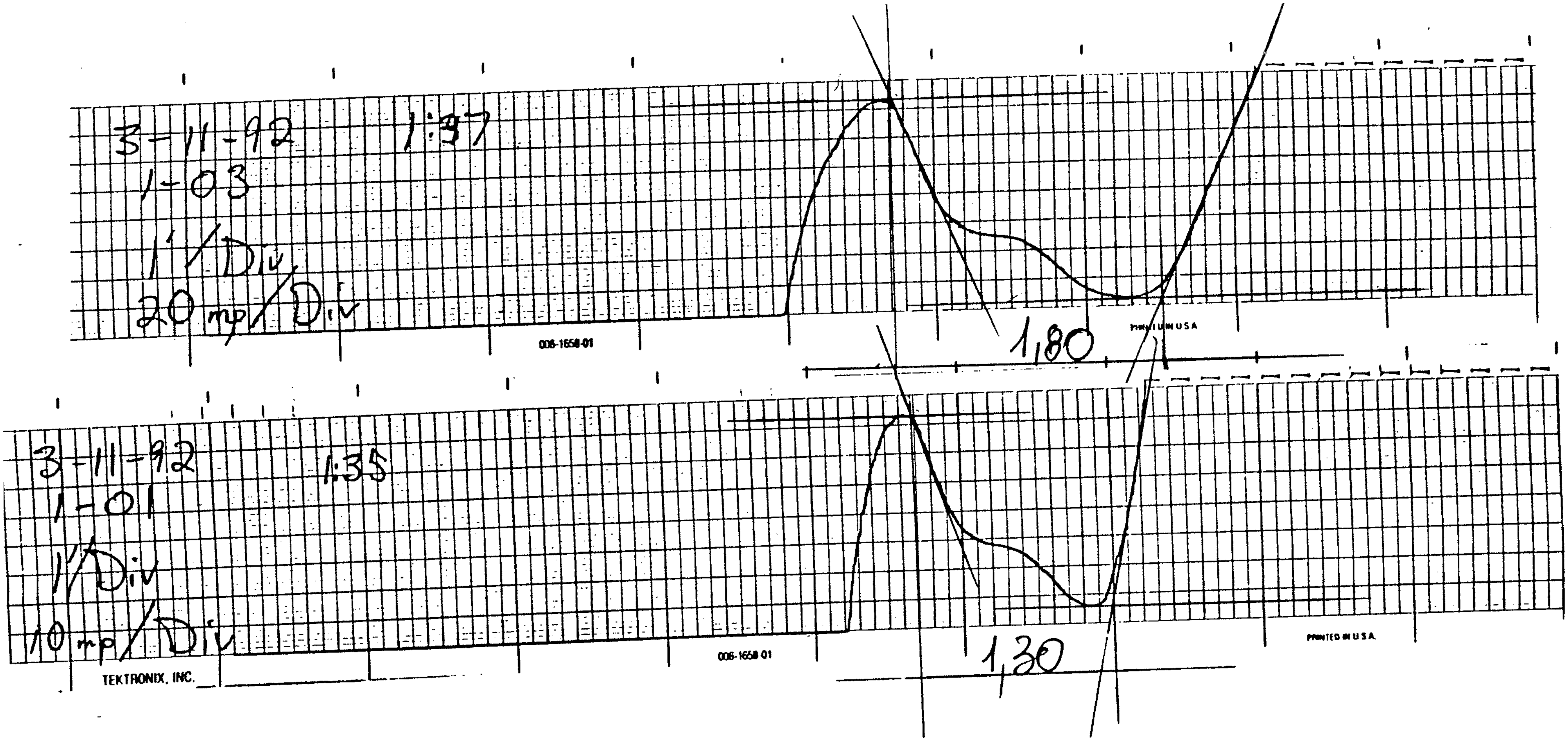
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006-1658-01
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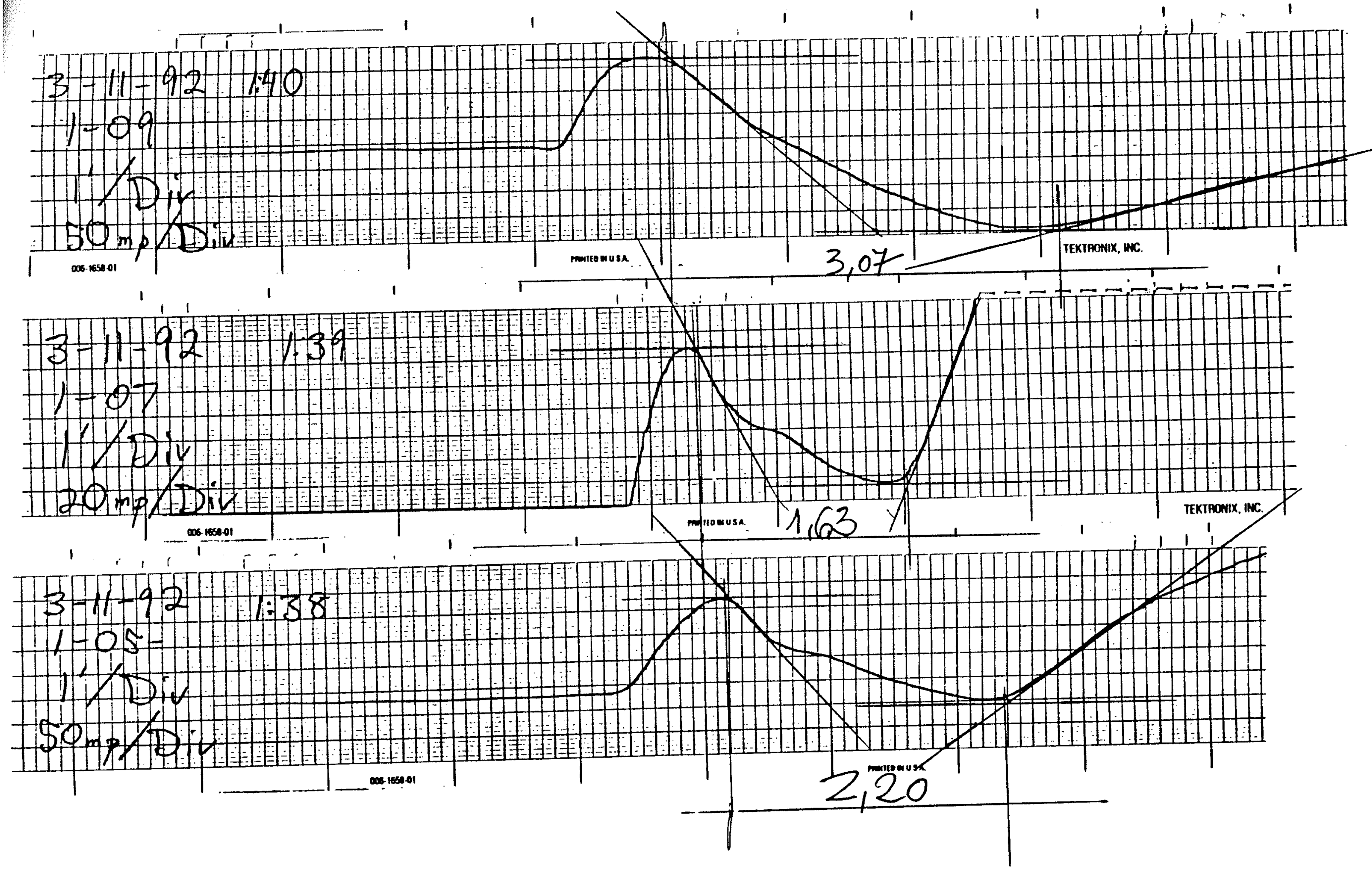
1,30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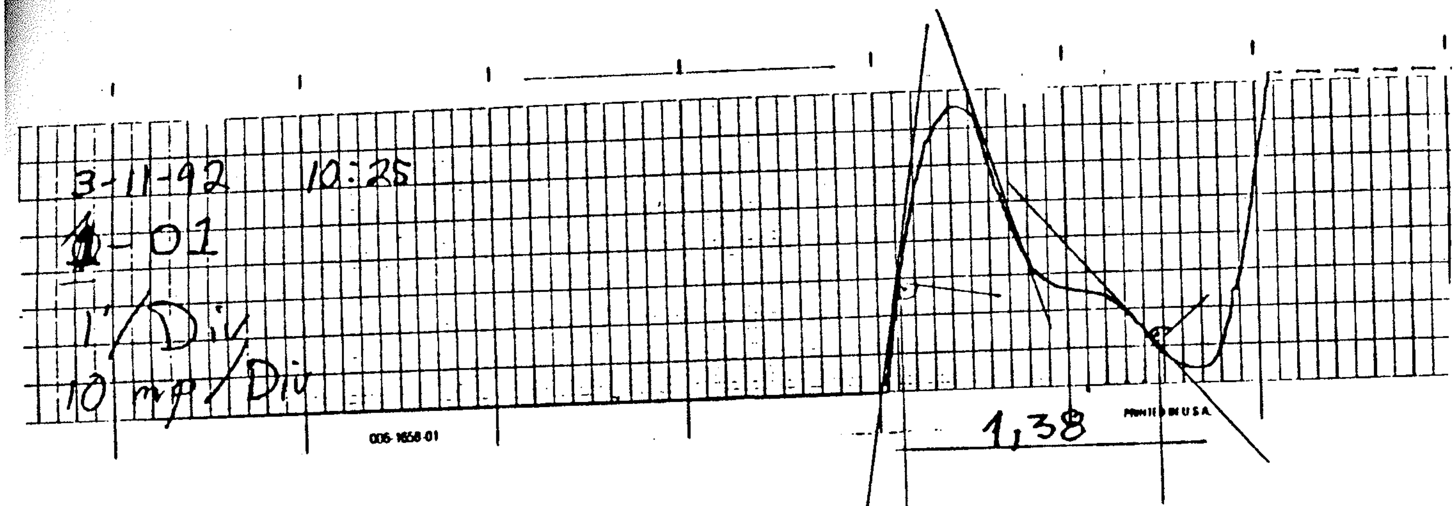
1,63
TEKTRONIX, INC.
3-11-92 1:38
1-05
1'/Div
50 mp/Div
006-1658-01
PRINTED IN U.S.A.
2,20

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

bbb) "Method of Diverging Lines", Borehole #2/March 1992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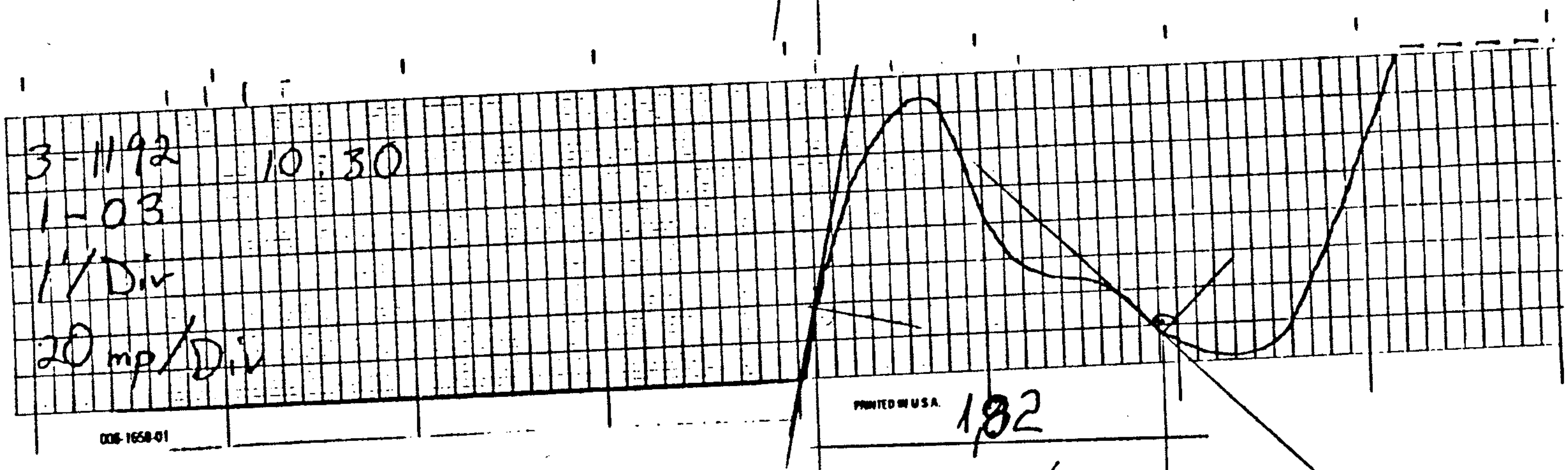
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006-1658-01
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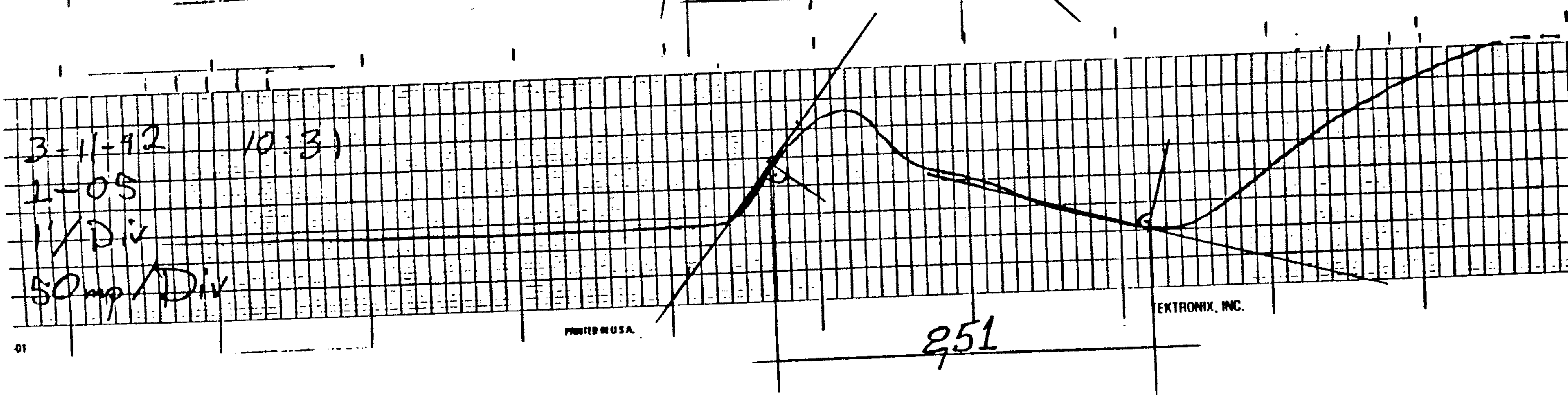
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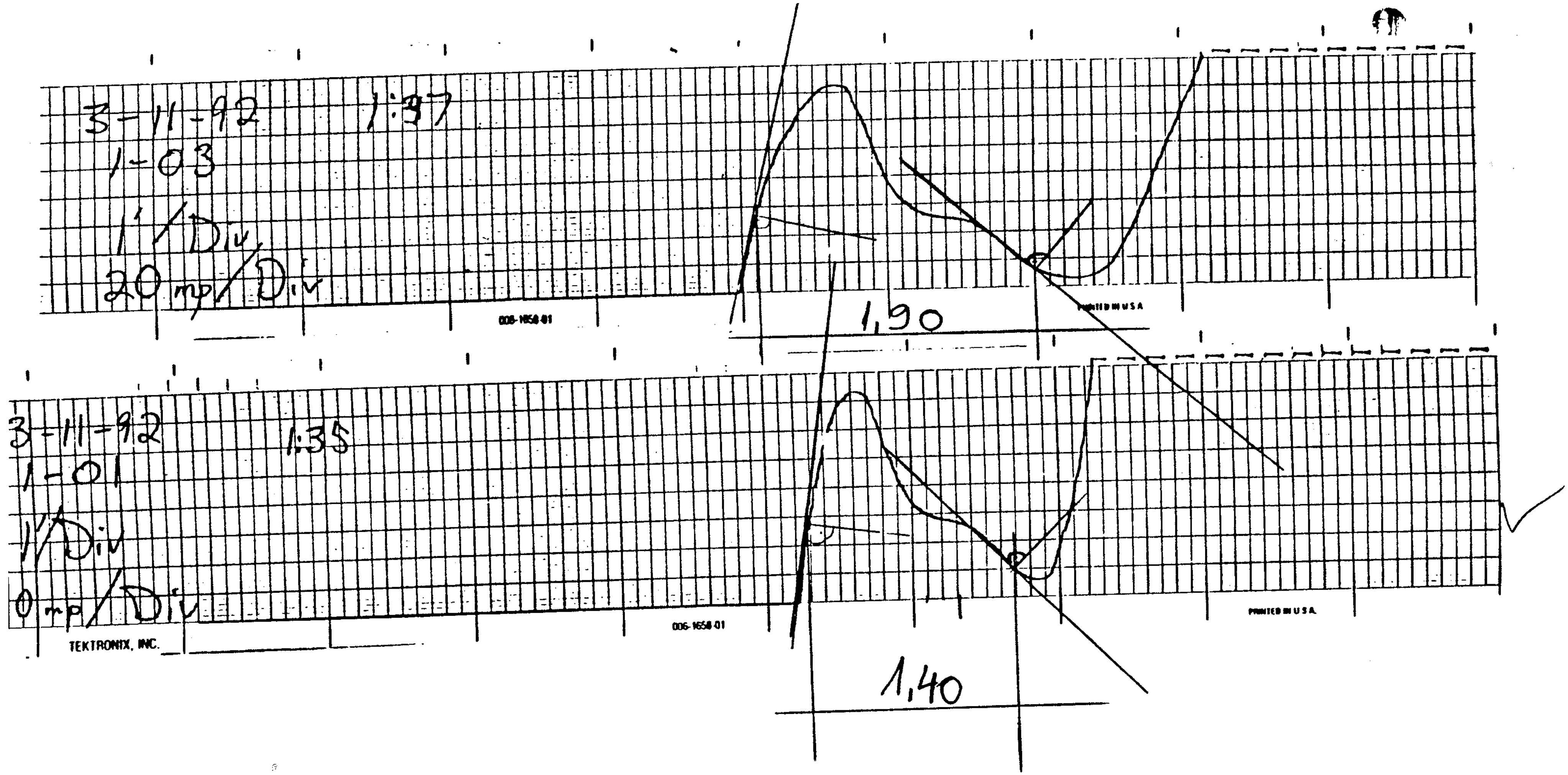
TEKTRONIX, INC.
2,51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006-1658-01
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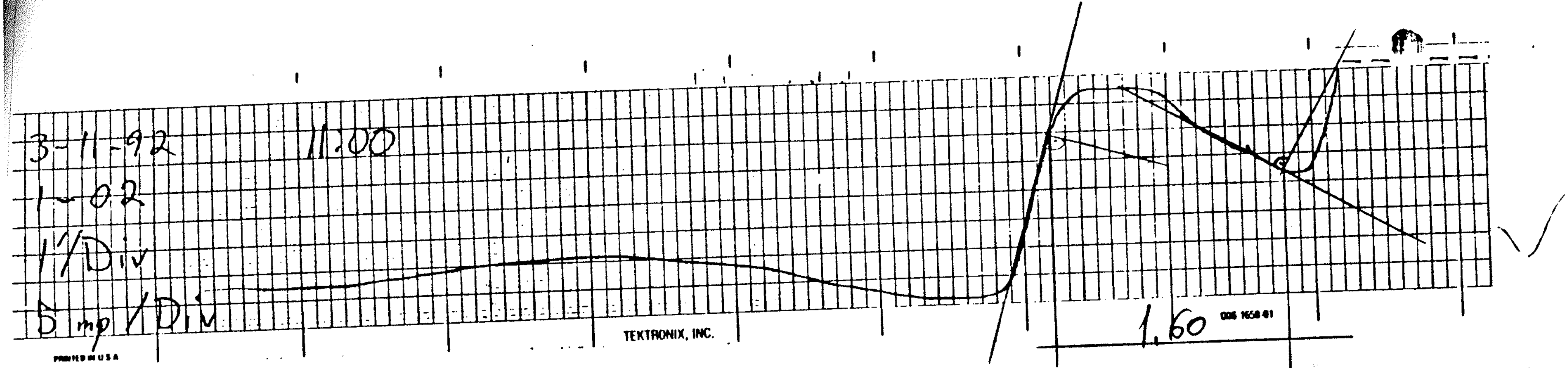
1,40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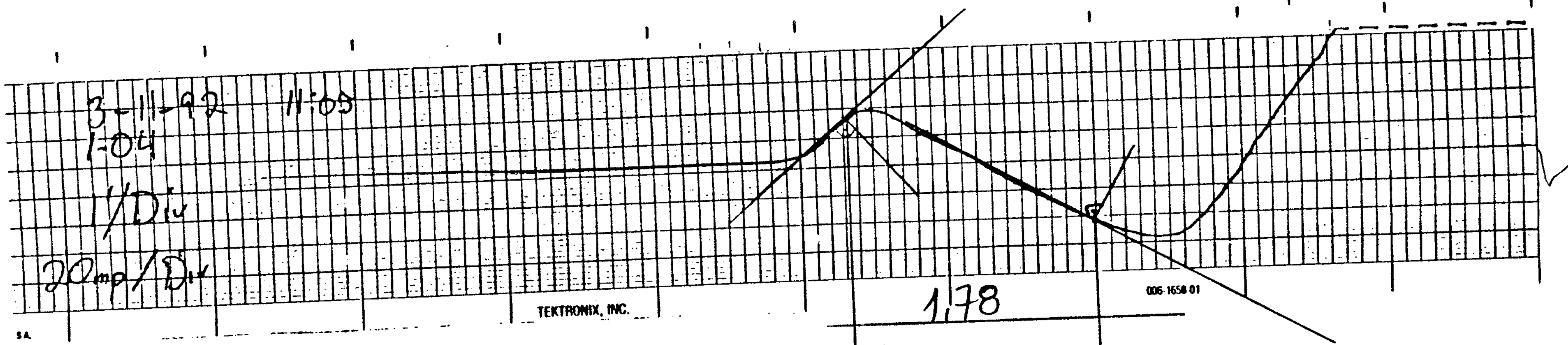
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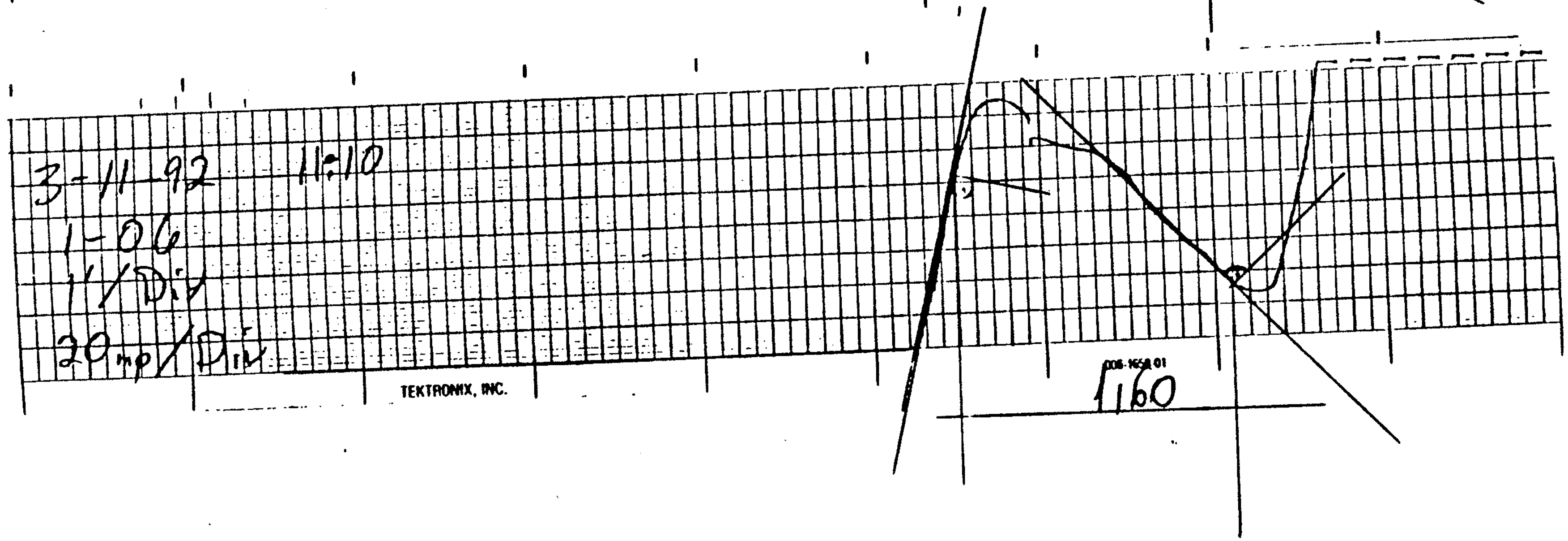
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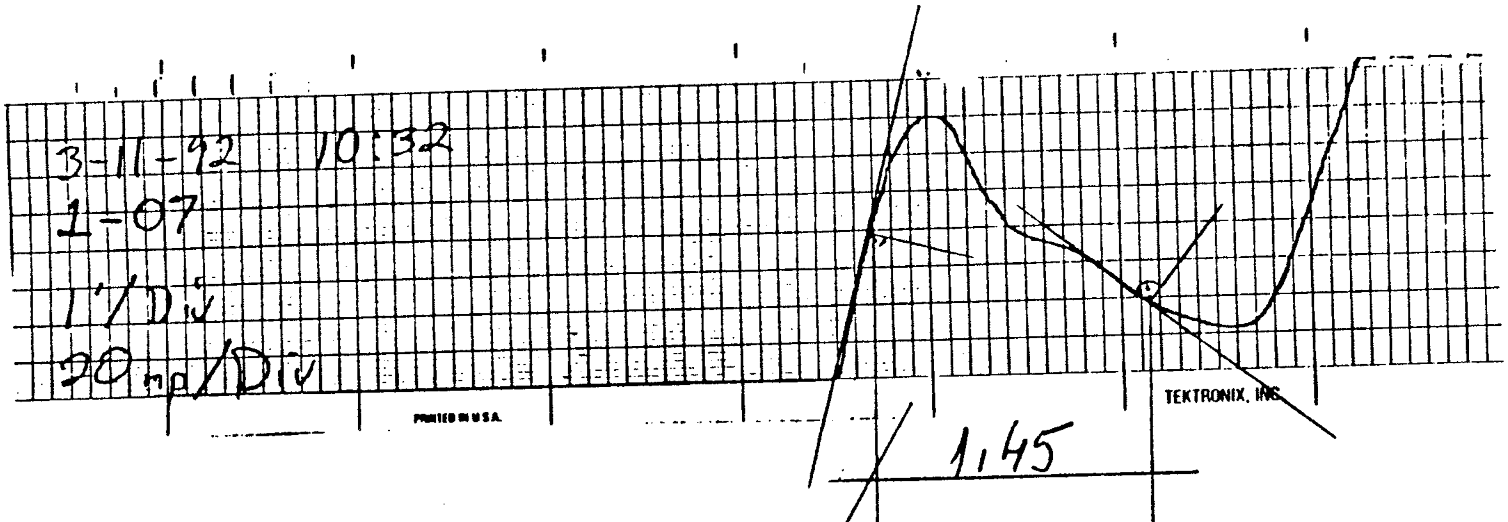
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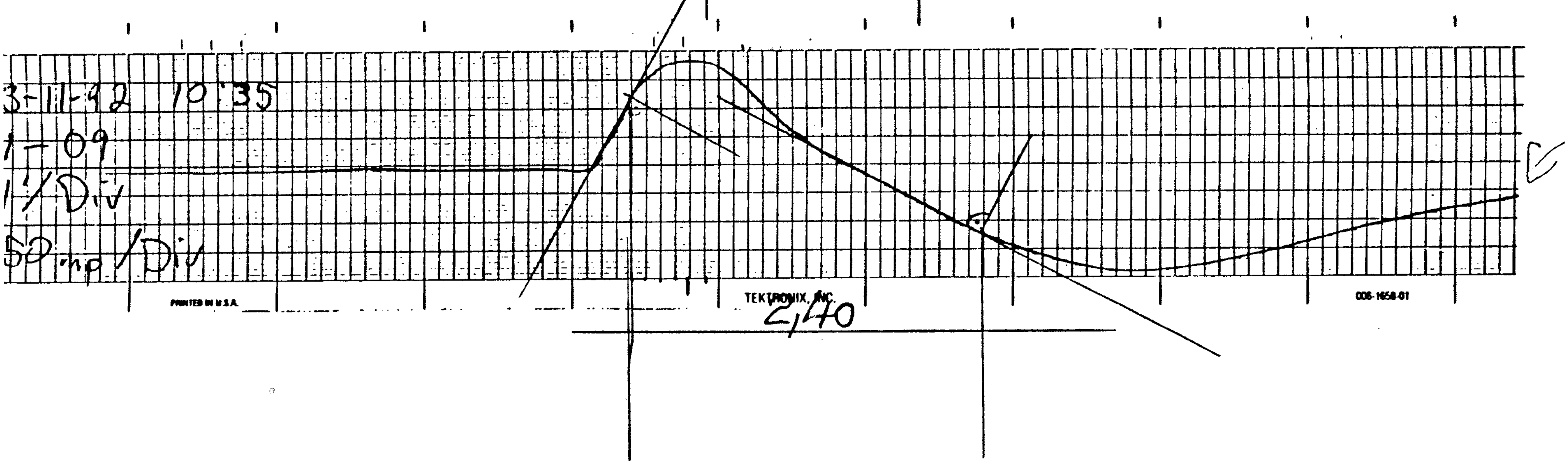
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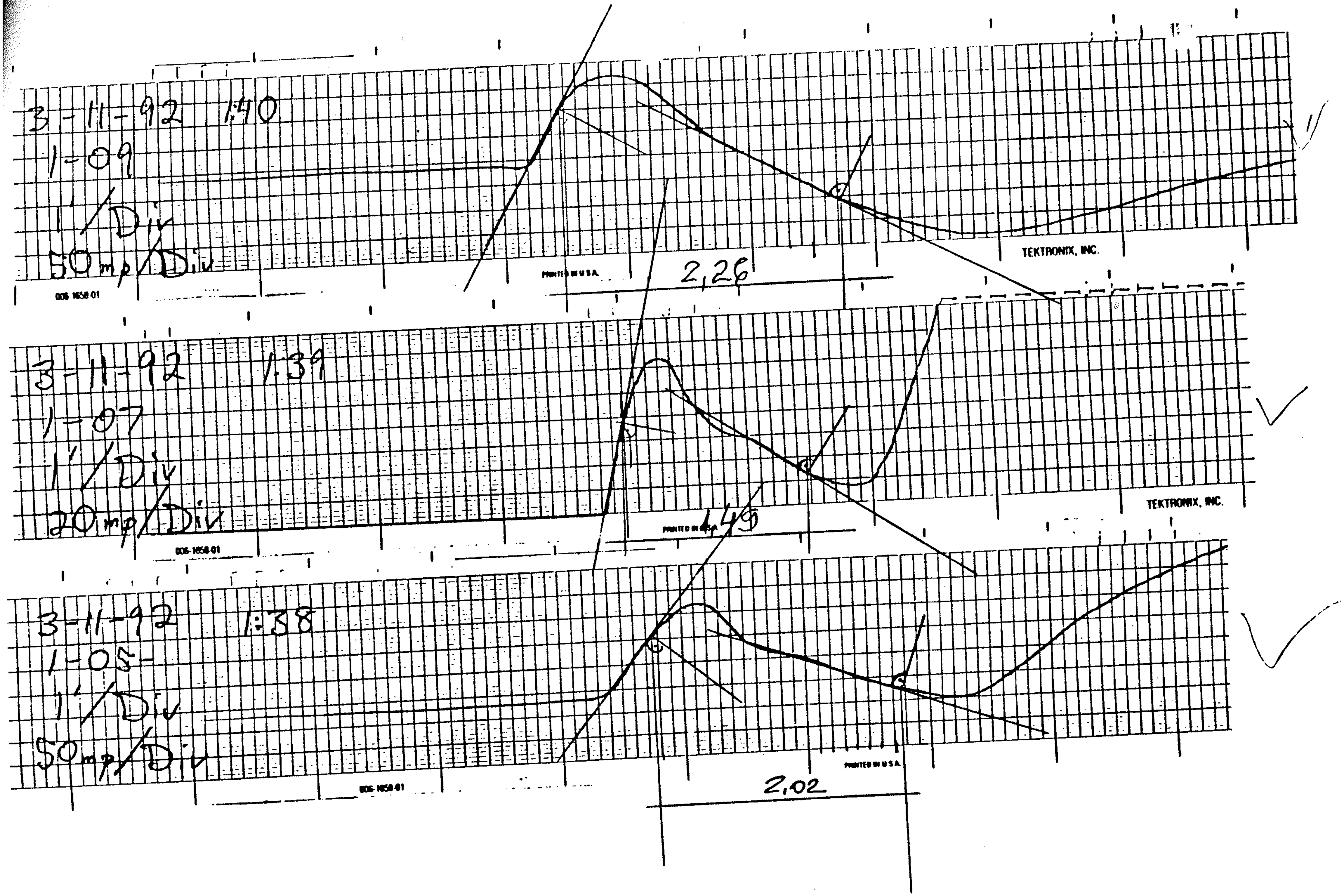
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PRINTED IN U.S.A.
006-1658-01
TEKTRONIX, INC.
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1-05
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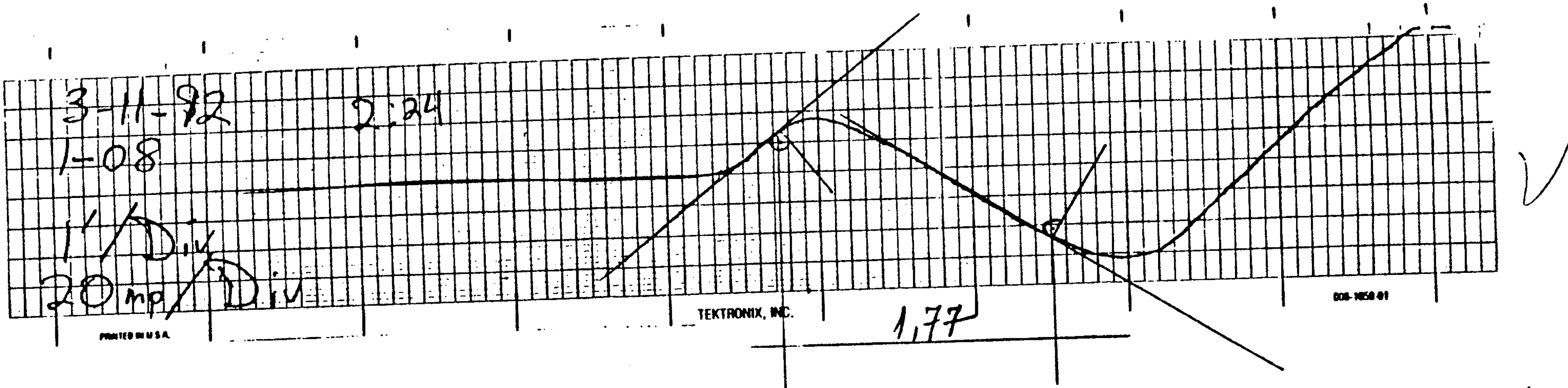
50 mp/Div
2,02
PRINTED IN U.S.A.
006-1658-01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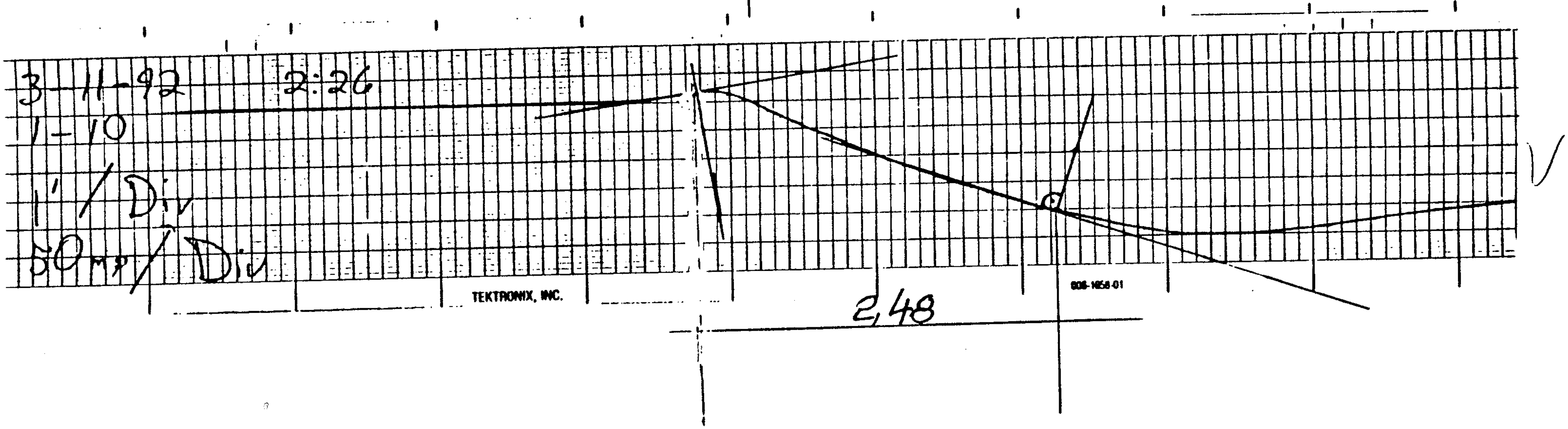
TEKTRONIX, INC.
1,77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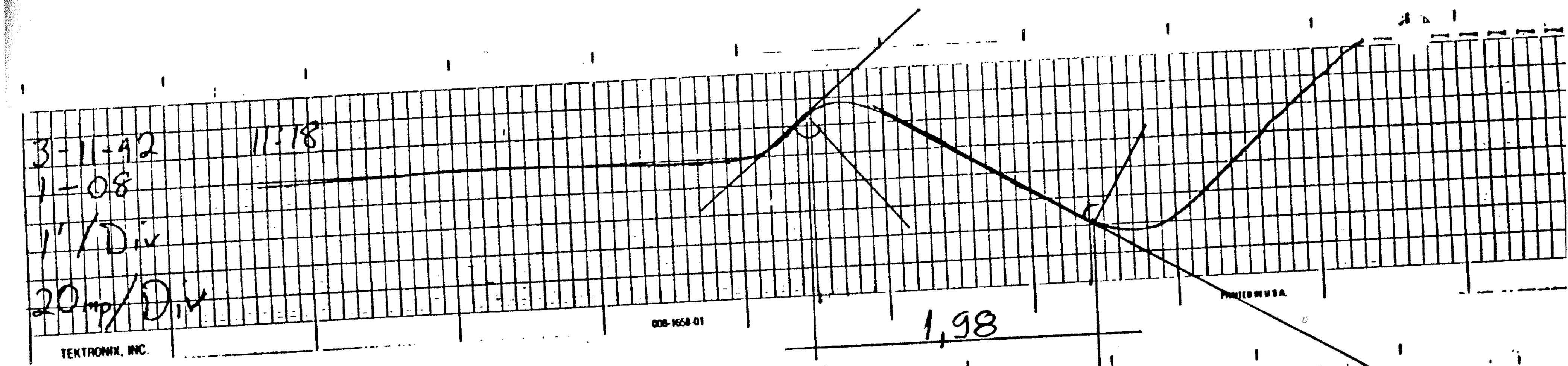
2,48



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006-1658-01
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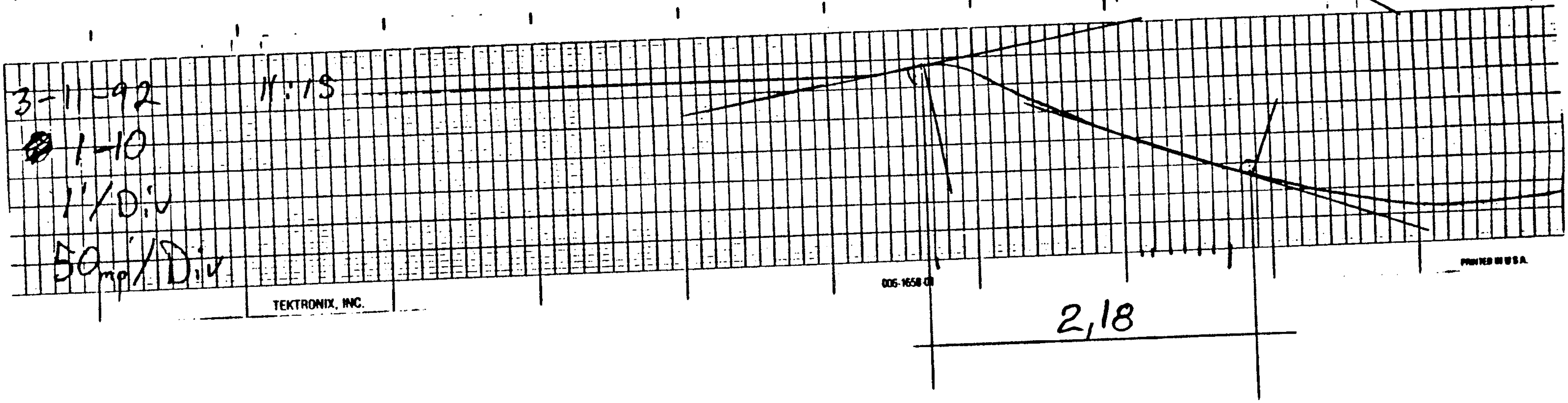
1,98

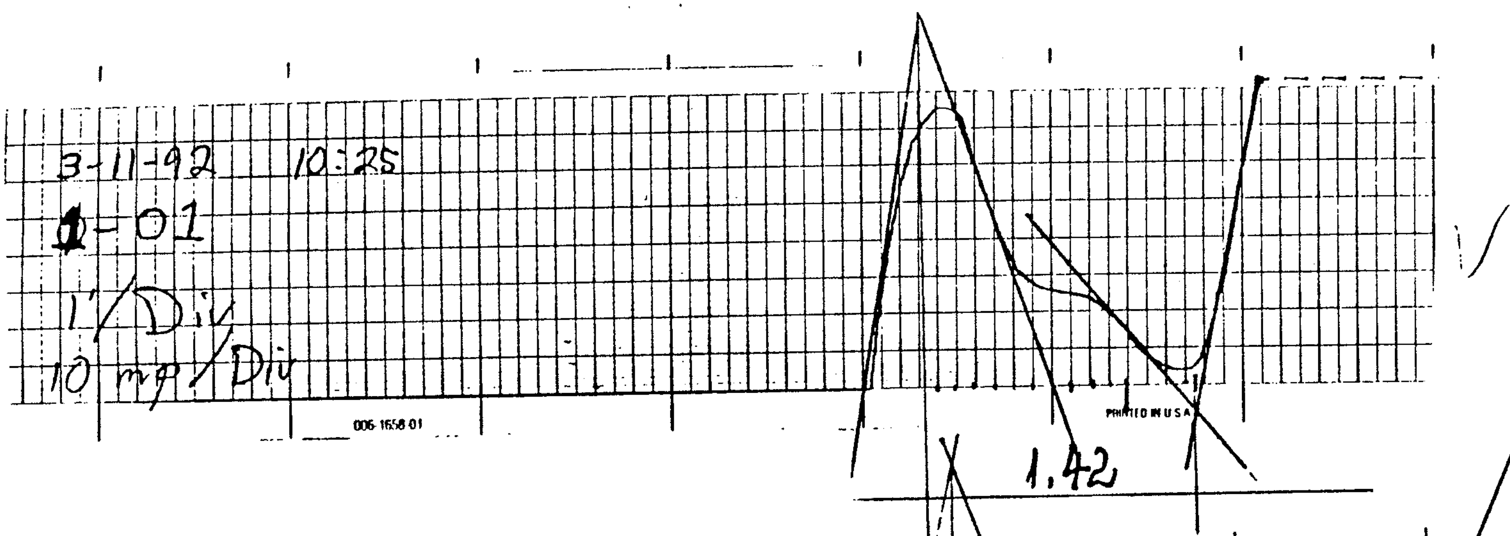


3-11-92
11:15
1-10
1'/Div
50mp/Div
TEKTRONIX, INC.
006-1658-01
PRINTED IN U.S.A.
2,18

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

ccc) "Method of Peaks", Borehole #2/March 1992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006-1658-01
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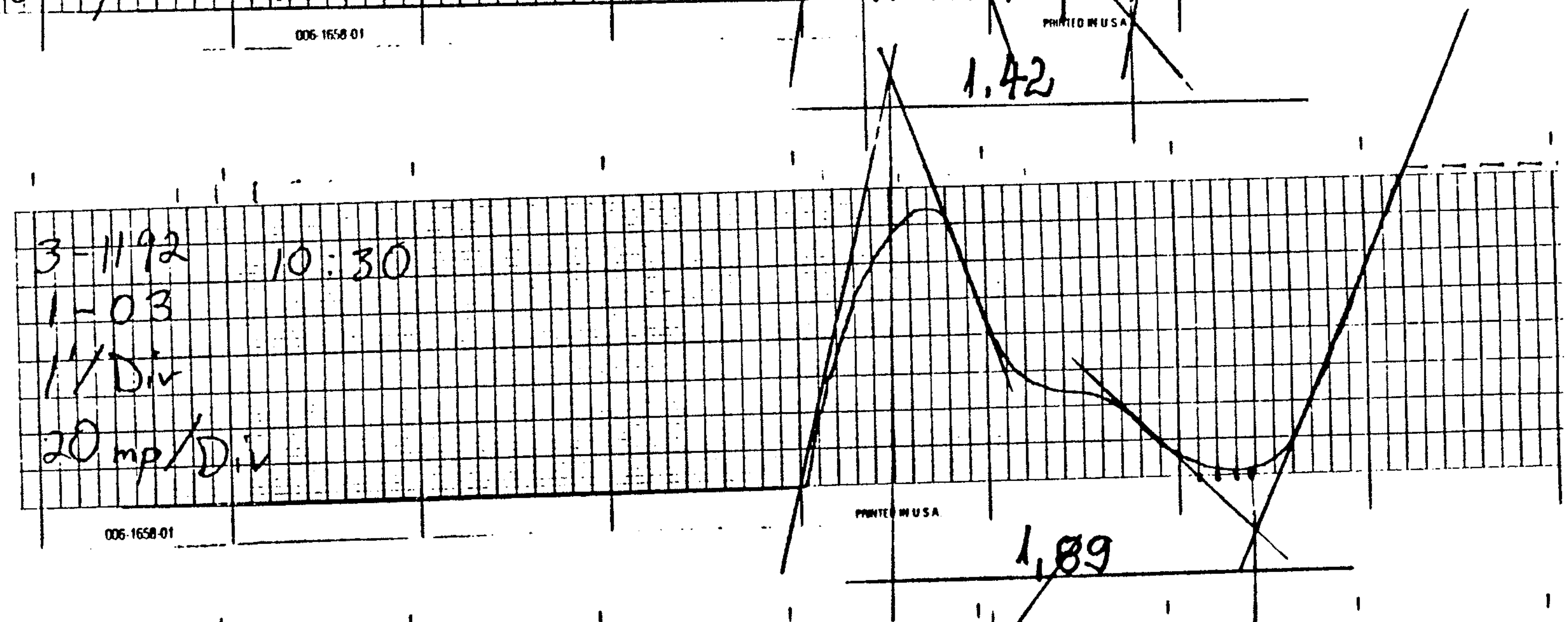
1,42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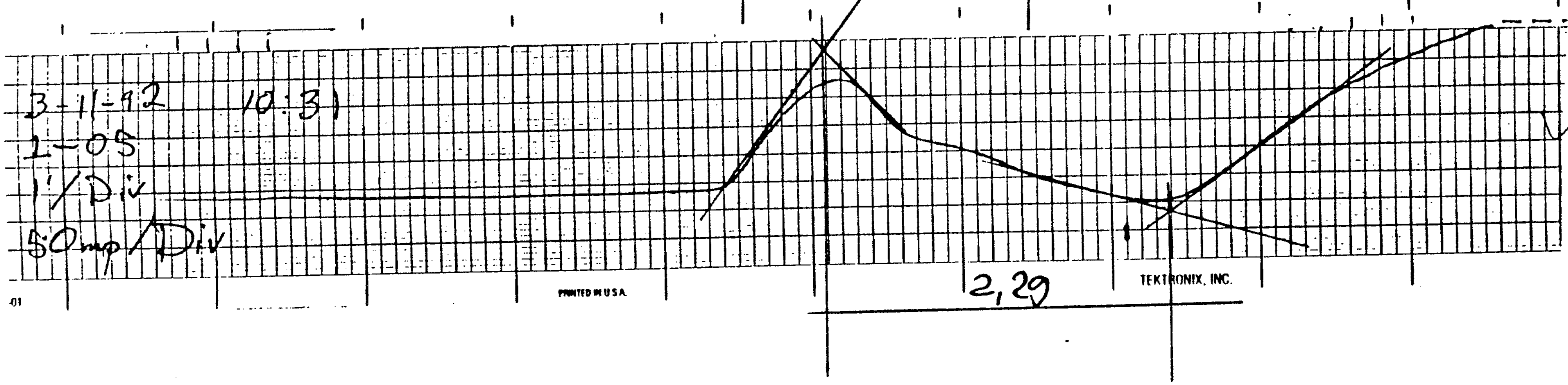
1,89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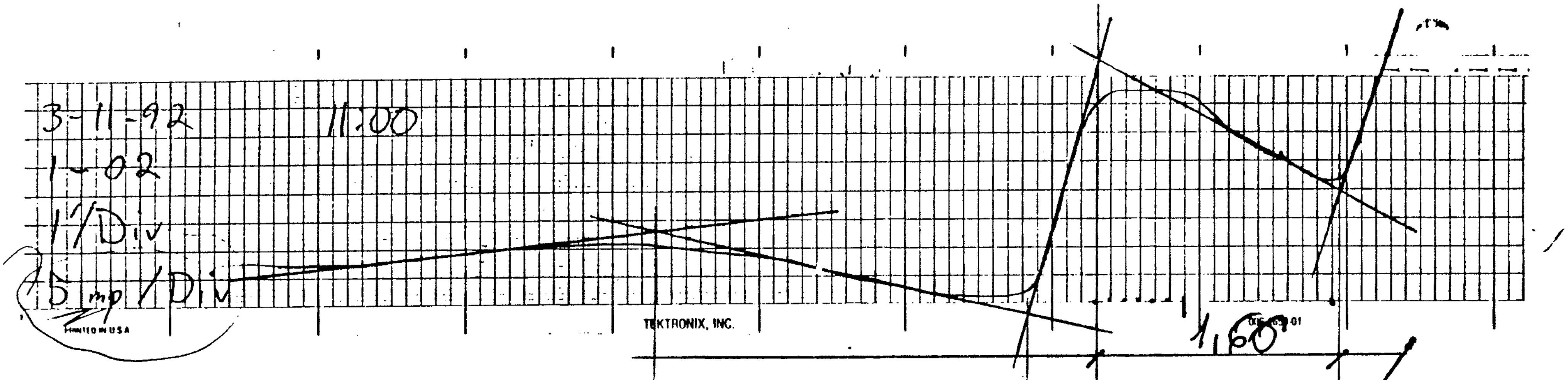
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,29



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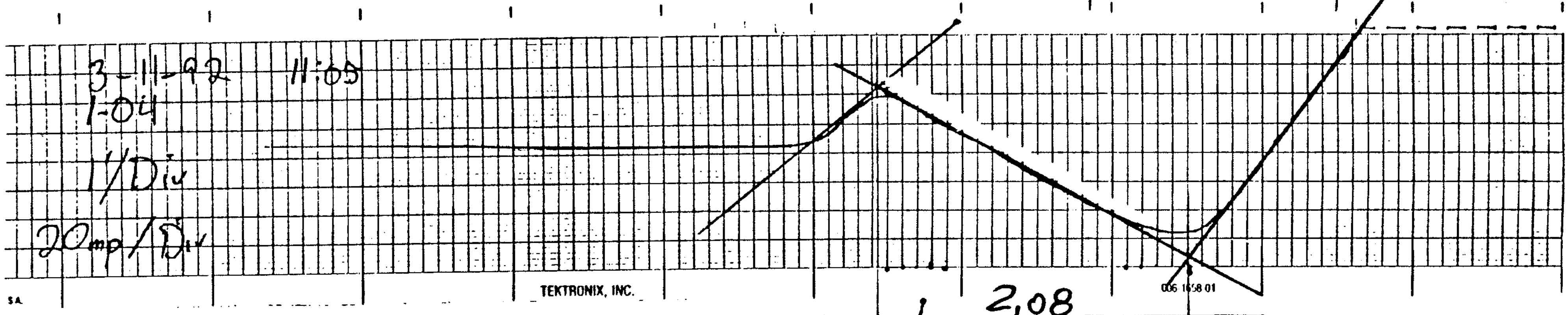
TEKTRONIX, INC.
006 1658 01
1,60



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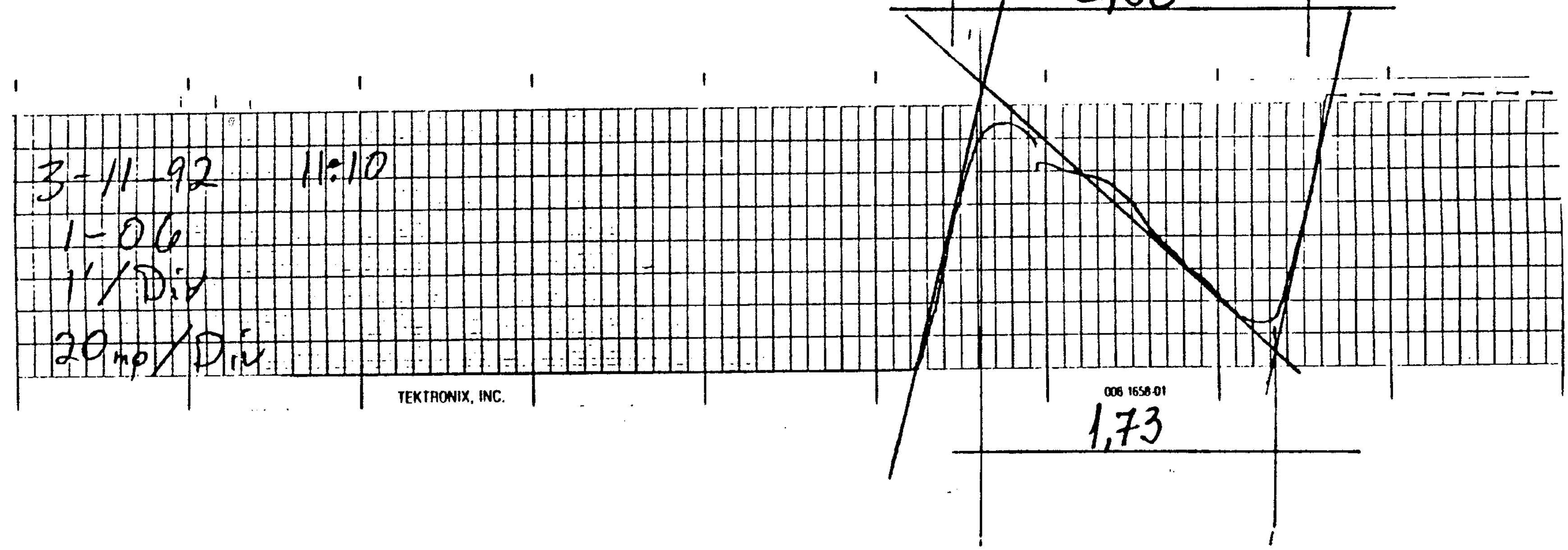
006 1658 01
2,08



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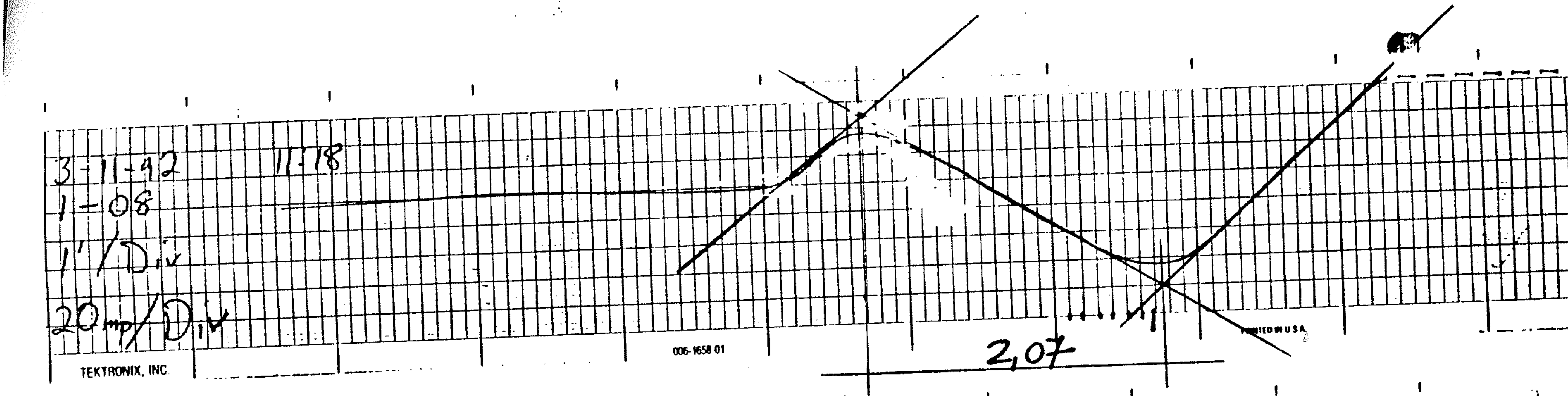
006 1658-01
1,73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1-08
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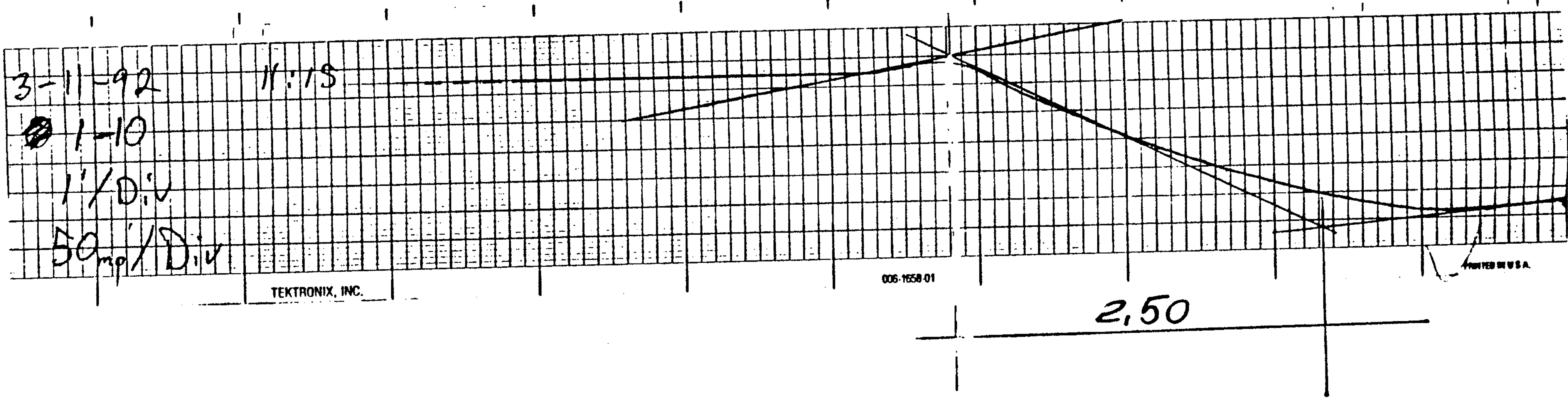
TEKTRONIX, INC.
006-1658-01
2,07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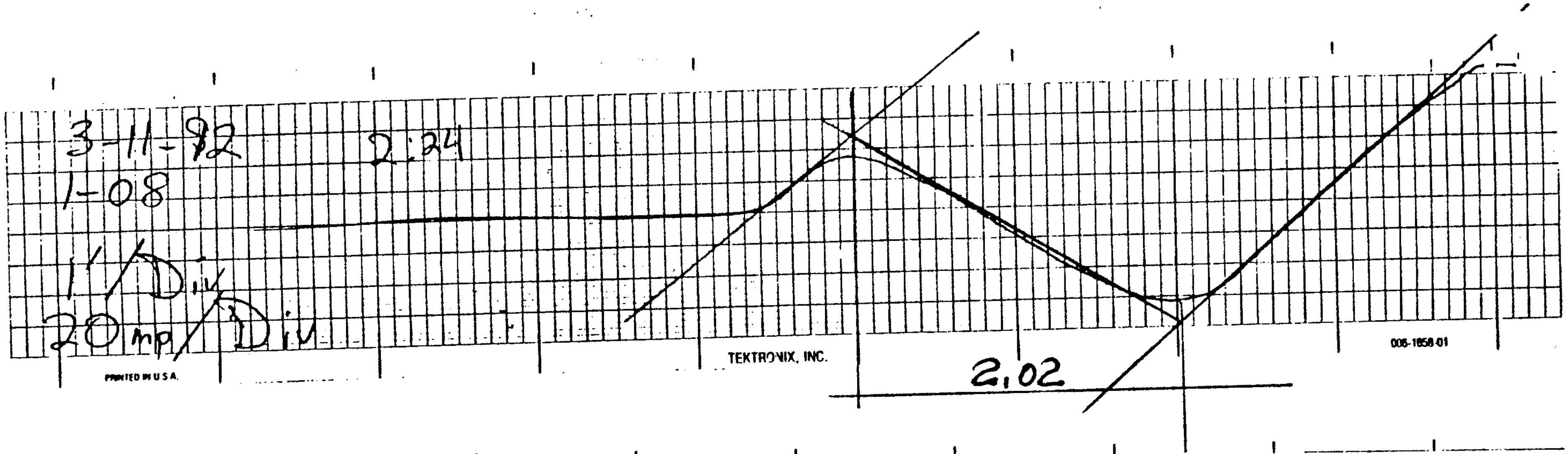
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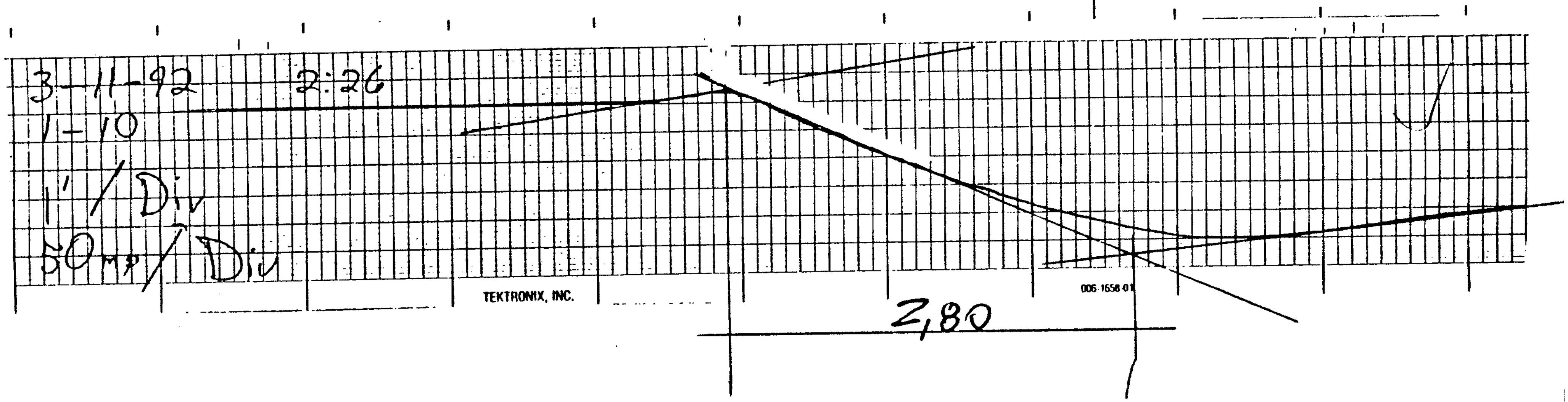
1-08
1'/Div
20 mp/Div
PRINTED IN U.S.A.
TEKTRONIX, INC.
006-1658-01
2.02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1-10
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006-1658-01
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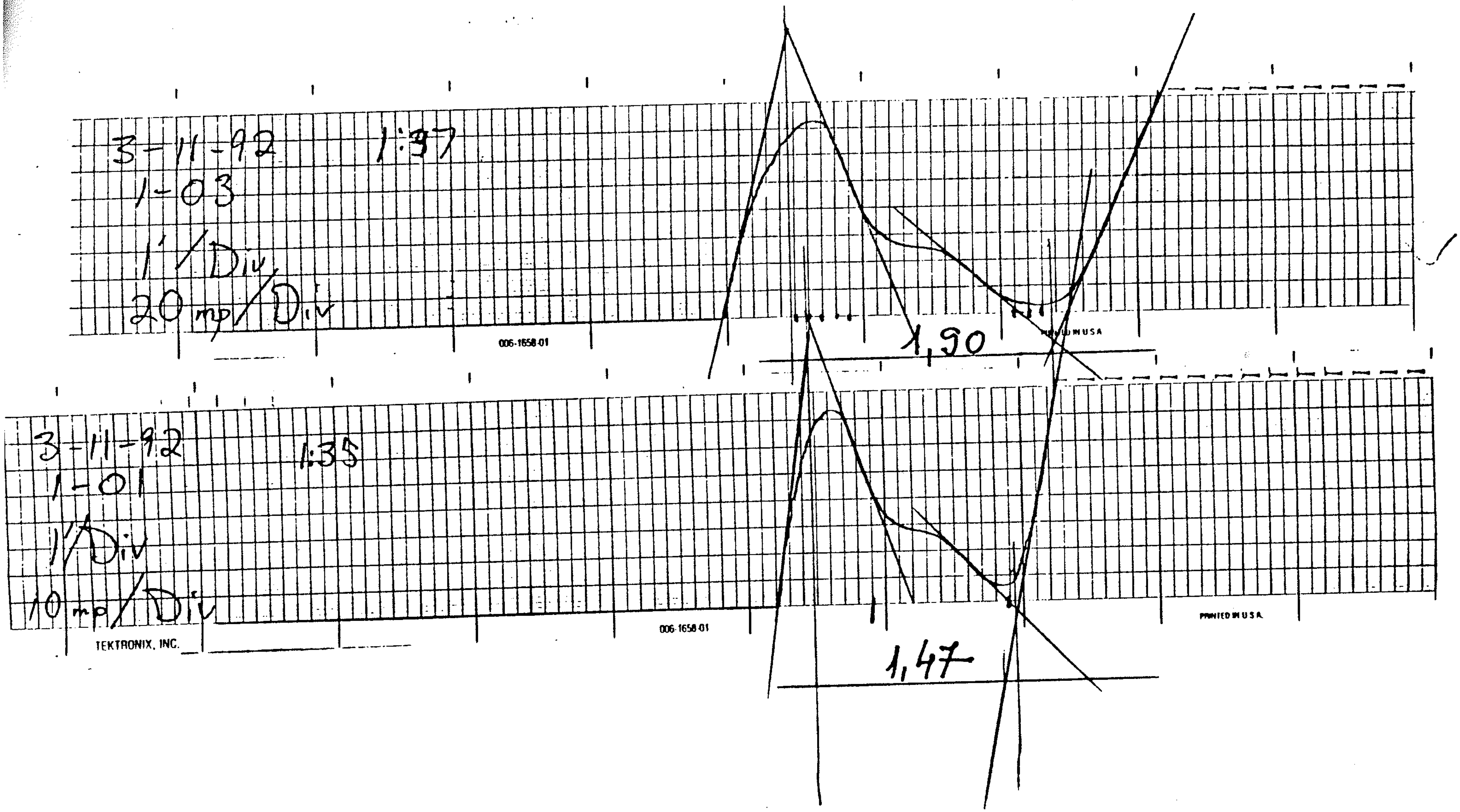
3-11-92
1:37
1-03
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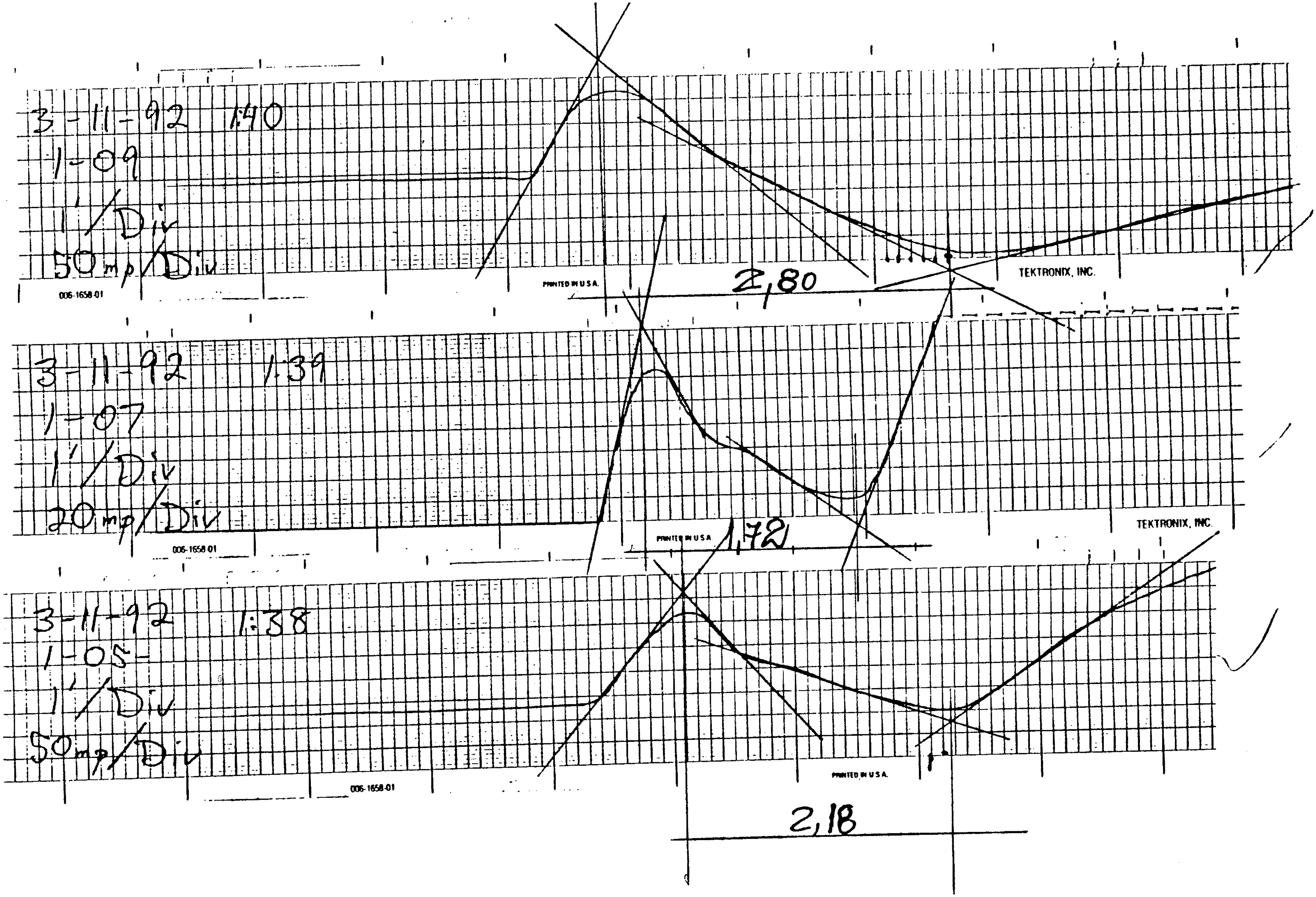
20 mp/Div
006-1658-01
PRINTED IN USA
1,90
3-11-92
1:35
1-01
1'/Div
10 mp/Div
TEKTRONIX, INC.
006-1658-01
PRINTED IN U.S.A.
1,47


3-11-92 1:40
1-09
1'/Div
50 mp/Div
006-1658-01
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,80
3-11-92 1:39
1-07
1'/Div
20mp/Div
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PRINTED IN U.S.A.
TEKTRONIX, INC.
1,72
3-11-92 1:38
1-05
1'/Div
50mp/Div
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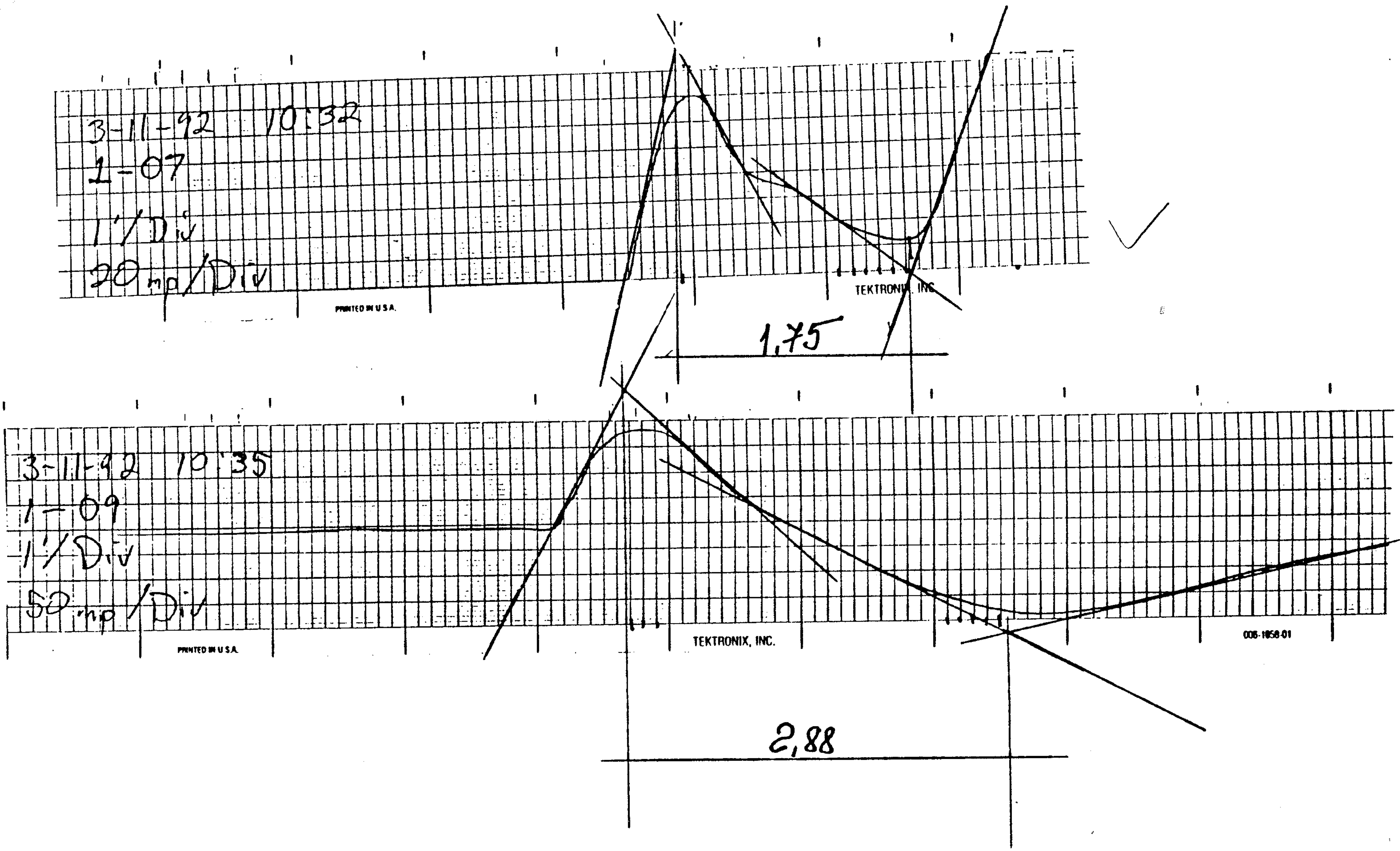
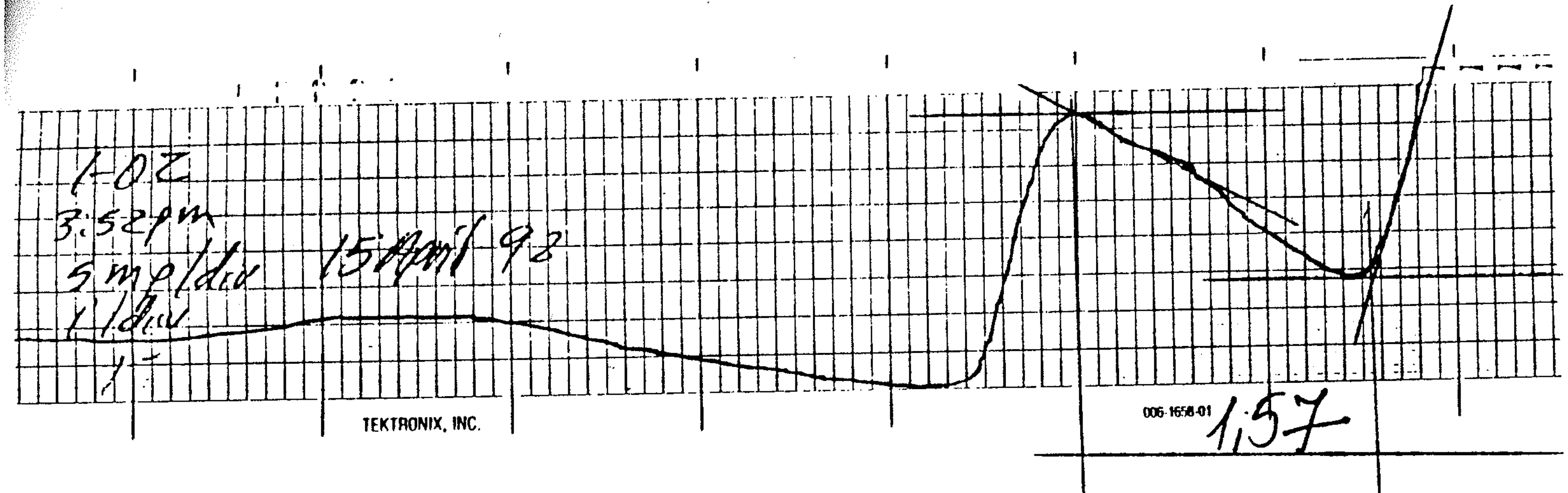
PRINTED IN U.S.A.
2,18


3-11-92 10:32
1-07
1'/Div
20 mp/Div
PRINTED IN U.S.A.
TEKTRONIX, INC.
1,75
3-11-92 10:35
1-09
1'/Div
50 mp/Div
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,88

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

aaa) "Method of Tangents", Borehole #2, April 1992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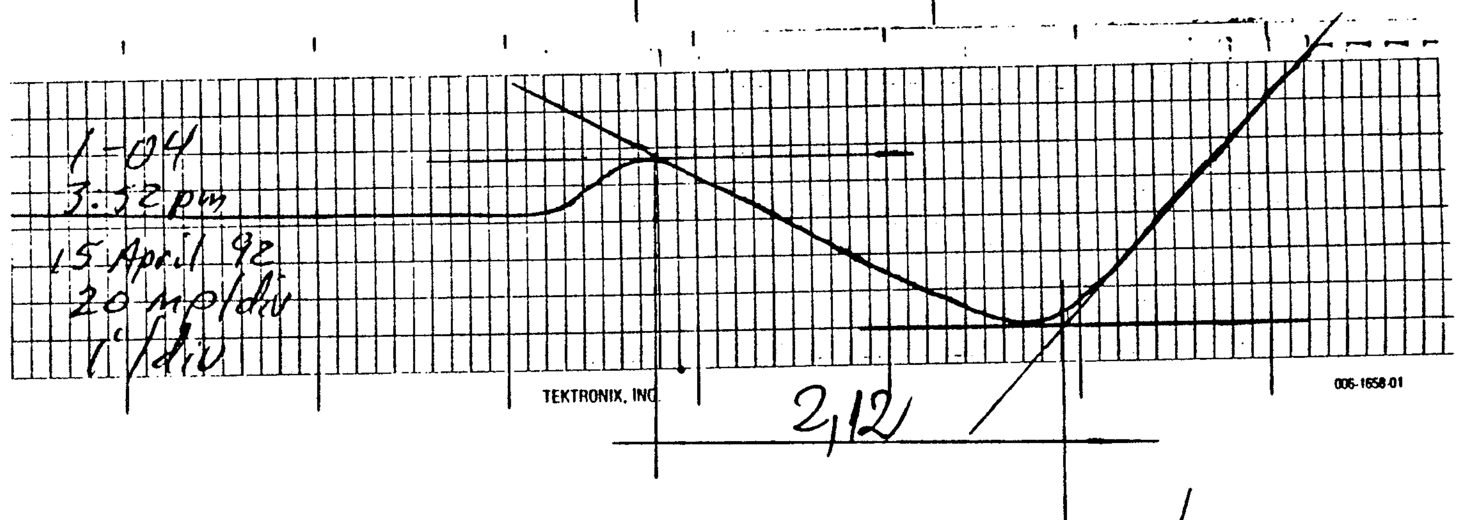
3:52pm
5 mp/div
1'/div
15 April 92
TEKTRONIX, INC.
006-1658-01
1,57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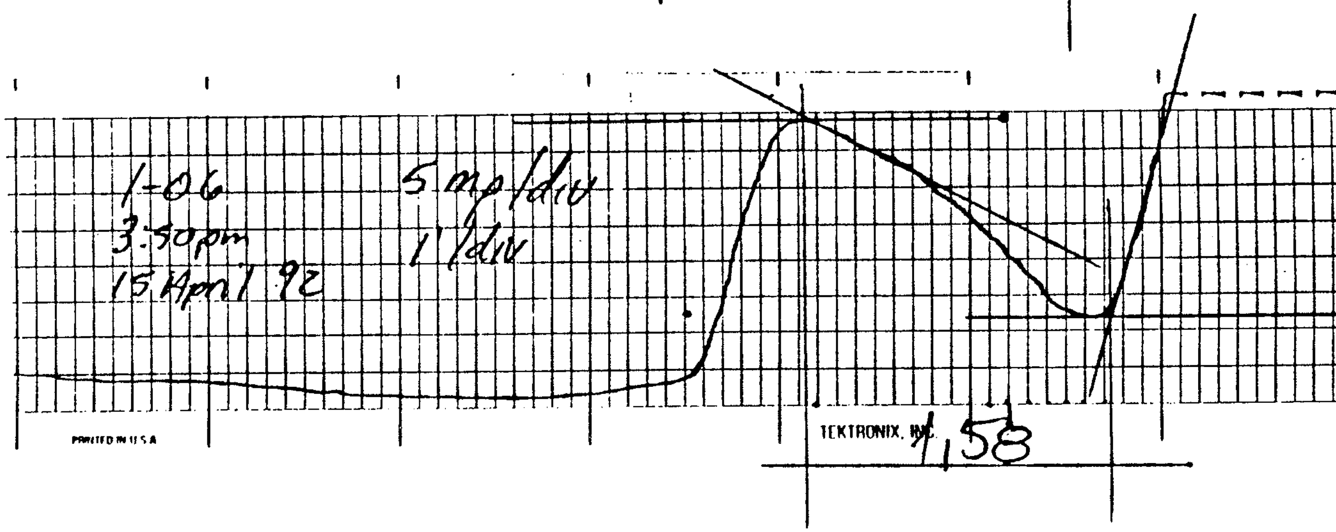
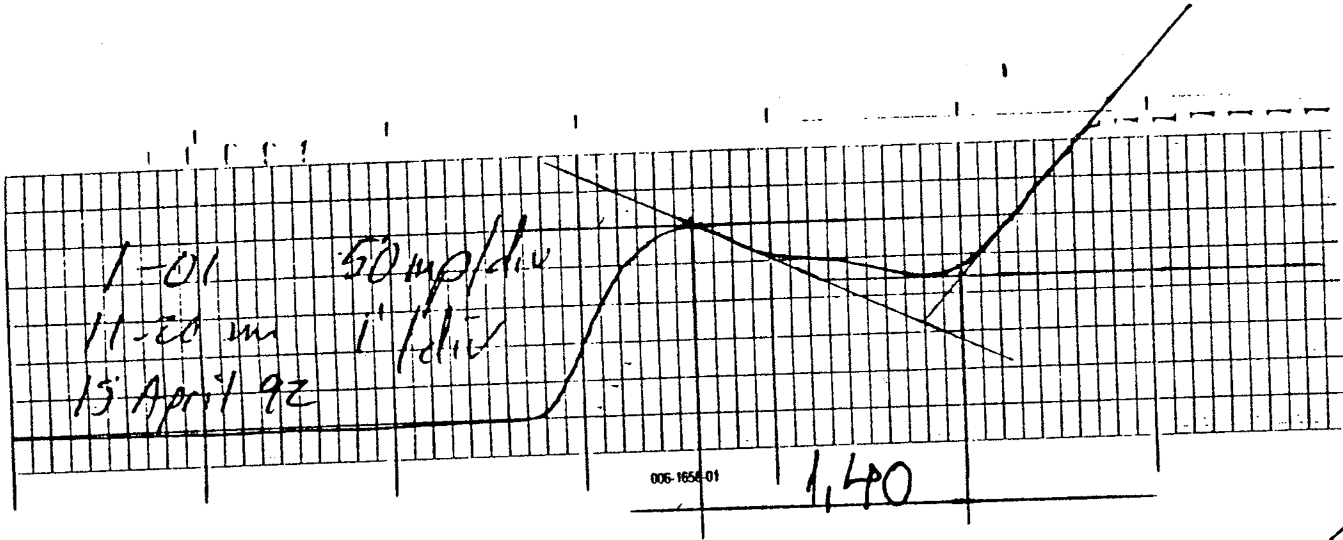
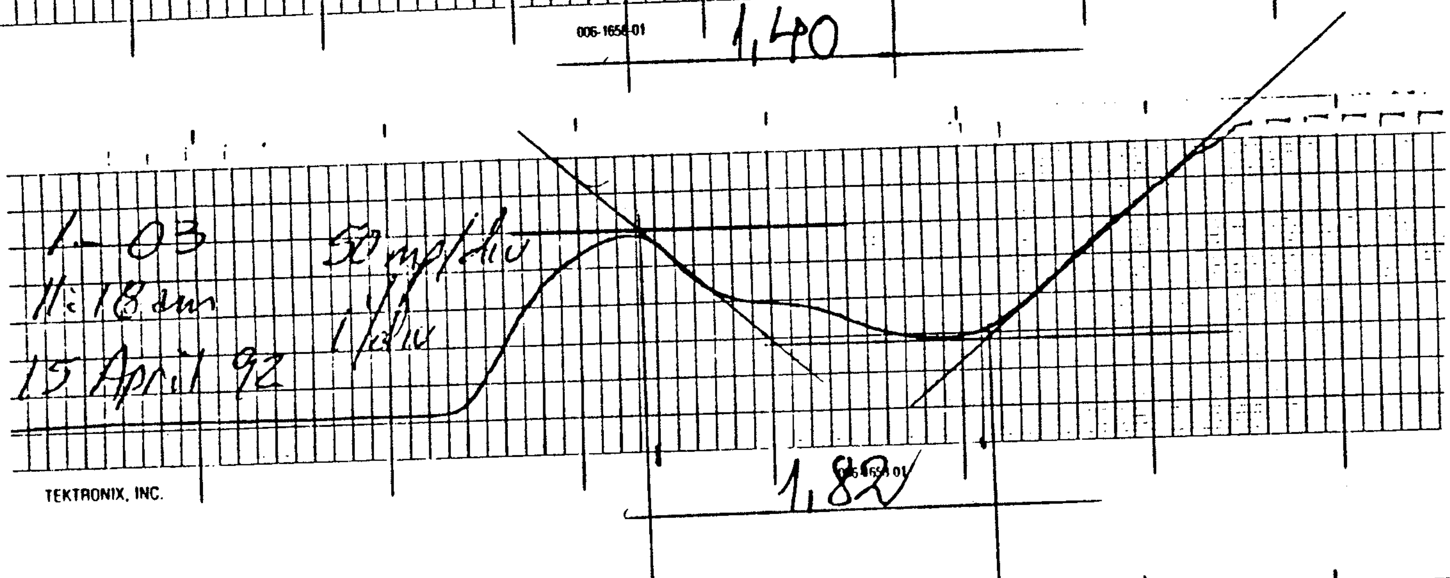
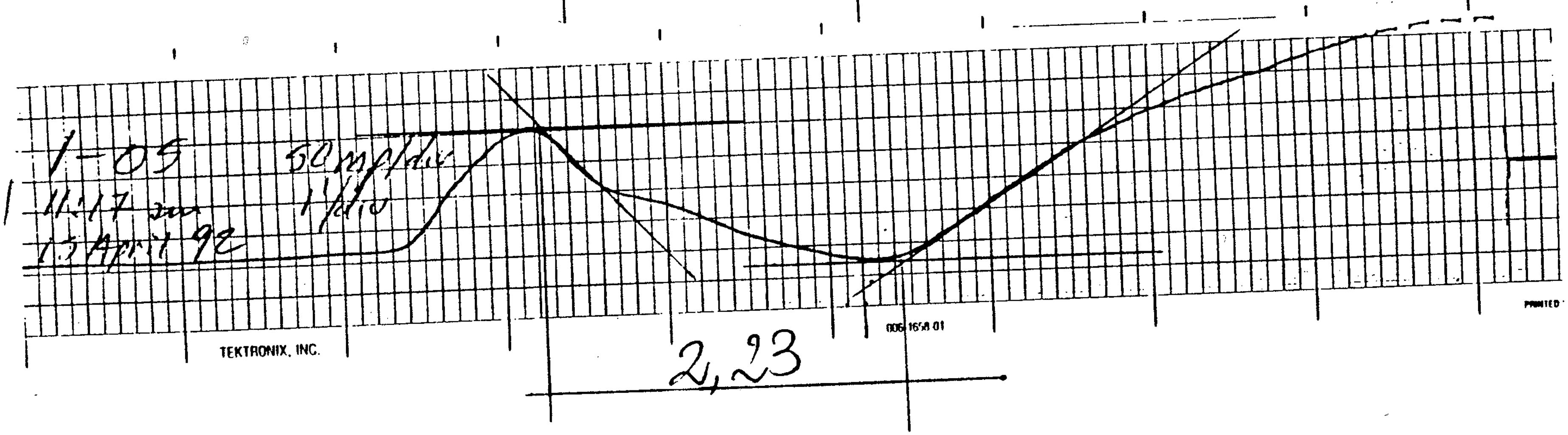
TEKTRONIX, INC.
006-1658-01
2,12


1-06
3:50pm
15 April 92
5 mp/div
1'/div
PRINTED IN U.S.A.
TEKTRONIX, INC.
1,58


1-01
15 April 92
006-1658-01
1,40


1-03
15 April 92
TEKTRONIX, INC.
1,82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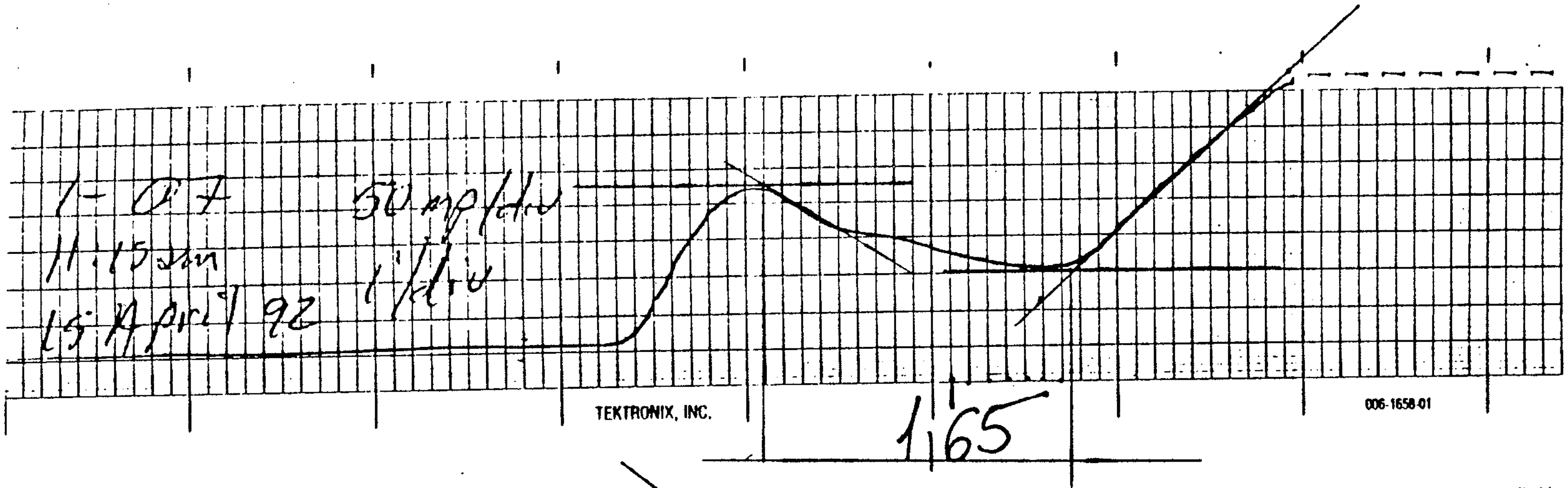
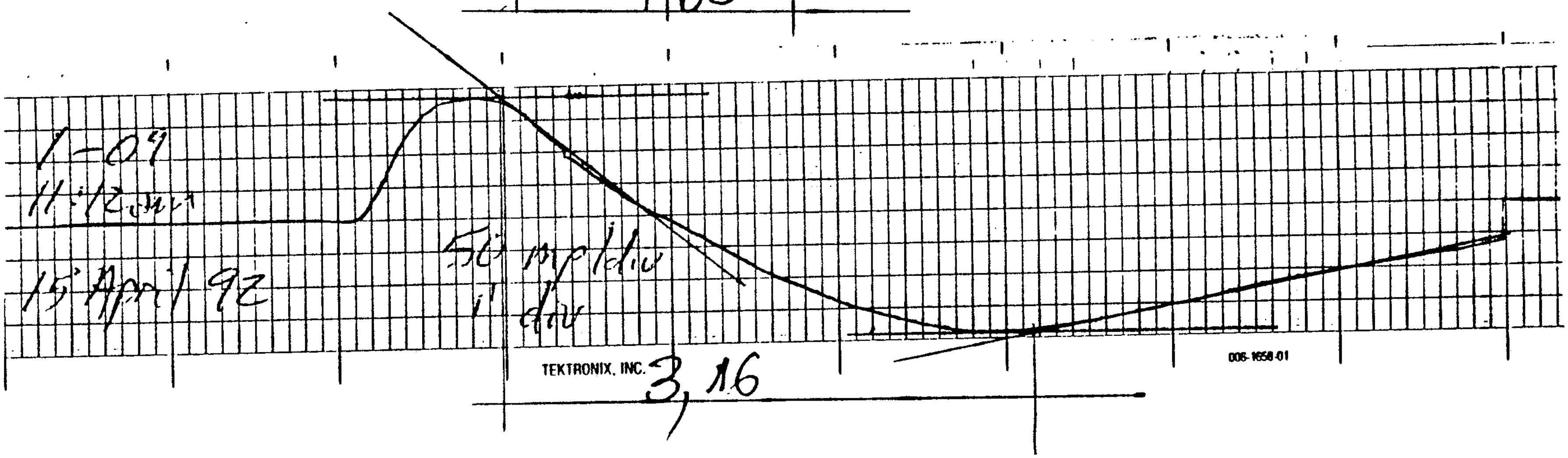
15 April 92
TEKTRONIX, INC.
006-1658-01
2,23


1-07
11:15am
15 April 92
50 mp/div
1'/div
TEKTRONIX, INC.
1,65
006-1658-01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11:12am
15 April 92
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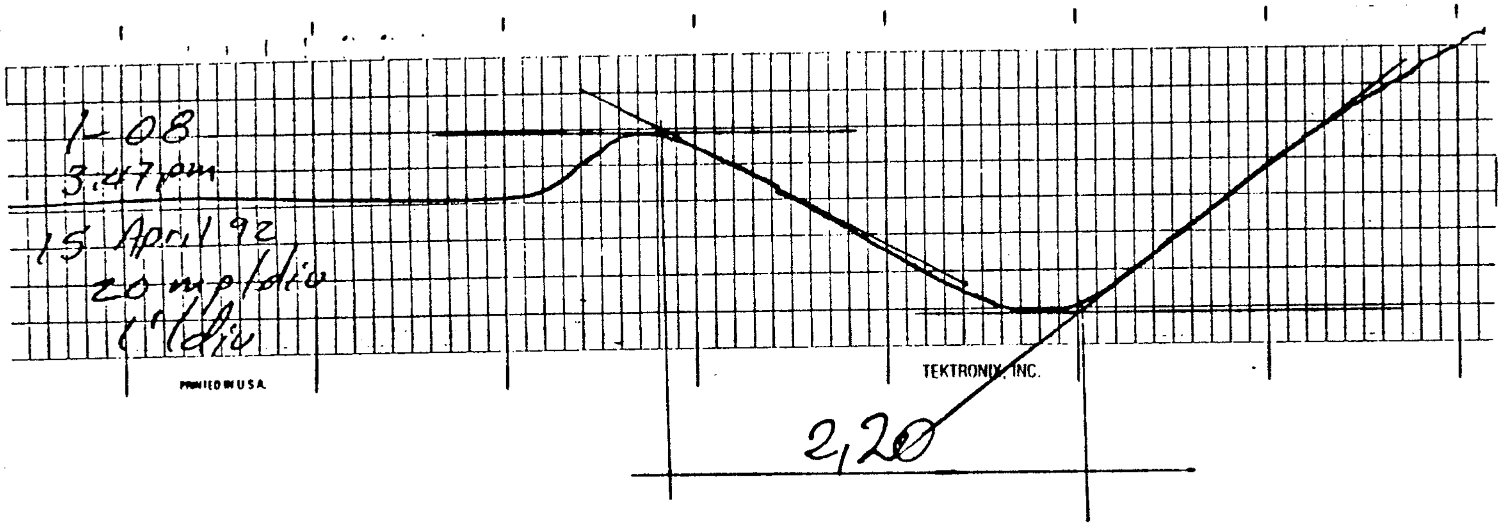
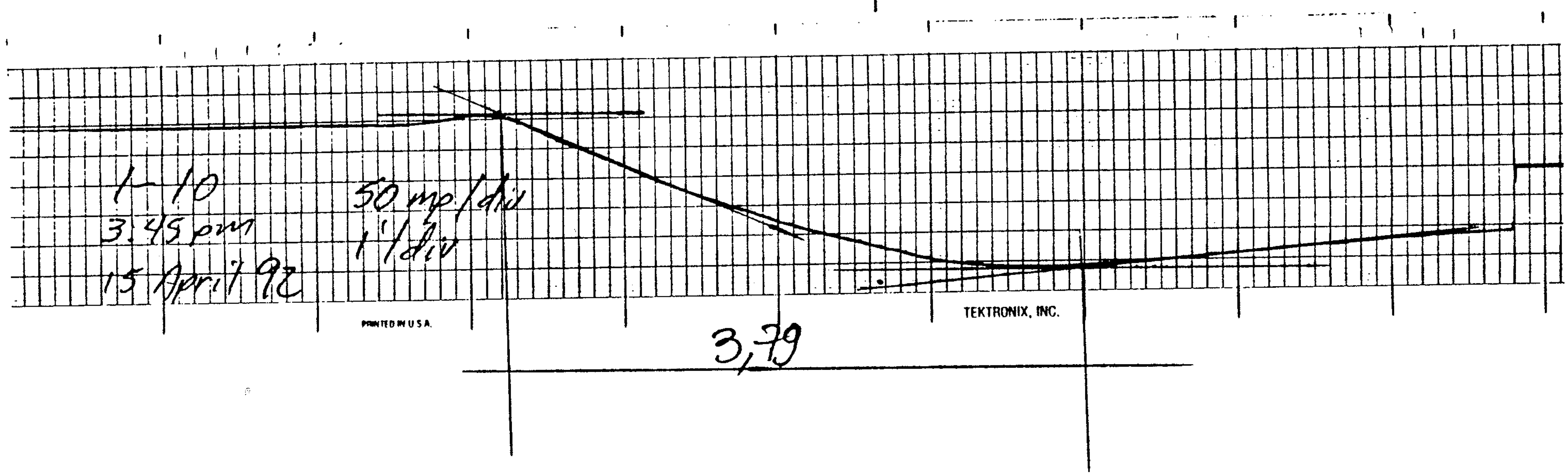
1' div
TEKTRONIX, INC.
3,16
006-1658-01

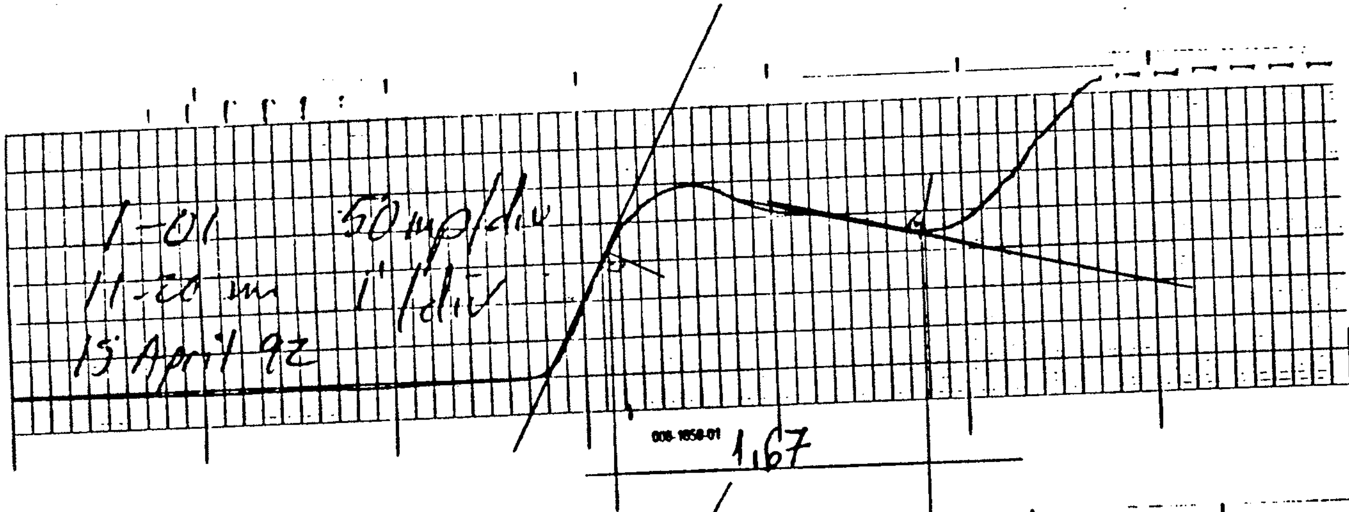

1-08
3:47 pm
15 April 92
20 mp/div
1"/div
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,20


1-10
3.45 pm
15 April 92
50 mp/div
1"/div
PRINTED IN U.S.A.
TEKTRONIX, INC.
3,79

Appendix 1.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

bbb) "Method of Diverging Lines", Borehole #2/April 1992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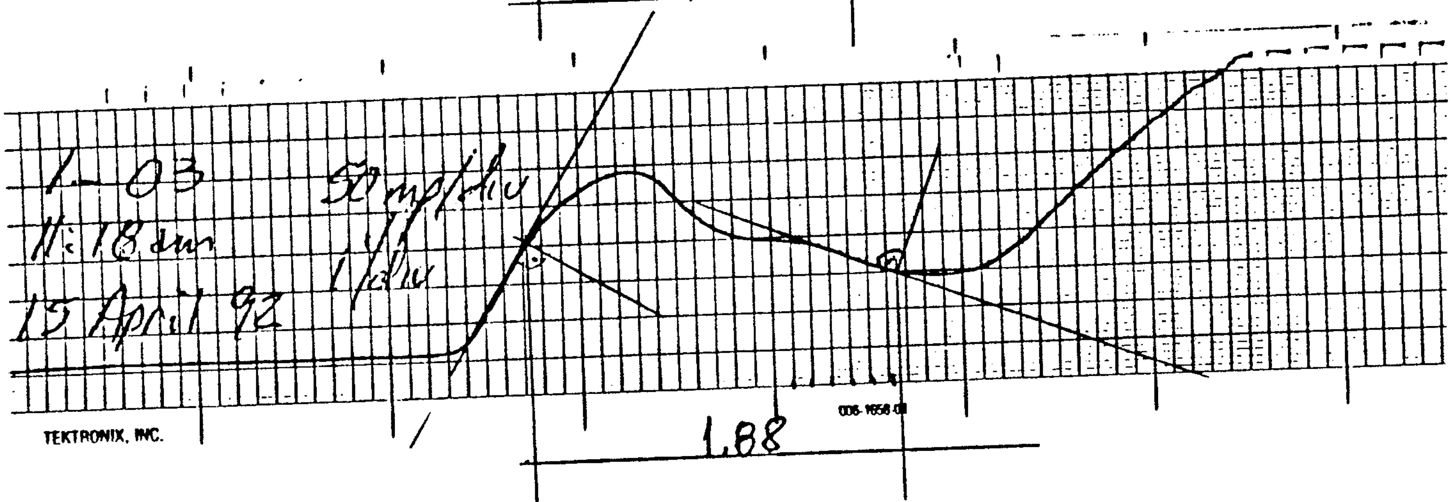
006-1658-01
1,67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11:18 am
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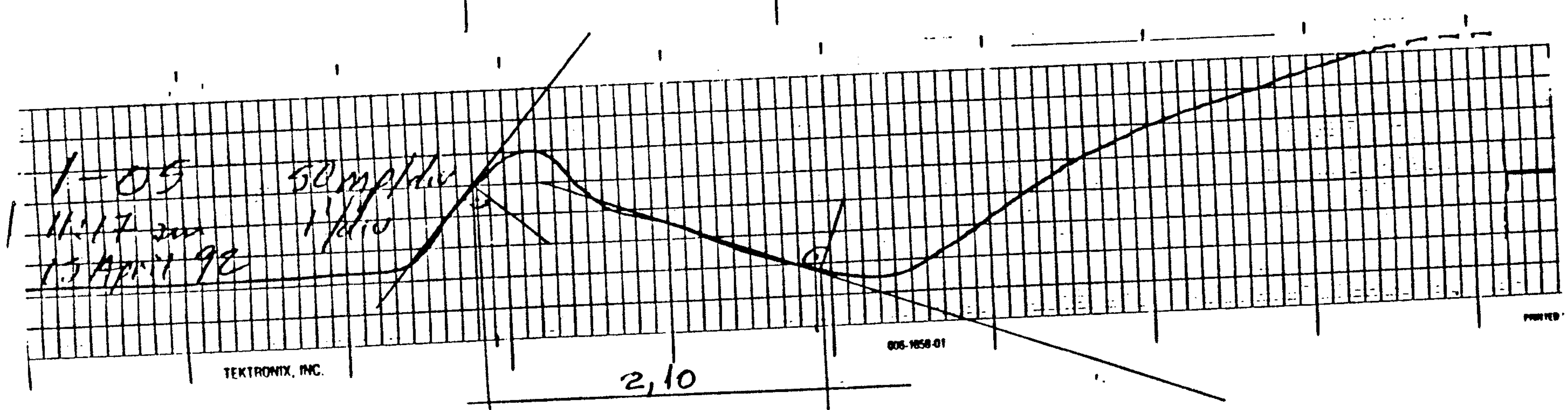
15 April 92
TEKTRONIX, INC.
006-1658-01
1,88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11:17 am
1"/div
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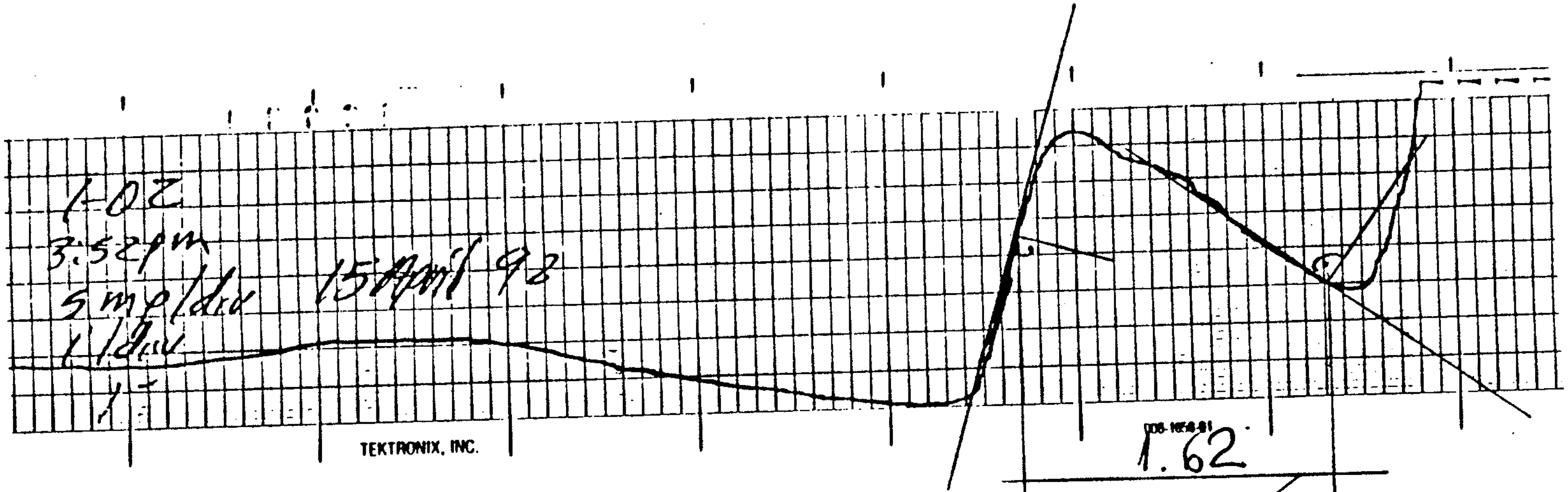
TEKTRONIX, INC.
006-1658-01
2,10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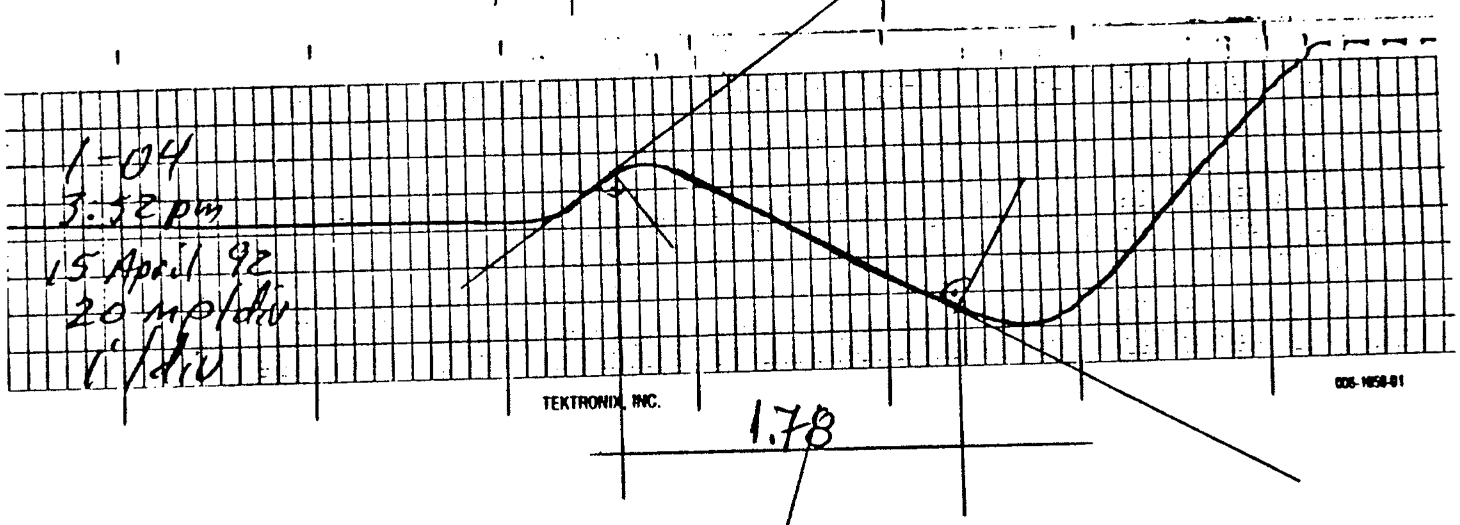
TEKTRONIX, INC.
1.62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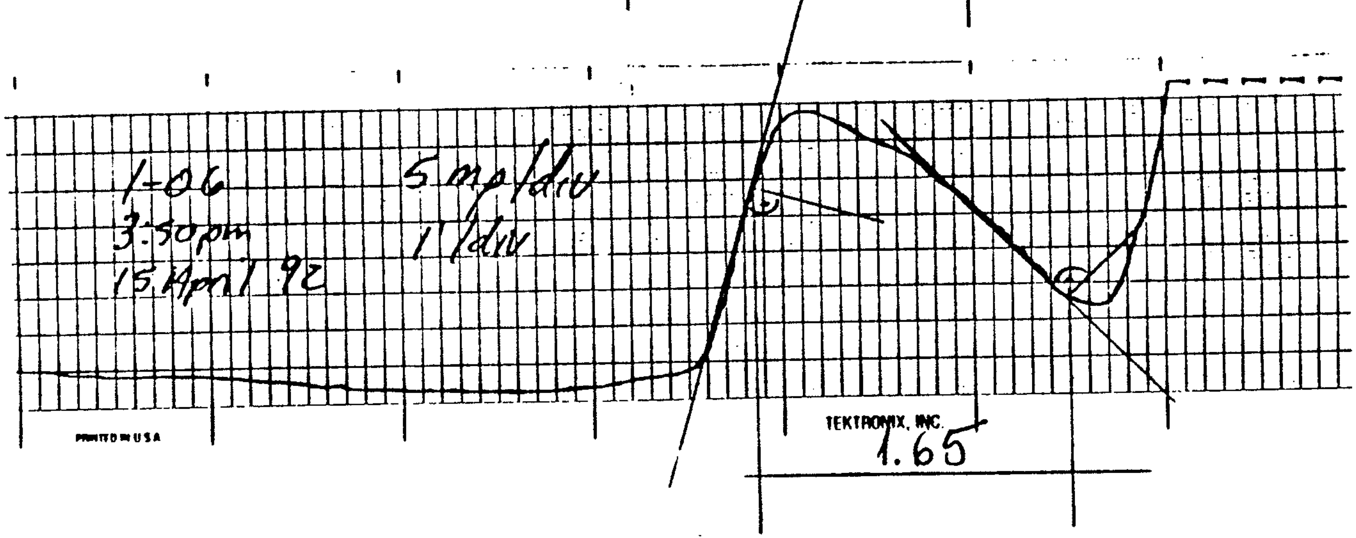
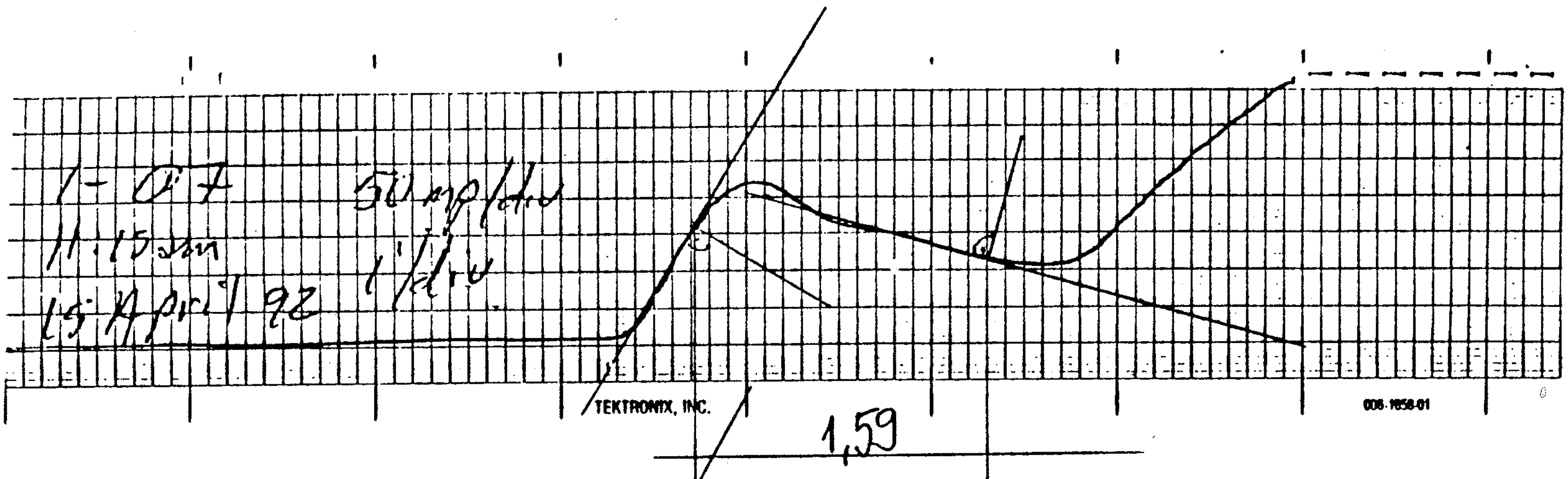
1.78


1-06
3:50pm
15 April 92
5 mp/div
1'/div
TEKTRONIX, INC.
1.65
PRINTED IN USA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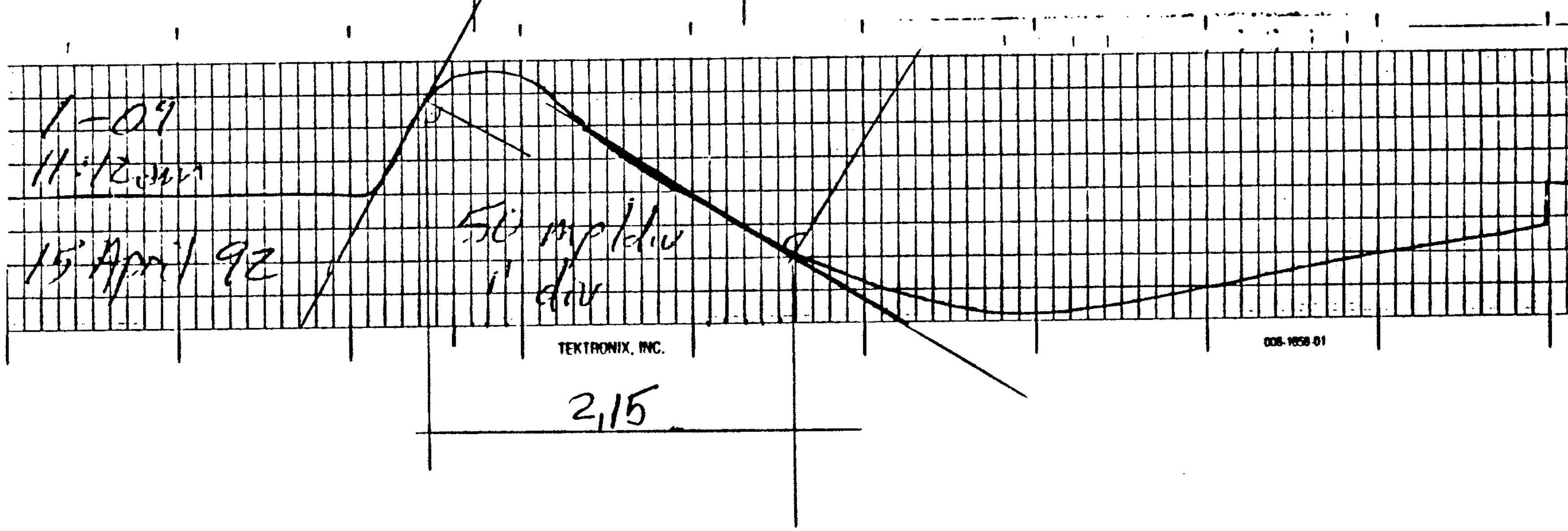
15 April 92
TEKTRONIX, INC.
006-1658-01
1,59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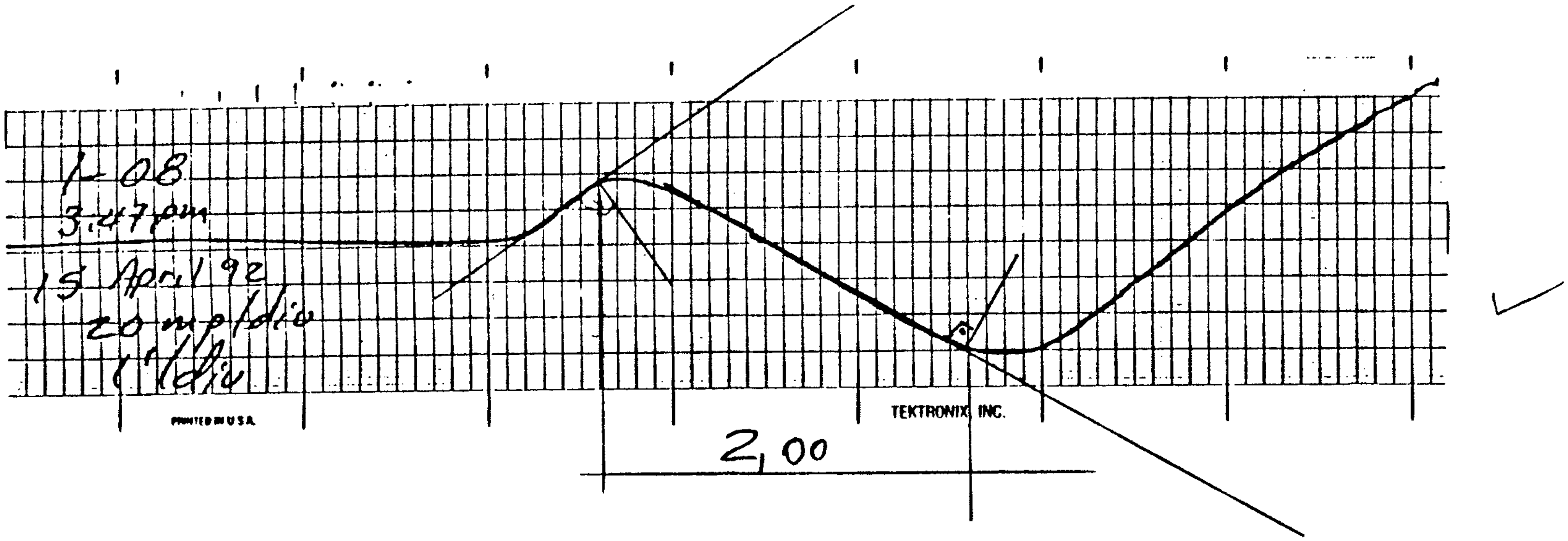
15 April 92
TEKTRONIX, INC.
006-1658-01
2,15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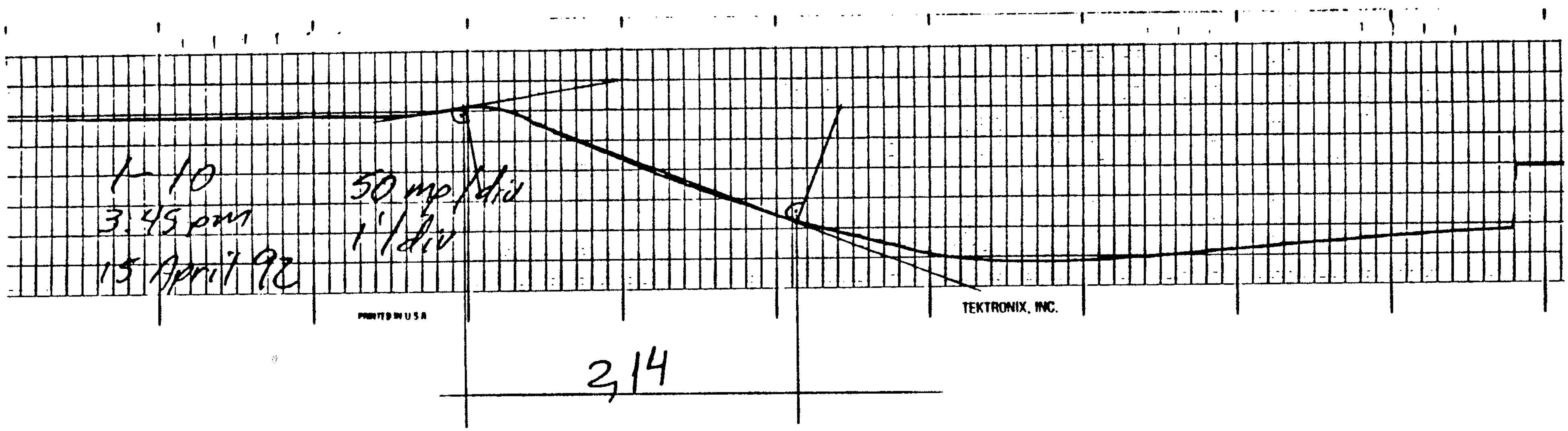
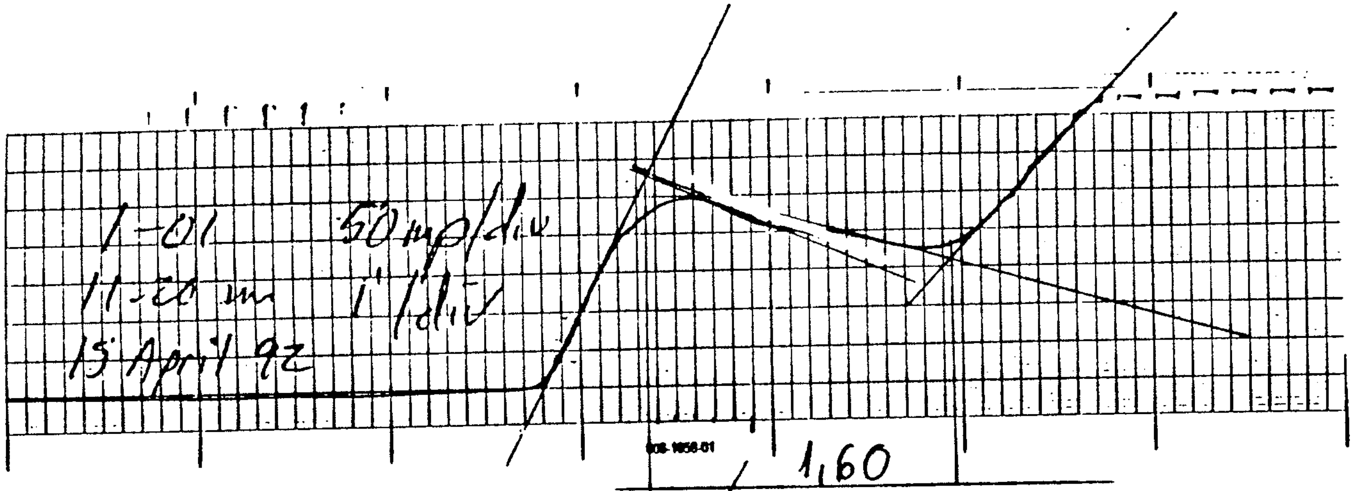
3:47pm
15 April 92
20 mp/div
1'/div
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,00

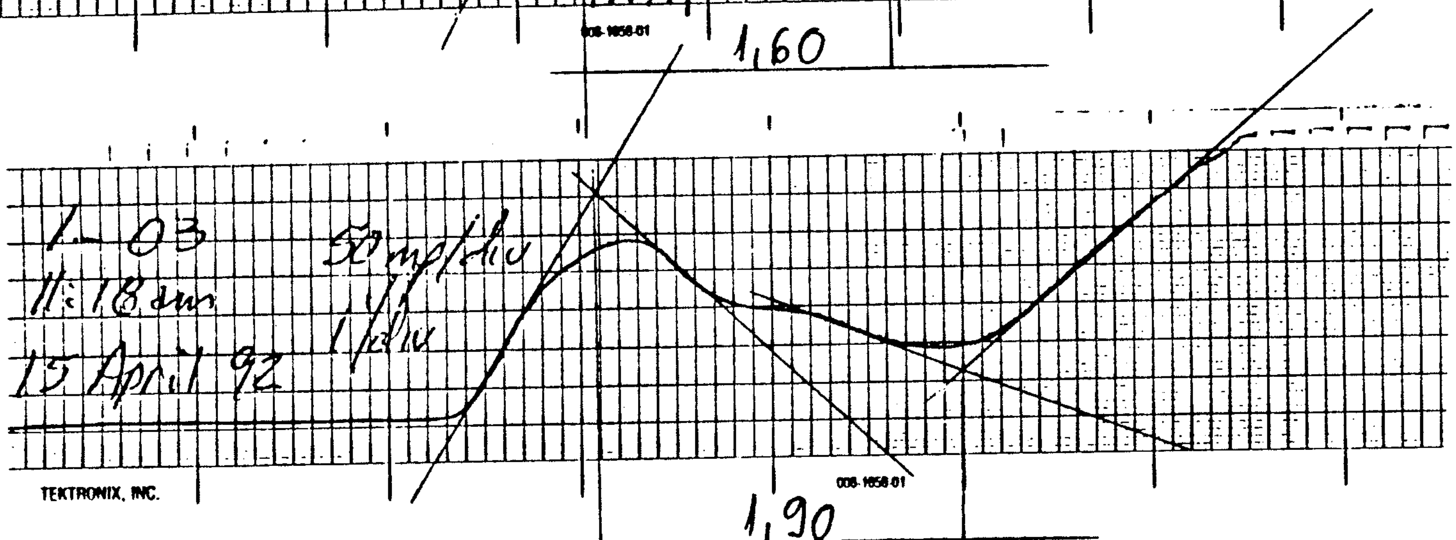

1-10
3:45pm
15 April 92
50 mp/div
1'/div
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,14

Appendix 1.

Plots of TDR traces obtained from field seasonal Measurements in February, March, and April 1992 in Borehole #2, using flat prongs 7.5" long and different graphical methods of reading.

ccc) "Method of Peaks", Borehole #2/April 1992


1-01
15 April 92
1,60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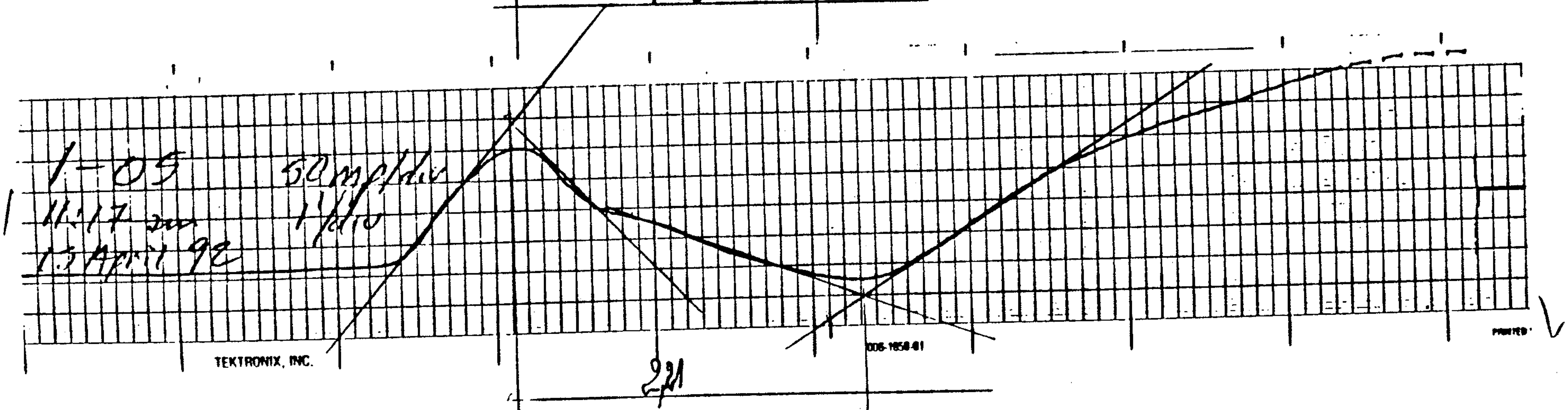
TEKTRONIX, INC.
1,90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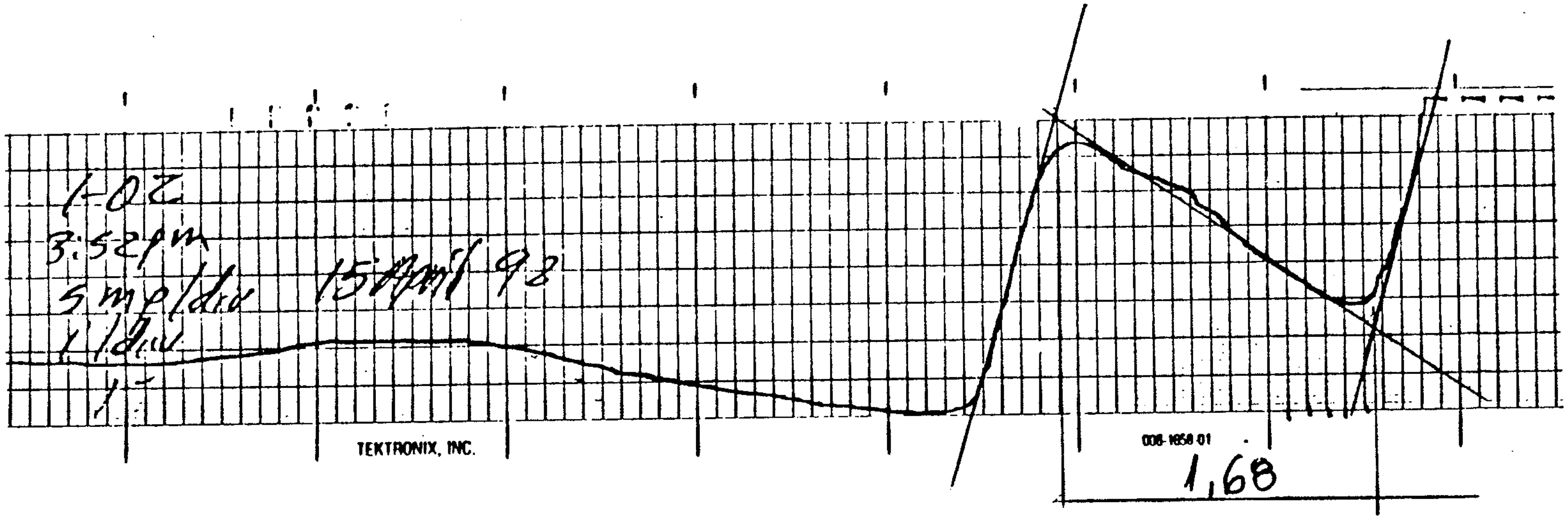
TEKTRONIX, INC.



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3:52pm
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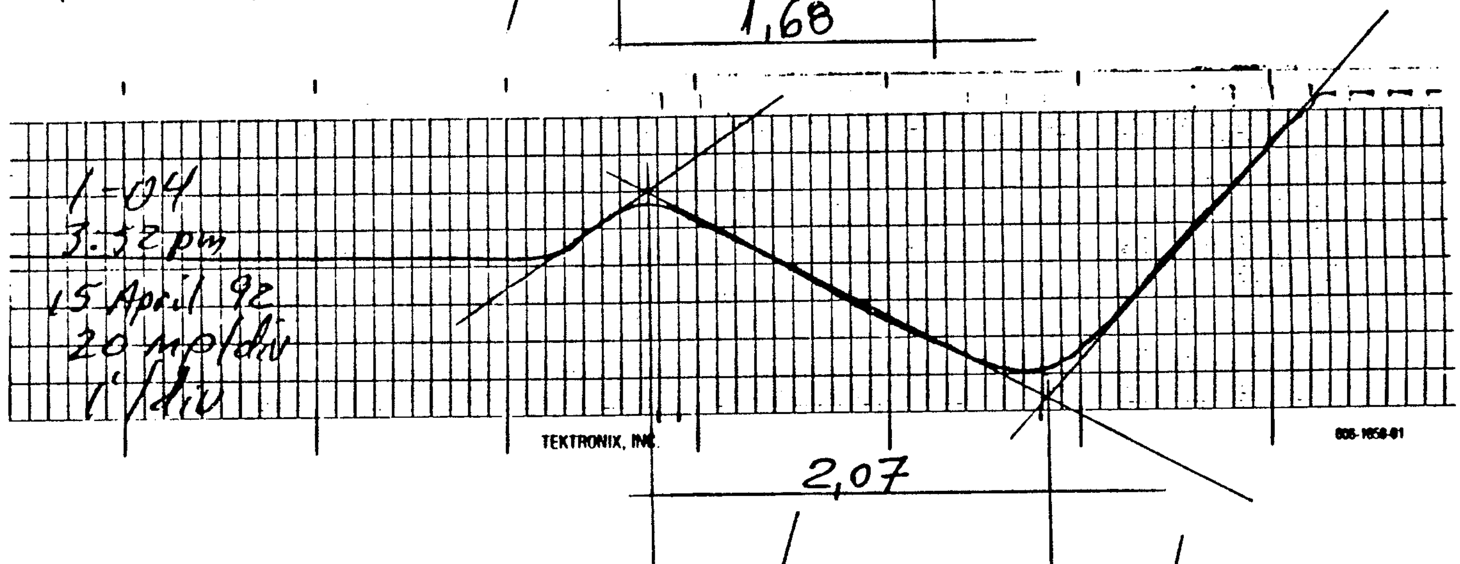
15 April 92
1'/div
TEKTRONIX, INC.
006-1658-01
1,68



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3:52pm
15 April 92
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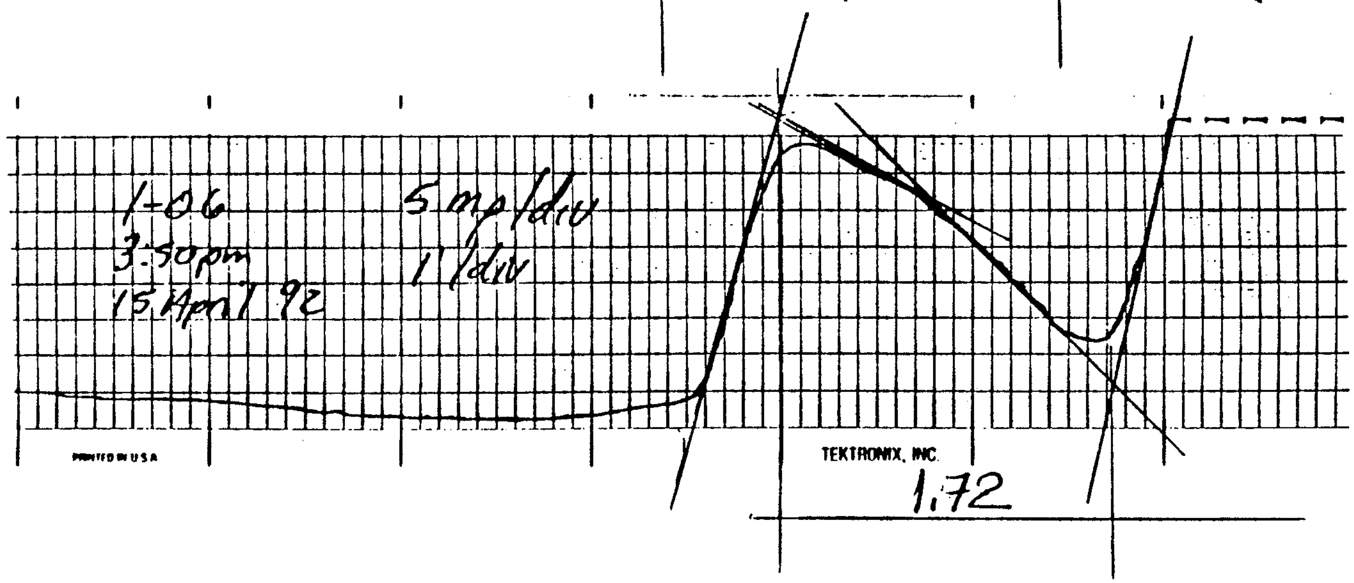
1'/div
TEKTRONIX, INC.
006-1658-01
2,07



1-06
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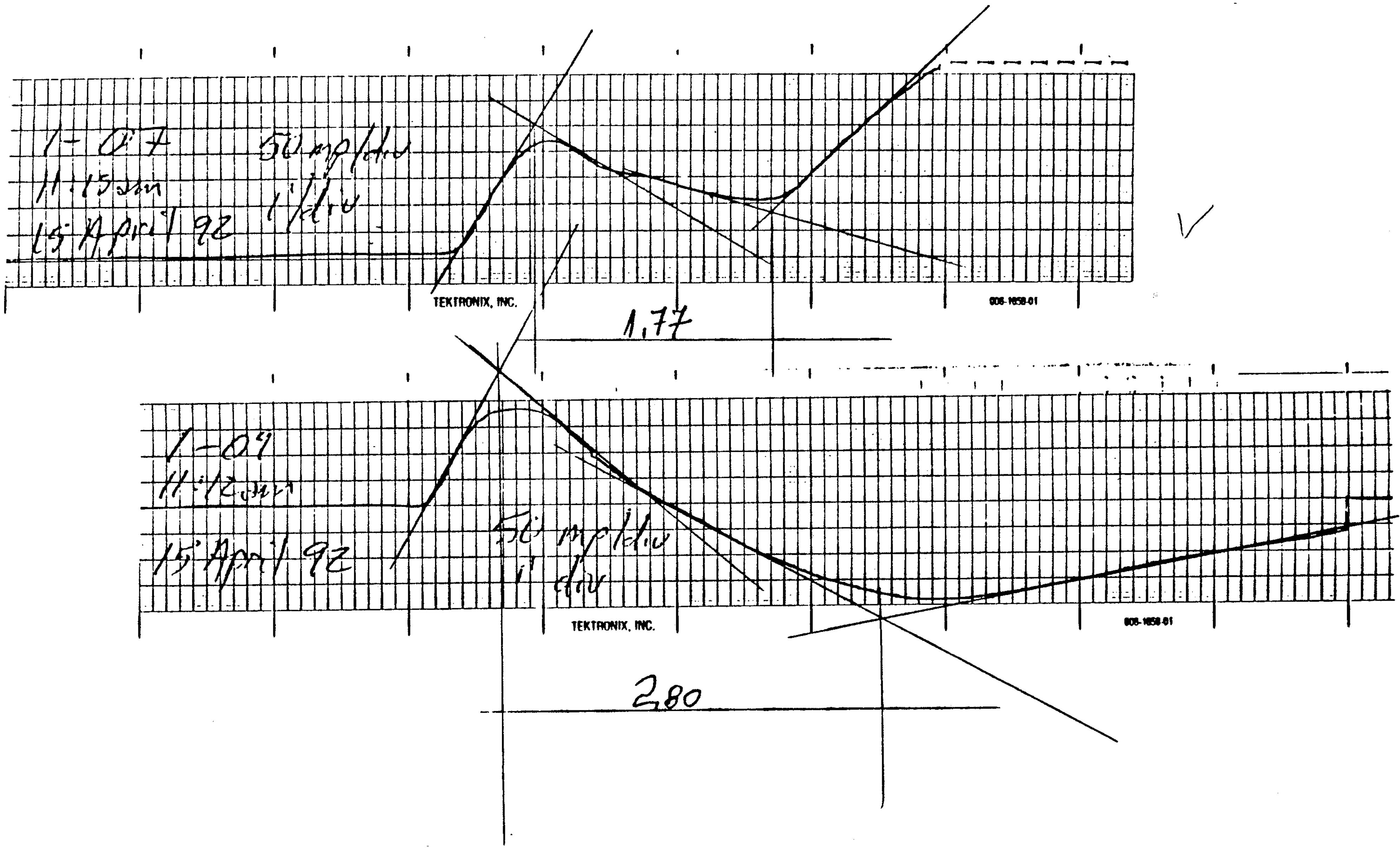
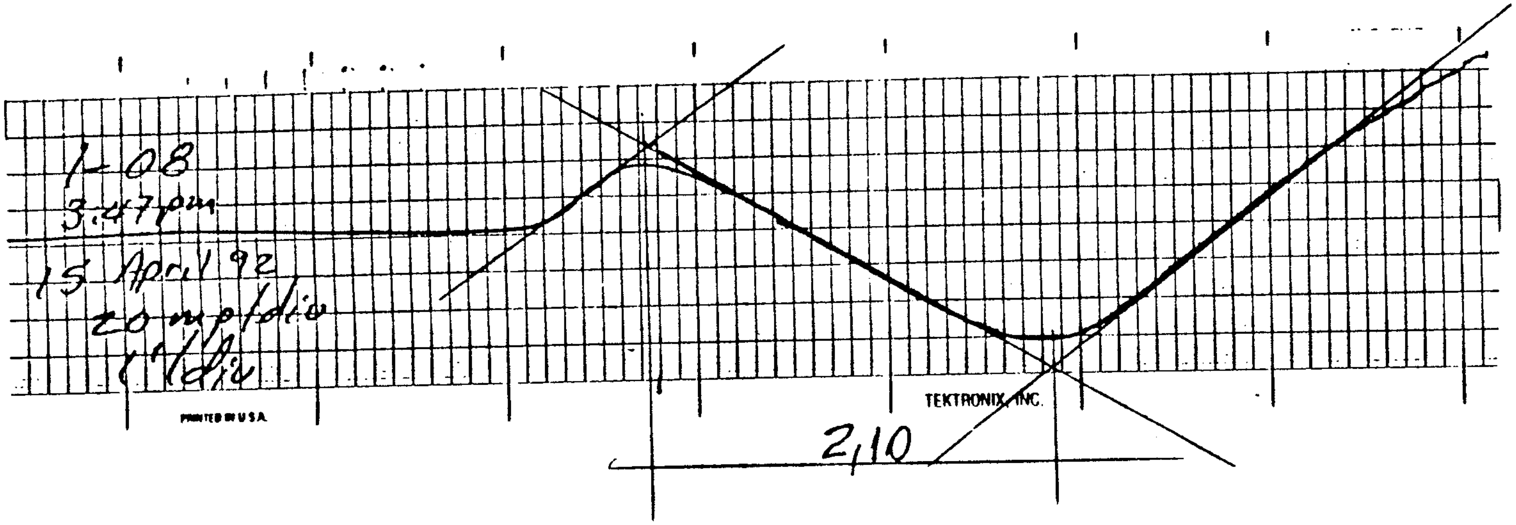
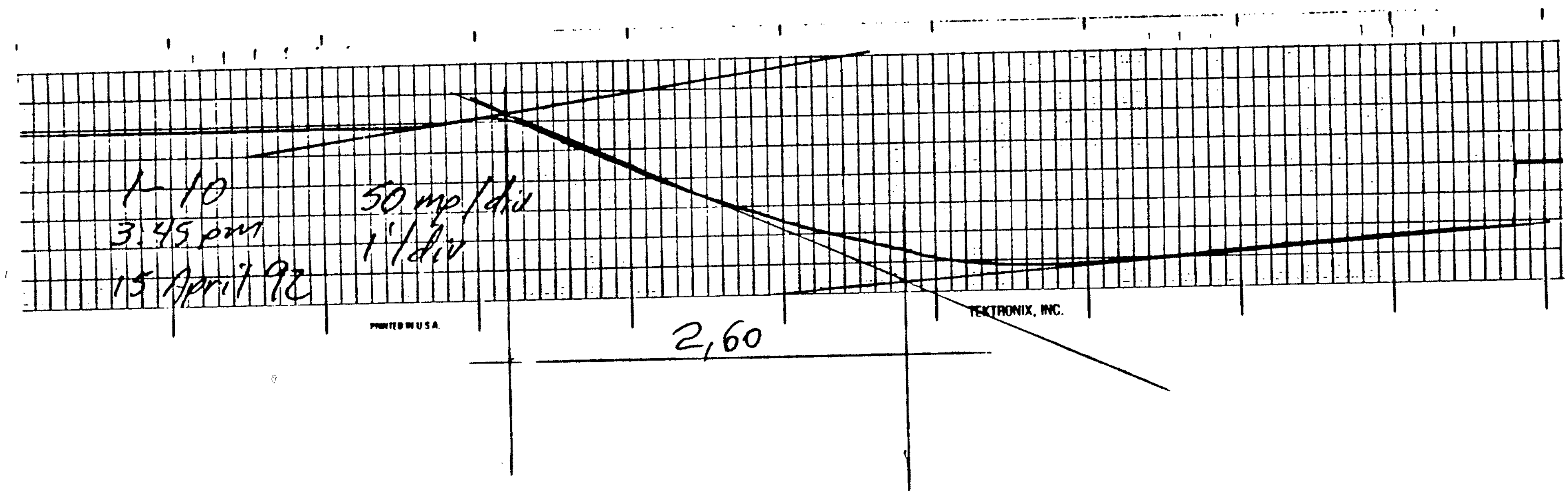
15 April 92
5 mp/div
1'/div
PRINTED IN U.S.A
TEKTRONIX, INC.
1,72



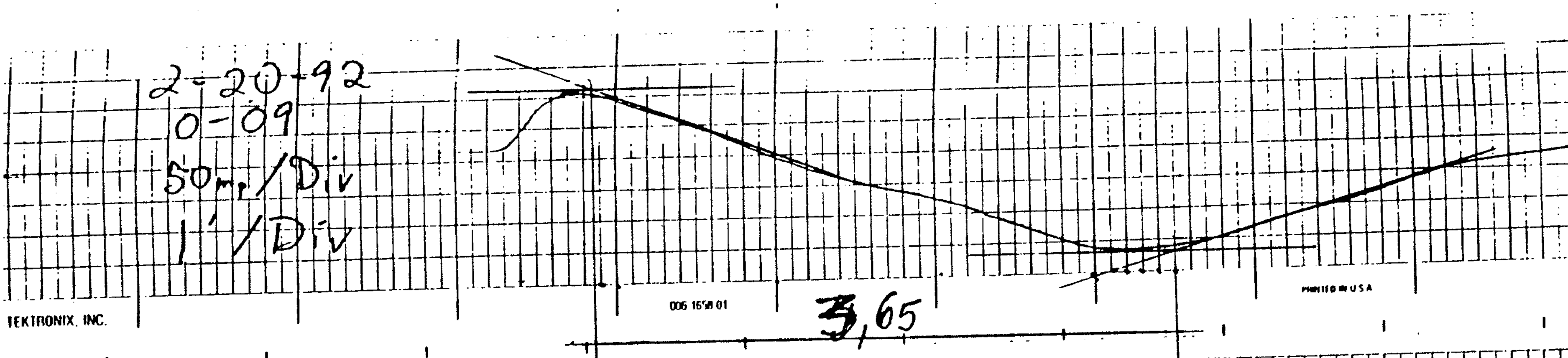
1-07
11:15am
15 April 92
TEKTRONIX, INC.
1.77
1-09
11:12am
15 April 92
TEKTRONIX, INC.
2.80

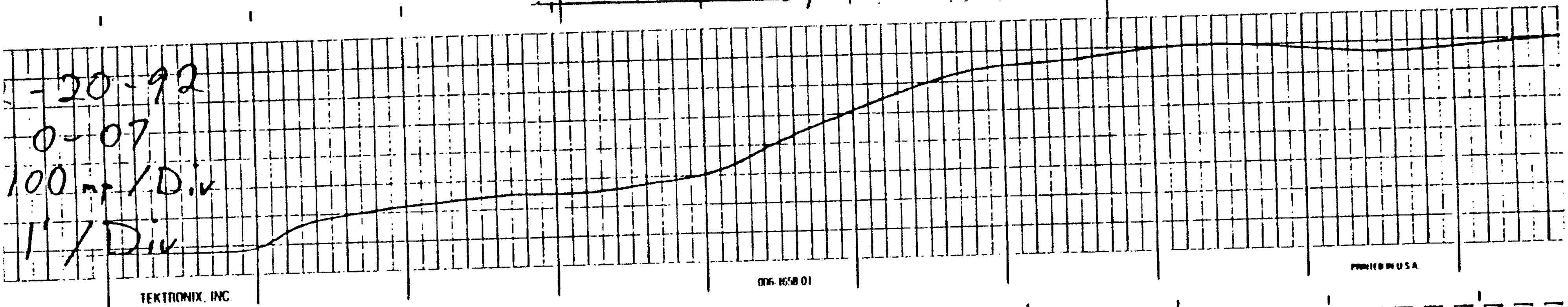

1-08
3:47 pm
15 April 92
20 mp/div
1'/div
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,10

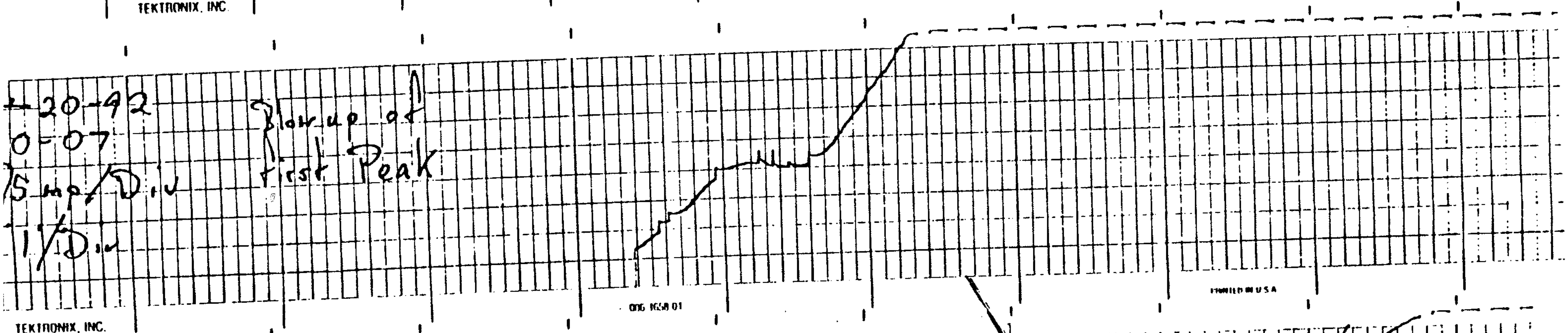

1-10
3:45 pm
15 April 92
50 mp/div
1'/div
PRINTED IN U.S.A.
TEKTRONIX, INC.
2,60

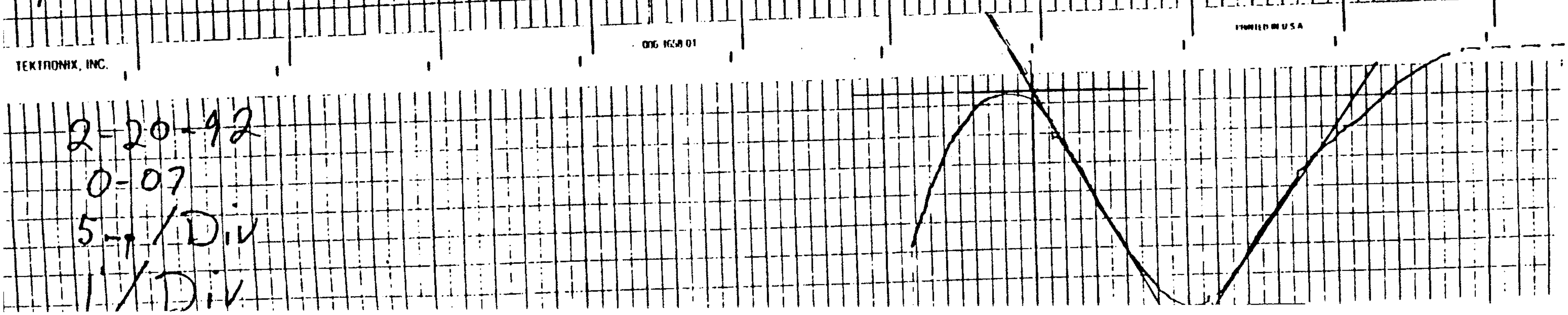
Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

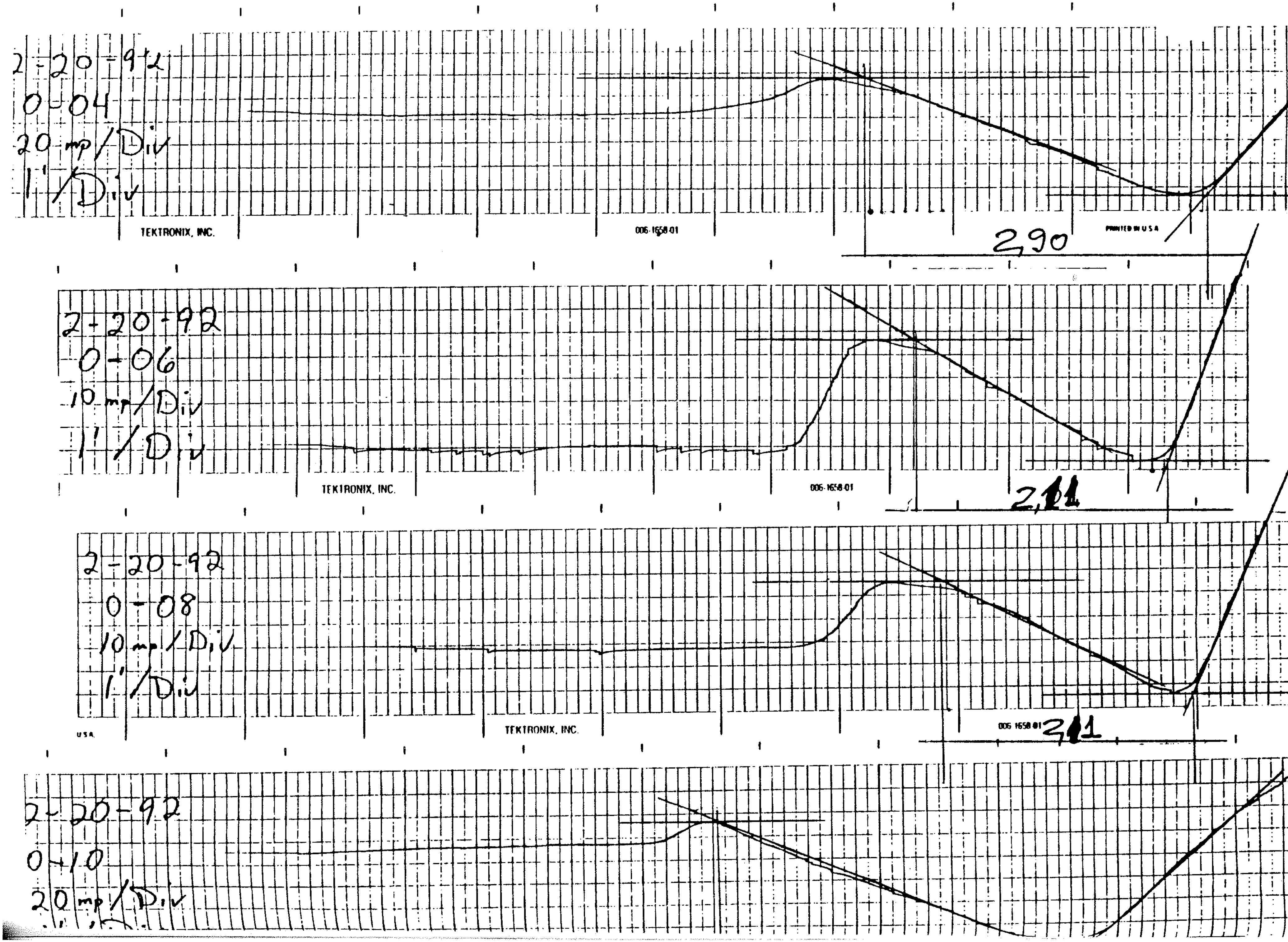
aaa) “Method of Tangents”, Borehole #3, February 1992


2-20-92
0-09
50mp/Div
1'/Div
TEKTRONIX, INC.
006 1658 01
PRINTED IN USA
3,65


-20-92
0-07
100mp/Div
1'/Div
TEKTRONIX, INC.
006 1658 01
PRINTED IN USA


-20-92
0-07
5mp/Div
1'/Div
Blowup of first Peak
TEKTRONIX, INC.
006 1658 01
PRINTED IN USA


2-20-92
0-07
5mp/Div
1'/Div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20 mp/Div
1'/Div
TEKTRONIX, INC.
006-1658-01
PRINTED IN U.S.A.
2,90
2-20-92
0-06
10 mp/Div
1'/Div
TEKTRONIX, INC.
006-1658-01
2,11
2-20-92
0-08
10 mp/Div
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TEKTRONIX, INC.
006-1658-01
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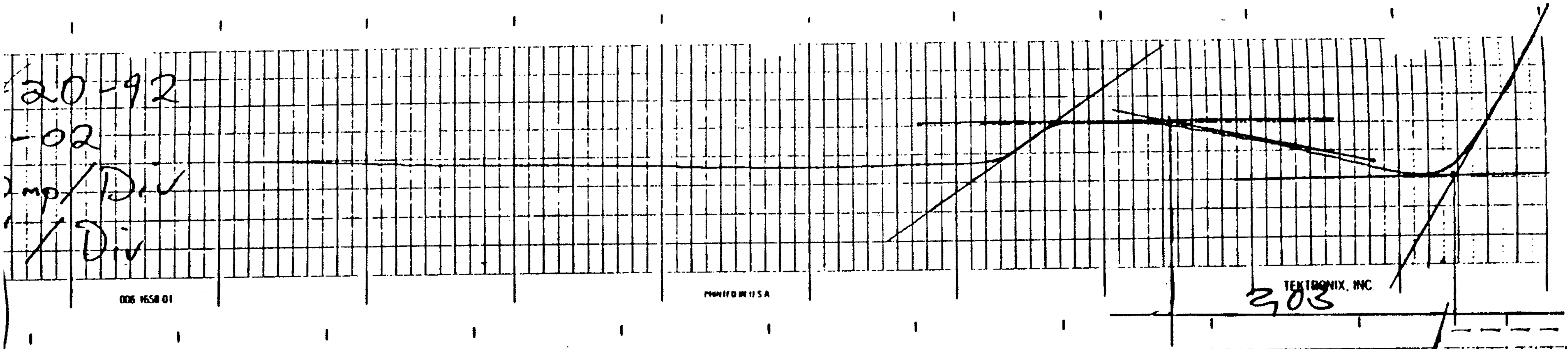
0-10
20 mp/Div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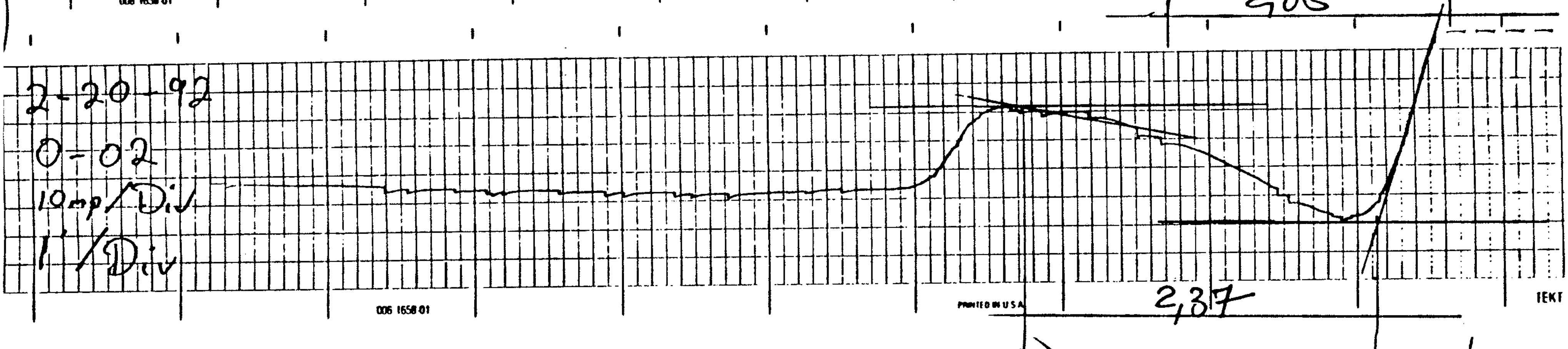
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006 1658 01
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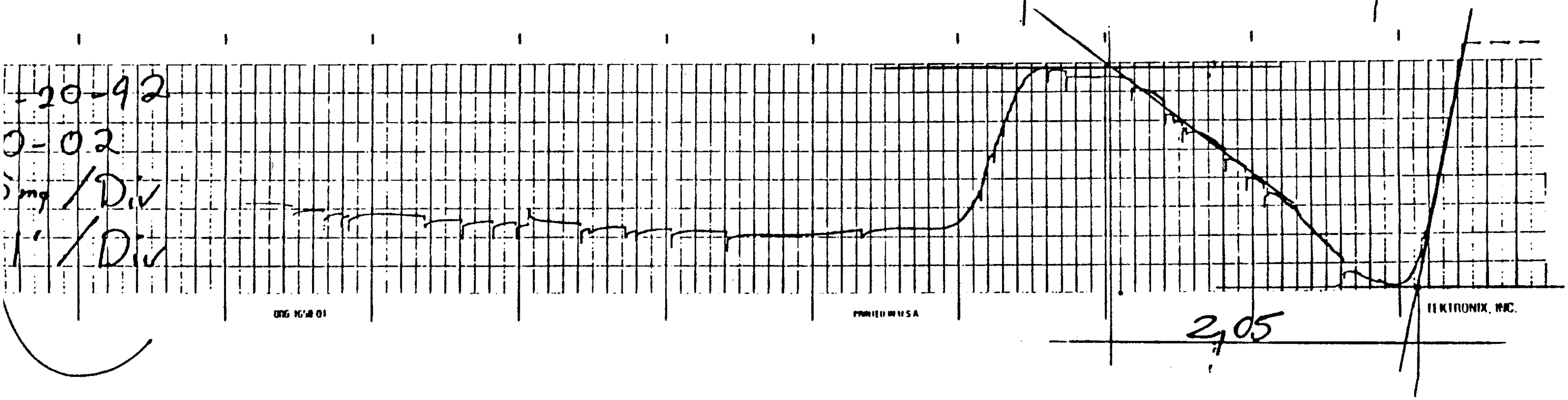
TEKT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1'/Div
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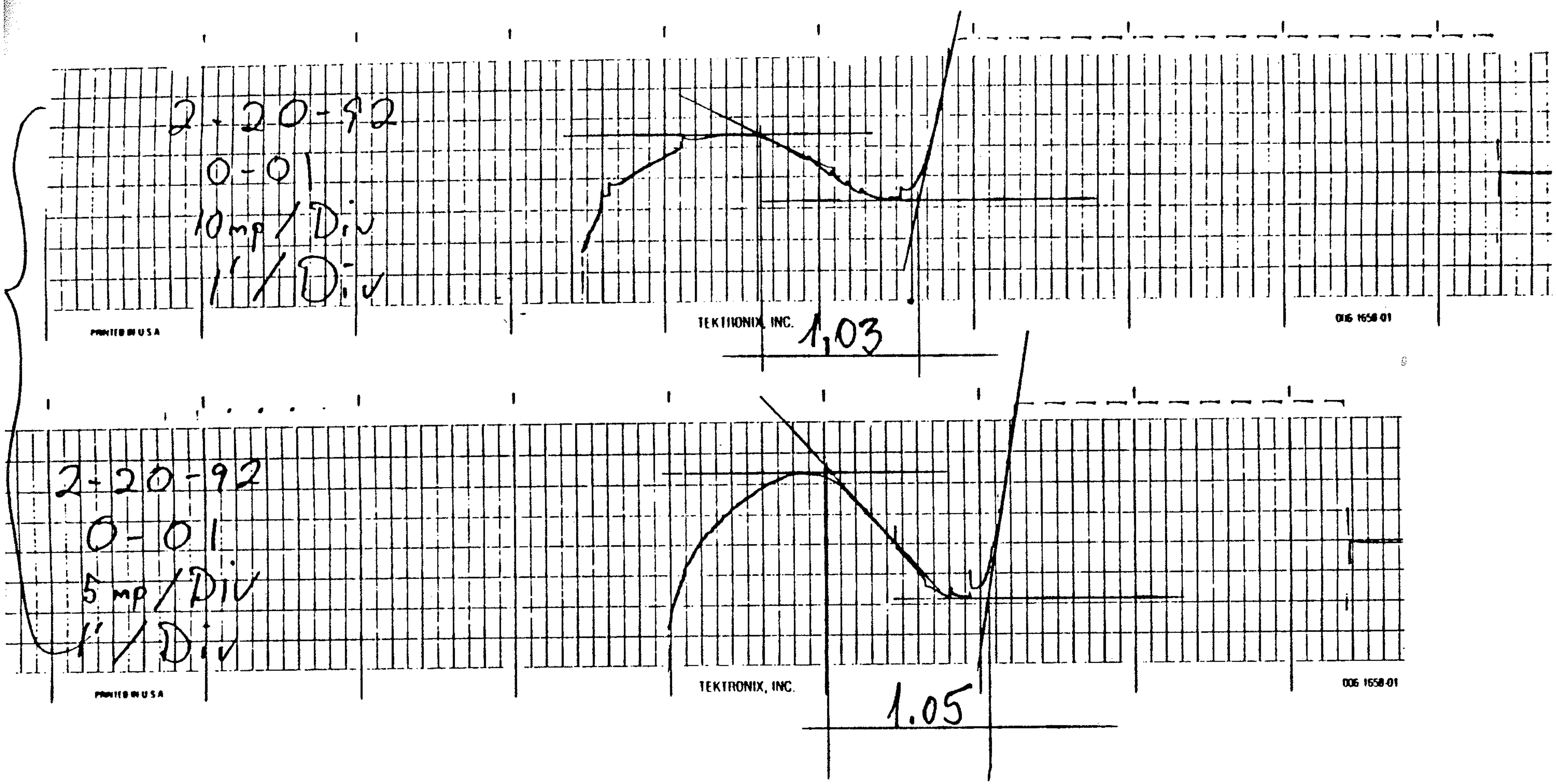
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TEKTRONIX, INC.
006 1658-01
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TEKTRONIX, INC.
006 1658-01
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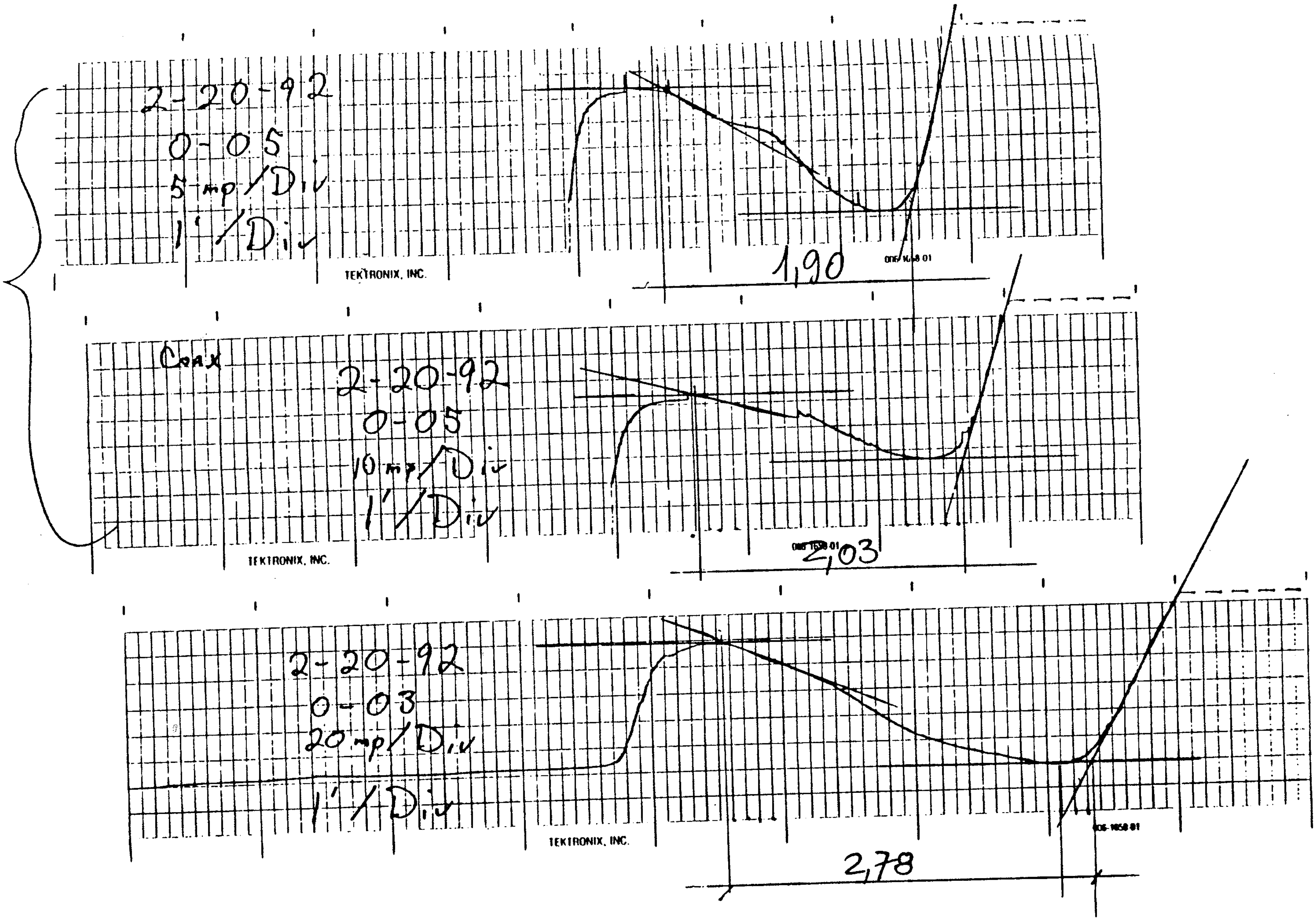

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1,90
TEKTRONIX, INC.
Coax
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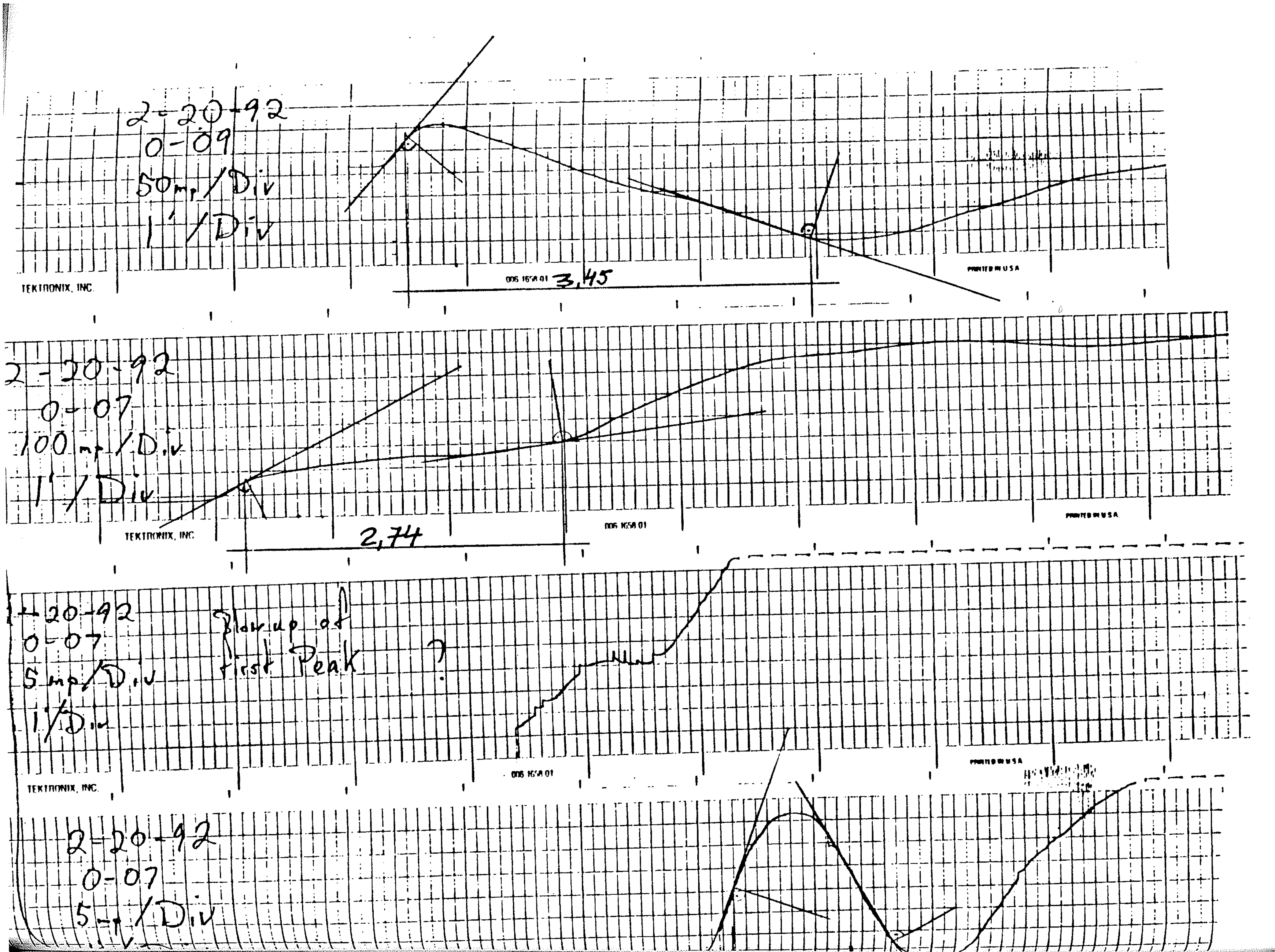
0-05
10 mp/Div
1'/Div
2,03
TEKTRONIX, INC.
2-20-92
0-03
20 mp/Div
1'/Div
2,78
TEKTRONIX, INC.

Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

bbb) "Method of Diverging Lines", Borehole #3, February 1992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TEKTRONIX, INC.
006 1658 01
3.45
PRINTED IN U.S.A.
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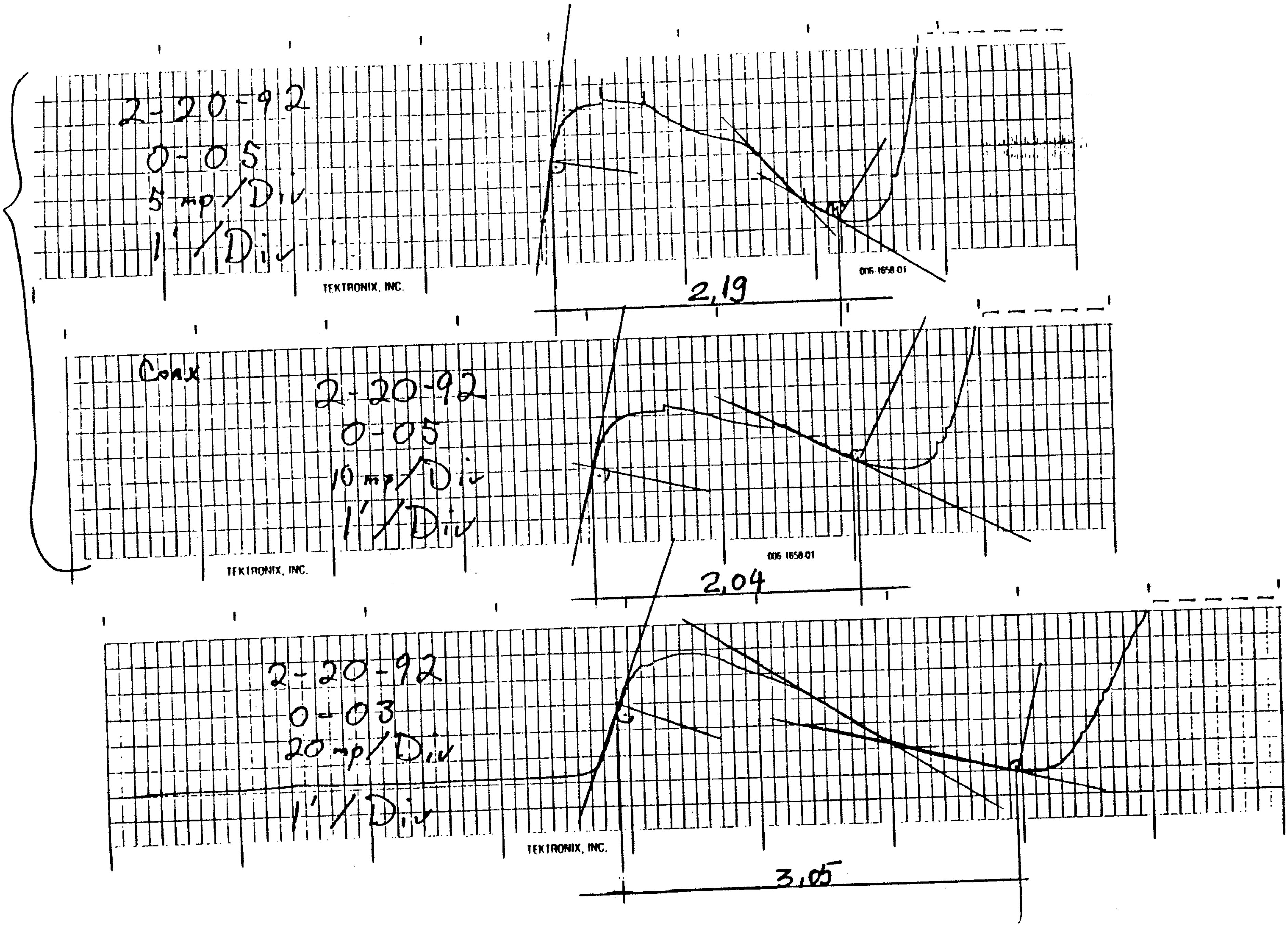
0-07
100 mp/Div
1'/Div
TEKTRONIX, INC
006 1658 01
2.74
PRINTED IN U.S.A.
2-20-92
0-07
5 mp/Div
1'/Div
Blow up of first Peak
?
TEKTRONIX, INC.
006 1658 01
PRINTED IN U.S.A.
2-20-92
0-07
5 -/Div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006-1658-01
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Coax
2-20-92
0-05
10 mp/Div
1'/Div
TEKTRONIX, INC.
006 1658-01
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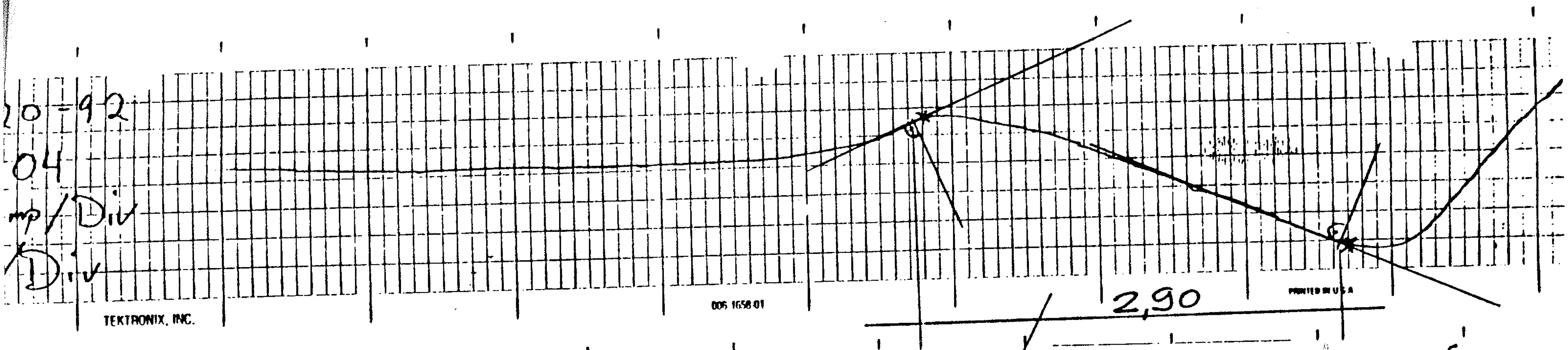
0-03
20 mp/Div
1'/Div
TEKTRONIX, INC.
3,05



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mp/Div
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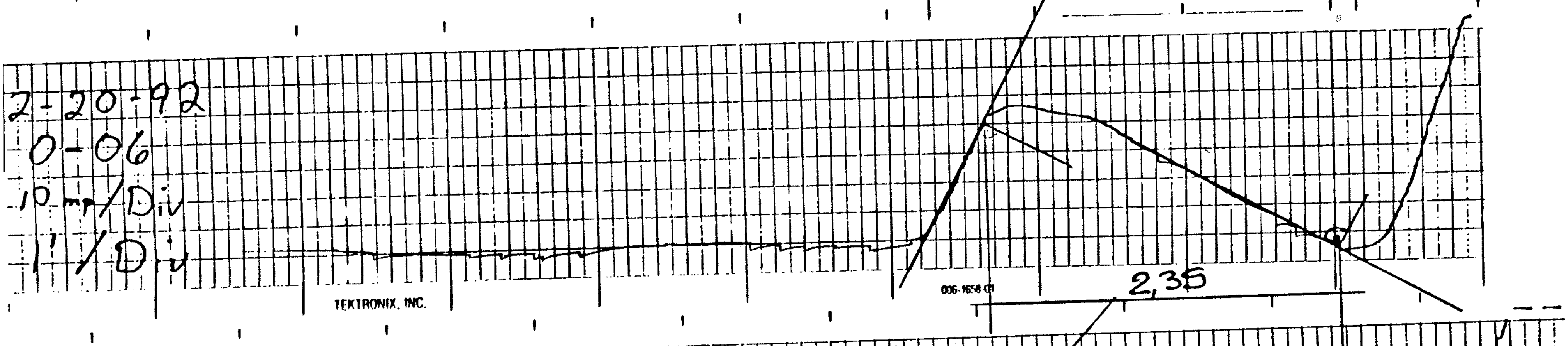

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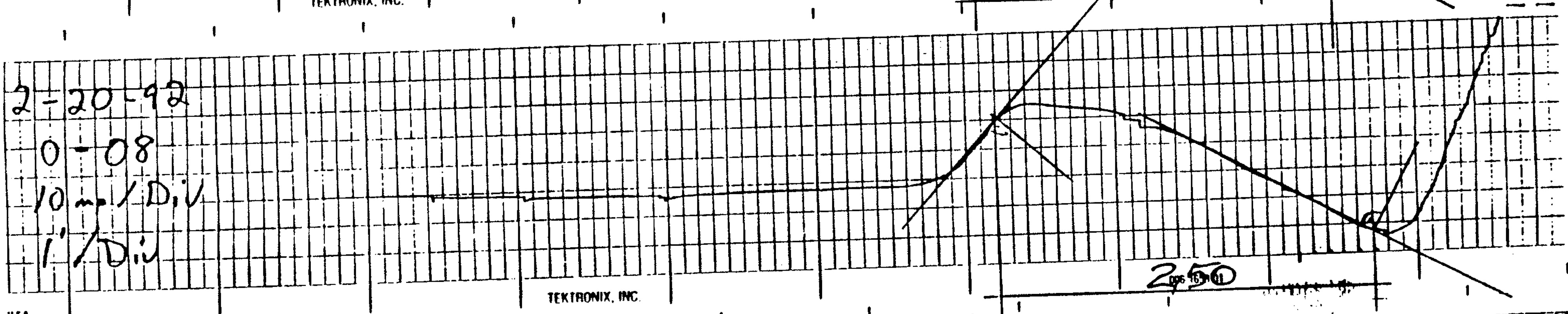

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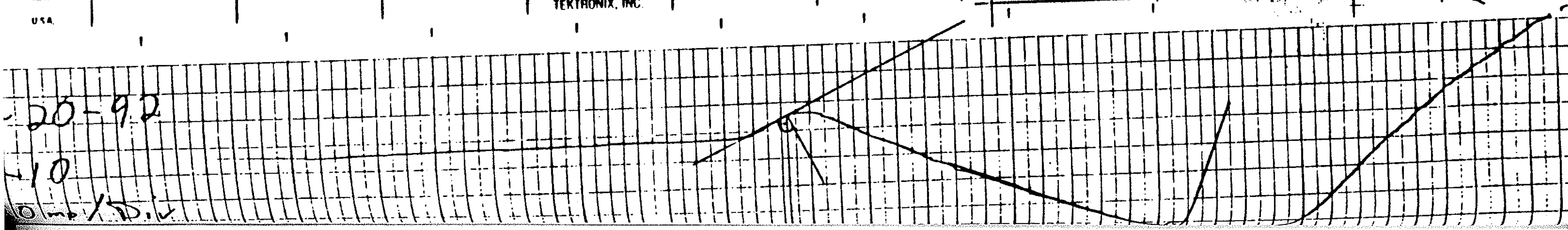
2,50
U.S.A.



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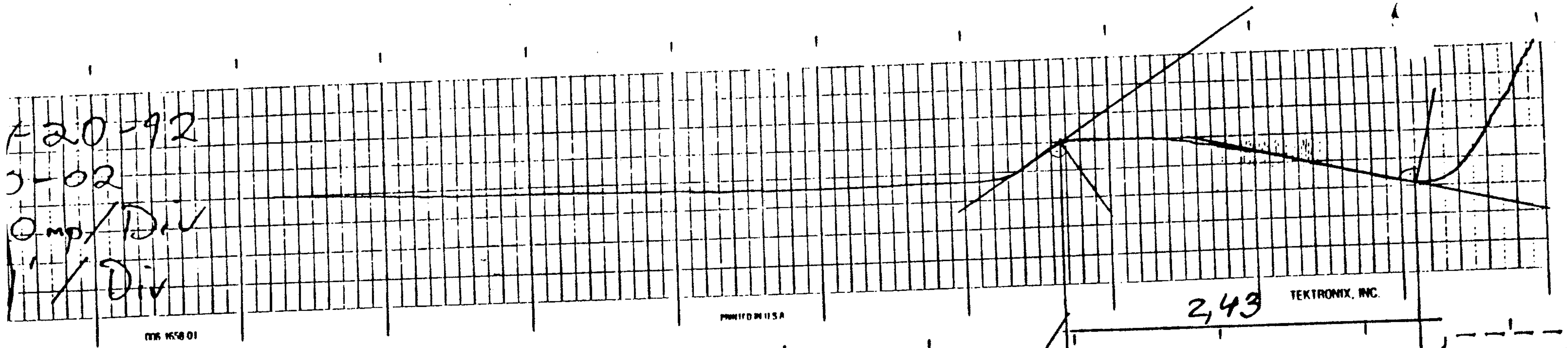

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1'/Div
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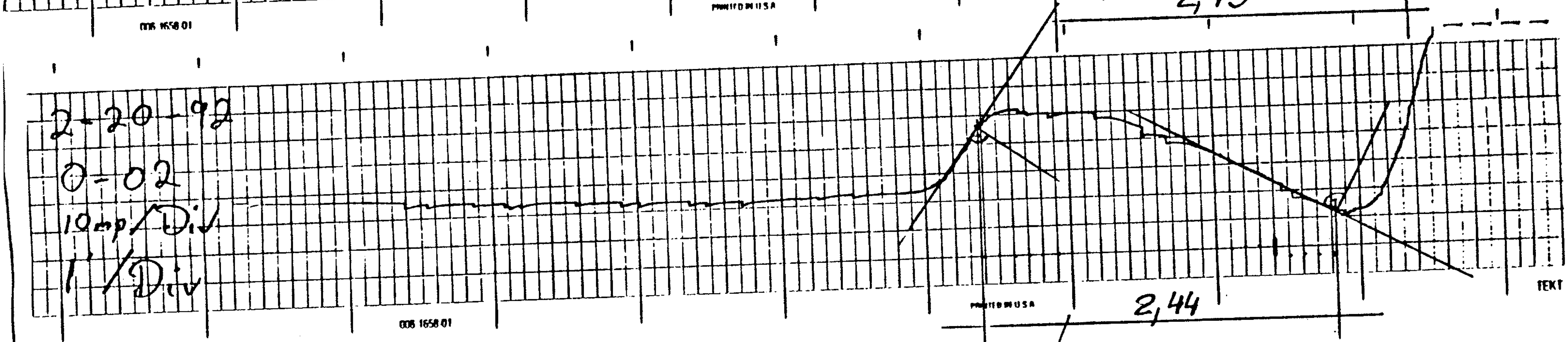
PRINTED IN U.S.A
006 1658 01



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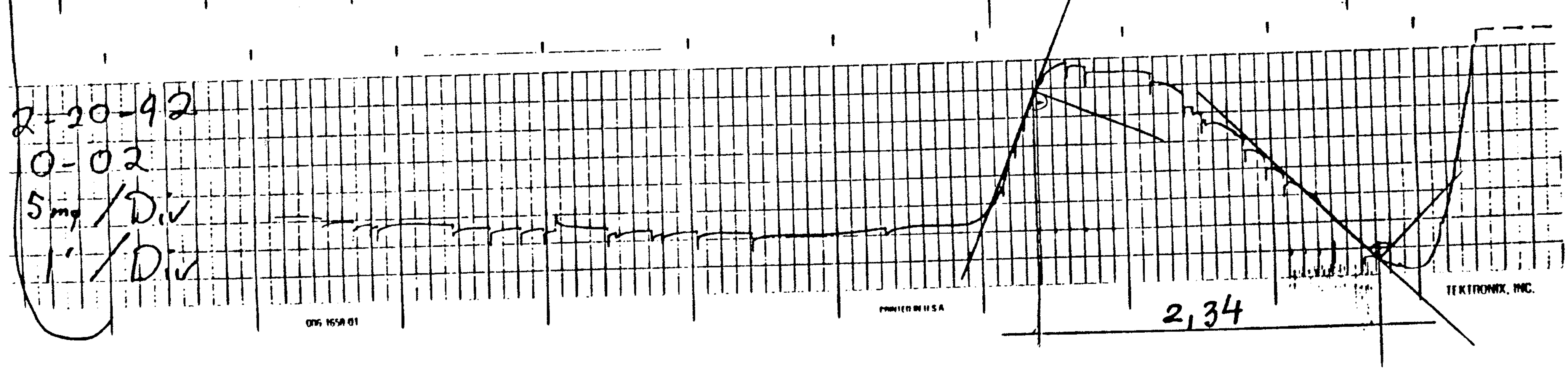
006 1658 01



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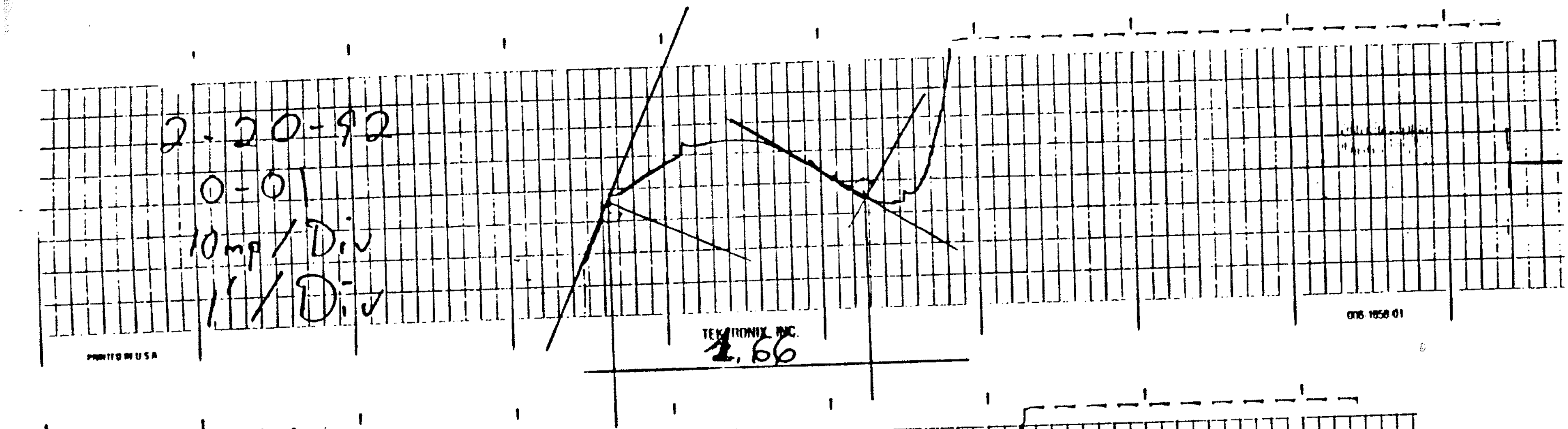
006 1658 01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1' / Div
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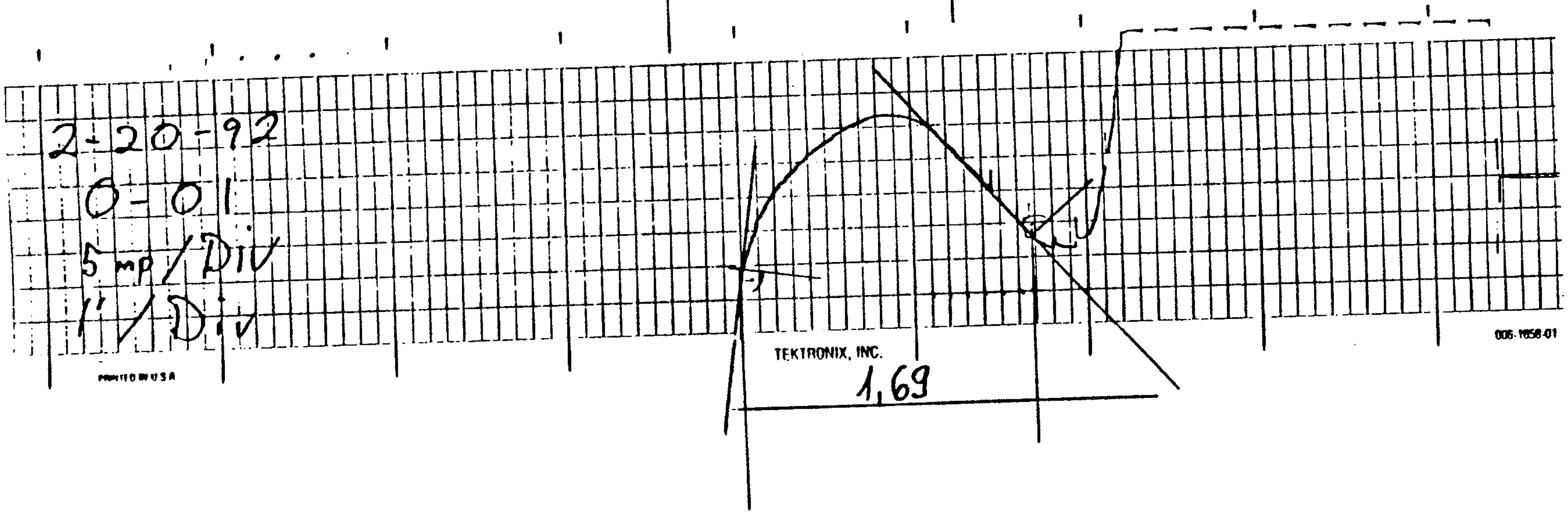
2-20-92
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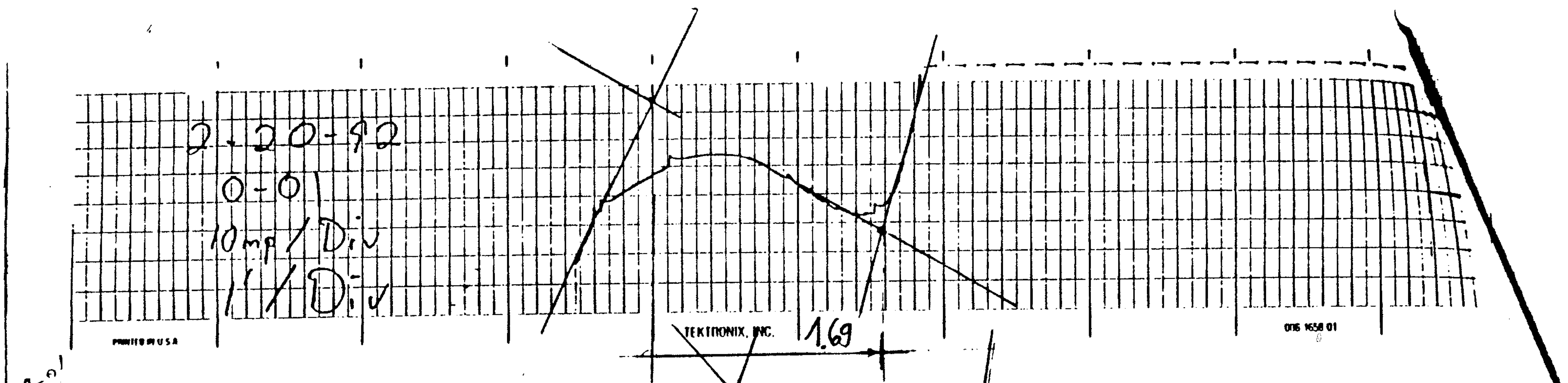
5 mp / DIV
1' / Div
TEKTRONIX, INC.
1,69
006-1658-01
PRINTED IN U.S.A.

Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

ccc) "Method of Peaks", Borehole #3, February 1992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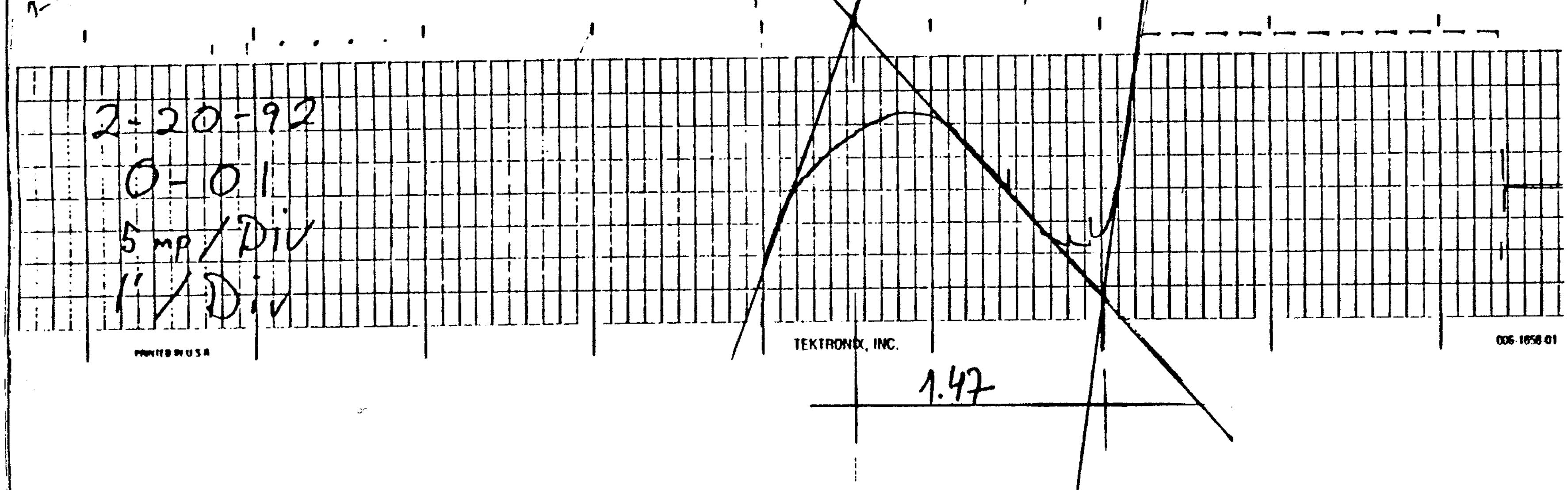
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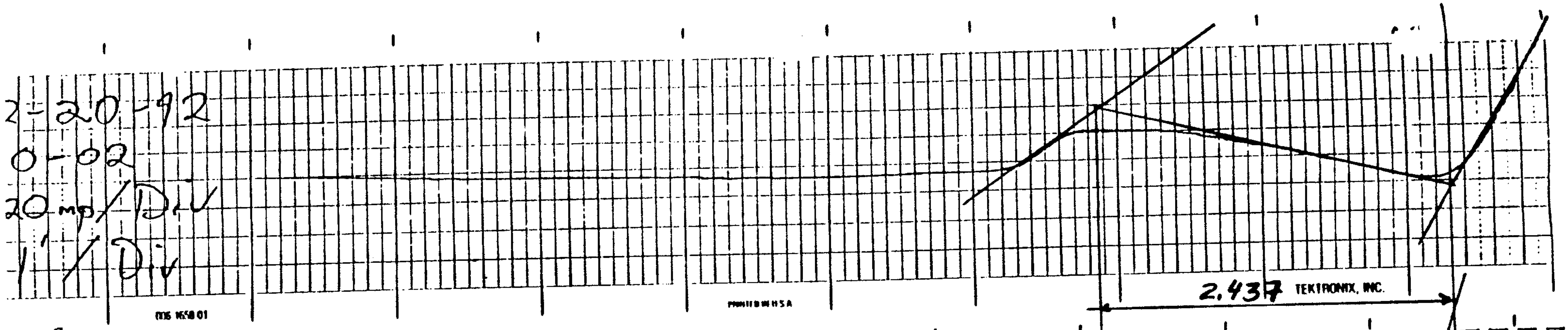
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006 1658 01
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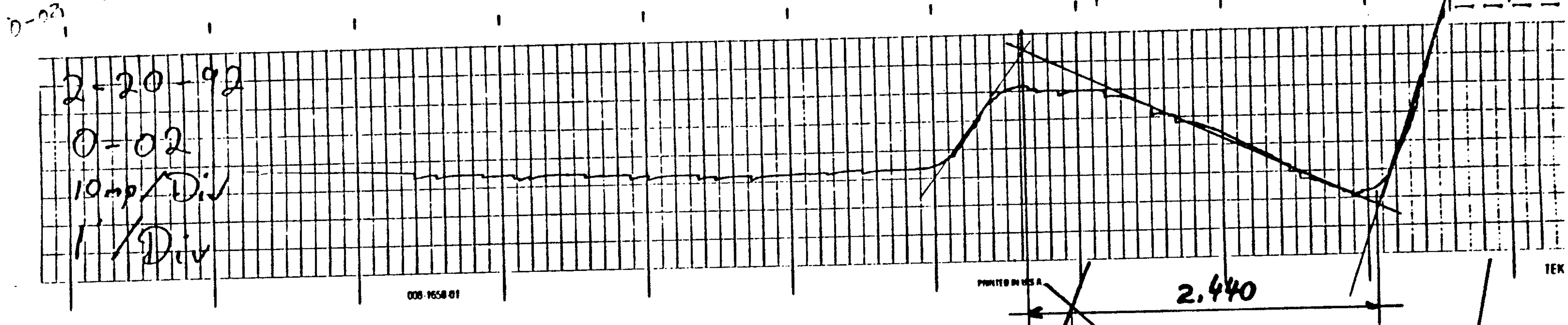
2,437
TEKTRONIX, INC.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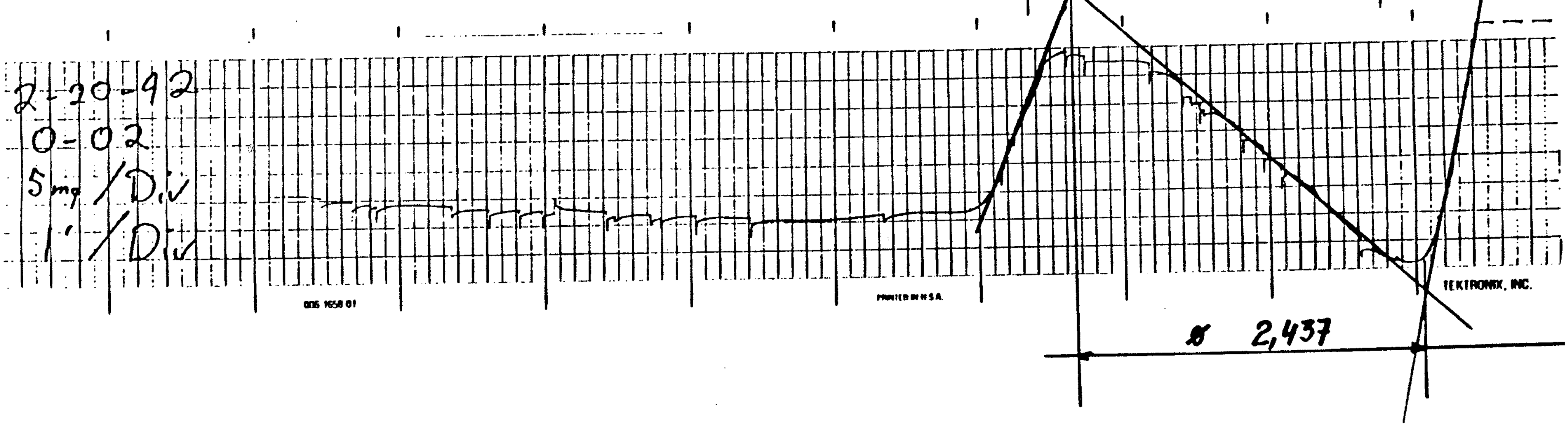
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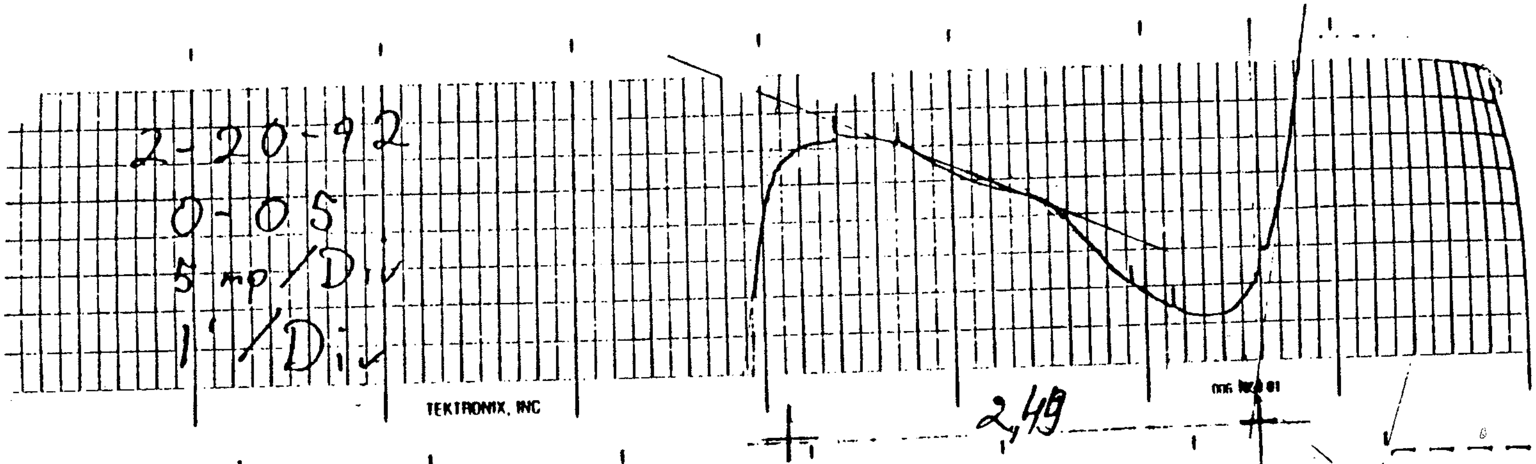
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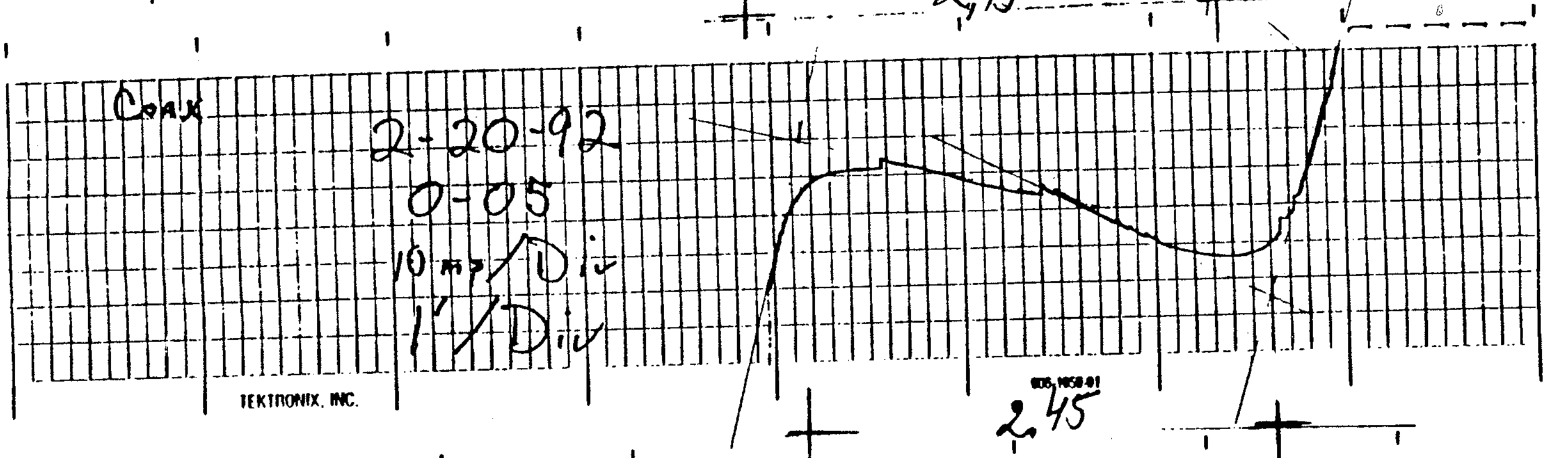
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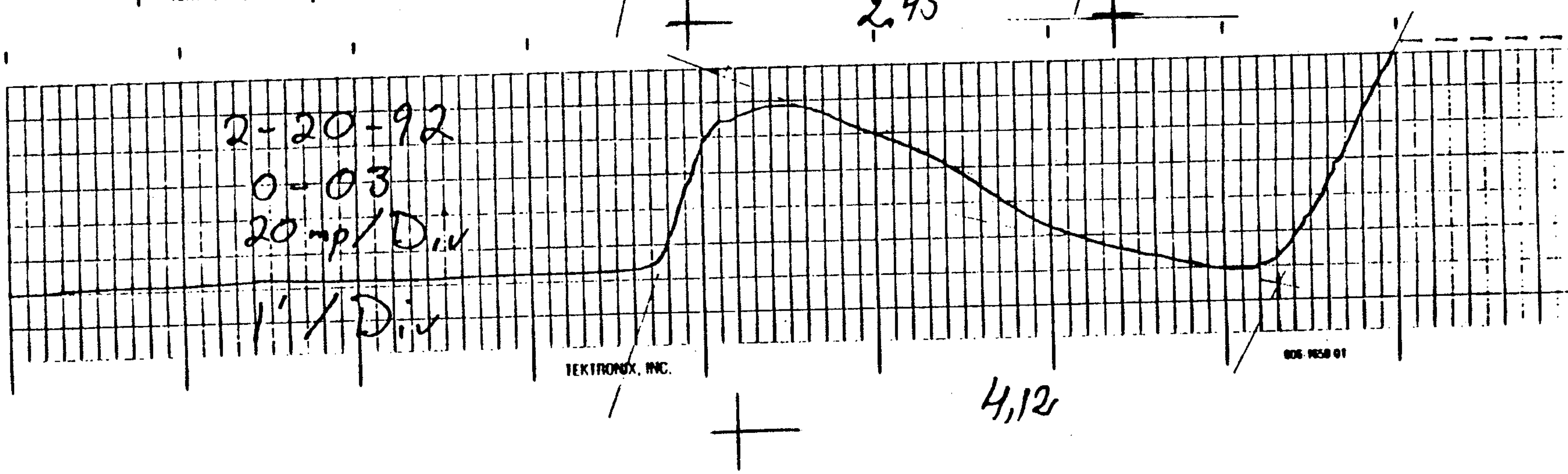
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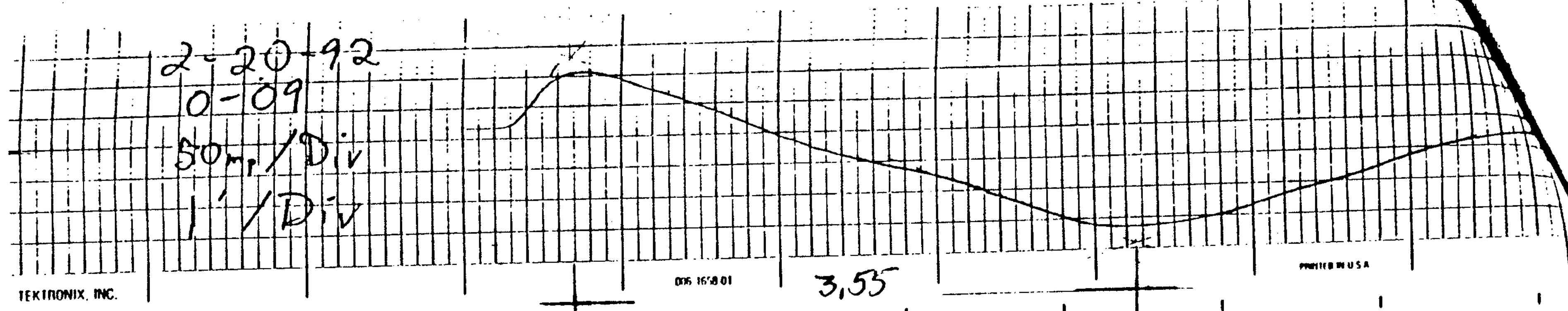
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1'/Div
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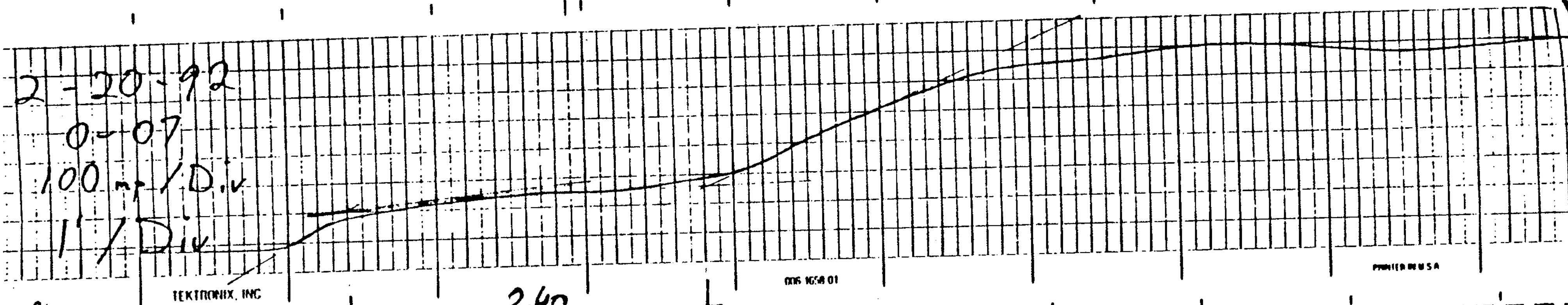
3.55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100 mv/Div
1'/Div
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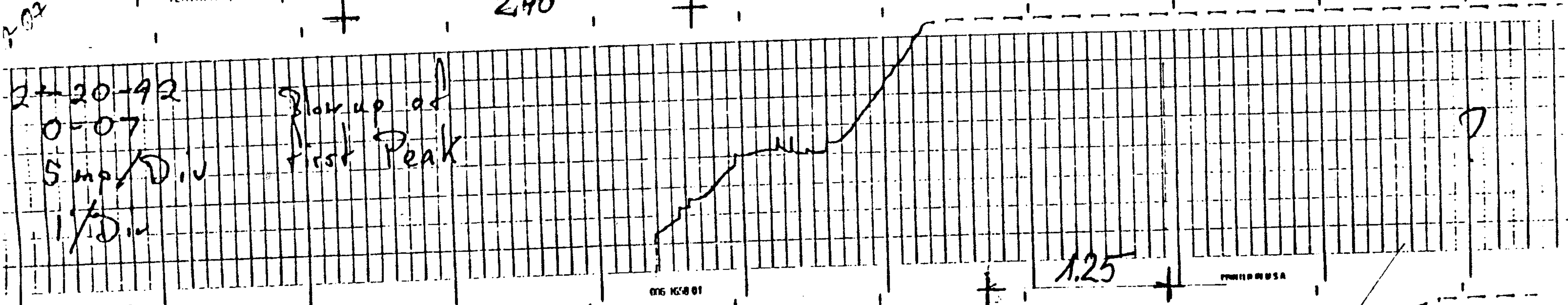
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Blow up of first Peak
?
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1.25
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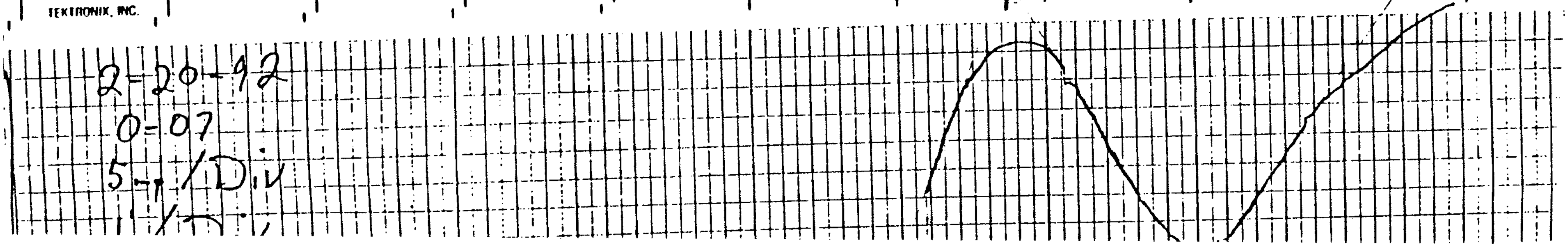
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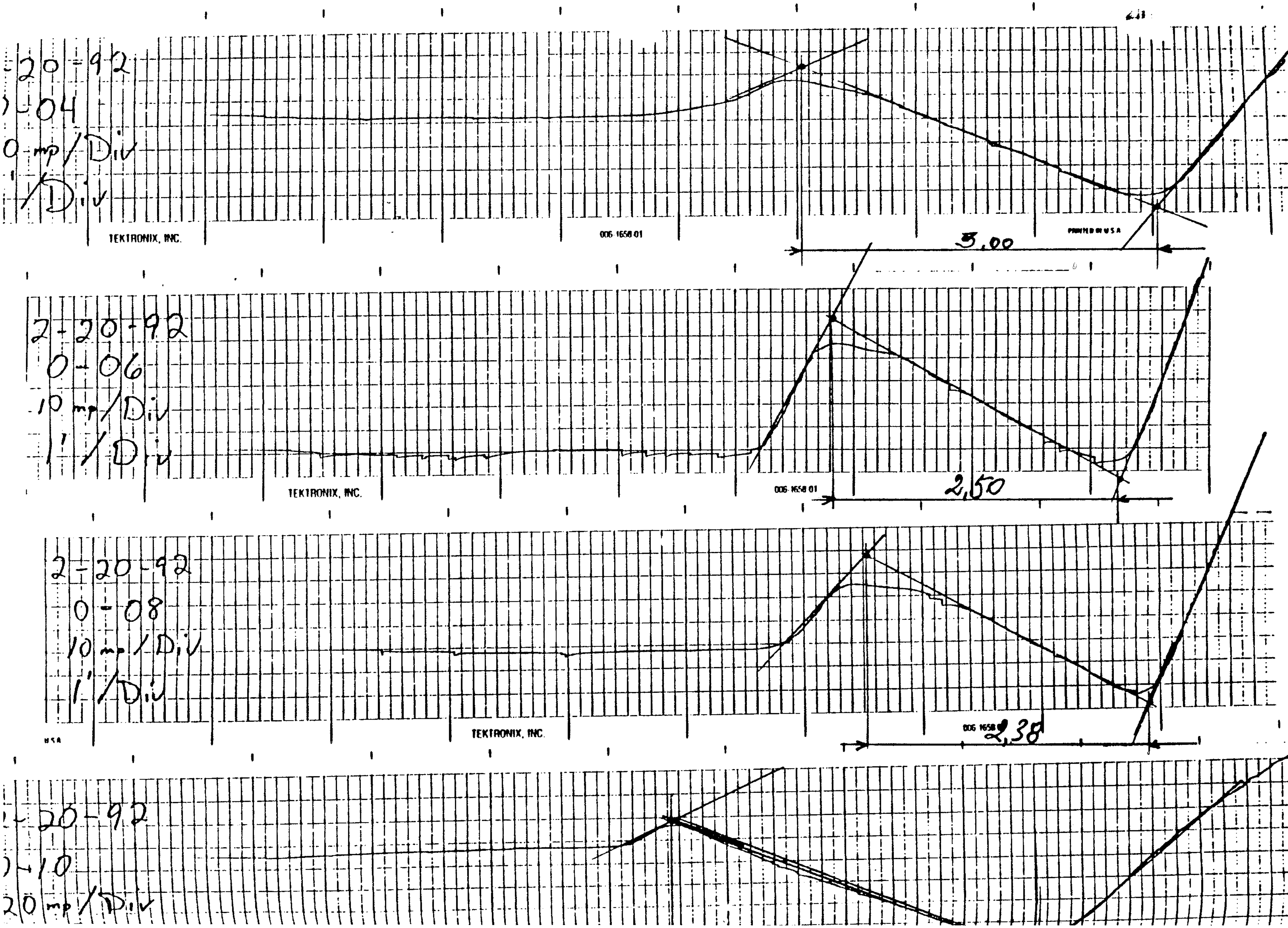

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2.50
2-20-92
0-08
10 mp/Div
1'/Div
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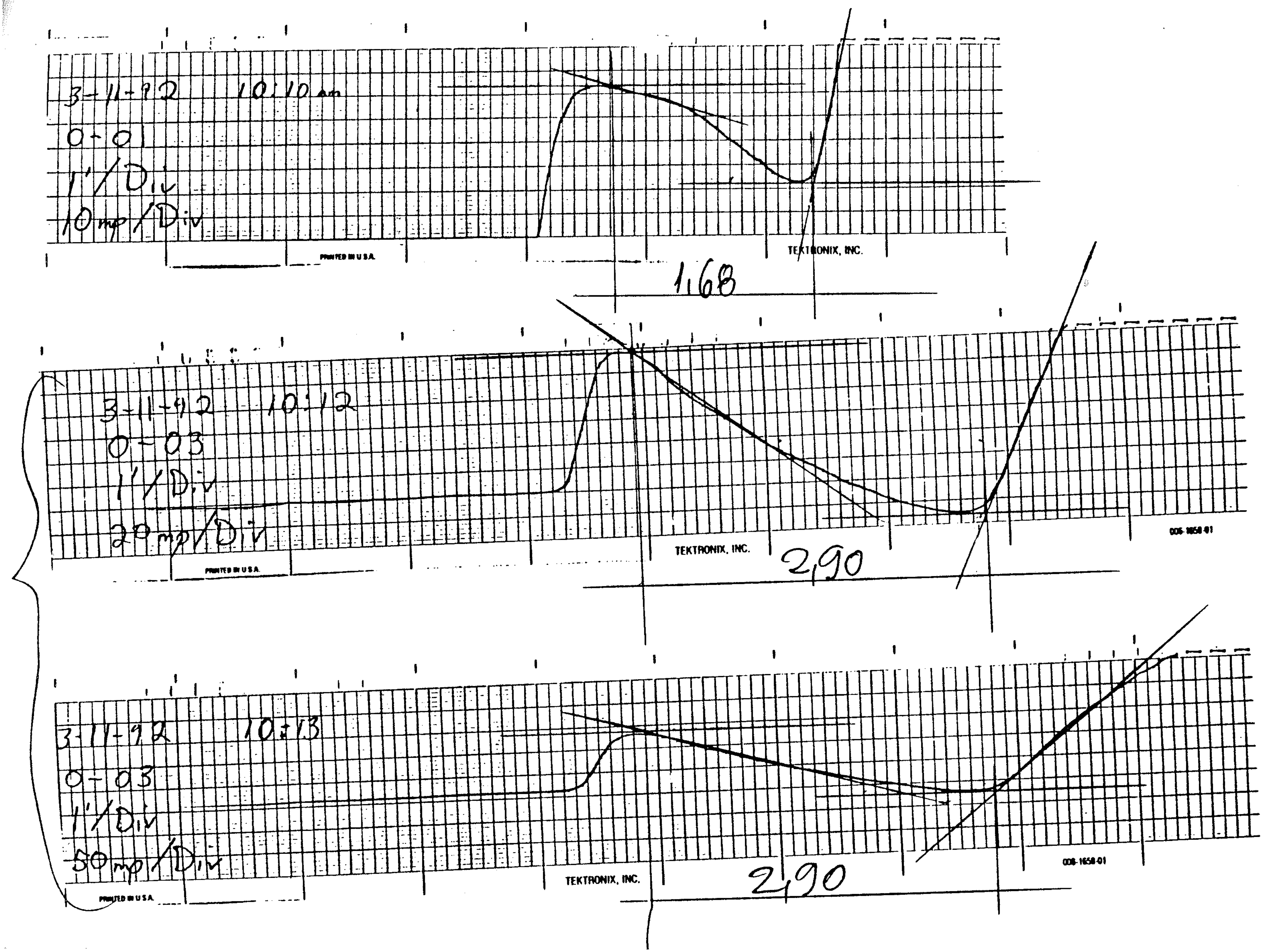
2.38
20-92
10
20 mp/Div

Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

aaa) "Method of Tangents", Borehole #3, March 1992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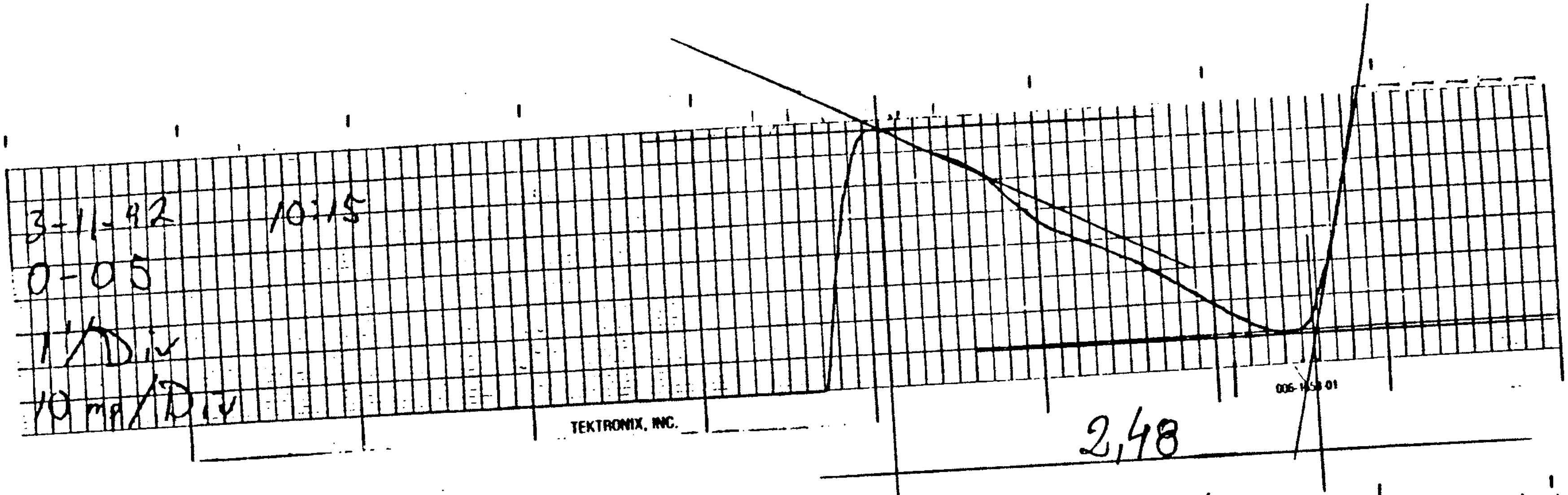
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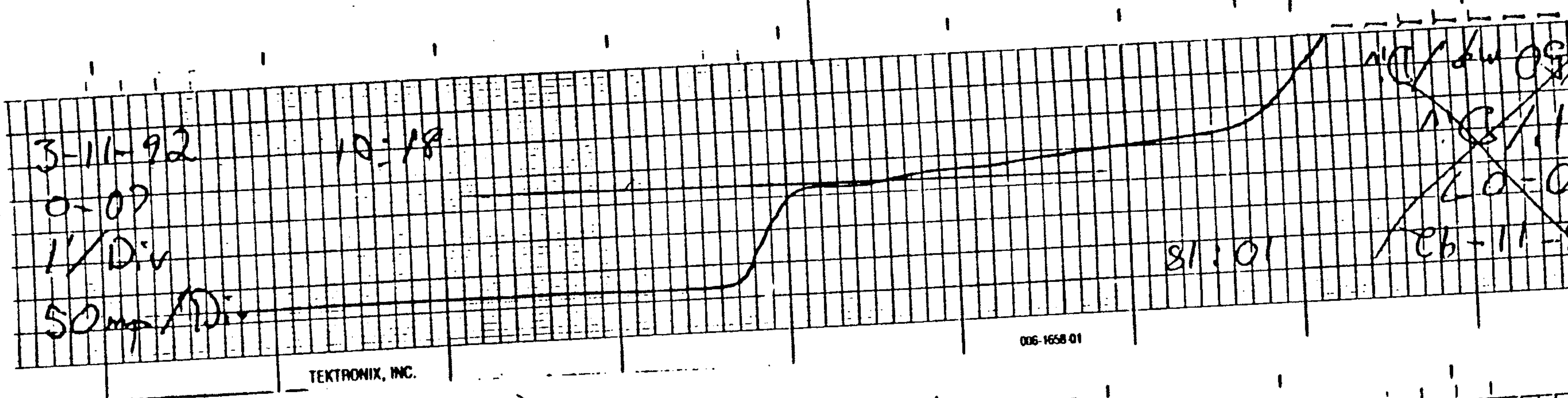
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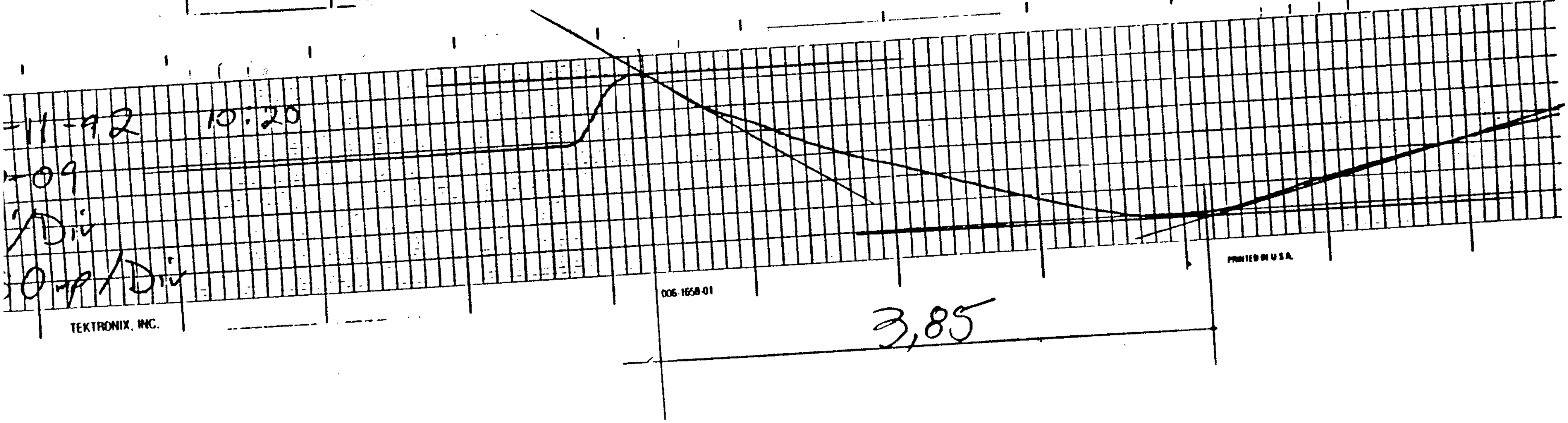
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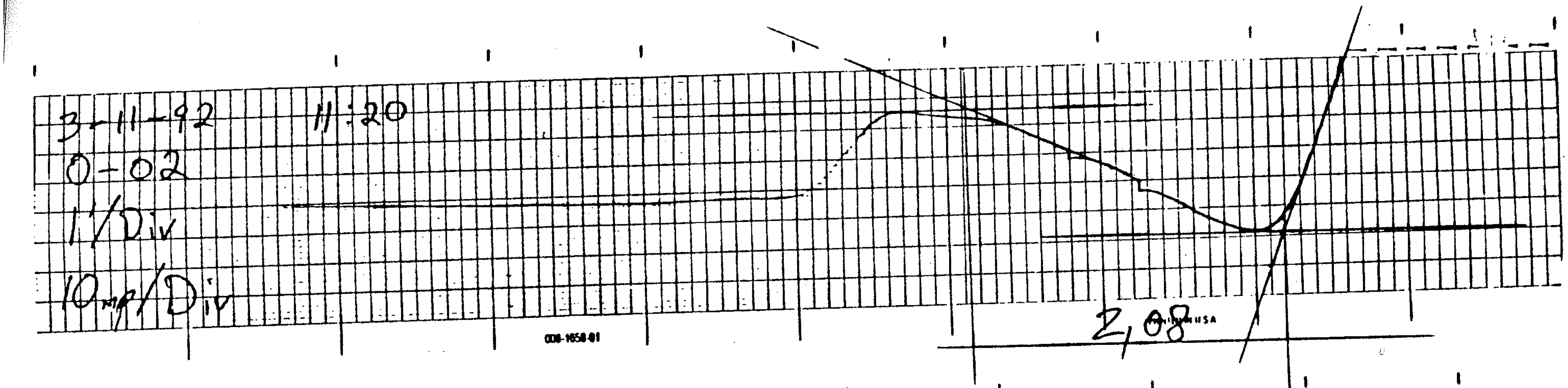
3,85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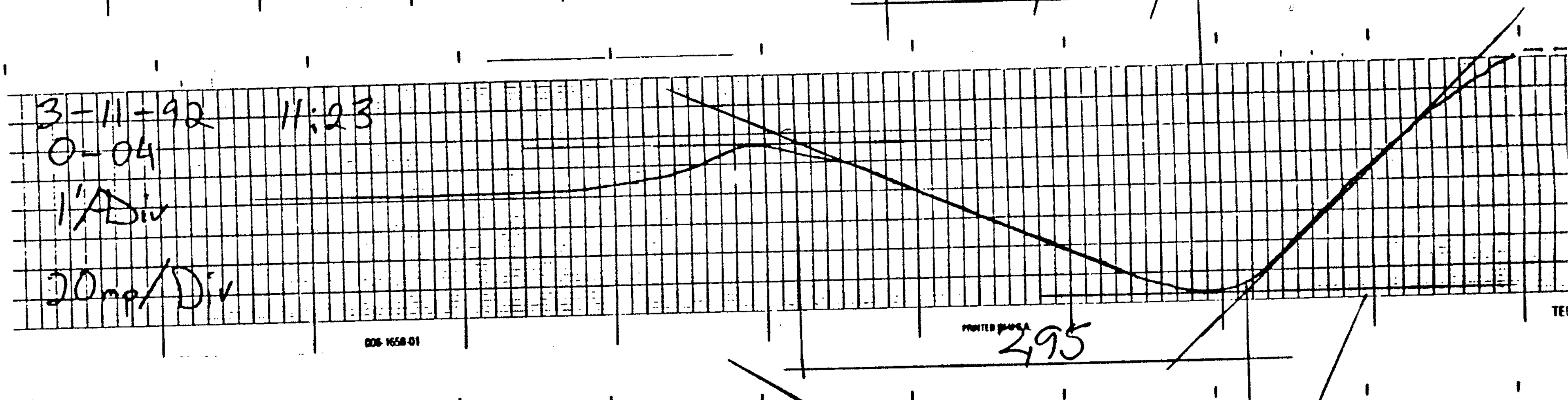
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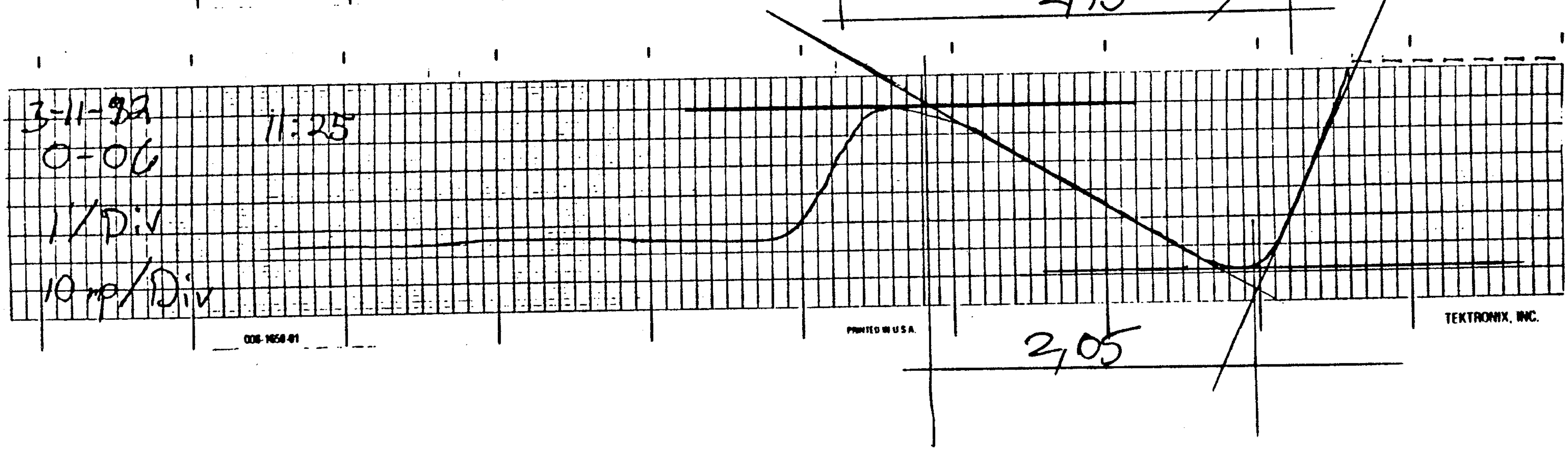
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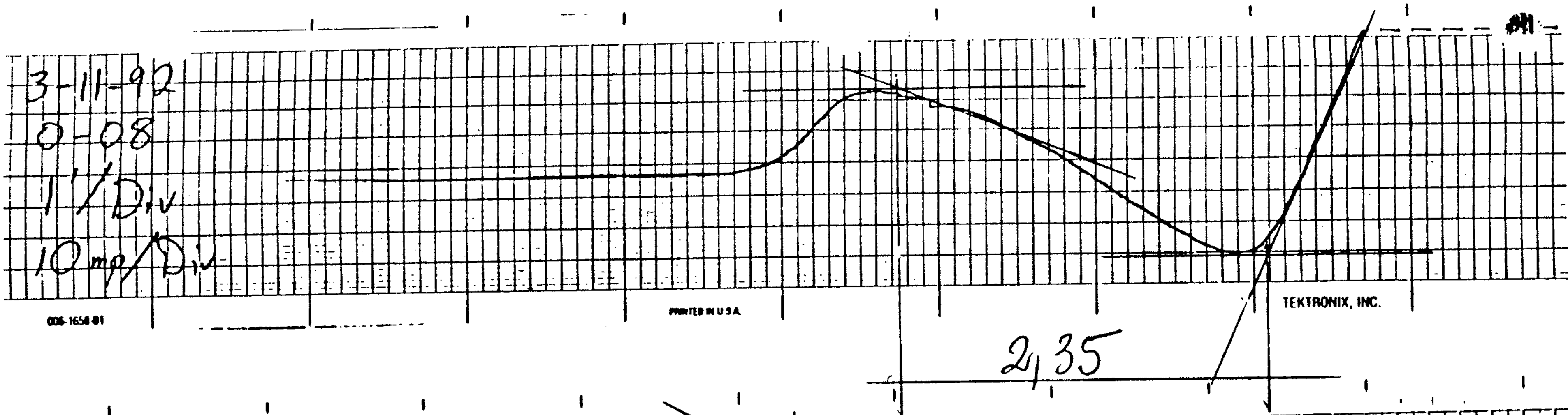

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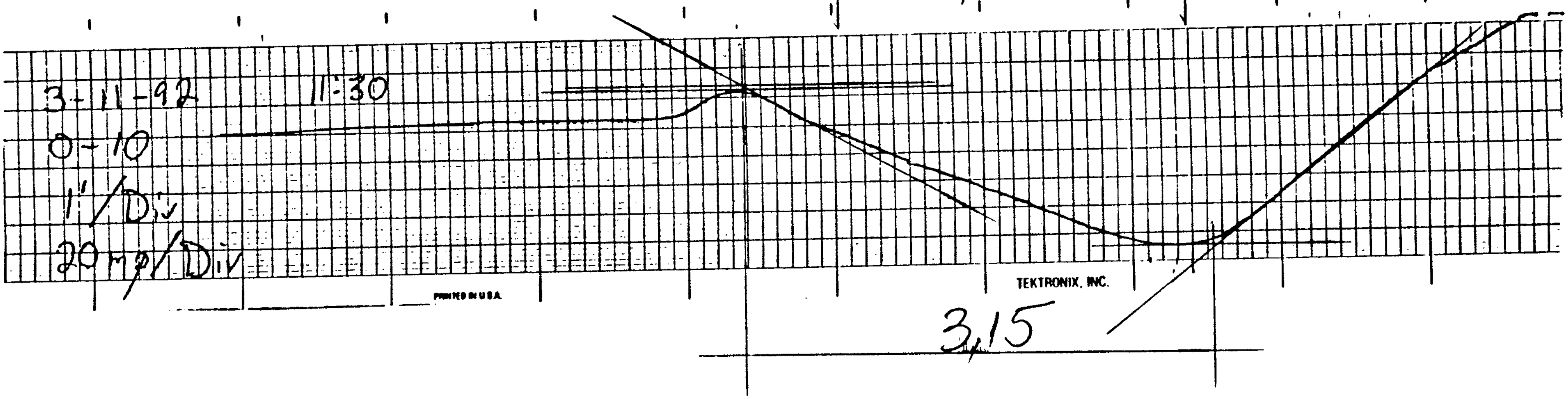
PRINTED IN U.S.A.
006-1658-01
#11



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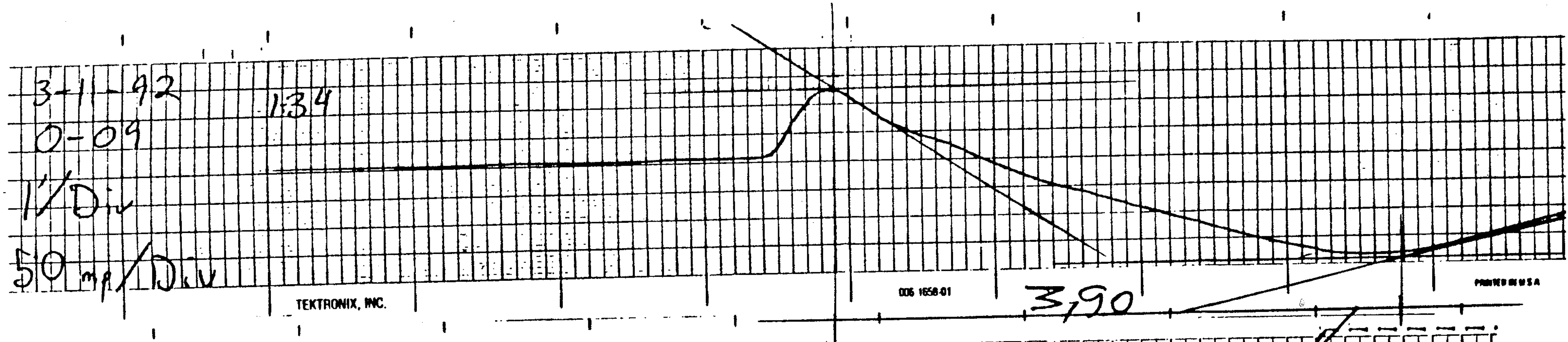
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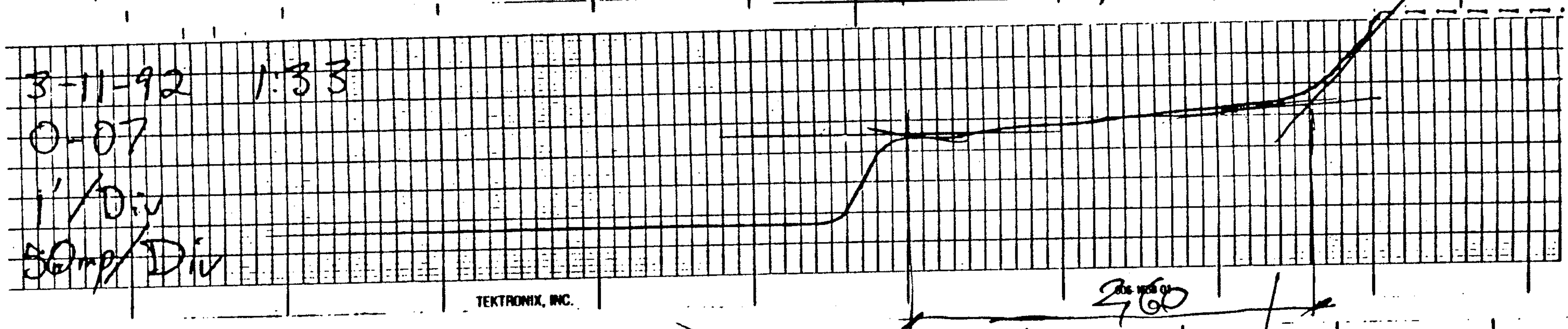
006 1658-01
PRINTED IN U.S.A.
3,90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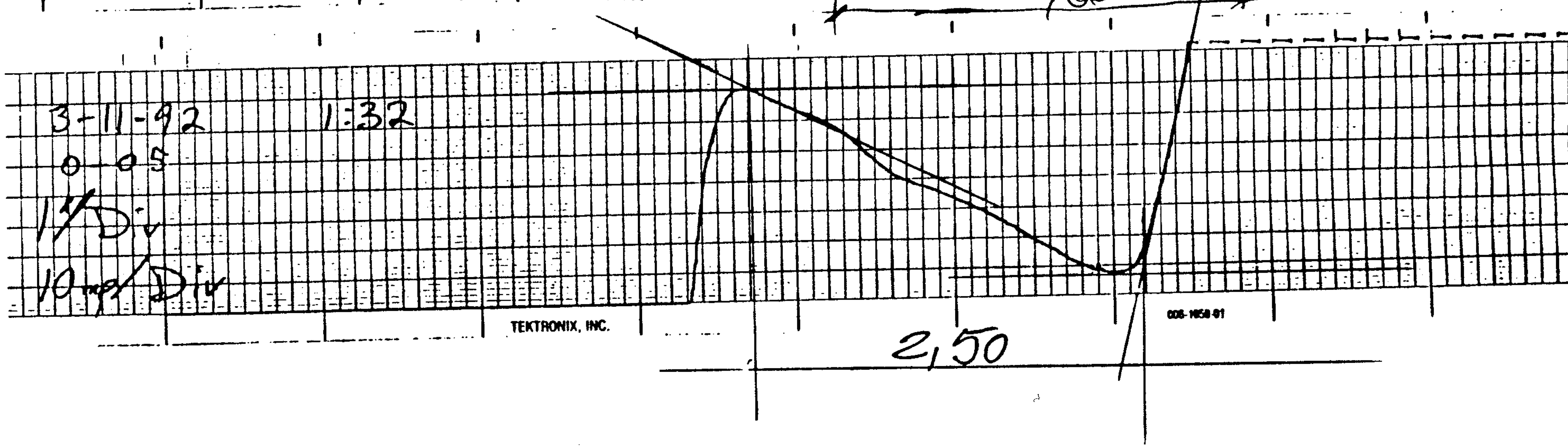
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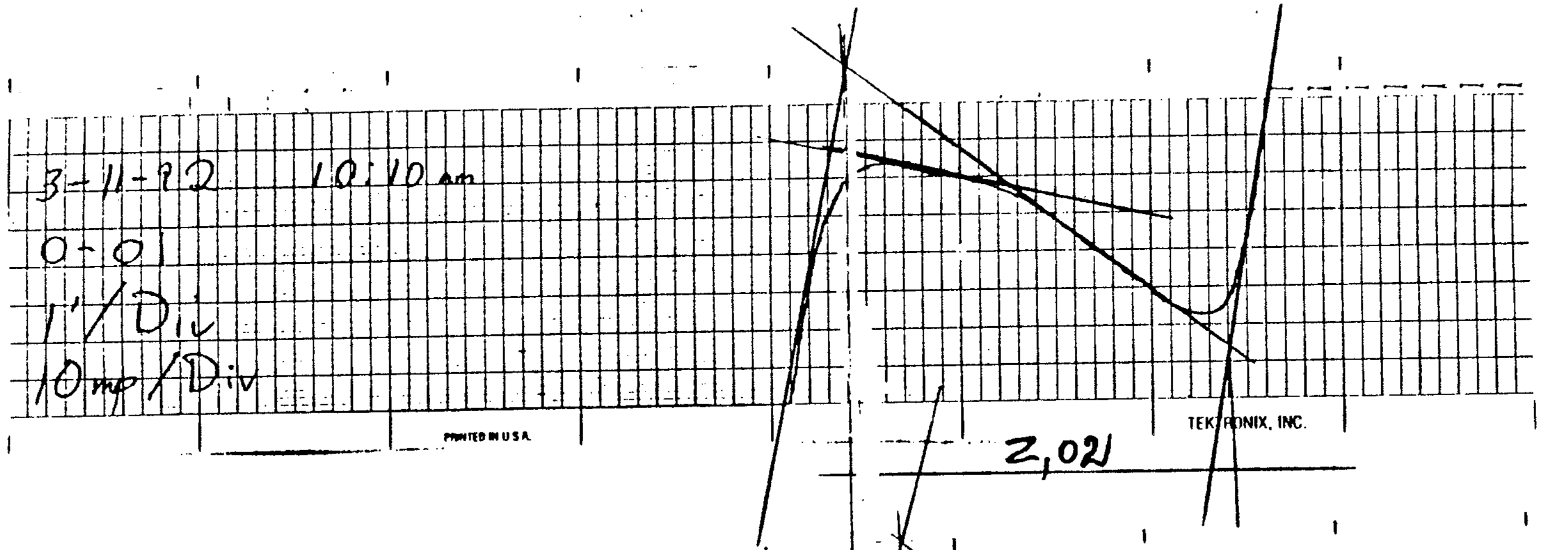
006-1658-01
2,50

Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

bbb) "Method of Diverging Lines", Borehole #3, March 1992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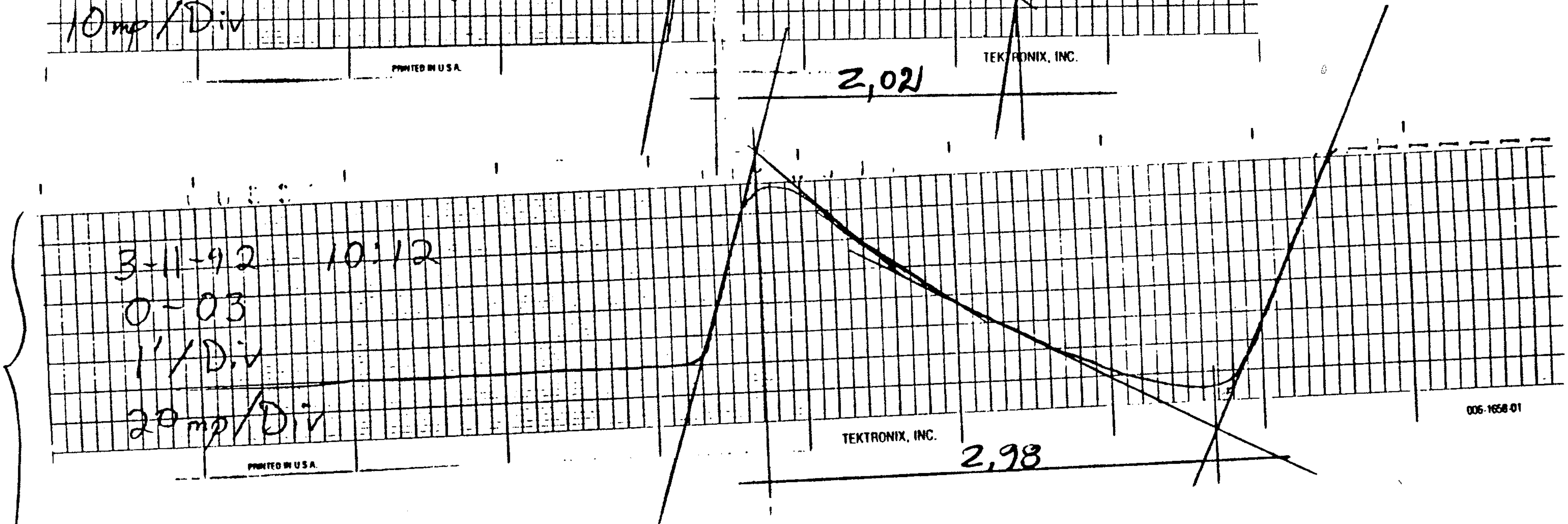
TEKTRONIX, INC.
2,021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006-1658-01
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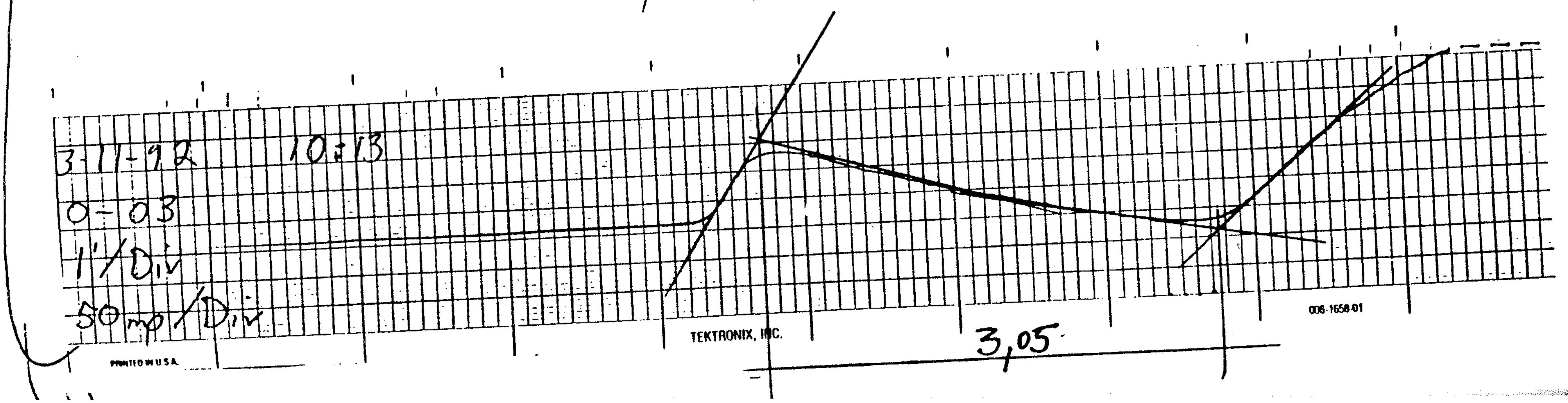
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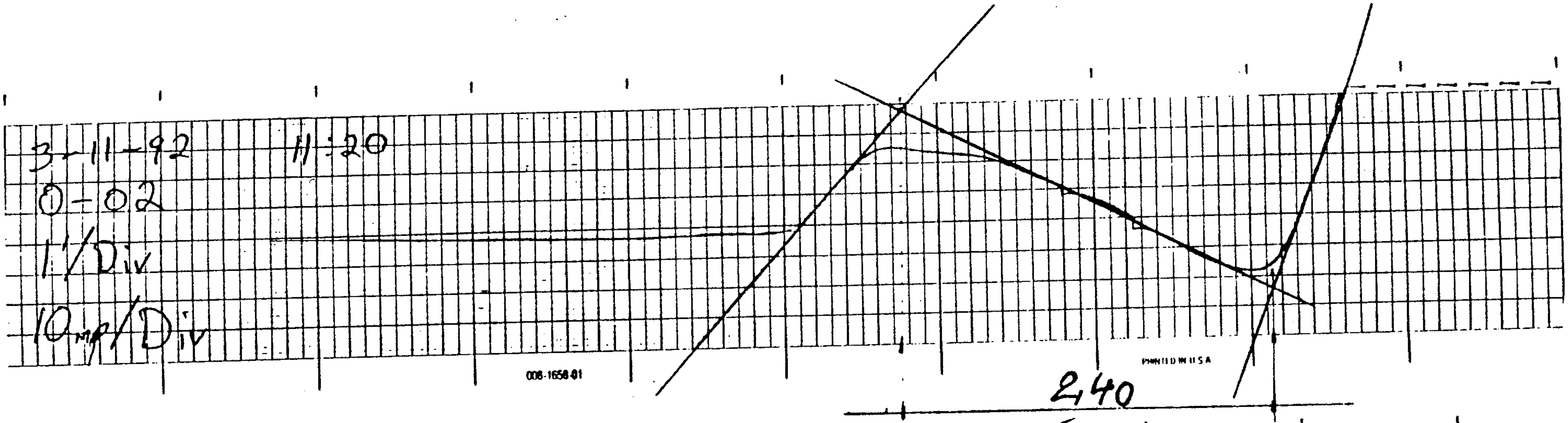
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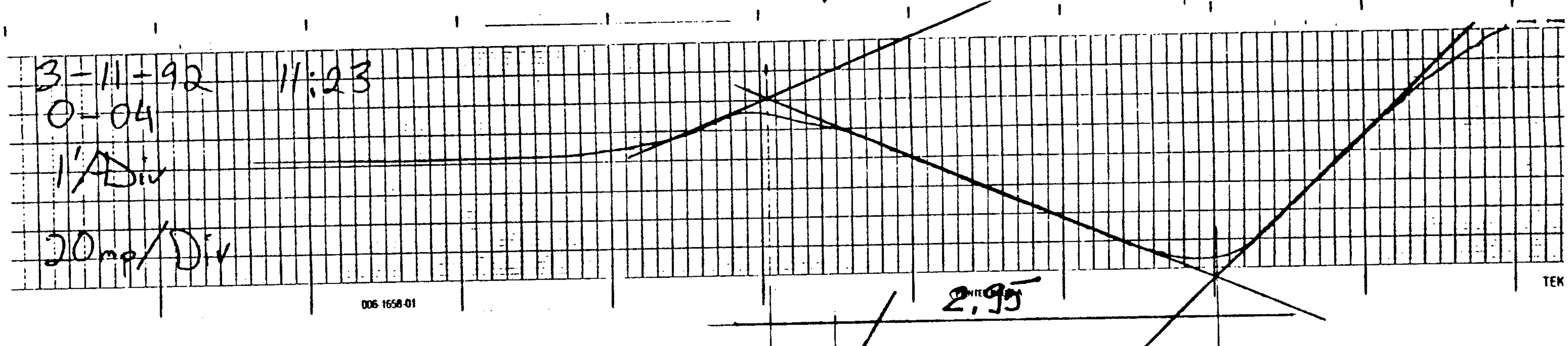
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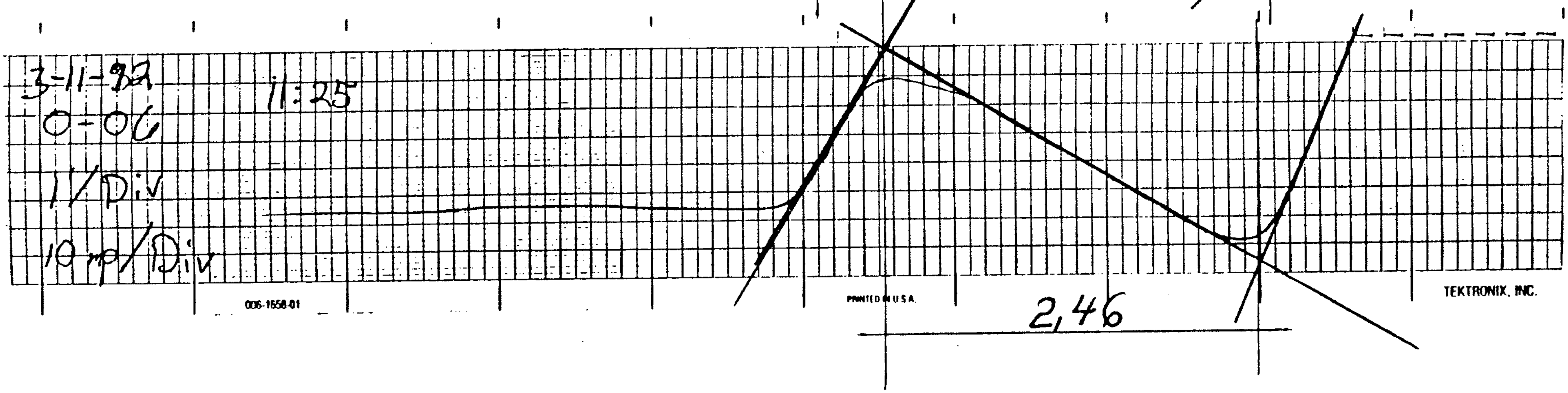
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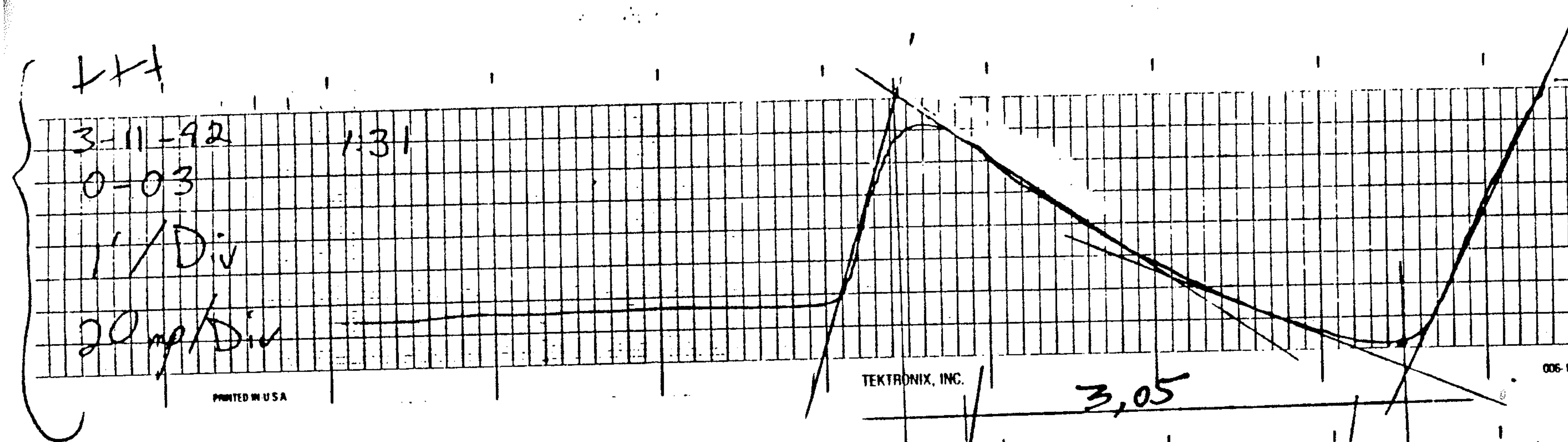
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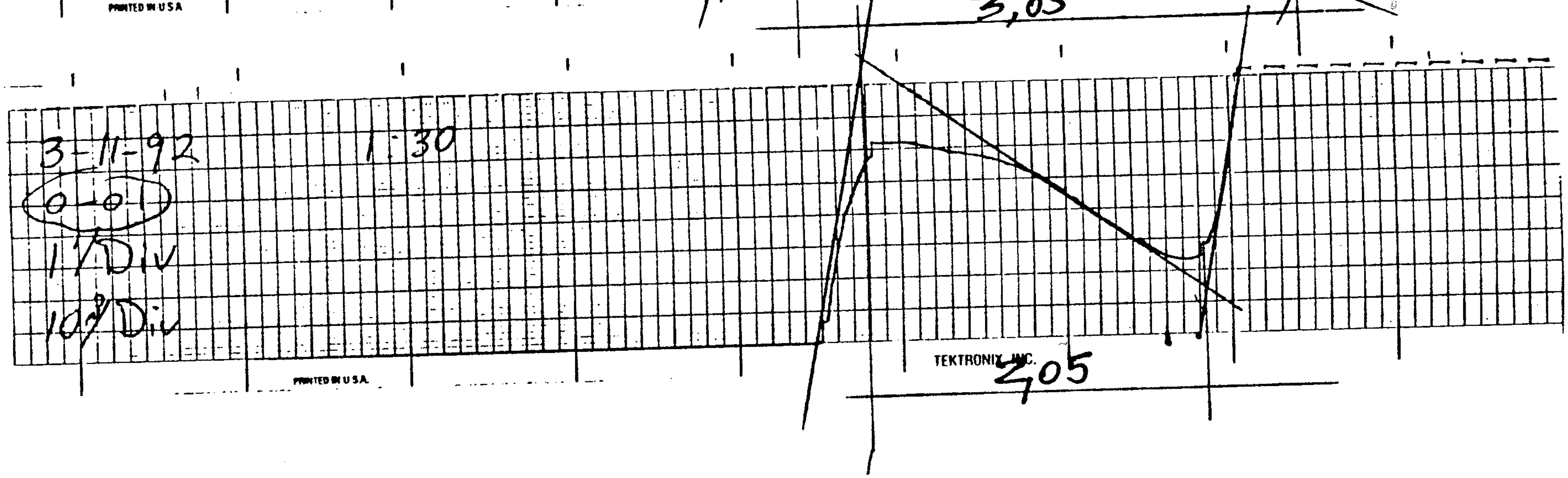
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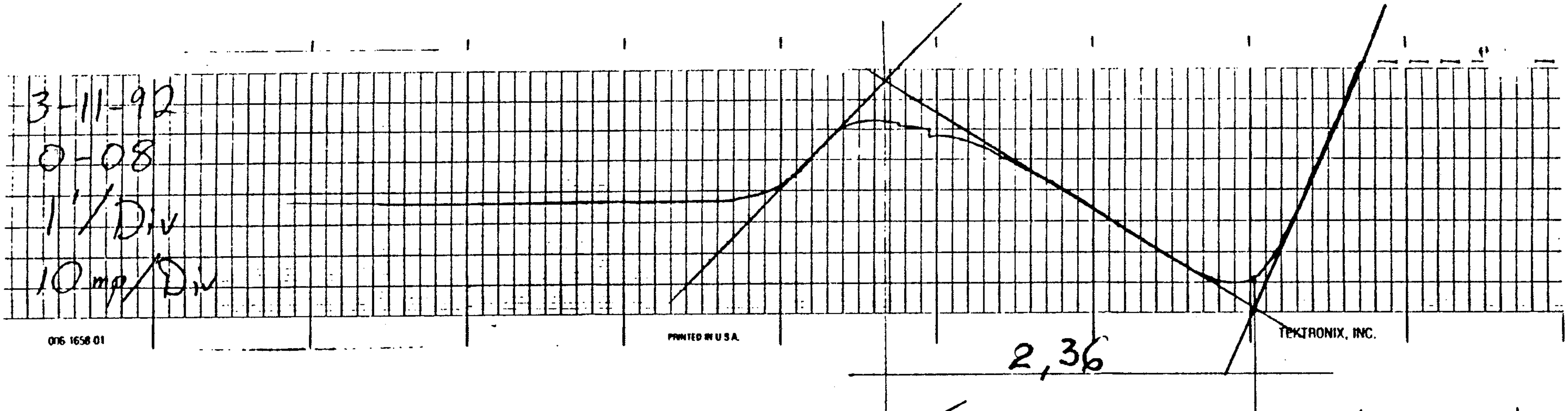
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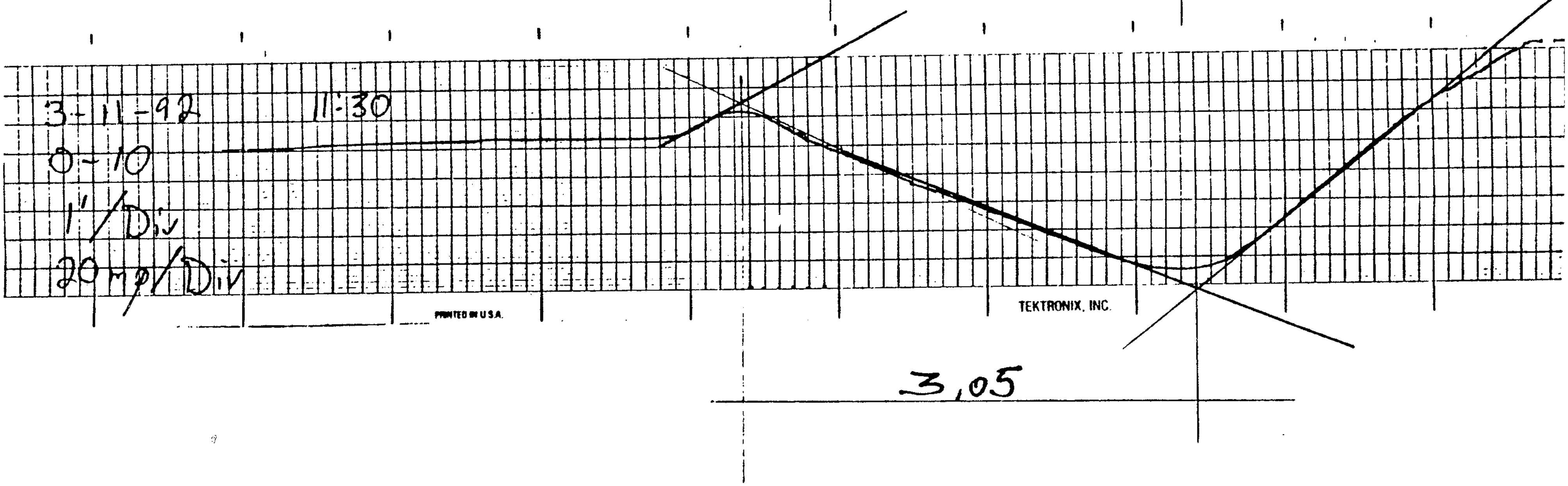
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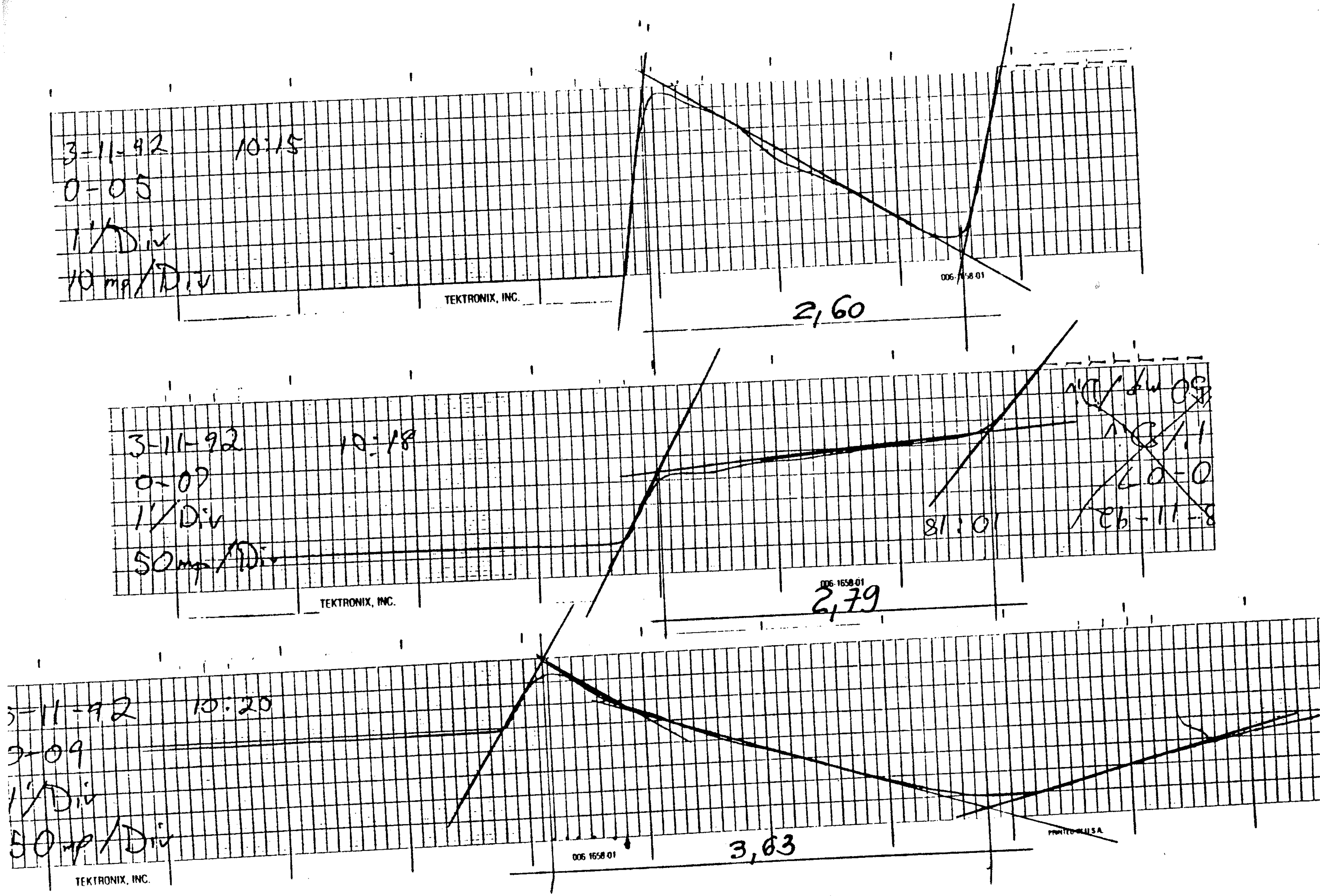

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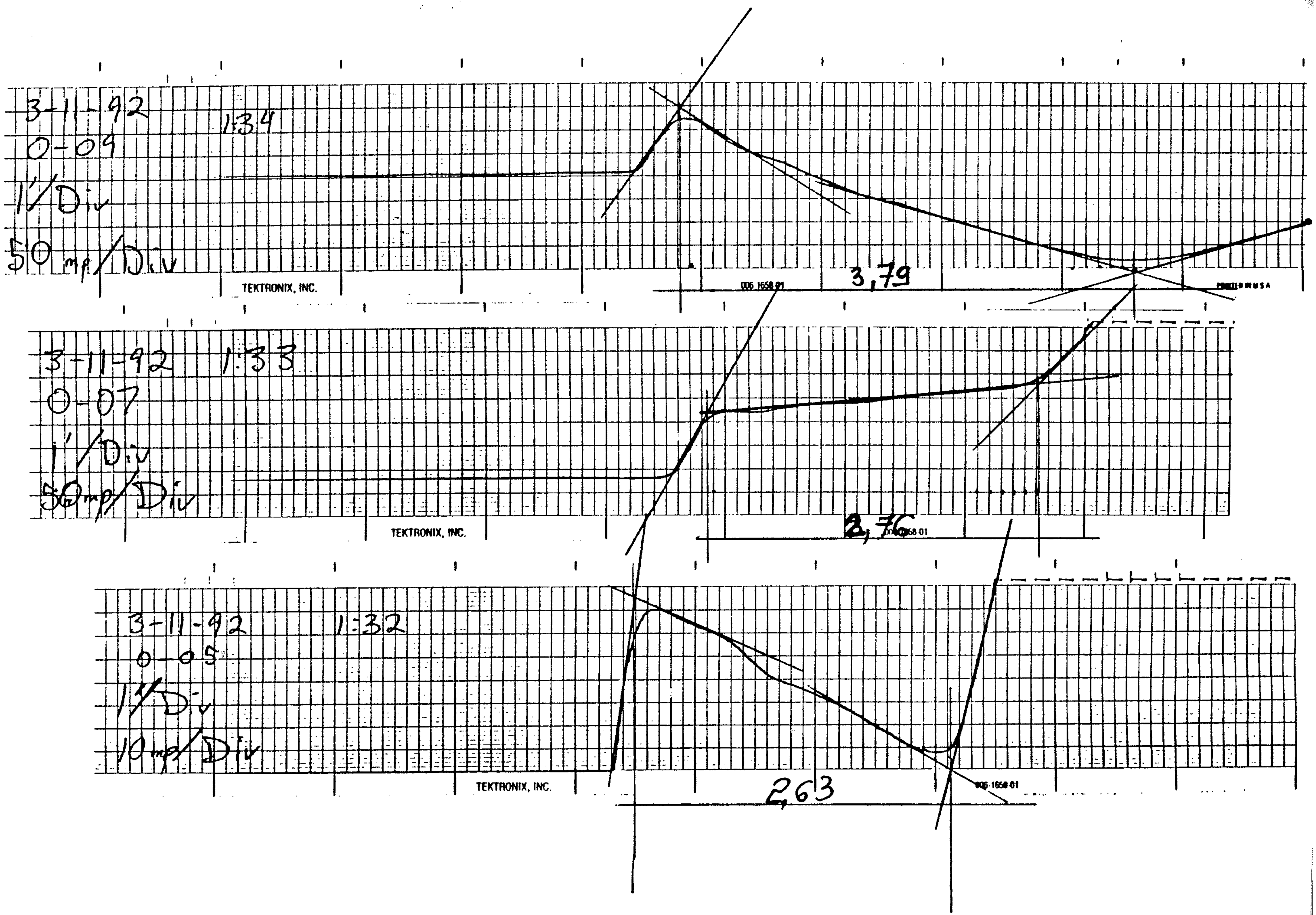
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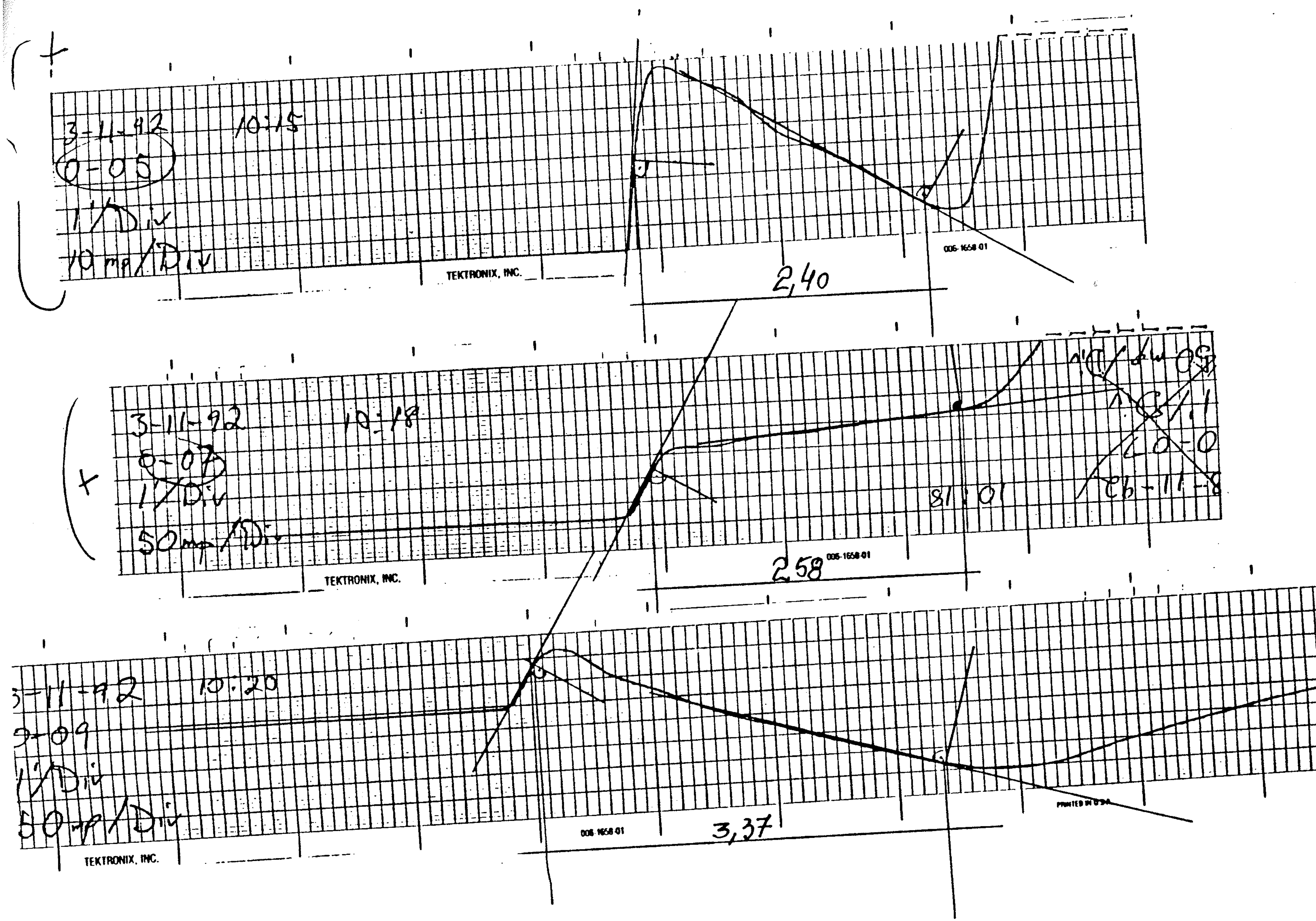
Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

ccc) "Method of Peaks", Borehole #3, March 1992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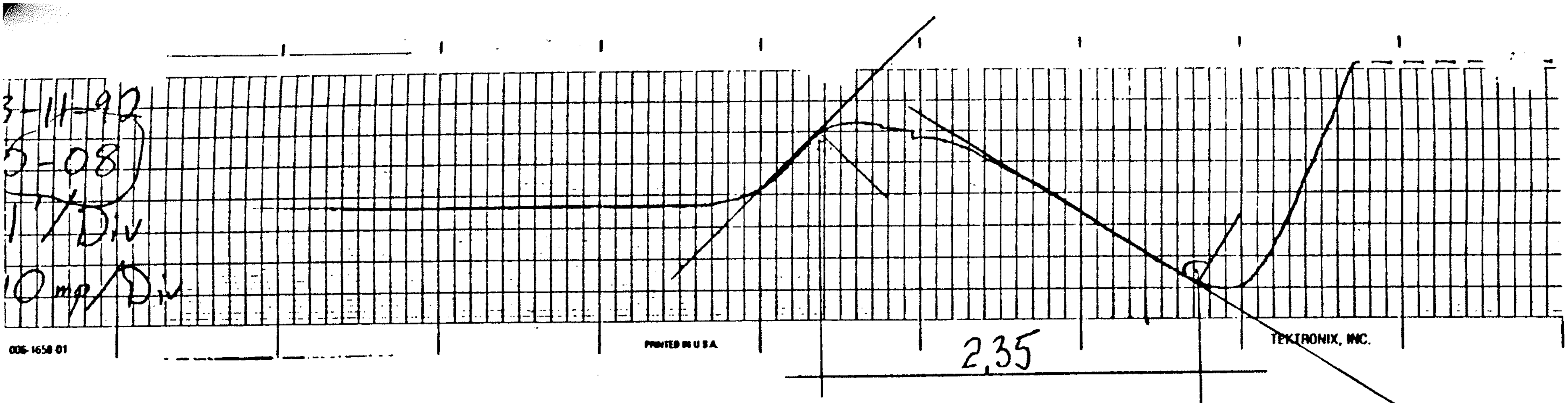
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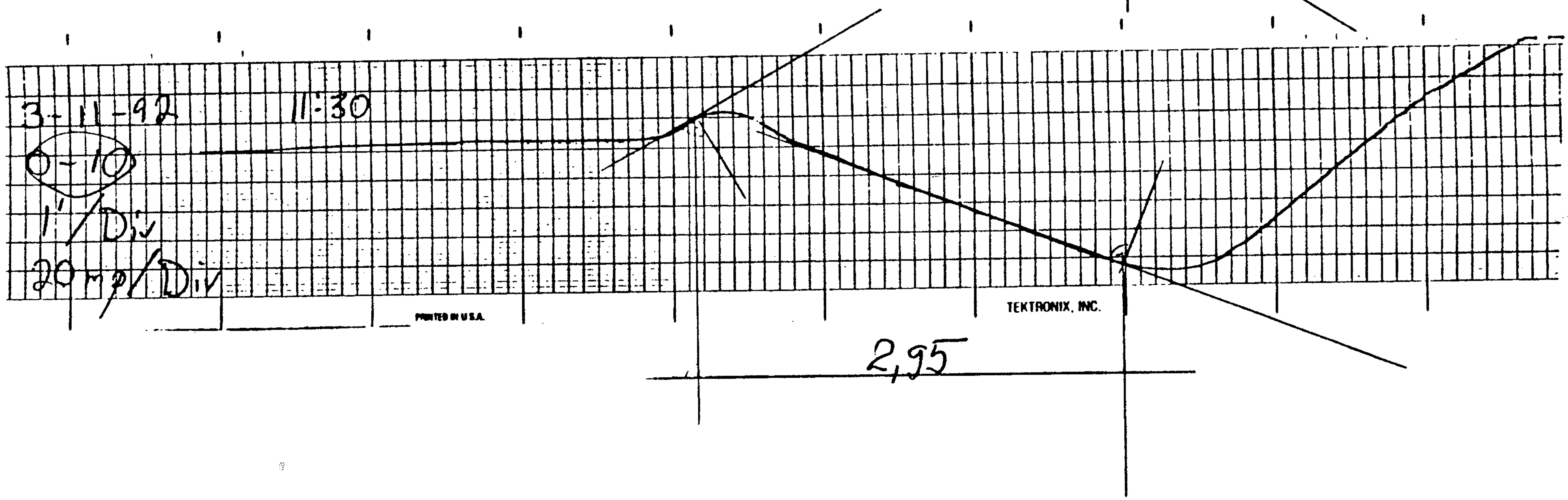
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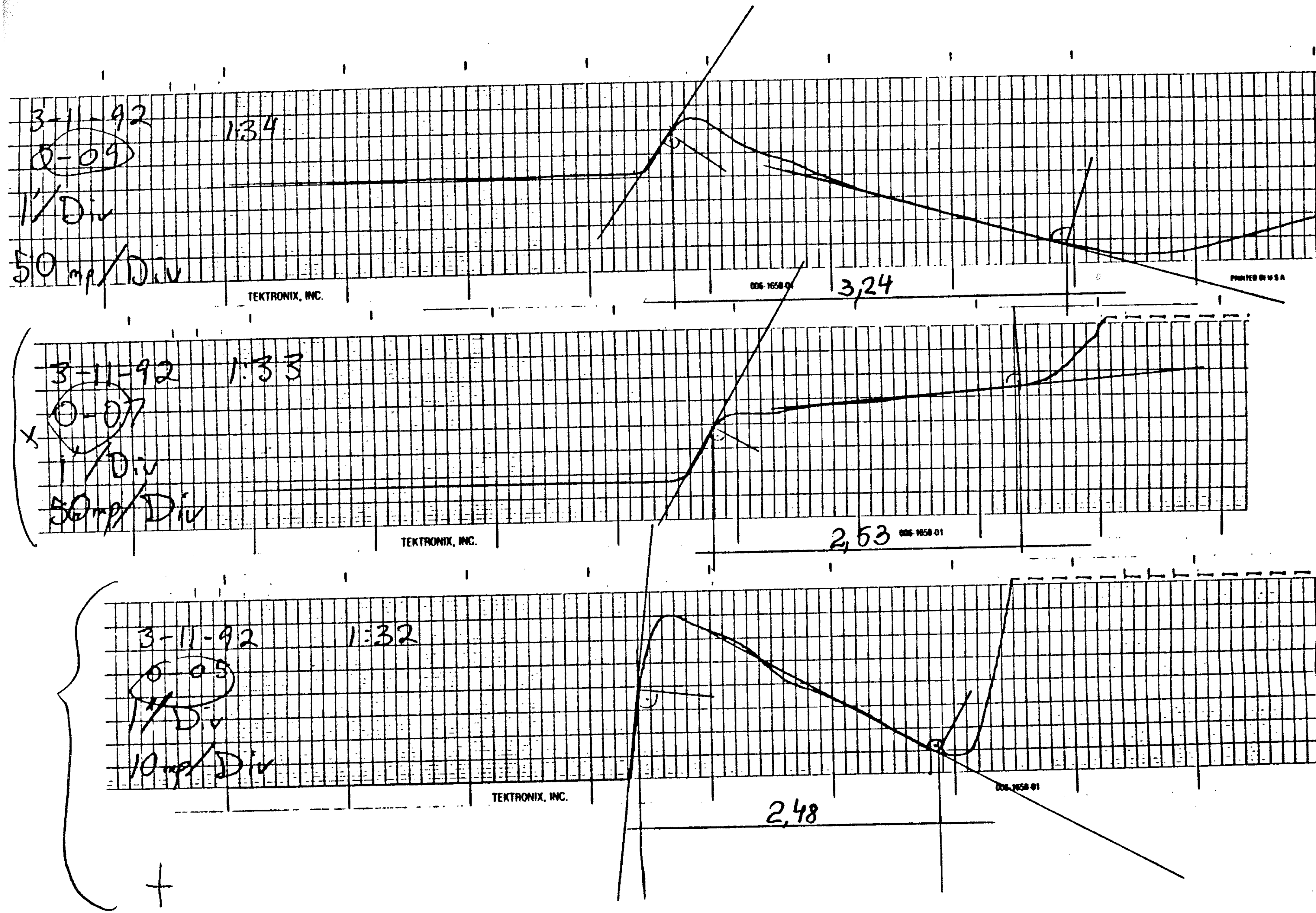
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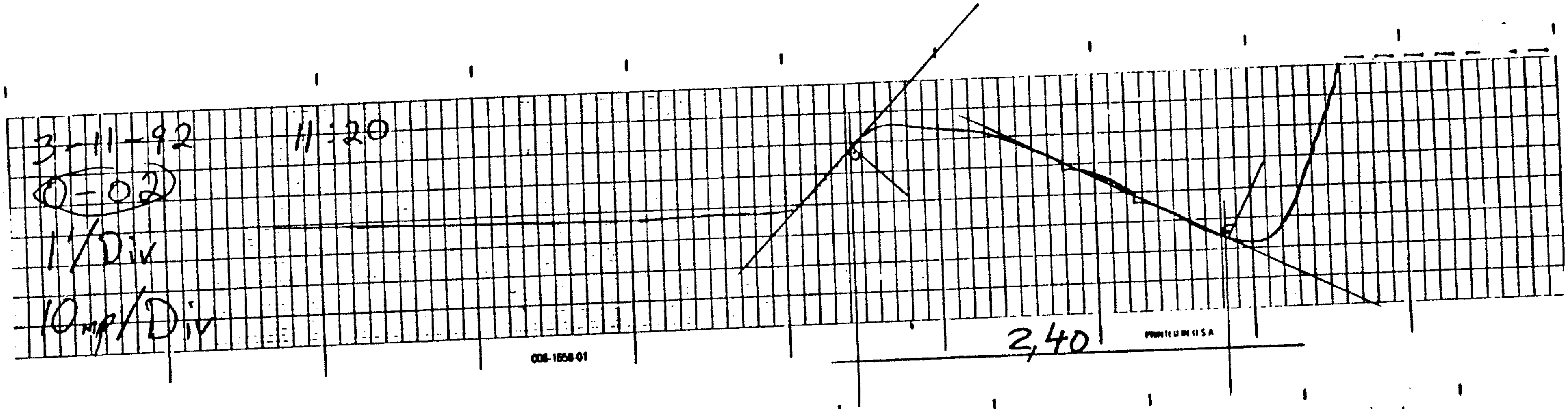
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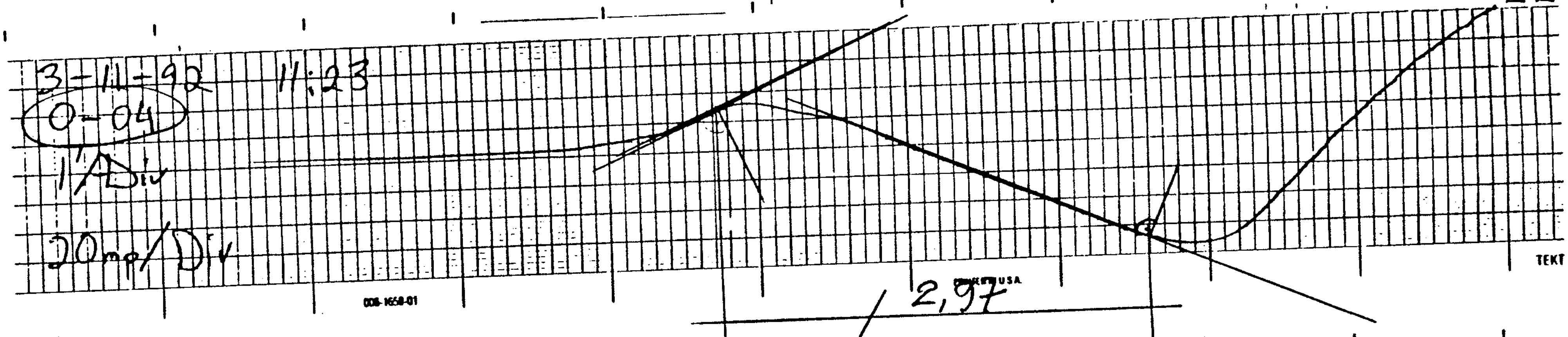
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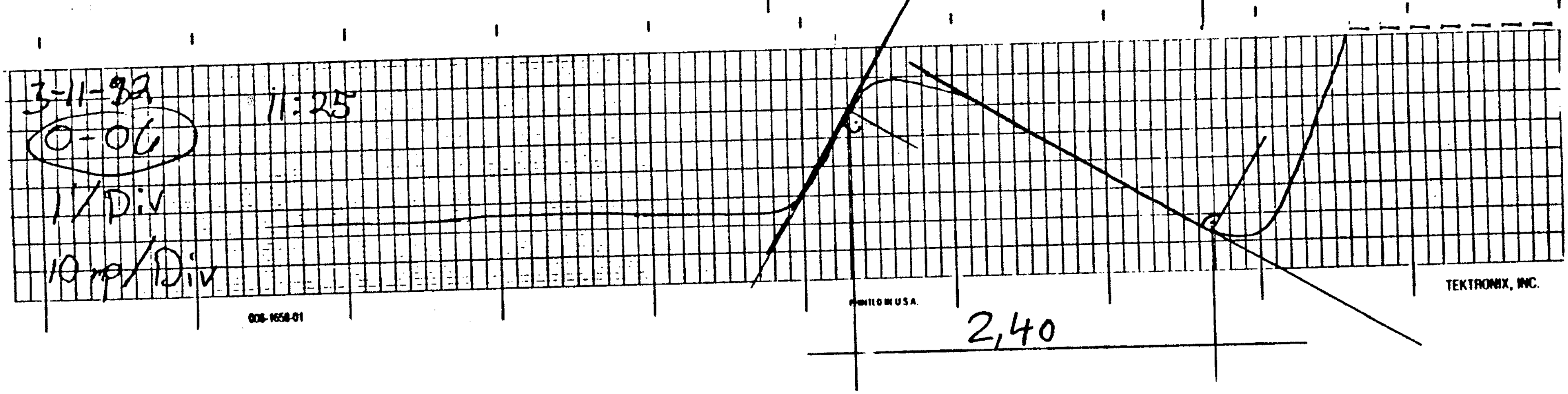
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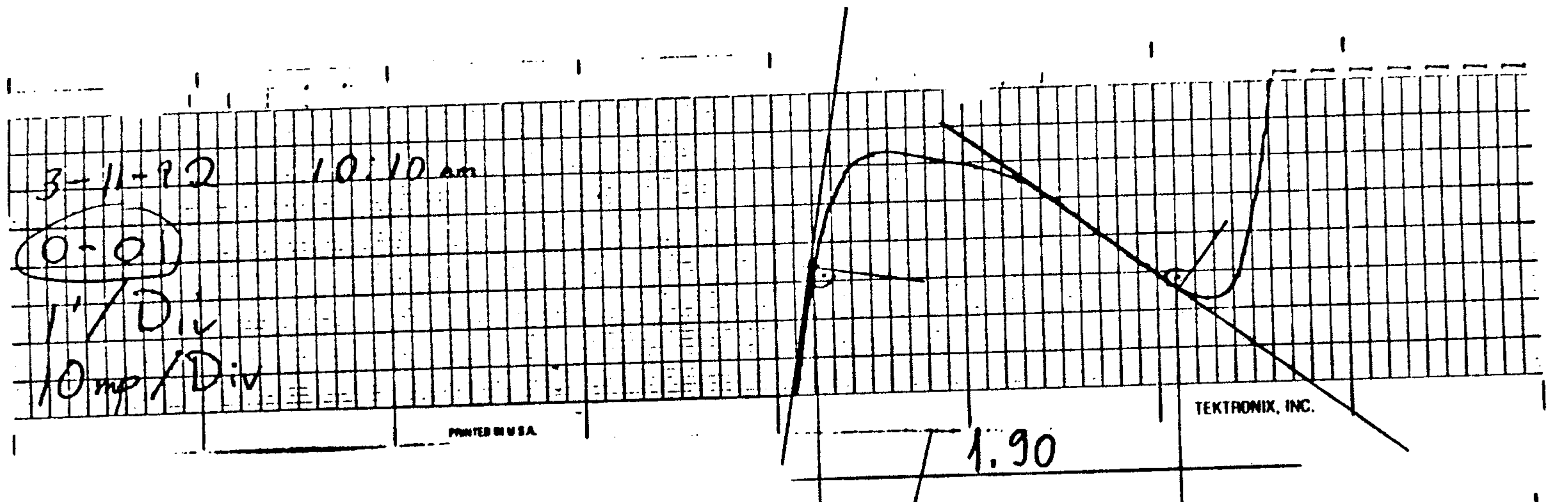
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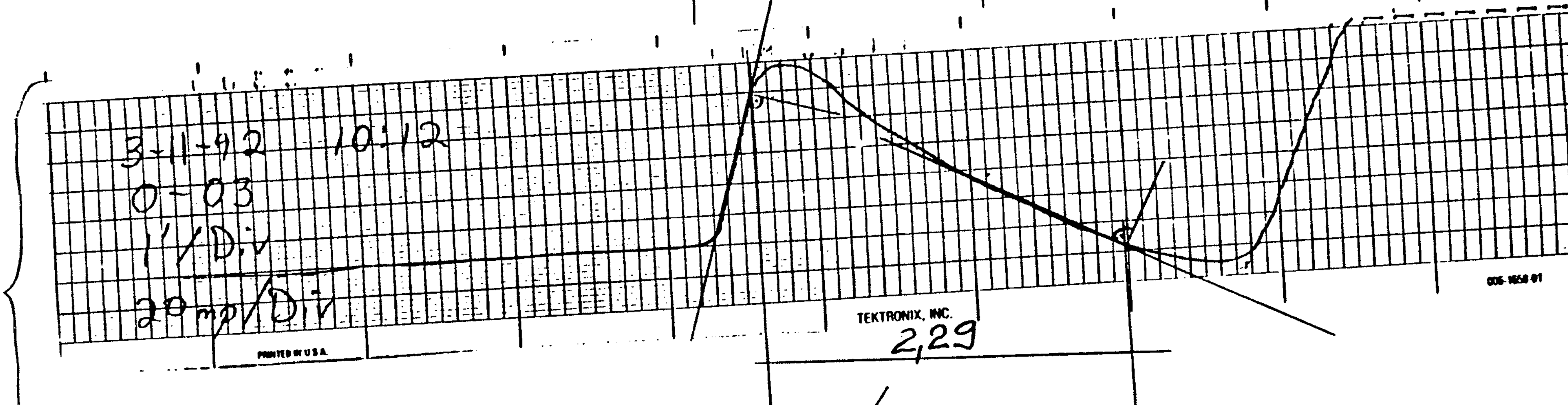
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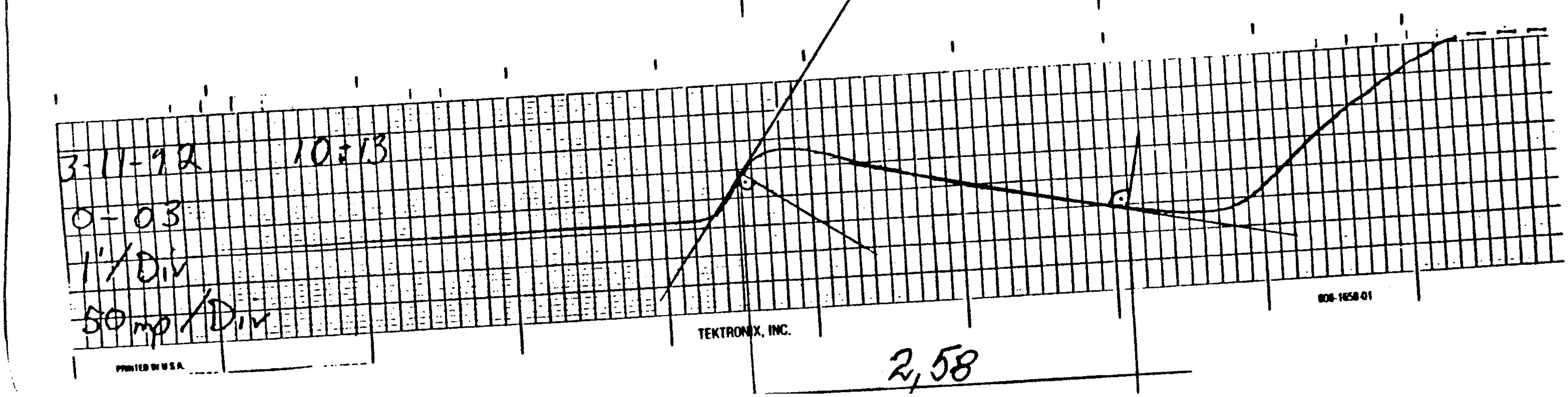
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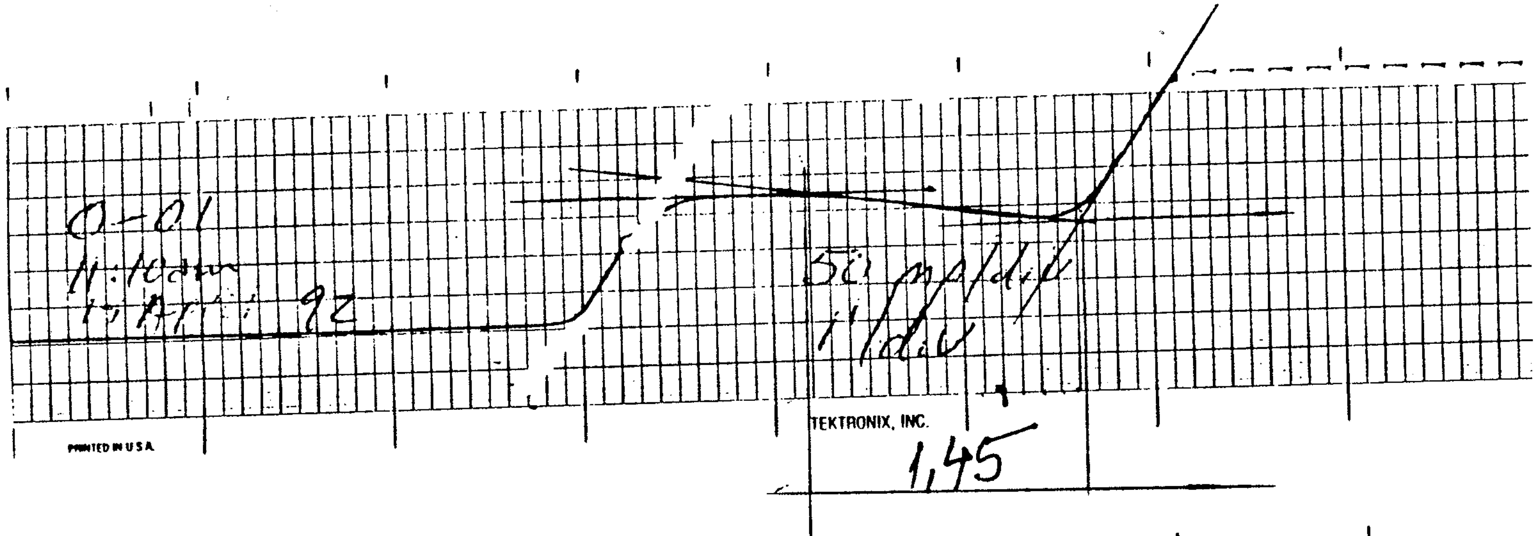
Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

aaa) "Method of Tangents", Borehole #3, April 1992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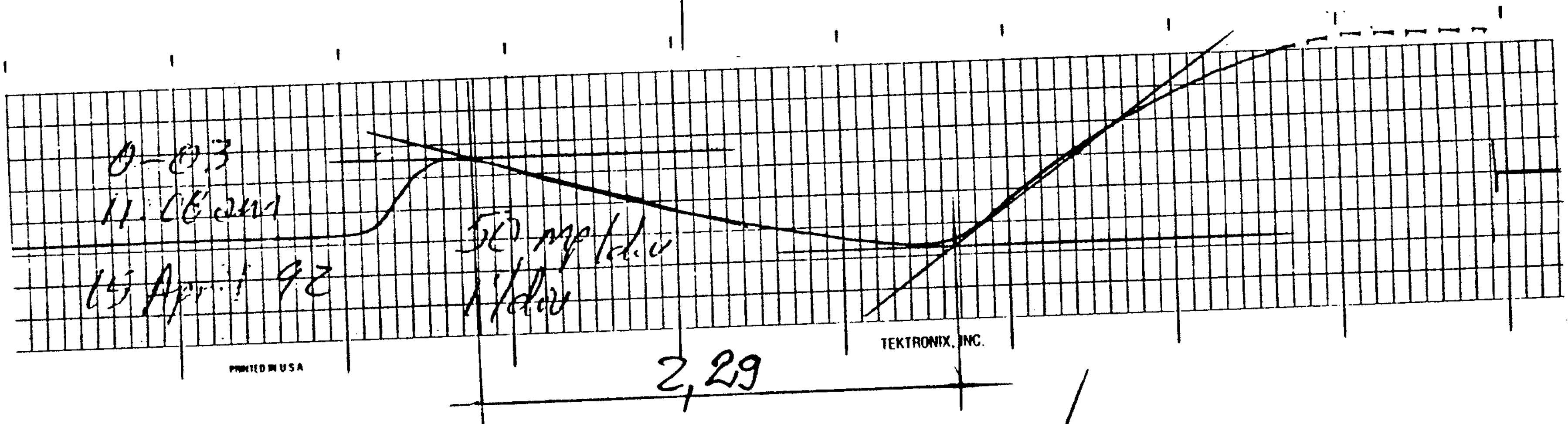
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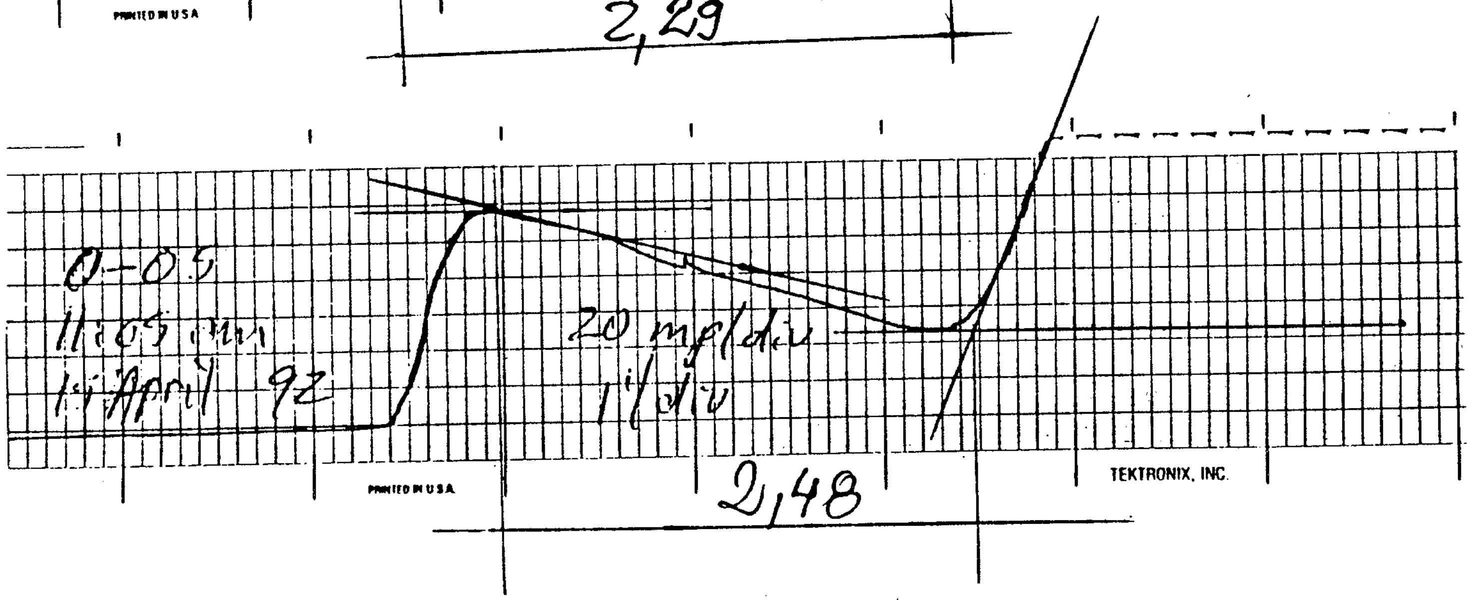
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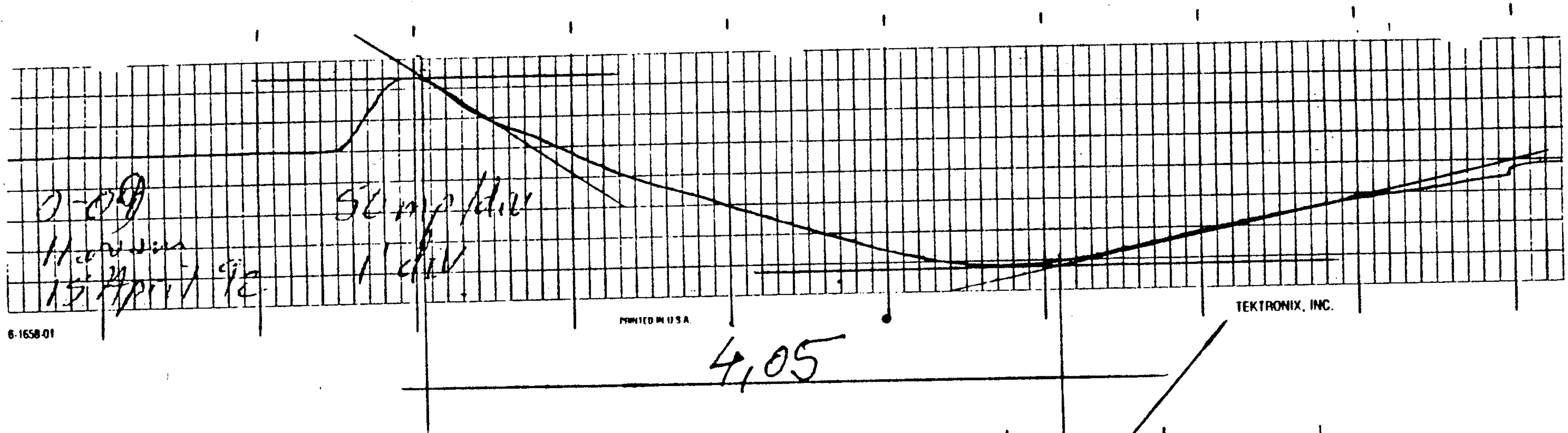
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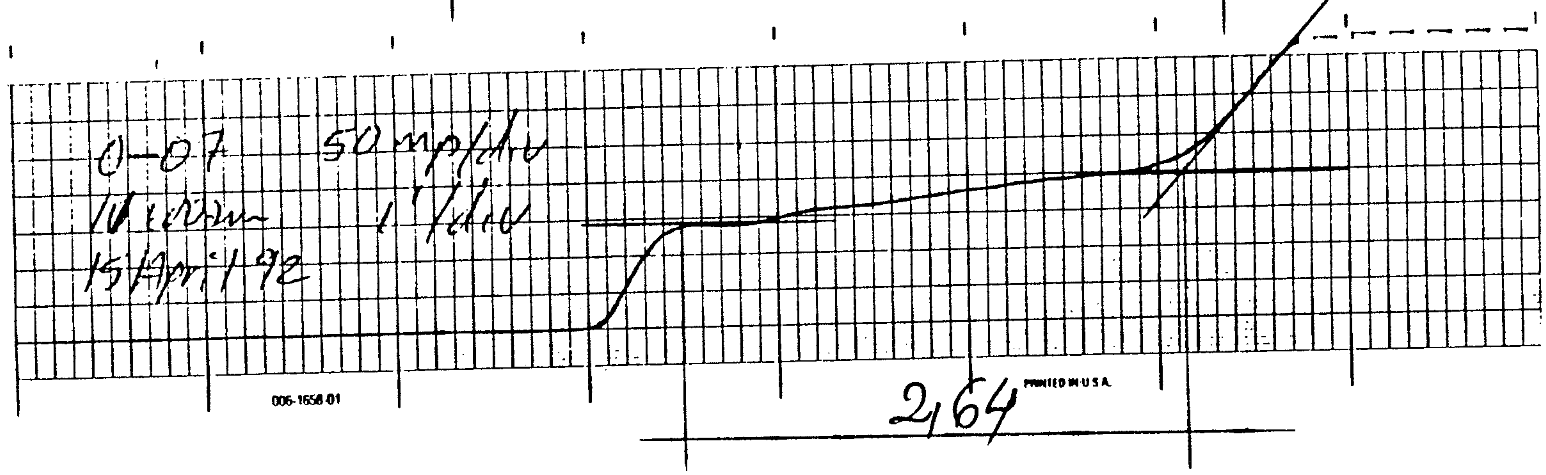
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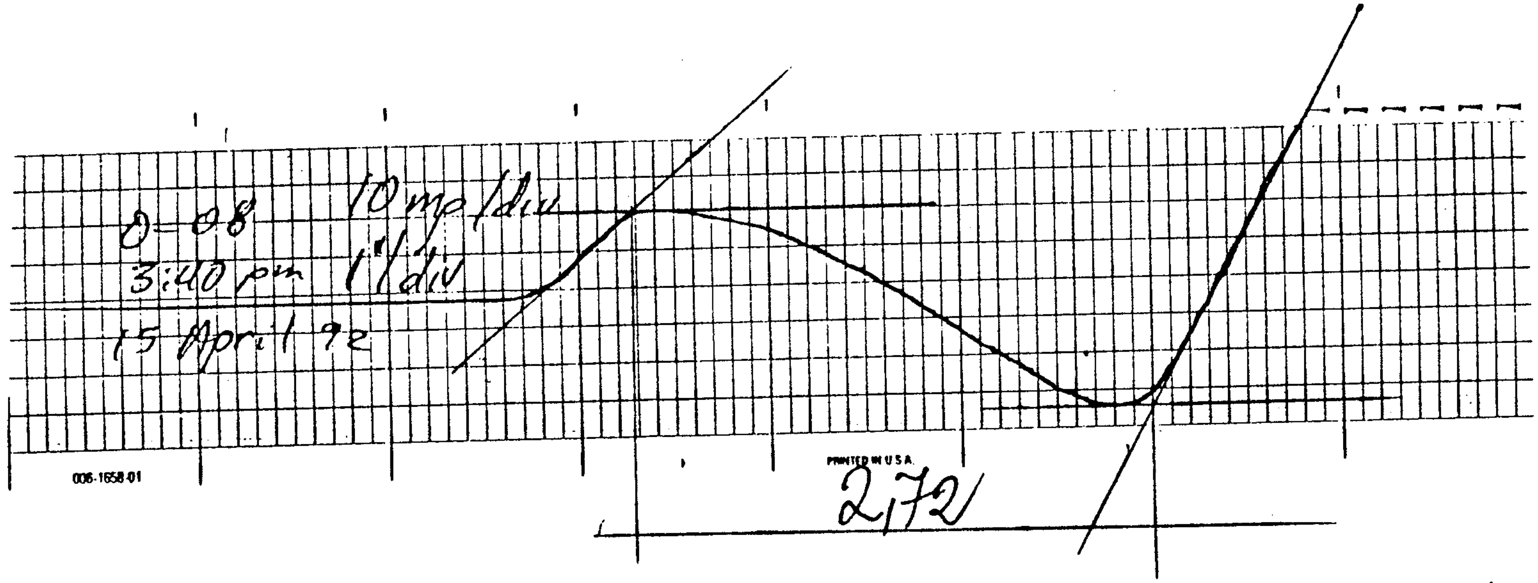
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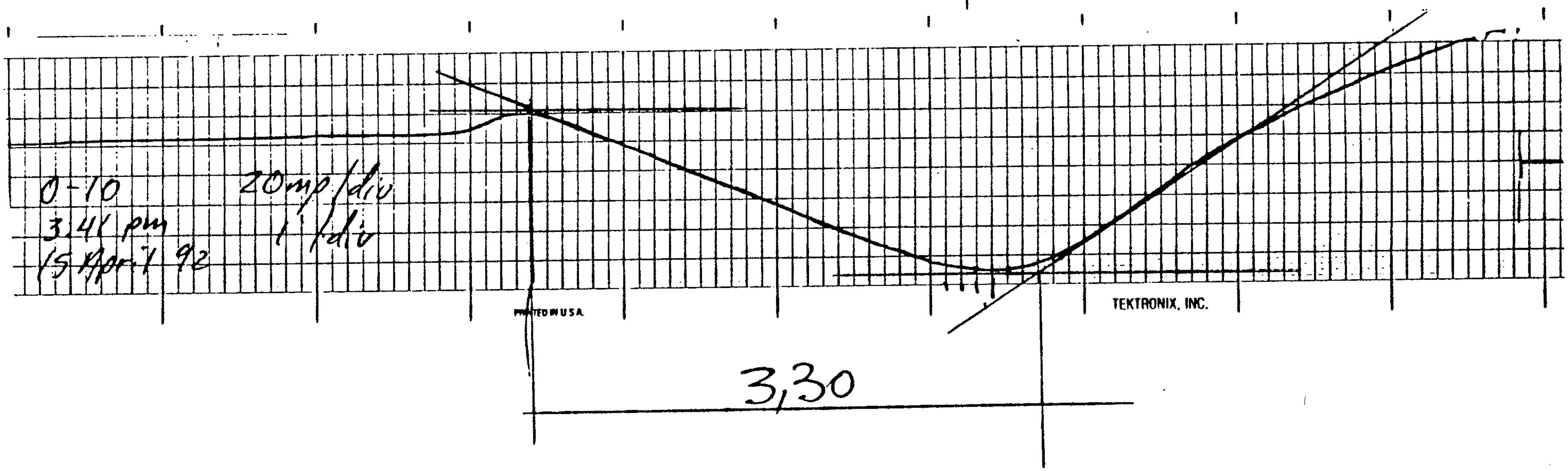
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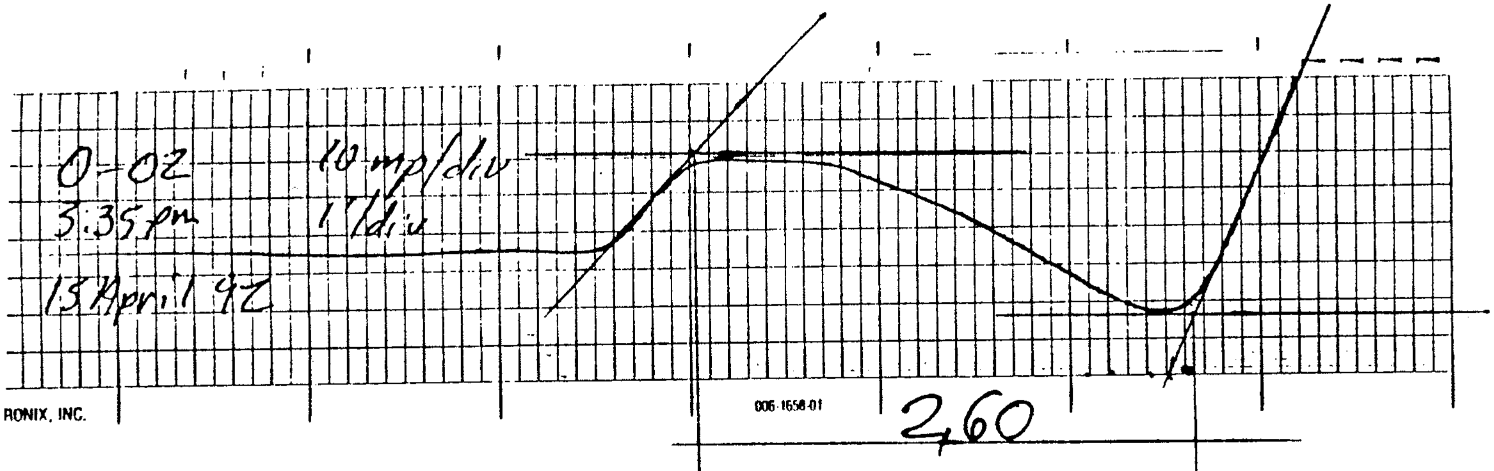
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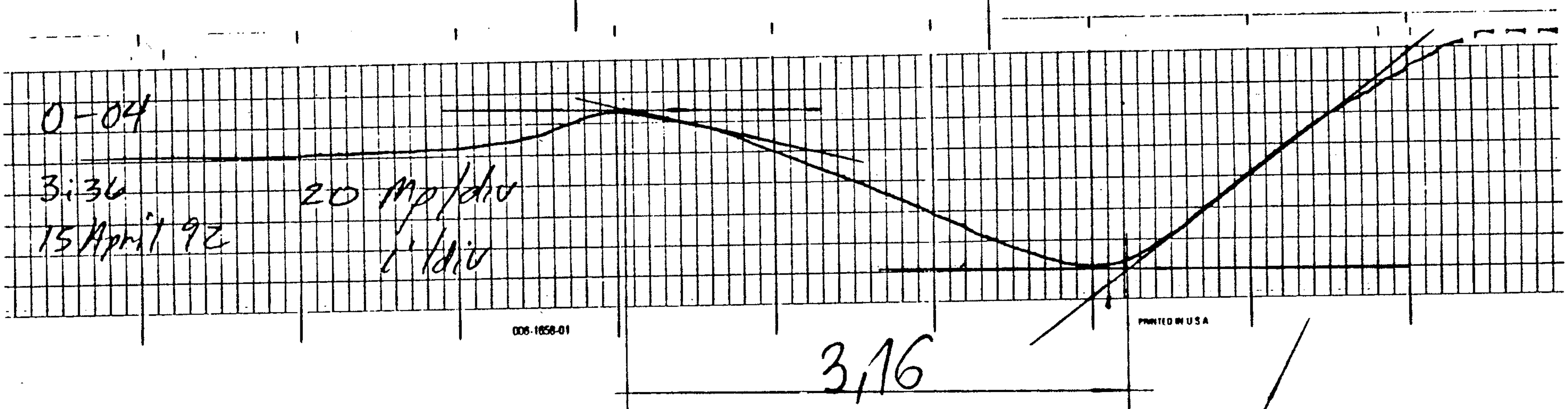
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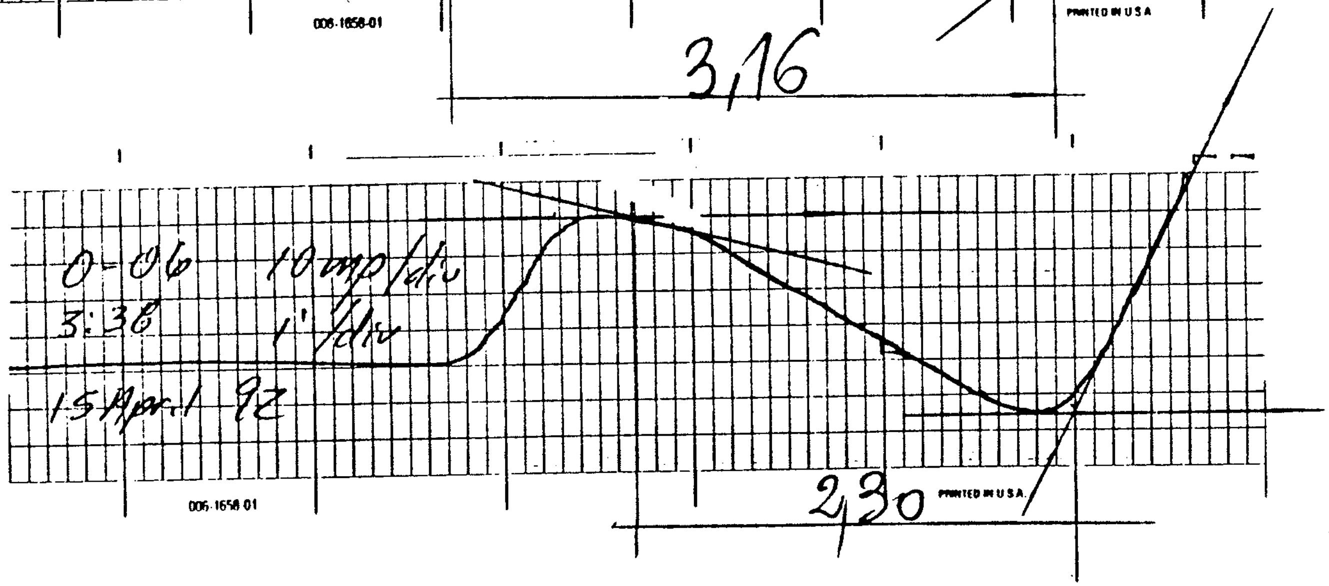
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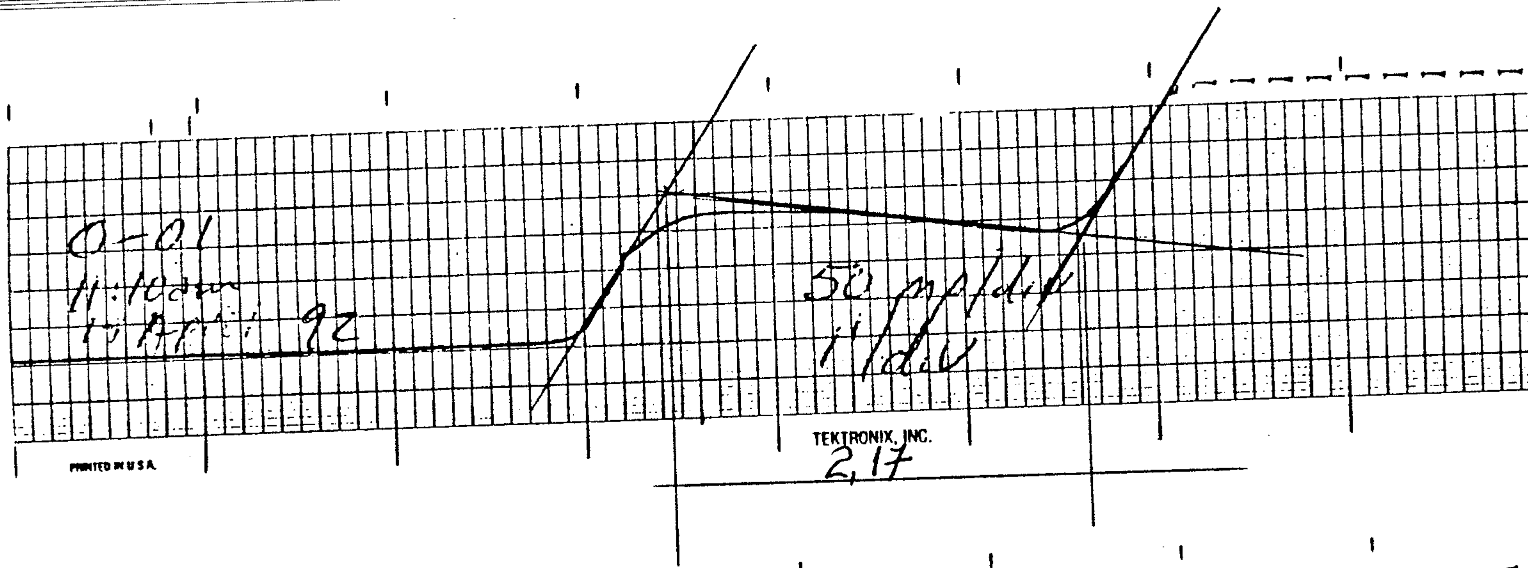
Appendix 2.
Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

bbb) "Method of Diverging Lines", Borehole #3, April 1992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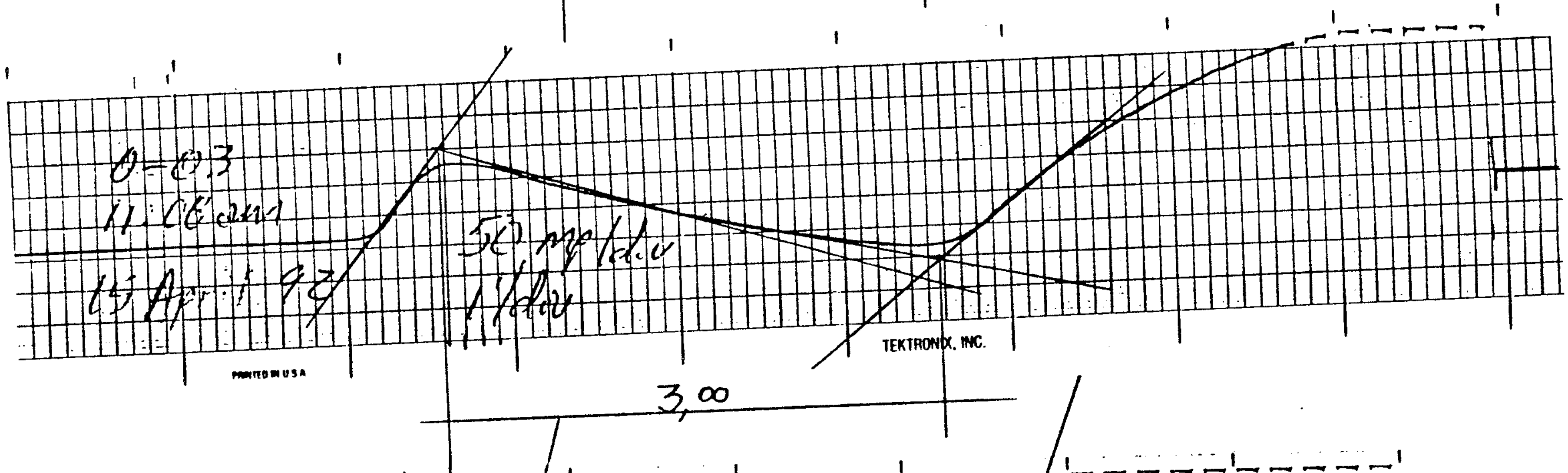
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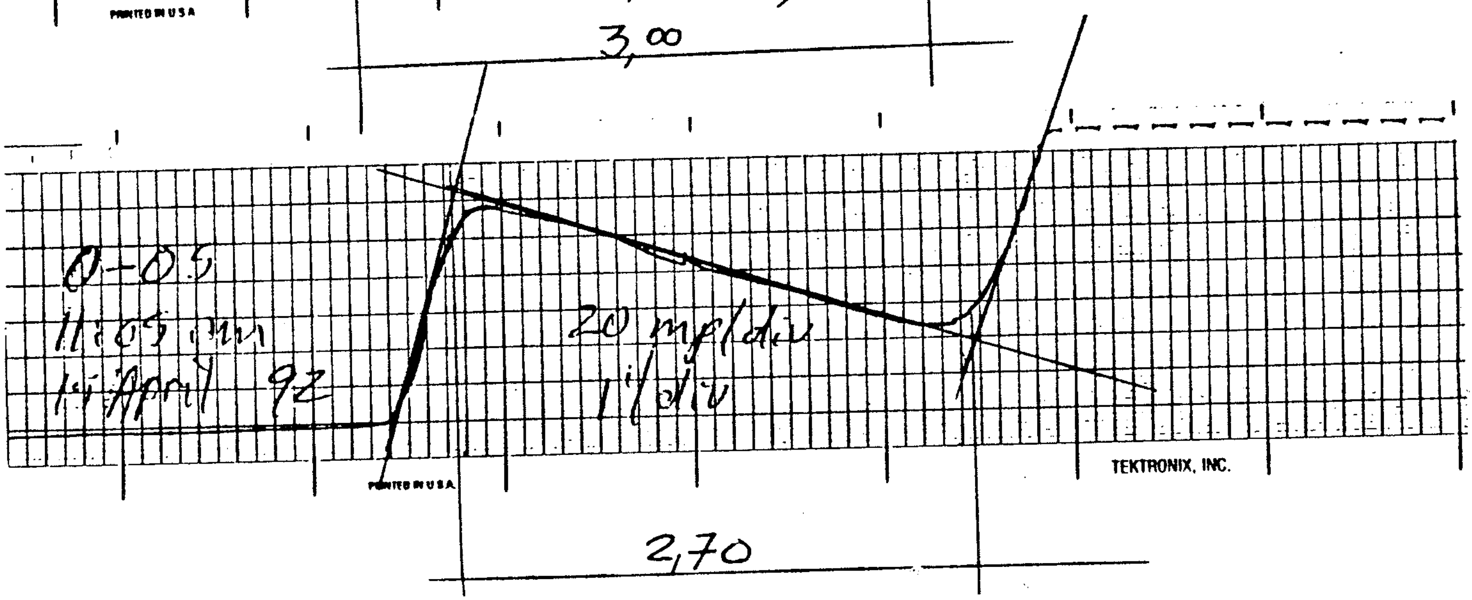
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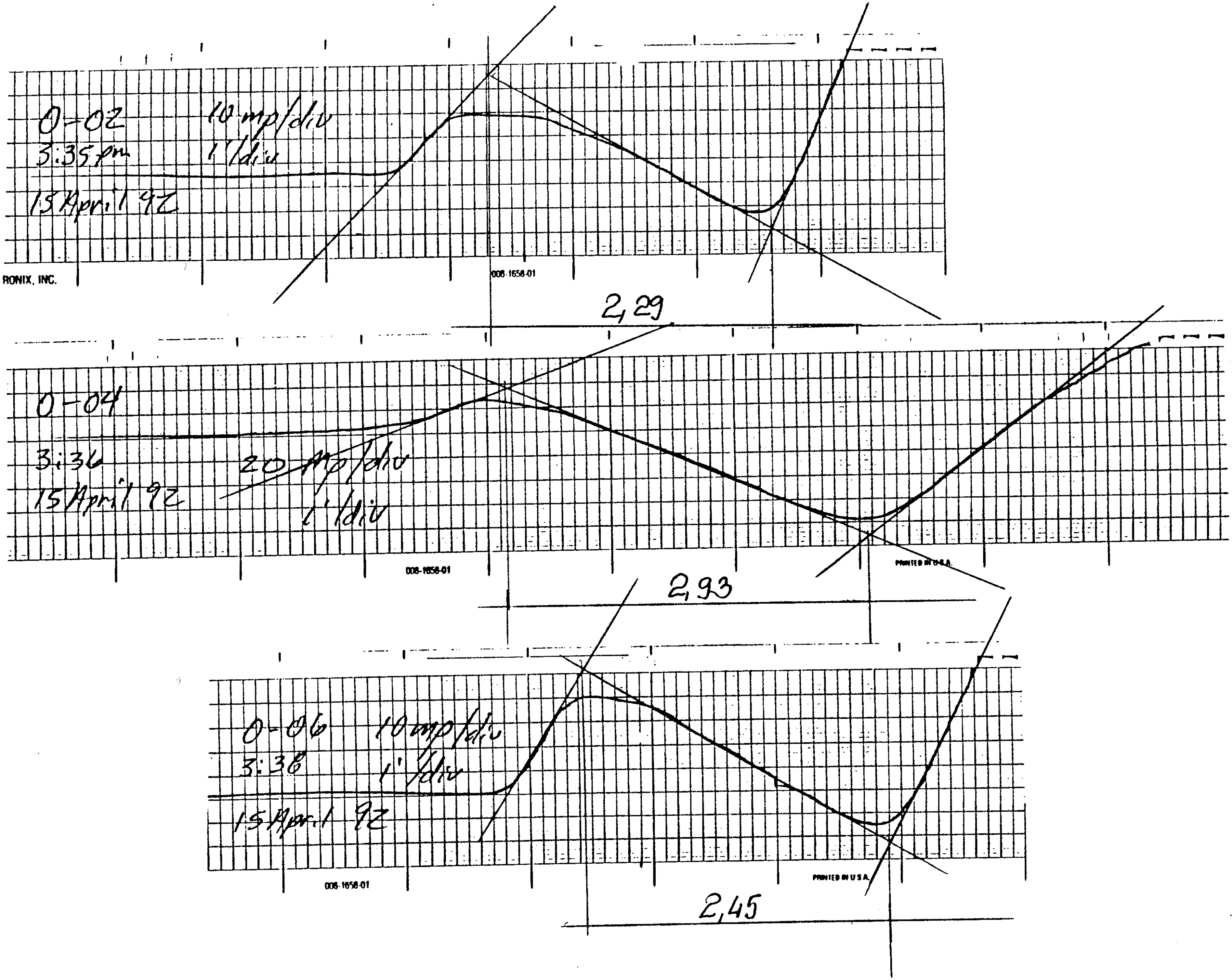

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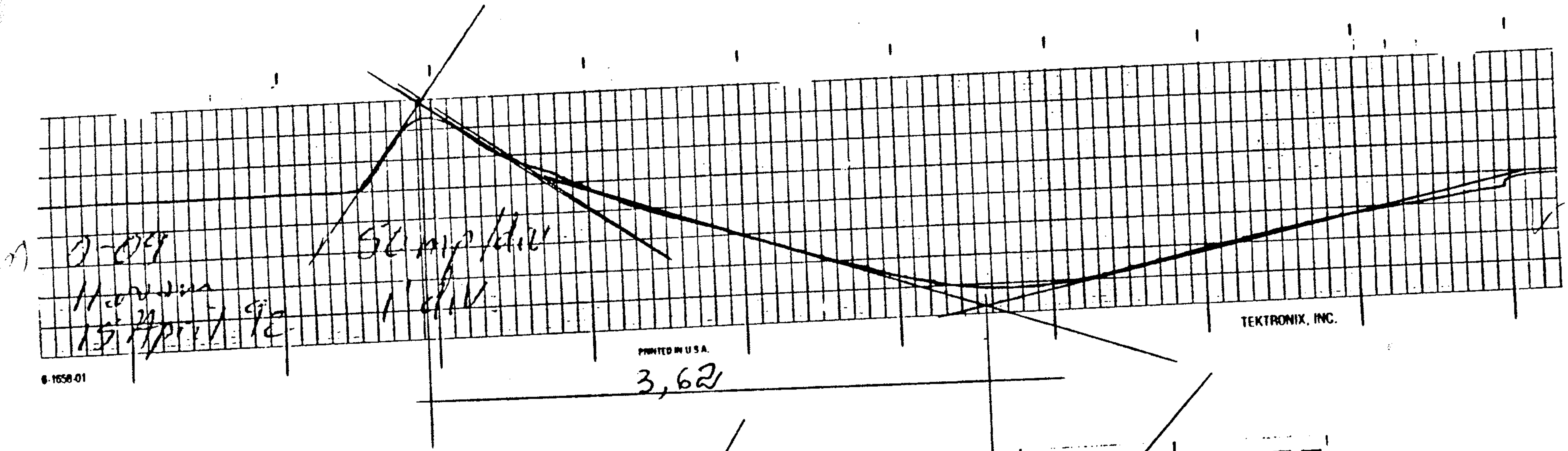
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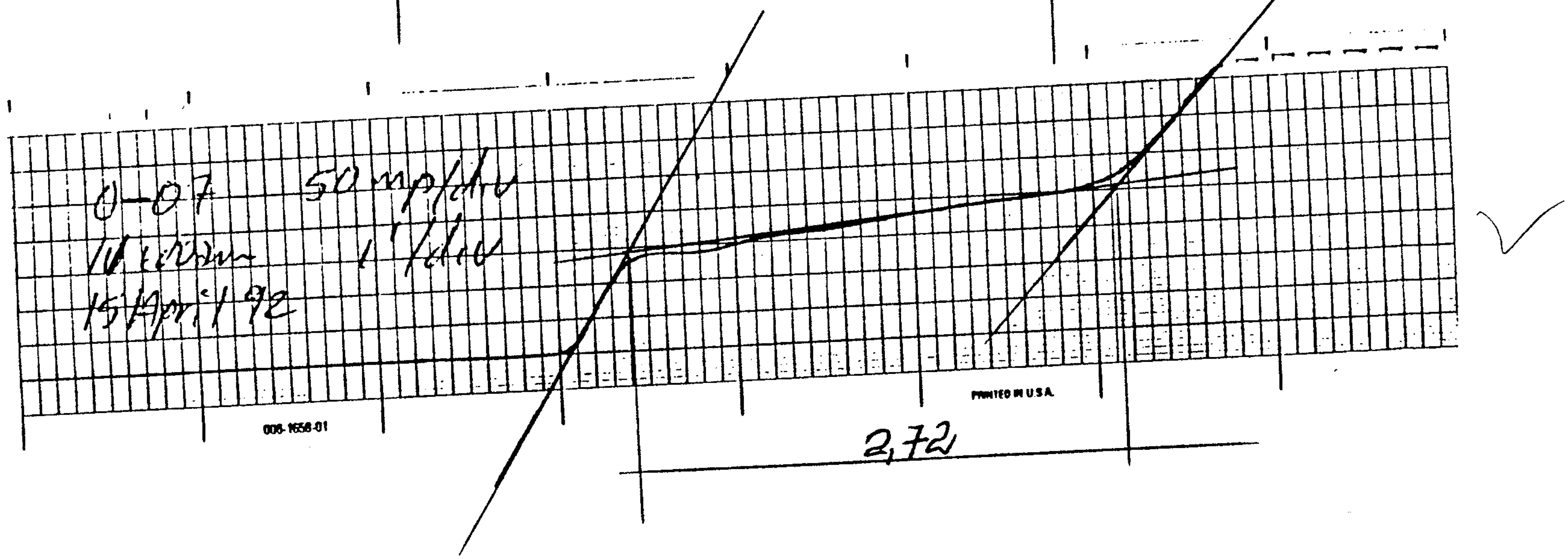
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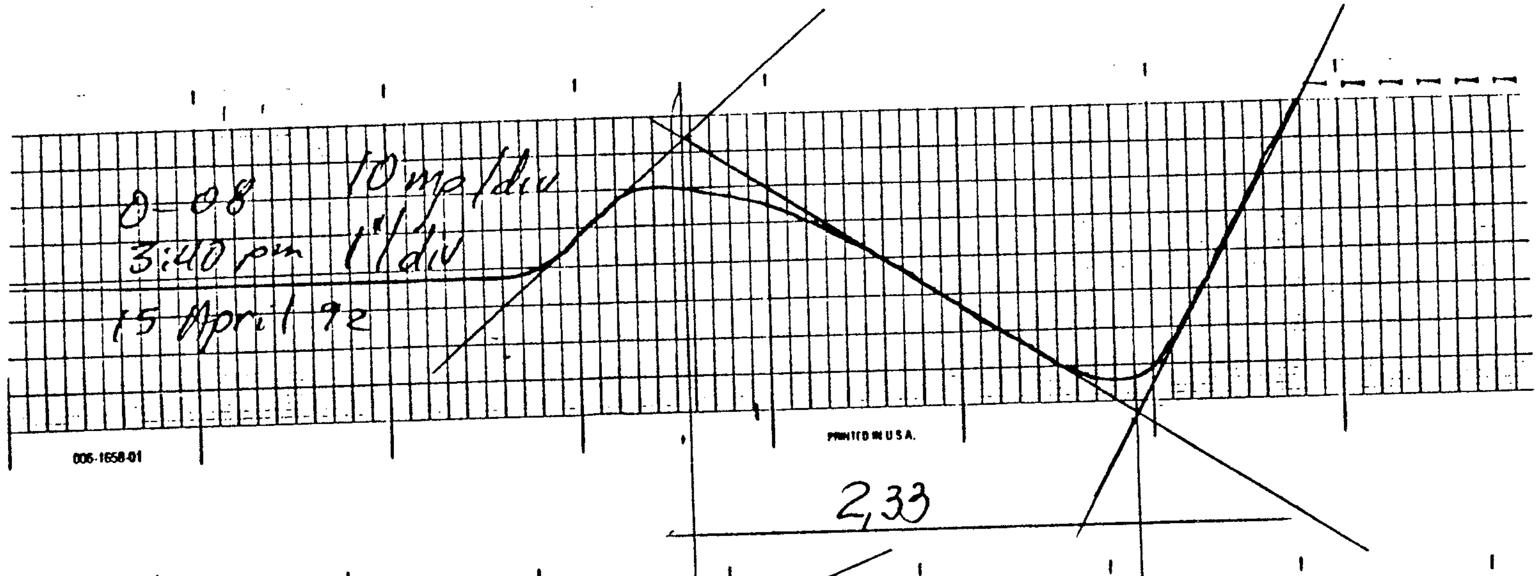
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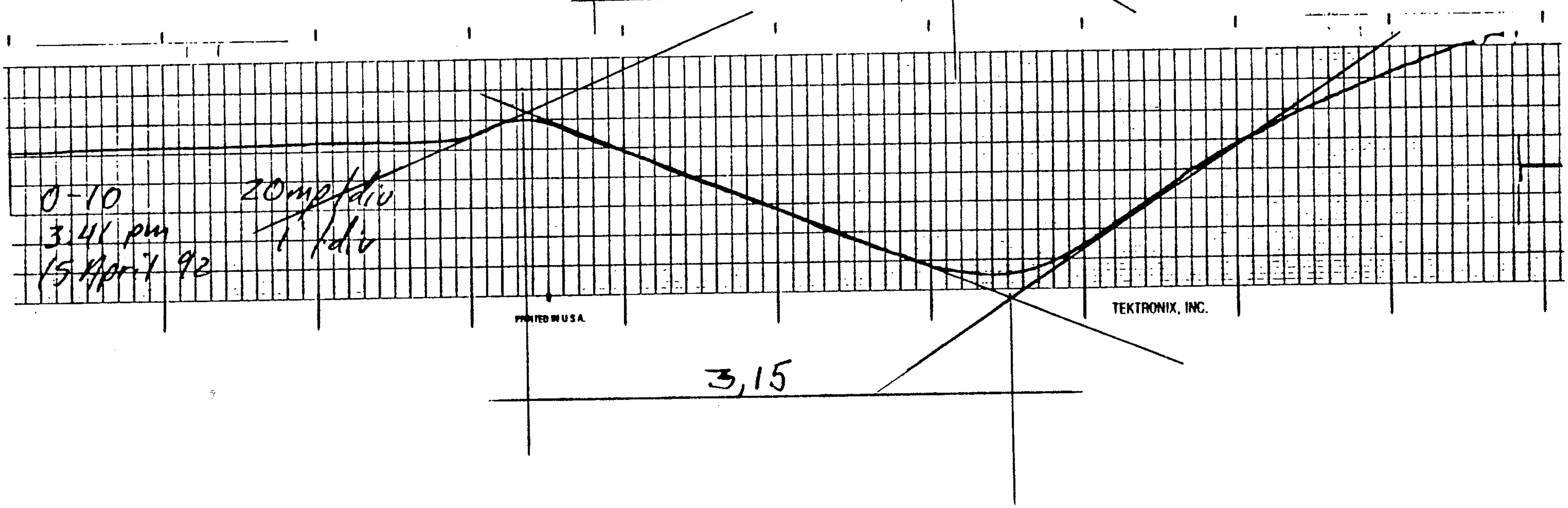
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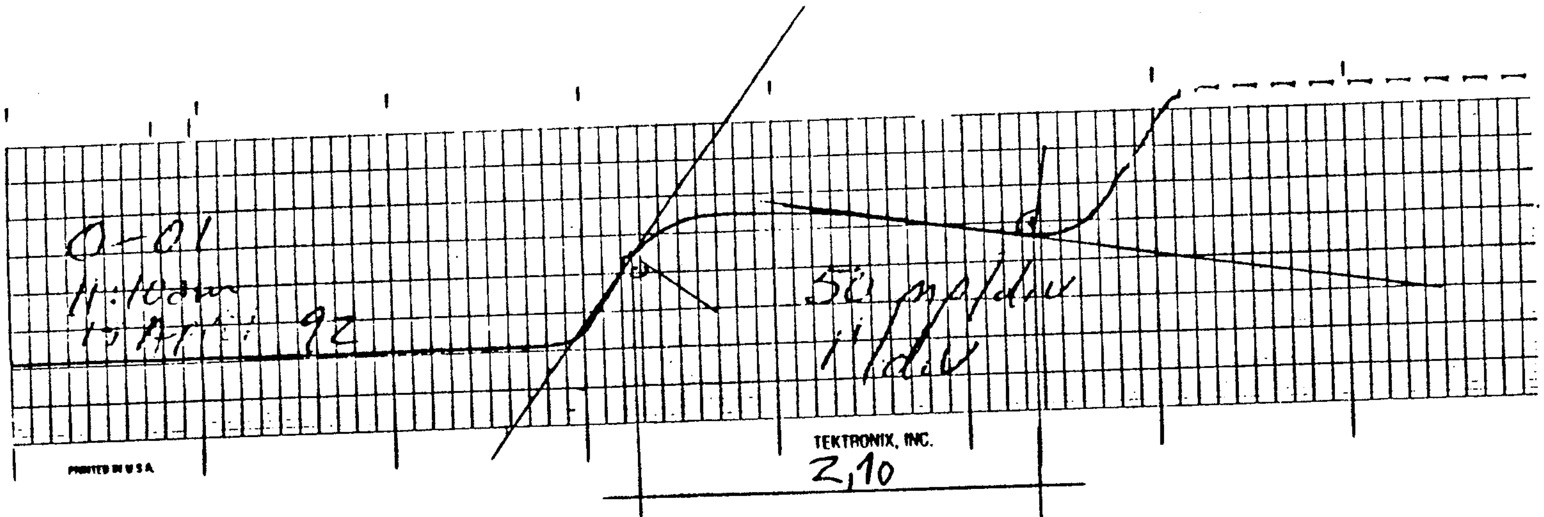
Appendix 2.

Plots of TDR traces obtained from field seasonal measurements in February, March, and April 1992, in Borehole #3, using curved prongs 12" long and different graphical methods of reading.

ccc) "Method of Peaks", Borehole #3, April 1992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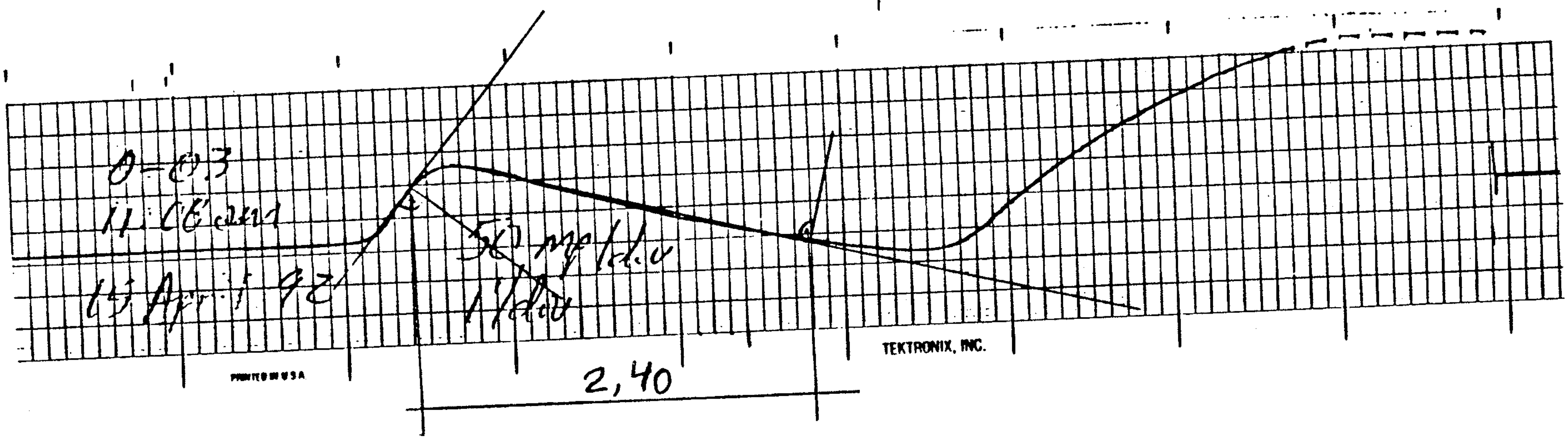
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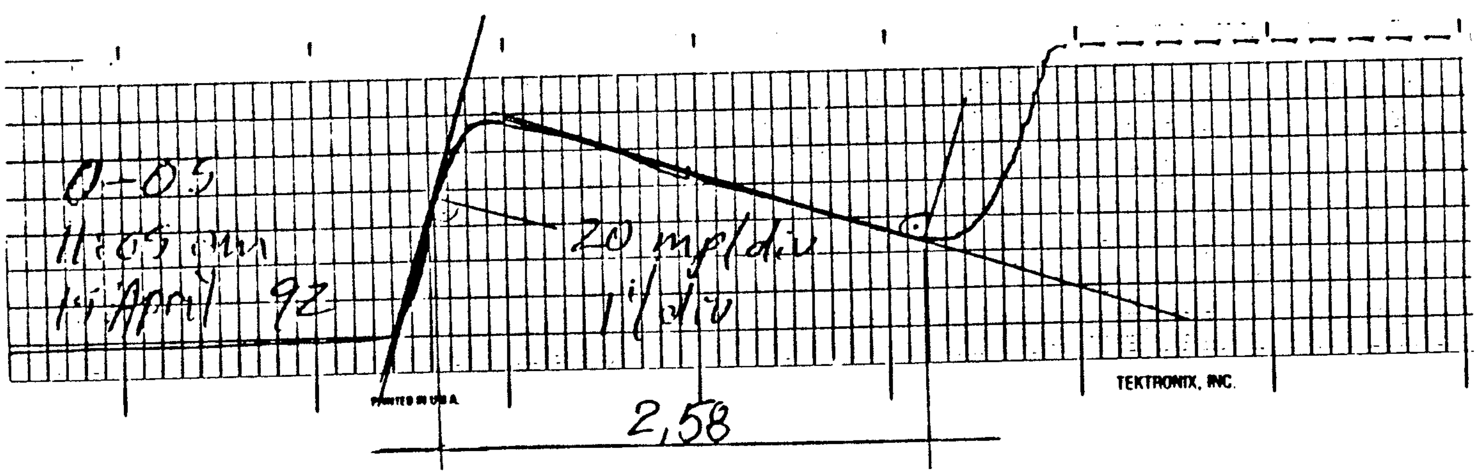
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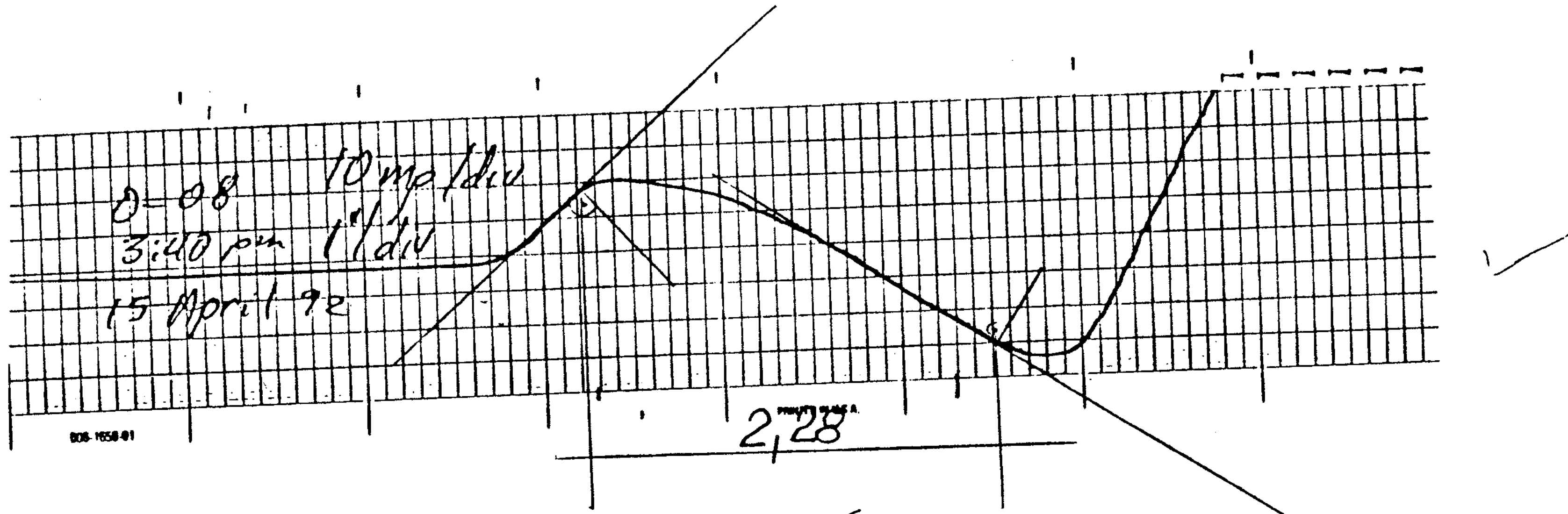
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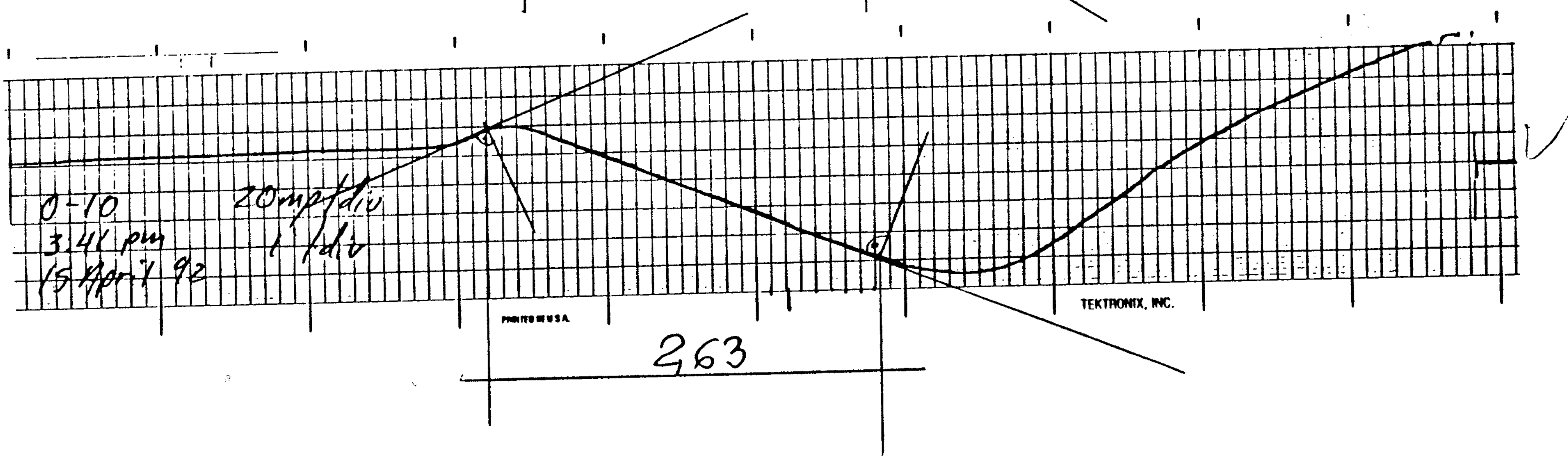
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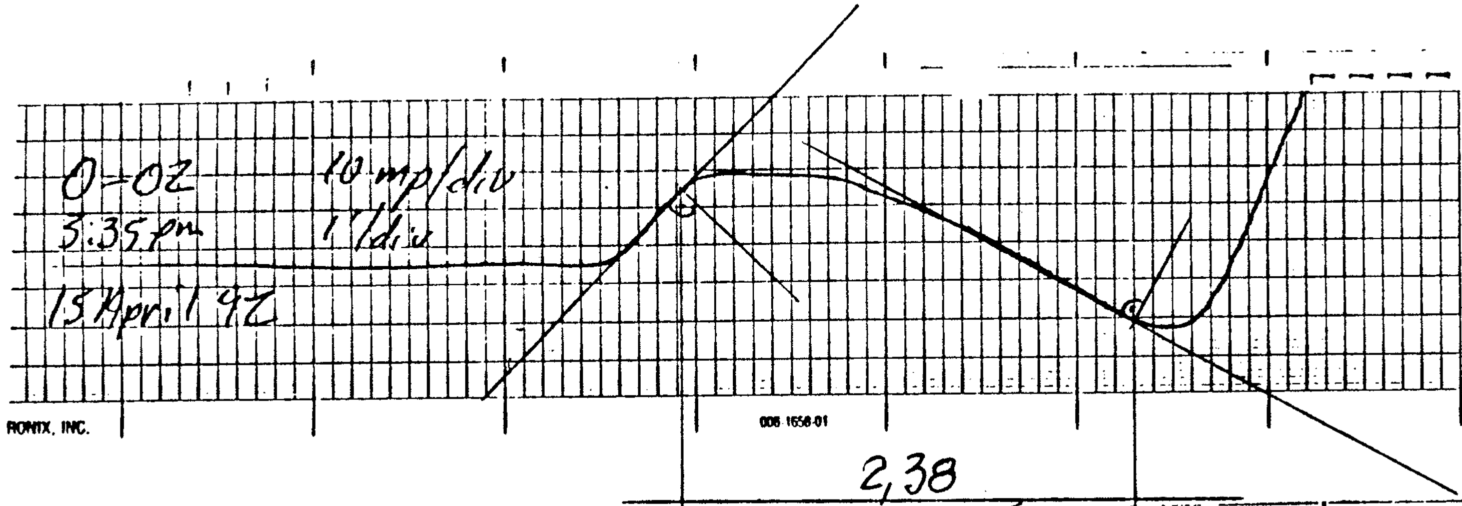
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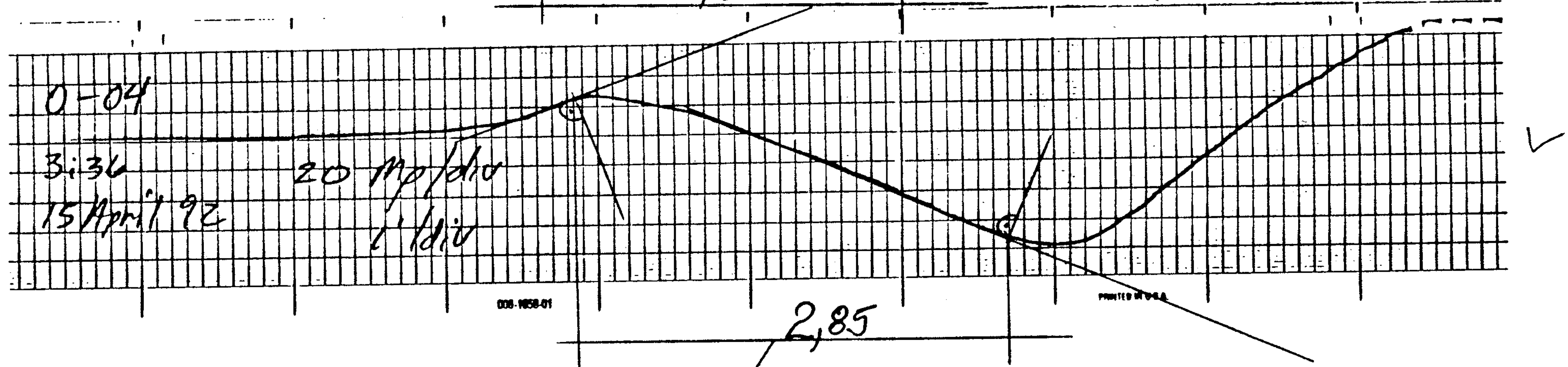
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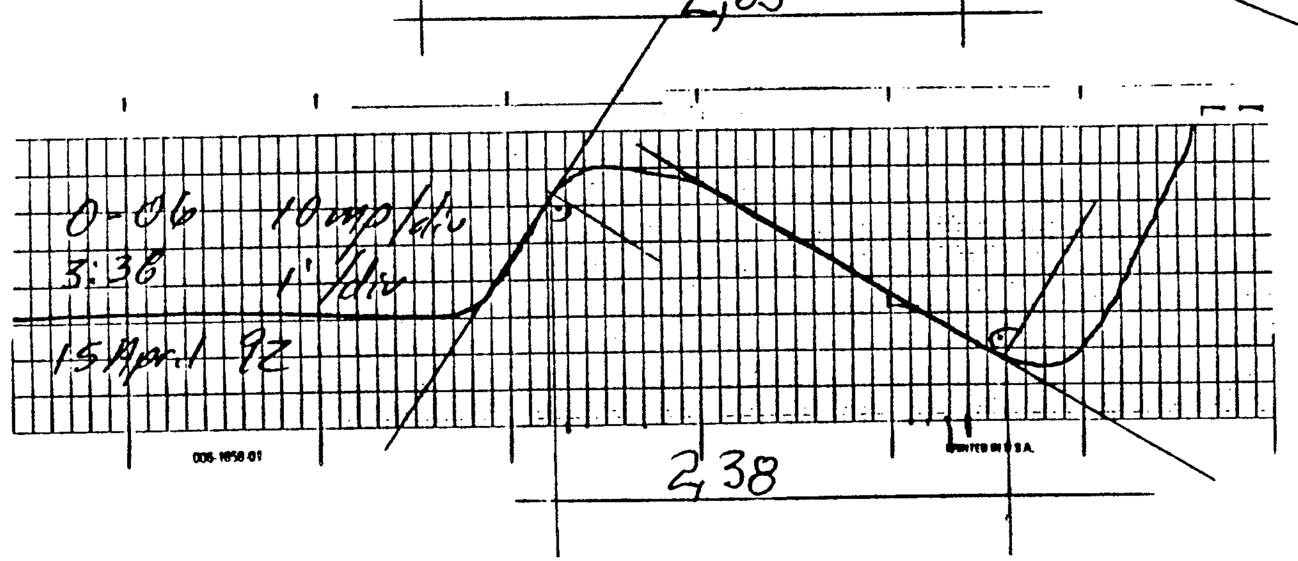
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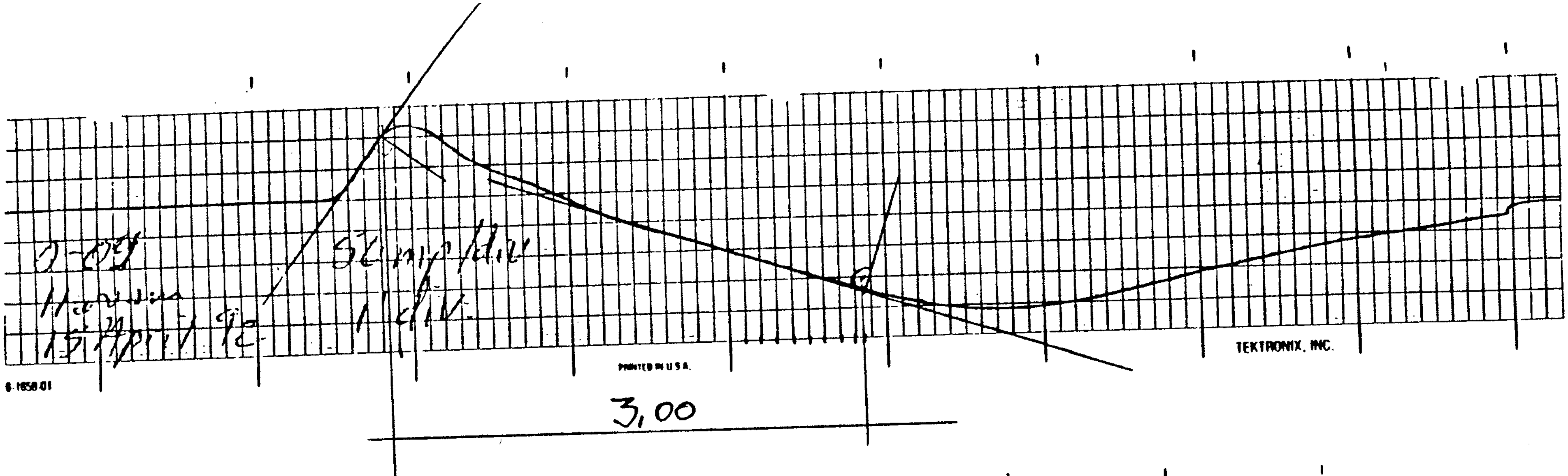
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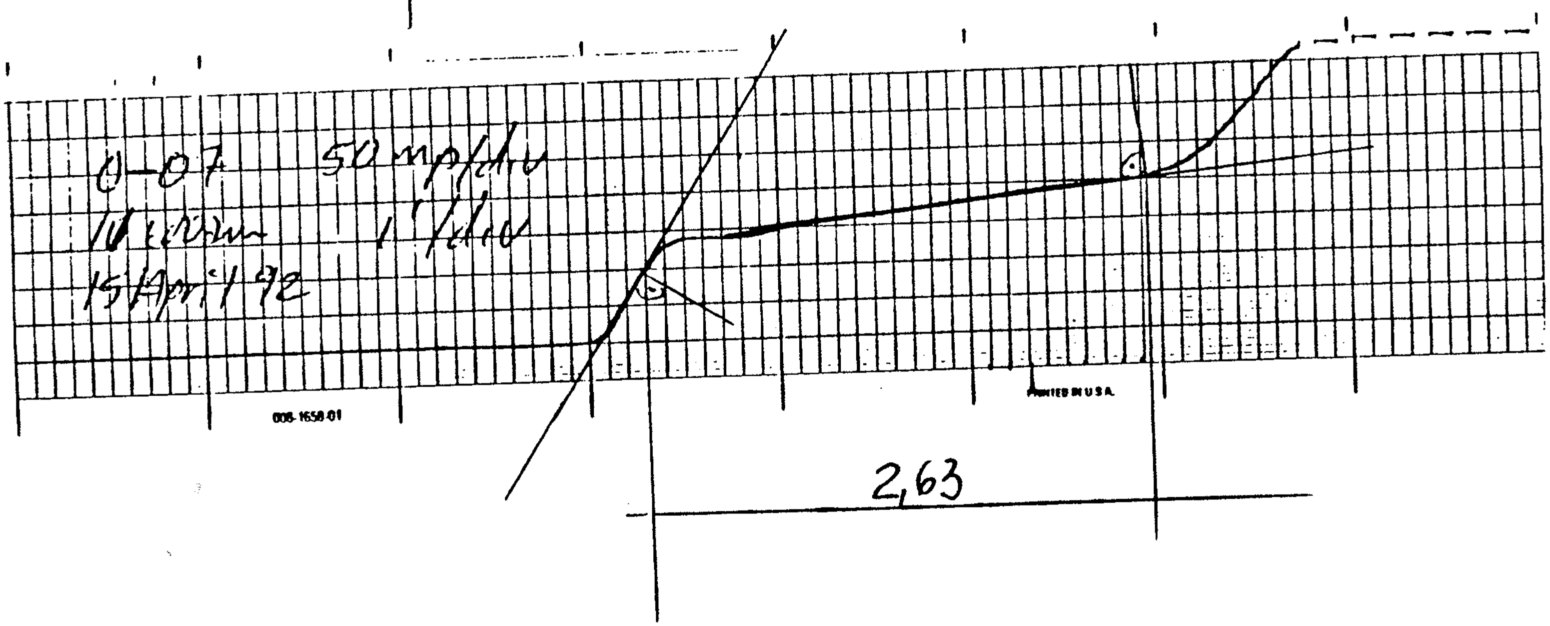
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Appendix 3.

Application of the Wilcox's Signed-Rank Test, for pairs of TDR readings, obtained in Borehole #2, by using different graphical methods of reading.

ANNEXE.31

Statistical comparison between pairs of TDR readings obtained by using :
(1) method of peaks
(2) method of diverted lines

A) DATA: Idaho,Boise, Test Section 163023, Borehole #2, flat prongs /7.5"length, February 1992. Values of TDR lengths are those ,given in the Annexe 1 and Table 1.

B) THE PROBLEM:

Using the Wilcoxon singned-rank test for paired observations ,test the hypothesis ,at the 0.05 level of significance ,to determine wether the two different methods used for reading the TDR experimental traces ,lead to simillar or different results.

C) SOLUTION:

Let m1 and m2 represent the mean TDR readings ,obtained by using the methods (1) and (2),respectively. Following the six step procedure one may have:

1.The null hypothesis, H0: m1=m2

2.The alternative hypothesis, H1: m1><m2

3.The significance level, a: 0.05

4.The critical region: Since the number of pairs of observations is 15, from the Table A16 (Walpole, 1985),for n=15 and a =0.05,one may find the critical region to be : **Wcr<=25**

5.Computations:

Pair	Method(1)	Method(2)	Di/Rank	Wi+	Wi-
1	1.40	1.40	0/1	1	
2	1.37	1.48	-0.11/8		8
3	1.72	1.60	0.12/9	9	
4	1.75	1.79	-0.04/4		4
5	1.65	1.67	-0.02/2.5		2.5
6	2.16	1.88	0.28/12.5	12.5	
7	2.25	2.00	0.25/11	11	
8	1.68	1.60	0.08/6	6	
9	1.65	1.67	-0.02/2.5		2.5
10	1.72	1.81	-0.09/7		7
11	1.65	1.60	0.05/5	5	
12	2.12	1.84	0.28/12.5	12.5	
13	2.77	2.45	0.32/14	14	
14	2.60	2.39	0.21/10	10	
15	3.10	2.22	0.88/15	15	
Total				96	24

W+=96

W-=24

W=24

6.Conclusion:

As the computed value, W=24 is less than the critical value Wcr<=25,one have to reject the null hypothesis, H0, and to conclude that the TDR readings,obtained by using the two investigated methods mentioned above are significantly different. Also one may observe that this assertion is not decissive,because the computed value, W is situated very near the up-limit of the critical region .

ANNEXE.32

Statistical comparison between pairs of TDR readings obtained by using :

(1) method of peaks

(2) method of diverted lines

A) DATA: Idaho,Boise, Test Section 163023, Borehole #2, flat prongs /7.5"length, March 1992. Values of TDR lengths are those ,given in the Annexe 1 and Table 1.

B) THE PROBLEM:

Using the Wilcoxon singned-rank test for paired observations ,test the hypothesis ,at the 0.05 level of significance ,to determine wether the two different methods used for reading the TDR experimental traces ,lead to simillar or different results.

C) SOLUTION:

Let m1 and m2 represent the mean TDR readings ,obtained by using the methods (1) and (2),respectively. Following the six step procedure one may have:

1.The null hypothesis, H0: m1=m2

2.The alternative hypothesis, H1: m1><m2

3.The significance level, a: 0.05

4.The critical region: Since the number of pairs of observations is 17, from the Table A16 (Walpole, 1985),for n=17 and a =0.05,one may find the critical region to be : **Wcr<=35**

5.Computations:

Pair	Method(1)	Method(2)	Di/Rank	Wi+	Wi-
1	1.42	1.38	0.04/1	1	
2	1.47	1.40	0.07/2.5	2.5	
3	1.60	1.60	0		
4	1.89	1.82	0.07/2.5	2.5	
5	1.90	1.90	0		
6	2.08	1.78	0.30/10.5	10.5	
7	2.29	2.51	-0.22/7		7
8	2.18	2.02	0.16/6	6	
9	1.73	1.60	0.13/5	5	
10	1.72	1.49	0.23/8	8	
11	1.75	1.45	0.30/10.5	10.5	
12	2.02	1.77	0.25/9	9	
13	2.07	1.98	0.09/4	4	
14	2.88	2.40	0.44/14	14	
15	2.80	2.26	0.54/15	15	
16	**2.80**	2.48	0.32/12.5	12.5	
17	2.50	2.18	0.32/12.5	12.5	
Total				111	7 ==

W+=111

W-=7

W=7

6.Conclusion:

As the computed value, W=7 is less than the critical value Wcr<=35, one have to reject the null hypothesis, H0, and to conclude that the TDR readings,obtained by using the two investigated methods mentioned above are significantly different.

ANNEXE.33

Statistical comparison between pairs of TDR readings obtained by using :
(1) method of peaks
(2) method of diverted lines

A) DATA: Idaho,Boise, Test Section 163023, Borehole #2, flat prongs /7.5"length, April 1992. Values of TDR lengths are those ,given in the Annexe 1 and Table 1.

B) THE PROBLEM:

Using the Wilcoxon singned-rank test for paired observations ,test the hypothesis ,at the 0.05 level of significance ,to determine wether the two different methods used for reading the TDR experimental traces ,lead to simillar or different results.

C) SOLUTION:

Let m1 and m2 represent the mean TDR readings ,obtained by using the methods (1) and (2),respectively. Following the six step procedure one may have:

1.The null hypothesis, H0: m1=m2

2.The alternative hypothesis, H1: m1><m2

3.The significance level, a: 0.05

4.The critical region: Since the number of pairs of observations is 10, from the Table A16 (Walpole, 1985),for n=10 and a =0.05,one may find the critical region to be : **Wcr<=8**

5.Computations:

Pair	Method(1)	Method(2)	Di/Rank	Wi+	Wi-
1	1.60	1.67	-0.07/3.5		3.5
2	1.68	1.62	0.06/2	2	
3	1.90	1.88	0.02/1	1	
4	2.04	1.78	0.26/8	8	
5	2.21	2.10	0.11/6	6	
6	1.72	1.65	0.07/3.5	3.5	
7	1.77	1.59	0.18/7	7	
8	2.10	2.00	0.10/5	5	
9	2.80	2.15	0.65/10	10	
10	2.60	2.14	0.46/9	9	
Total:				51.5	3.5

W+=51.5

W-=3.5

W=3.5

6.Conclusion:

As the computed value, W=3.5 is less than the critical value Wcr<=8, one have to reject the null hypothesis, H0, and to conclude that the TDR readings,obtained by using the two investigated methods mentioned above are significantly different.

Appendix 4.

Application of the Wilcox's Signed-Rank Test, for pairs of TDR readings, obtained on Borehole #3, by using different graphical methods of reading.

ANNEXE.41

Statistical comparison between pairs of TDR readings obtained by using :

(1) method of peaks

(2) method of diverted lines

A) DATA: Idaho,Boise, Test Section 163023, Borehole #3,curved prongs /12"length, February 1992. Values of TDR lengths are those ,given in the Annexe 2 and Table 2.

B) THE PROBLEM:

Using the Wilcoxon singned-rank test for paired observations ,test the hypothesis ,at the 0.05 level of significance ,to determine wether the two different methods used for reading the TDR experimental traces ,lead to simillar or different results.

C) SOLUTION:

Let m1 and m2 represent the mean TDR readings ,obtained by using the methods (1) and (2),respectively. Following the six step procedure one may have:

1.The null hypothesis, H0: m1=m2

2.The alternative hypothesis, H1: m1><m2

3.The significance level, a: 0.05

4.The critical region: Since the number of pairs of observations is 15, from the Table A16 (Walpole, 1985),for n=15 and a =0.05,one may find the critical region to be : **Wcr<=25**

5.Computations:

Pair	Method(1)	Method(2)	Di/Rank	Wi+	Wi-
1	1.69	1.66	0.03/2	2	
2	1.47	1.69	-0.22/9		9
3	2.44	2.43	0.01/1	1	
4	2.44	2.44	0		
5	2.44	2.34	0.10/4.5	4.5	
6	4.12	3.05	1.07/14	14	
7	3.00	2.90	0.10/4.5	4.5	
8	2.49	2.19	0.30/10	10	
9	2.45	2.04	0.41/12	12	
10	2.50	2.39	0.11/7	7	
11	2.40	2.74	-0.34/11		11
12	1.25	1.35	-0.10/4.5		4.5
13	2.38	2.50	-0.12/8		8
14	3.55	3.45	0.10/4.5	4.5	
15	3.30	2.66	0.64/13	13	
Total				62.5	32.5

W+=62.5

W-=32.5

W=32.5

6.Conclusion:

As the computed value, W=32.5 is higher than the critical value Wcr<=25,one have to accept the null hypothesis, H0, and to conclude that the TDR readings,obtained by using the two investigated methods mentioned above are not significantly different.

ANNEX4.42

Statistical comparison between pairs of TDR readings obtained by using :

(1) method of peaks

(2) method of diverted lines

A) DATA: Idaho,Boise, Test Section 163023, Borehole #3, curved prongs /12"length, March 1992. Values of TDR lengths are those ,given in the Annexe 2 and Table 2.

B) THE PROBLEM:

Using the Wilcoxon singned-rank test for paired observations ,test the hypothesis ,at the 0.05 level of significance ,to determine wether the two different methods used for reading the TDR experimental traces ,lead to similar or different results.

C) SOLUTION:

Let m1 and m2 represent the mean TDR readings ,obtained by using the methods (1) and (2),respectively. Following the six step procedure one may have:

1.The null hypothesis, H0: m1=m2

2.The alternative hypothesis, H1: m1><m2

3.The significance level, a: 0.05

4.The critical region: Since the number of pairs of observations is 13, from the Table A16 (Walpole, 1985),for n=13 and a =0.05,one may find the critical region to be : **Wcr<=17**

5.Computations:

Pair	Method(1)	Method(2)	Di/Rank	Wi+	Wi-
1	2.02	1.90	0.12/5	5	
2	2.40	2.40	0		
3	2.98	2.29	0.69/12	12	
4	3.05	2.58	0.47/11	11	
5	2.95	2.97	-0.02/2		2
6	2.60	2.48	0.14/6	6	
7	2.63	2.40	0.06/8	8	
8	2.46	2.40	0.06/3	3	
9	2.79	2.53	0.26/9.5	9.5	
10	2.76	2.58	0.18/7	7	
11	2.36	2.35	0.01/1	1	
12	3.63	3.37	0.26/9.5	9.5	
13	3.05	2.35	0.10/4	4	
	Total			76	2

W+=76

W-=2

W=2

===============

6.Conclusion:

As the computed value, W=2 is less than the critical value Wcr<=17, one have to reject the null hypothesis, H0, and to conclude that the TDR readings,obtained by using the two investigated methods mentioned above are significantly different.

ANNEX.43

Statistical comparison between pairs of TDR readings obtained by using :
(1) method of peaks
(2) method of diverted lines

A) DATA: Idaho,Boise, Test Section 163023, Borehole #3, curved prongs /12"length, April 1992. Values of TDR lengths are those ,given in the Annexe 2 and Table 2.

B) THE PROBLEM:

Using the Wilcoxon singned-rank test for paired observations ,test the hypothesis ,at the 0.05 level of significance ,to determine wether the two different methods used for reading the TDR experimental traces ,lead to simillar or different results.

C) SOLUTION:

Let m1 and m2 represent the mean TDR readings ,obtained by using the methods (1) and (2),respectively. Following the six step procedure one may have:

1.The null hypothesis, H0: m1=m2

2.The alternative hypothesis, H1: m1><m2

3.The significance level, a: 0.05

4.The critical region: Since the number of pairs of observations is 10, from the Table A16 (Walpole, 1985),for n=10 and a =0.05,one may find the critical region to be : **Wcr<=8**

5.Computations:

Pair	Method(1)	Method(2)	Di/Rank	Wi+	Wi-
1	2.17	2.10	0.07/2.5	2.5	
2	2.29	2.38	-0.09/5.5		5.5
3	3.00	2.40	0.60/8	9	
4	2.93	2.85	0.08/4	4	
5	2.70	2.58	0.12/10	7	6
2.45	2.38	0.07/2.5	2.5		
7	2.72	2.63	0.09/5.5	5.5	
8	2.33	2.28	0.05/1	1	
9	3.62	3.00	0.62/9	10	
10	3.15	2.63	0.52/7	8	
	Total			40.5	5.5

W+=40.5

W-=5.5

W=5.5

==============

6.Conclusion:

As the computed value, W=5.5 is less than the critical value Wcr<=8, one have to reject the null hypothesis, H0, and to conclude that the TDR readings,obtained by using the two investigated methods mentioned above are significantly different.

Appendix 5.

Testing the significance of the differences between pairs of TDR readings, obtained by using different methods of reading.

Annexe 5 Testing the differences between the TDR values obtained by using different methods for reading TDR traces

Crt. No.	Brhl#3 Annexe 41	Annexe 42	annexe 43	Brhl#2 Annexe 31	Annexe 32	Annexe 33
1.00	0.03	0.12	0.07	0.00	0.04	-0.07
2.00	-0.22	0.00	-0.09	-0.11	0.07	0.06
3.00	0.01	0.69	0.60	0.12	0.00	0.02
4.00	0.00	0.74	0.08	-0.04	0.07	0.26
5.00	0.10	-0.02	0.12	-0.02	0.00	0.11
6.00	1.07	0.14	0.07	0.28	0.30	0.07
7.00	0.10	0.06	0.09	0.25	-0.22	0.18
8.00	0.30	0.26	0.05	0.08	0.16	0.10
9.00	0.41	0.18	0.62	-0.02	0.13	0.65
10.00	0.11	0.01	0.52	-0.09	0.23	0.46
11.00	-0.34	0.26		0.05	0.30	
12.00	-0.10	0.10		0.28	0.25	
13.00	-0.12	0.47		0.32	0.09	
14.00	0.10			0.21	0.44	
15.00	0.64			0.88	0.54	
16.00					0.32	
17.00					0.32	
SUM (FT)	2.09	3.01	2.13	2.19	3.04	1.84
AVG (ft)	0.14	0.23	0.21	0.15	0.18	0.18
STD	0.34	0.24	0.25	0.24	0.18	0.21
N	15.00	13.00	10.00	15.00	17.00	10.00
t	1.58	3.45	2.73	2.35	4.09	2.80
Sign.						

Appendix 6.

Evaluation of the confidence levels for the average differences, di, between pairs of TDR readings, obtained by using different methods of reading.

Test 31 : Analysys of the average differences,related with data from Annex 31 :n=15; Avg=0.15, Std=0.24

P%	t	t*s/n^0.5	dmin	dmax
60	0.54	0.0334	0.1165	0.1834
80	0.87	0.0539	0.0960	0.2039
90	1.35	0.0836	0.0663	0.2336
95	1.76	0.1090	0.0409	0.2590
97.5	2.15	0.1332	0.0167	0.2832
99	2.62	0.1623	-0.012	0.3123
99.5	2.98	0.1846	-0.034	0.3346
99.95	4.14	0.2565	-0.106	0.4065

Test 32:Analysis of the average differencese,related with data from Annexe 32:n=17; Avg=0.18;Std=0.18

P%	t	t*s/n^0.5	dmin	dmax
60	0.535	0.0233	0.1566	0.2033
80	0.865	0.0377	0.1422	0.2177
90	1.337	0.0583	0.1216	0.2383
95	1.746	0.0762	0.1037	0.2562
97.5	2.12	0.0925	0.0874	0.2725
99	2.583	0.1127	0.0672	0.2927
99.5	2.921	0.1275	0.0524	0.3075
99.95	4.015	0.1752	0.0047	0.3552

Test 33: Analysis of average differences,related with data fom Annexe 33 : n=10; Avg=0.18; Std=0.21

P%	t	t*s/n^0.5	dmin	dmax
60	0.543	0.0360	0.1439	0.2160
80	0.893	0.0593	0.1206	0.2393
90	1.383	0.0918	0.0881	0.2718
95	1.833	0.1217	0.0582	0.3017
97.5	2.262	0.1502	0.0297	0.3302
99	2.821	0.1873	-0.007	0.3673
99.5	3.25	0.2158	-0.035	0.3958
99.95	4.781	0.3174	-0.137	0.4974

Test 41:Analysis of average differences related with data from Annexe 41 : n=15; Avg=0.14; Std=0.34

P%	t	t*s/n^0.5	dmin	dmax
60	0.537	0.0471	0.0928	0.1871
80	0.868	0.0761	0.0638	0.2161
90	1.345	0.1180	0.0219	0.2580
95	1.761	0.1545	-0.014	0.2945
97.5	2.145	0.1883	-0.048	0.3283
99	2.624	0.2303	-0.090	0.3703
99.5	2.977	0.2613	-0.121	0.4013
99.95	4.14	0.3634	-0.223	0.5034

Test 42: Analysis of average differences related with da from Annexe 42: n=13; Avg=0.23; Std=0.24

P%	t	t*s/n^0.5	dmin	dmax
60	0.538	0.0358	0.1941	0.2658
80	0.87	0.0579	0.1720	0.1979
90	1.35	0.0898	0.1401	0.2298
95	1.771	0.1178	0.1121	0.2578
97.5	2.16	0.1437	0.0862	0.2837
99	2.65	0.1763	0.0536	0.3163
99.5	3.012	0.2004	0.0295	0.3404
99.95	4.221	0.2809	-0.050	0.5109

Test 43. Analysis of the average differences related with data from Annexe 43: n=10; Avg=0.21; Std=0.25

P%	t	t*s/n^0.5	dmin	dmax
60	0.543	0.0429	0.1670	0.2529
80	0.893	0.0705	0.1394	0.2805
90	1.383	0.1093	0.1006	0.3193
95	1.833	0.1449	0.0650	0.3549
97.5	2.262	0.1788	0.0311	0.3888
99	2.821	0.2230	-0.013	0.4330
99.5	3.25	0.2569	-0.046	0.4669
99.95	4.781	0.3779	-0.167	0.5879

Appendix 7.

Synthesis of confidence limits for the average differences, di, between the length values of TDR traces, evaluated according the procedures from Appendix 6.

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Table 4. Confidence levels for the average differences , di, between the lenth values of TDR traces obtained by using different methods of reading

P%:	60	80	90	95	97.5	99	99.5	99.95	n	Avg	Std
dmin test:											
31	0.12	0.1	0.07	0.04	0.02	0	0	-0.1	15	0.15	0.24
32	0.16	0.14	0.12	0.1	0.08	0.07	0.05	0	17	0.18	0.18
33	0.14	0.12	0.09	0.06	0.03	0	-0.04	-0.13	10	0.18	0.21
41	0.09	0.06	0.02	-0.01	-0.05	-0.09	-0.12	-0.22	15	0.14	0.34
42	0.19	0.17	0.14	0.12	0.09	0.05	0.02	-0.05	13	0.23	0.24
43	0.17	0.13	0.1	0.07	0.03	-0.01	-0.04	-0.17	10	0.21	0.25
Avg	0.145	0.12	0.09	0.063	0.033	0.003	-0.02	-0.11	13.33	0.181	0.243
Std	0.033	0.034	0.038	0.041	0.045	0.050	0.054	0.072	2.624	0.031	0.049
Cv%	22.78	28.46	42.55	66.15	137.1	1526.	-250.	-65.2	19.68	17.23	20.22
dmax test											
31	0.18	0.2	0.23	0.26	0.28	0.31	0.33	0.41			
32	0.2	0.22	0.24	0.25	0.27	0.29	0.31	0.36			
33	0.22	0.23	0.27	0.3	0.33	0.37	0.4	0.5			
41	0.19	0.22	0.25	0.29	0.32	0.37	0.4	0.5			
42	0.27	0.2	0.22	0.26	0.28	0.32	0.34	0.51			
43	0.25	0.28	0.32	0.35	0.38	0.43	0.46	0.59			
Avg	0.218	0.225	0.255	0.285	0.31	0.348	0.373	0.478			
Std	0.032	0.026	0.033	0.034	0.038	0.047	0.051	0.074			
Cv%	14.82	11.96	12.95	11.94	12.35	13.50	13.80	15.52			

Appendix 8.

Estimation of the influence of the differences in reading TDR traces on the final moisture content.

Annexe8
Influence of differences in reading TDR traces

TDR	flat	prongs			curvedprongs			
TDR	k	VMC	MC	DifMC	k	VMC	MC	DifMC
ft		V%	wt%	%		V%	wt%	%
1	2.612	-0.04	-0.02	3.104	1.02	-0.5	-0.25	-2.46
1.25	4.081	5.934	3.082	2.843	1.54	-5.24	-2.72	2.155
1.5	5.877	11.40	5.925	2.445	2.296	-1.09	-0.56	0.566
1.75	7.995	16.11	8.371	2.309				
2	10.44	20.56	10.68	2.259	4.081	5.939	3.085	1.849
2.25	13.22	24.91	12.94	2.256	5.165	9.5	4.935	1.623
2.5	16.32	29.25	15.19	2.273	6.377	12.62	6.558	1.517
2.75	19.75	33.63	17.47	2.311	7.716	15.54	8.075	1.460
3	23.50	38.08	19.78	2.365	9.183	18.35	9.536	1.429
3.25	27.58	42.63	22.14	2.434	10.70	21.10	10.96	1.413
3.5	31.99	47.32	24.58	2.520	12.49	23.82	12.37	1.423
3.75	36.37	52.17	27.10	2.625	14.38	26.57	13.80	1.394
4	41.79	57.22	29.72		16.32	29.25	15.19	
Avg				2.479				1.124
Std				0.251				1.191
Cv%				10.14				105.9

For an average differance di=0.18 ft,the average difMC is:

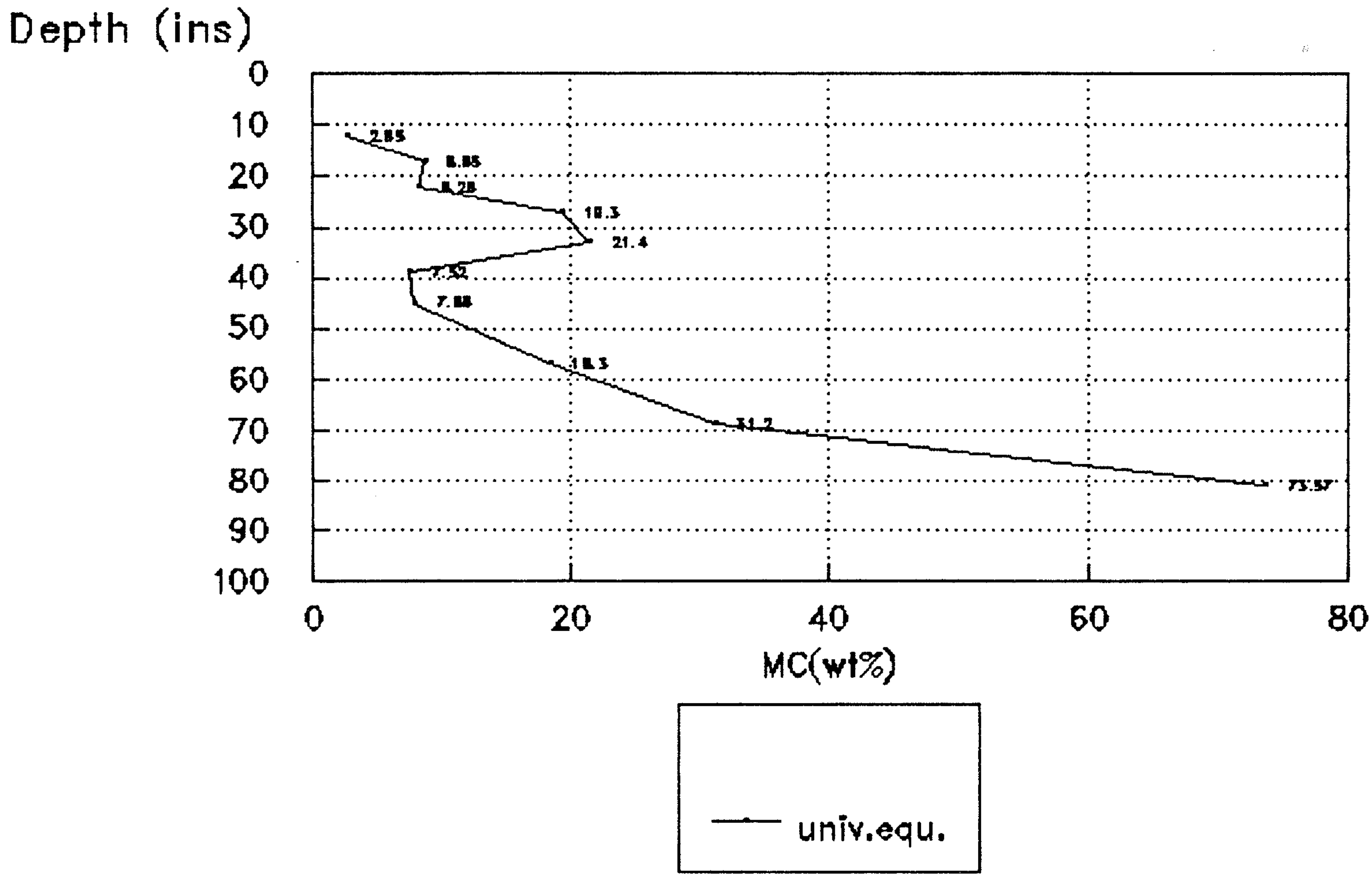
Avg	1.785	0.809

Appendix 9.

Evaluation of soil moisture contents from TDR readings, graphically determined, by Method of Diverging Lines. Worksheets and graphical representation.

Evaluation of Soil Moisture Content/TDR

Idaho,Test Section 163023,Borehole#2



February 1992

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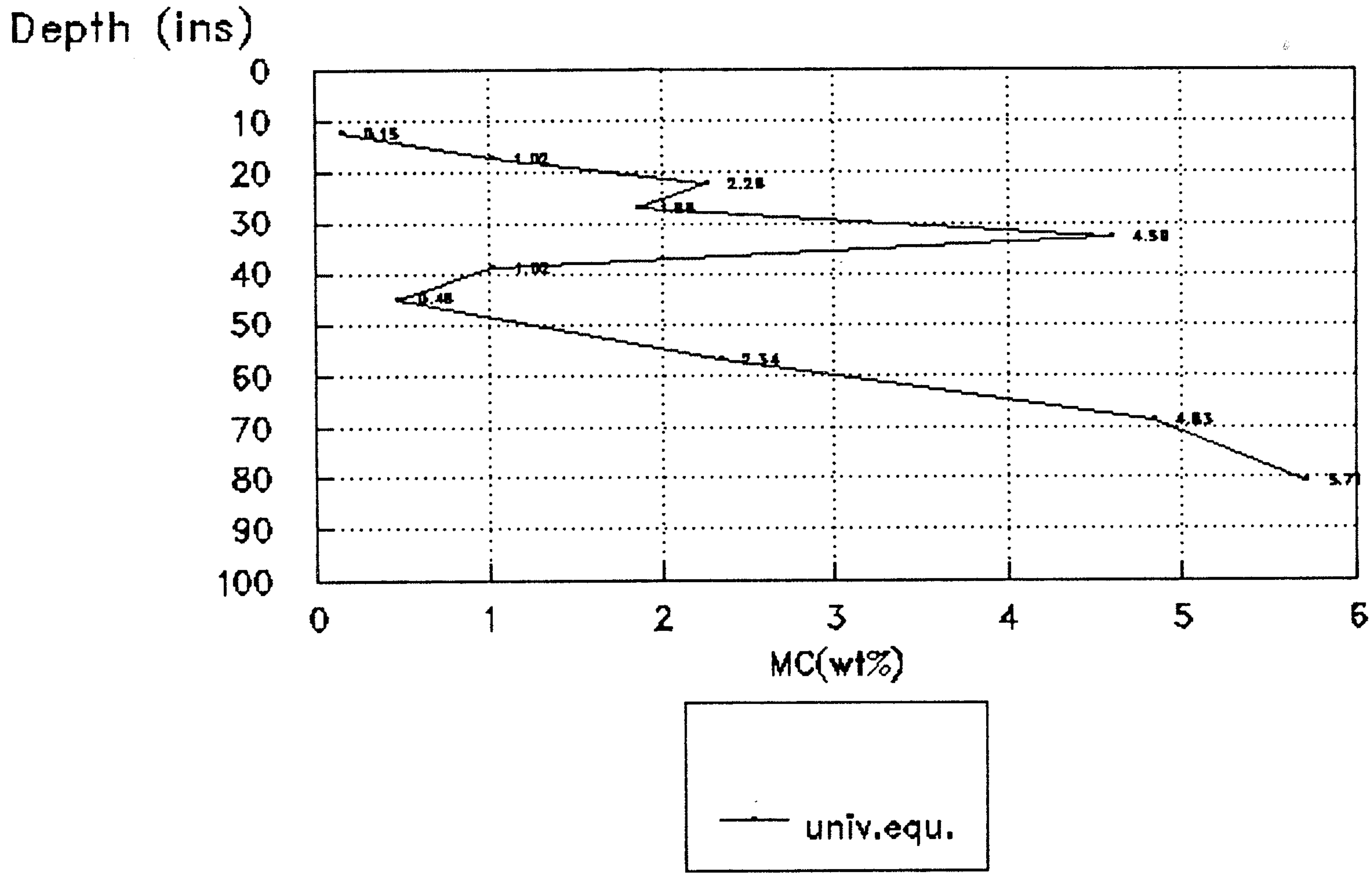
Evaluation of soil moisture content from TDR readin
Idaho,Boise,Test Section 163023,Borehole #2,February 1

Wire No.	Depth ins	Phase veloci ft/sec	Probe length ft	TDR readin ft	Dielec const. k	VMC Univers. Equation % vol.	Dry De g/cm3	MC %wt
o-01	12	0.99	0.625	1.4	3.919	5.326	2.163	2.462
		0.99	0.625	1.37	3.594	4.504	2.163	2.082
o-02	17	0.99	0.625	1.72	8.929	16.69	1.929	8.655
o-03	22	0.99	0.625	1.75	9.569	17.98	1.924	9.346
		0.99	0.625	1.65	7.562	13.82	1.924	7.184
o-04	27	0.99	0.625	2.16	22.20	37.13	1.924	19.30
o-05	33	0.99	0.625	2.25	26.14	41.13	1.921	21.41
o-06	39	0.99	0.625	1.68	8.127	15.03	1.923	7.816
		0.99	0.625	1.65	7.562	13.82	1.923	7.188
o-07	45	0.99	0.625	1.72	8.929	16.69	1.917	8.709
		0.99	0.625	1.65	7.562	13.82	1.917	7.210
o-08	57	0.99	0.625	2.12	20.60	35.28	1.927	18.30
o-09	69	0.99	0.625	2.77	60.06	64.84	1.918	33.80
		0.99	0.625	2.6	46.62	54.86	1.918	28.60
o-10	81	0.99	0.625	3.1	94.22	141.2	1.92	73.57

Evaluation of Soil Moisture Content/TDR

Idaho,Test Section 163023,Borehole#2



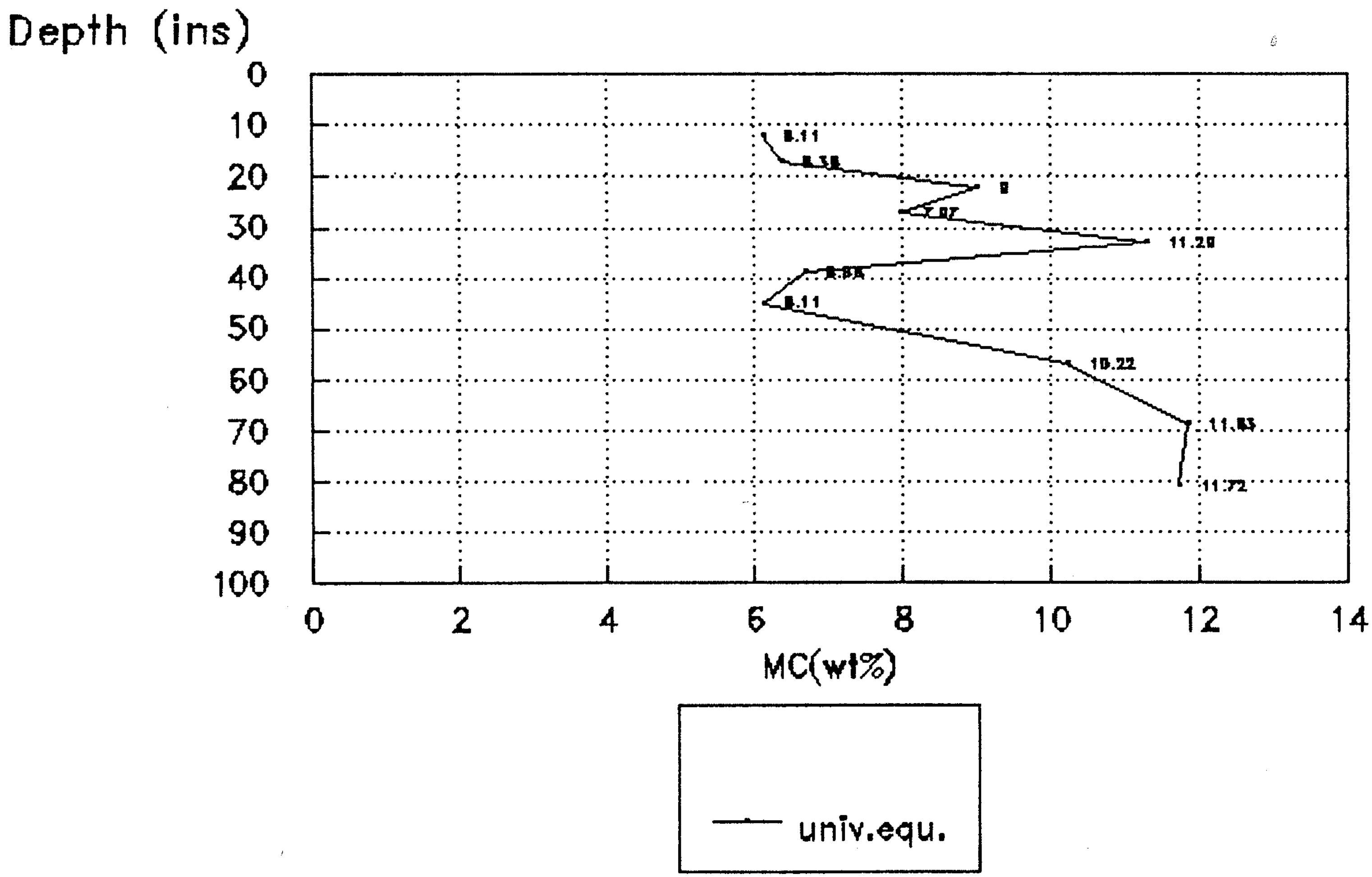
March 1992

R Evaluation of soil moisture content from TDR readings
Idaho ,Boise,Test Section 163023,Borehole #2,March 1992

Wire No.	Depth ins	Phase velocit ft/sec	Probe length ft	TDR reading ft	Dielect const. k	VMC Univers. Equation % vol.	Dry Den g/cm3	MC %wt
o-01	12.00	0.99	0.63	1.38	0.76	-3.12	2.16	-1.44
		0.99	0.63	1.40	2.00	0.32	2.16	0.15
o-02	17.00	0.99	0.63	1.60	2.61	1.96	1.93	1.02
o-03	22.00	0.99	0.63	1.82	3.38	3.96	1.92	2.06
		0.99	0.63	1.90	3.68	4.73	1.92	2.46
o-04	27.00	0.99	0.63	1.78	3.23	3.58	1.92	1.86
o-05	33.00	0.99	0.63	2.51	6.43	11.31	1.92	5.89
		0.99	0.63	2.02	4.16	5.93	1.92	3.09
o-06	39.00	0.99	0.63	1.60	2.61	1.96	1.92	1.02
o-07	45.00	0.99	0.63	1.49	2.27	1.04	1.92	0.54
		0.99	0.63	1.45	2.15	0.72	1.92	0.37
o-08	57.00	0.99	0.63	1.77	3.20	3.49	1.93	1.81
		0.99	0.63	1.98	4.00	5.53	1.93	2.87
o-09	69.00	0.99	0.00	2.40	5.88	10.05	1.92	5.24
		0.99	0.63	2.26	5.21	8.48	1.92	4.42
o-10	81.00	0.99	0.63	2.48	6.28	10.96	1.92	5.71

Evaluation of Soil Moisture Content/TDR

Idaho,Test Section 163023,Borehole#2



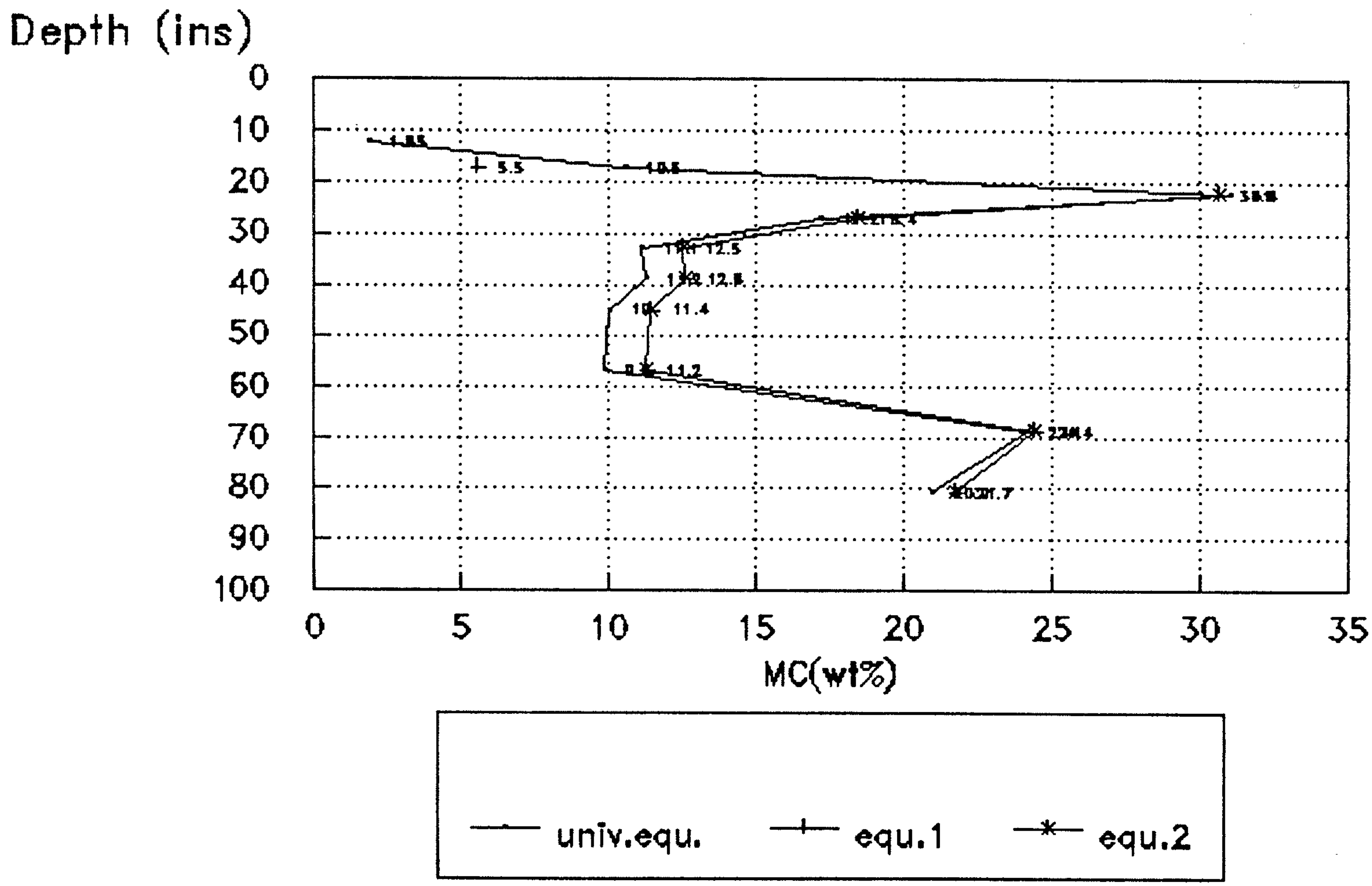
April 1992

R Evaluation of soil moisture content from TDR readings
Idaho,Boise,Test Section 163023,Borehole #2,April 1992

Wire No.	Depth ins	Phase velocit ft/sec	Probe length ft	TDR reading ft	Dielect const. k	VMC Univers. Equation % vol.	Dry Den g/cm3	MC %wt
o-01	12.00	0.99	0.63	1.67	7.28	13.22	2.16	6.11
o-02	17.00	0.99	0.63	1.62	6.85	12.27	1.93	6.36
o-03	22.00	0.99	0.63	1.88	9.23	17.31	1.92	9.00
o-04	27.00	0.99	0.63	1.78	8.28	15.34	1.92	7.97
o-05	33.00	0.99	0.63	2.10	11.52	21.69	1.92	11.29
o-06	39.00	0.99	0.63	1.65	7.11	12.84	1.92	6.68
o-07	45.00	0.99	0.63	1.59	6.60	11.71	1.92	6.11
o-08	57.00	0.99	0.63	2.00	10.45	19.69	1.93	10.22
o-09	69.00	0.99	0.63	2.15	12.07	22.69	1.92	11.83
o-10	81.00	0.99	0.63	2.14	11.96	22.49	1.92	11.72

Evaluation of Soil Moisture Content/TDR

Idaho,Test Section 163023,Borehole#3



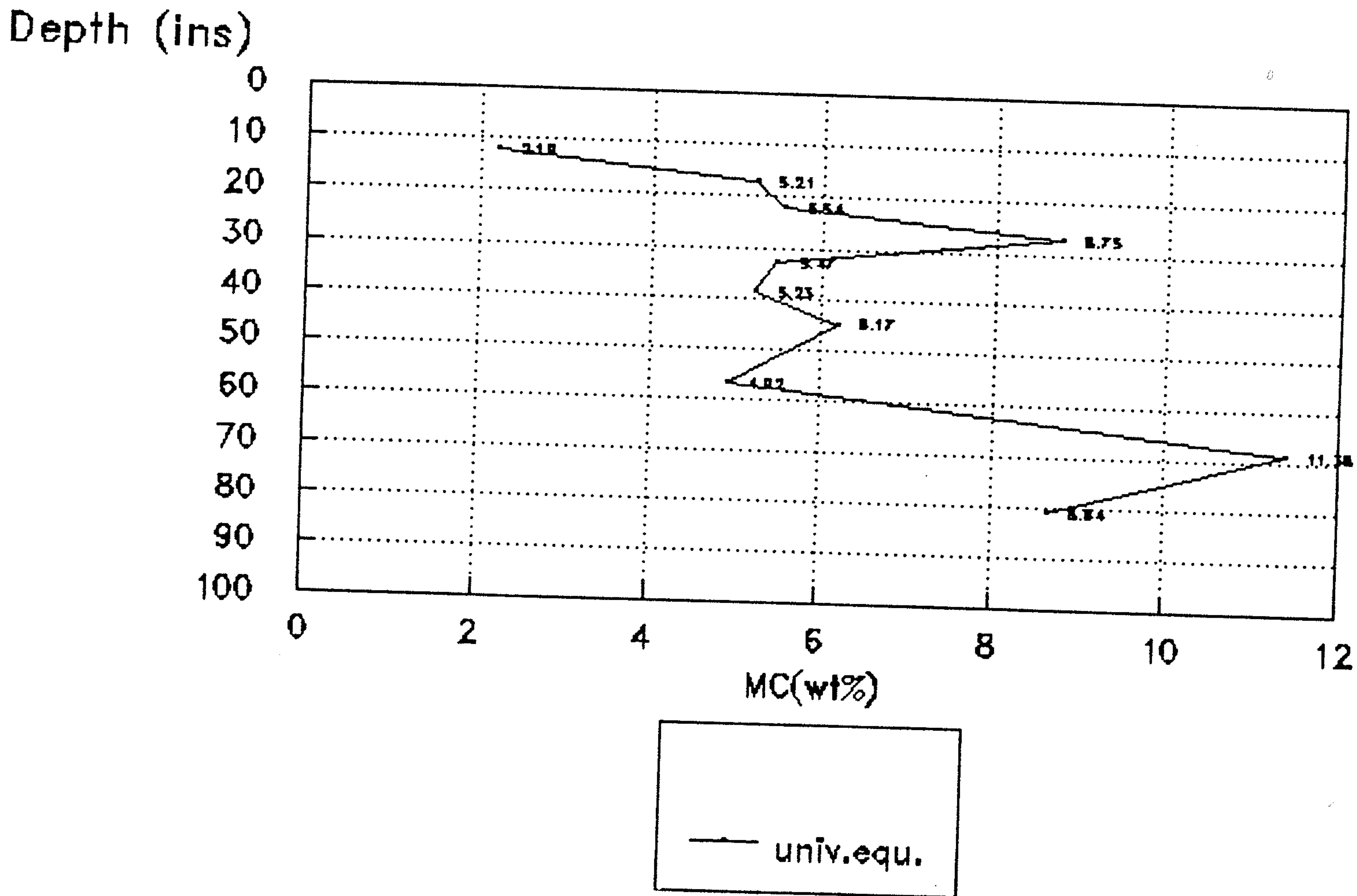
February 1992

Evaluation of soil moisture content from TDR readings
Idaho,Boise,Test Section 163023,Borehole #3,February 1992

Wire No.	Depth ins	Phase velocil ft/sec	Probe length ft	TDR readin ft	Dielec const. k	VMC Univer Equation % vol.	VMC Equ.1 %vol	VMC Equ.2 %vol	Dry De g/cm3	MC univ. equ. %wt	MC Equ.1 gran. %wt	MC Equ.2 soil %wt
o-01	12.0	1.0	1.0	1.7	2.9	2.8	-5.9		2.2	1.3	-2.70	
		1.0	1.0	1.5	2.2	0.9	-5.7		2.2	0.4	-2.64	
o-02	17.0	1.0	1.0	2.4	6.1	10.5	5.5		1.9	5.4	2.83	
		1.0	1.0	2.4	6.1	10.5	5.5		1.9	5.4	2.83	
		1.0	1.0	2.4	6.1	10.5	5.5		1.9	5.4	2.83	
o-03	22.0	1.0	1.0	4.1	17.3	31.0		30.6	1.9	16.1		15.88
o-04	27.0	1.0	1.0	3.0	9.2	17.2		18.4	1.9	8.9		9.54
o-05	33.0	1.0	1.0	2.5	6.3	11.1		12.5	1.9	5.8		6.51
		1.0	1.0	2.5	6.1	10.6		12.0	1.9	5.5		6.26
o-06	39.0	1.0	1.0	2.5	6.4	11.2		12.6	1.9	5.8		6.56
o-07	45.0	1.0	1.0	2.4	5.9	10.0		11.4	1.9	5.2		5.95
		1.0	1.0	1.3	1.6	-0.8		-5.5	1.9	-0.4		-2.87
o-08	57.0	1.0	1.0	2.4	5.8	9.8		11.2	1.9	5.1		5.79
o-09	69.0	1.0	1.0	3.6	12.9	24.1		24.4	1.9	12.5		12.71
o-10	81.0	1.0	1.0	3.3	11.1	20.9		21.7	1.9	10.9		11.28

Evaluation of Soil Moisture Content/TDR

Idaho,Test Section 163023,Borehole#3



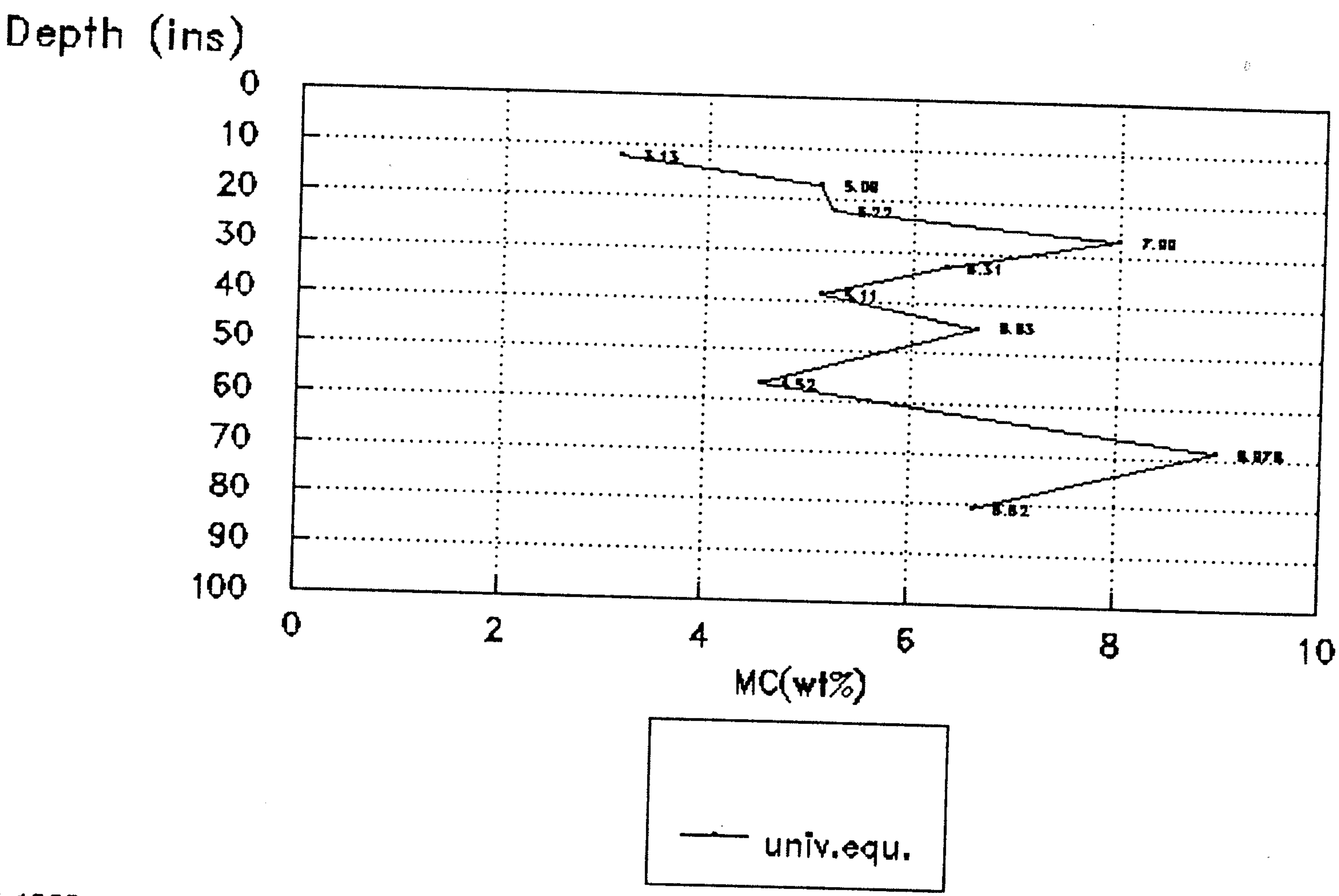
March 1992

Evaluation of soil moisture content from TDR readings
Idaho, Boise, Test Section 163023, Borehole #3, March 1992

Wire No.	Depth ins	Phase velocit ft/sec	Probe length ft	TDR reading ft	Dielect const. k	VMC Univers. Equation % vol.	Dry Den g/cm3	MC %wt
o-01	12.00	0.99	1.00	1.90	3.68	4.73	2.16	2.19
o-02	17.00	0.99	1.00	2.40	5.88	10.05	1.93	5.21
o-03	22.00	0.99	1.00	2.29	5.35	8.81	1.92	4.58
		0.99	1.00	2.58	6.79	12.13	1.92	6.32
o-04	27.00	0.99	1.00	2.97	9.00	16.84	1.92	8.75
o-05	33.00	0.99	1.00	2.48	6.28	10.96	1.92	5.71
		0.99	1.00	2.40	5.88	10.05	1.92	5.23
o-06	39.00	0.99	1.00	2.40	5.88	10.05	1.92	5.23
o-07	45.00	0.99	1.00	2.53	6.53	11.54	1.92	6.02
		0.99	1.00	2.58	6.79	12.13	1.92	6.33
o-08	57.00	0.99	1.00	2.35	5.63	9.48	1.93	4.92
o-09	69.00	0.99	1.00	3.37	11.59	21.82	1.92	11.38
o-10	81.00	0.99	1.00	2.95	8.88	16.59	1.92	8.64

Evaluation of Soil Moisture Content/TDR

Idaho,Test Section 163023,Borehole#3



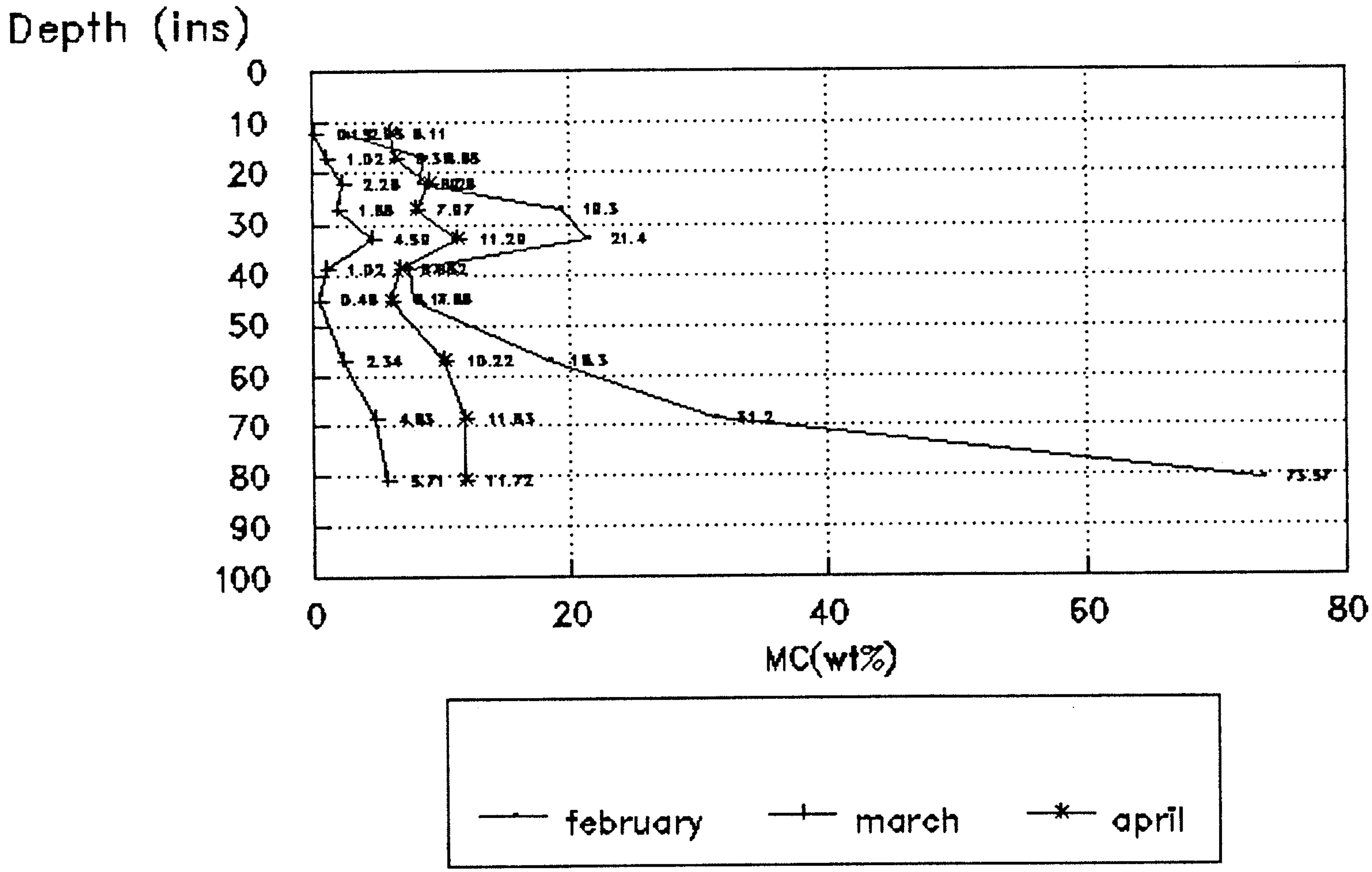
April 1992

Evaluation of soil moisture content from TDR readings
Idaho,Boise,Test Section 163023,Borehole #3,April 1992

Wire No.	Depth ins	Phase velocit ft/sec	Probe length ft	TDR reading ft	Dielect const. k	VMC Univers. Equation % vol.	Dry Den g/cm3	MC %wt
o-01	12.00	0.99	1.00	2.10	4.50	6.76	2.16	3.13
o-02	17.00	0.99	1.00	2.38	5.78	9.82	1.93	5.09
o-03	22.00	0.99	1.00	2.40	5.88	10.05	1.92	5.22
o-04	27.00	0.99	1.00	2.85	8.29	15.37	1.92	7.99
o-05	33.00	0.99	1.00	2.58	6.79	12.13	1.92	6.31
o-06	39.00	0.99	1.00	2.38	5.78	9.82	1.92	5.11
o-07	45.00	0.99	1.00	2.63	7.06	12.72	1.92	6.63
o-08	57.00	0.99	1.00	2.28	5.30	8.70	1.93	4.52
o-09	69.00	0.99	1.00	3.00	9.18	17.21	1.92	8.97
o-10	81.00	0.99	1.00	2.63	7.06	12.72	1.92	6.62

Evaluation of Soil Moisture Content/TDR

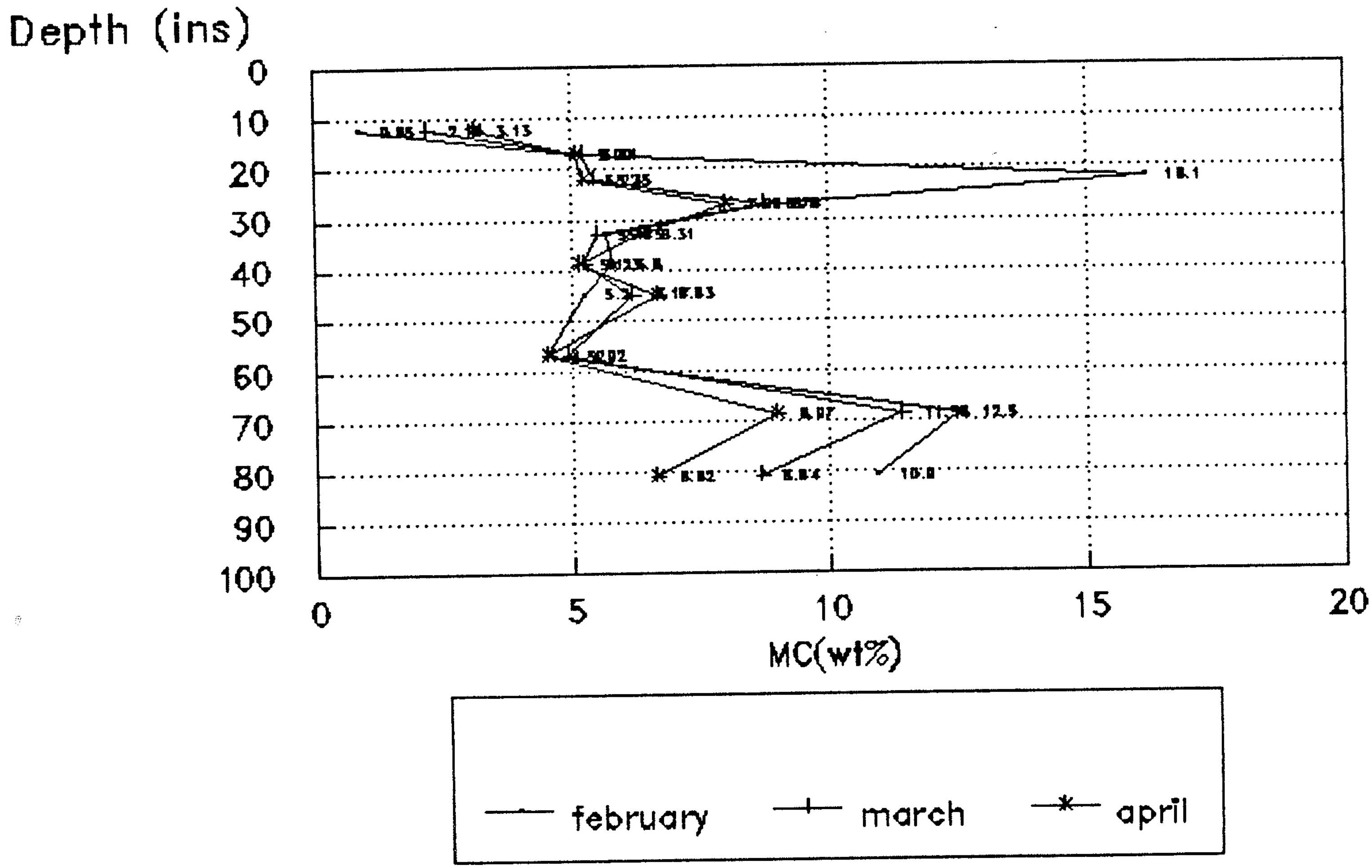
Idaho,Test Section 163023,Borehole#2



February, March, April

Evaluation of Soil Moisture Content/TDR

Idaho,Test Section 163023,Borehole #3



February, March, April

Seasonal "average " moisture content under road pavements

Depth Ins	Borehole #2 February	Borehole #2 March	Borehole #2 April	Borehole #3 February	Borehole #3 March	Borehole #3 April
12	2.65	0.15	6.11	0.85	2.19	3.13
17	8.65	1.02	6.36	5.4	5.21	5.09
22	8.26	2.26	9	16.1	5.45	5.22
27	19.3	1.86	7.97	8.9	8.75	7.99
33	21.41	4.59	11.29	5.65	5.47	6.31
39	7.5	1.02	6.68	5.8	5.23	5.11
45	7.86	0.46	6.11	5.2	6.17	6.63
57	18.3	2.34	10.22	5.1	4.92	4.52
69	31.2	4.83	11.83	12.5	11.38	8.97
81	73.57	5.71	11.72	10.9	8.64	6.62
Sum	198.7	24.24	87.29	76.4	63.41	59.59
Avg	19.9	2.4	8.7	7.6	6.3	5.9
Std	19.7	1.9	2.3	4.2	2.4	1.6
Cv%	98.9	79.1	26.4	55.3	38.1	27.1

Appendix 10.

Instructions for use of Lotus-based spreadsheet routine for calculation of volumetric moisture content (VMC) from TDR experimental data using various empirical equations.

Using the commercially available spreadsheet program Lotus 1-2-3 as a framework, a computer-aided process was developed for determining volumetric moisture content (X) using:

Option 1: Paterson's Gravel Equation

$$Ka = (L/VP)^2$$

$$X^3 - (198.8/417.3)X^2 - (30.08/417.3)X - (3.91\text{-}Ka)/417.3 = 0$$

Option 2: Topp's Universal Equation

$$Ka = (L/VP)^2$$

$$X = -0.053 + (0.0292)Ka - (0.00055)Ka^2 + (0.0000043)Ka^3$$

Option 3: Paterson's Soil Equation

$$K = (L/VP)^2$$

$$X^3 - (146/76.7)X^2 - (9.3/76.7)X - (3.03\text{-}K)/76.7 = 0$$

where L is the trace length, V is the phase velocity, P is the probe length, and $0>X>1$.

In this process, the user enters in the appropriate spreadsheet cells the trace length, the phase velocity, and the probe length. The resulting values of Ka and X (or Theta in the spreadsheet printout) appear immediately in their assigned cells.

Figure A10.1 shows a typical application of this process with a trace length of 2.200 feet, a phase velocity of 0.990 feet/second, and a probe length of 0.625 feet. Option 1 is a direct algebraic solution of Topp's Universal Equation and produces a dialectric constant of 12.642 and a volumetric moisture content of 0.237.

Option 2 is a solution of Paterson's gravel equation based on Newton's method[12], which seeks a solution of the equation $y = f(x) = 0$. Paired values of x and y are calculated from this equation and tabulated in columns E and F. The pairs in these columns are examined to select, as an initial approximation of a solution, a positive value of x for which y changes sign. In this example, $x = +0.2$ has been selected as the initial value and appears at the head of a column of figures in column G. An examination of this column, which tabulates succeeding trial values of x in this iterative method, shows that convergence to a volumetric moisture content of 0.169 is achieved in four cycles.

Option 3 is a solution of Paterson's soil equation also based on Newton's method. It follows the same form as Option 2 and shows convergence in five cycles to a volumetric moisture content of 0.240.

In Options 2 and 3, if the values of x in column G do not converge, the user can enter a different starting value at the head of column E, check succeeding values, and repeat the process (i.e., enter a new starting value) until convergence is achieved.

"TDR01": A VMC CALCULATOR BY A. ROBERT RAAB, SHRP SENIOR STAFF OFFICER

OPTION 1: SOIL VMC CALCULATION USING PETERSON'S GRAVEL EQUATION

		FUNCTION PLOT		SOLUTION BY NEWTON'S METHOD		
		x	y	X=Theta	Y=f(X)	dY/dX
INPUT		-0.50	-0.19	0.2000000000	-0.0045473872	-0.1426407860
Trace L=	2.200	-0.40	-0.09	0.1681200072	0.0000932223	-0.1474727812
Ph Vel=	0.990	-0.30	-0.03	0.1687521398	0.0000000114	-0.1474362283
Probe L=	0.625	-0.20	0.01	0.1687522173	0.0000000000	-0.1474362237
OUTPUT		-0.10	0.02	0.1687522173	-0.0000000000	-0.1474362237
Ka=	12.642	0.00	0.02	0.1687522173	-0.0000000000	-0.1474362237
		0.10	0.01	0.1687522173	-0.0000000000	-0.1474362237
SOLUTION		0.20	-0.00	0.1687522173	-0.0000000000	-0.1474362237
Theta=	0.169	0.30	-0.02	0.1687522173	-0.0000000000	-0.1474362237
Theta %=	16.88	0.40	-0.02	0.1687522173	-0.0000000000	-0.1474362237
		0.50	-0.01	0.1687522173	-0.0000000000	-0.1474362237
CHECK		0.60	0.02	0.1687522173	-0.0000000000	-0.1474362237
Kac=	12.642	0.70	0.08	0.1687522173	-0.0000000000	-0.1474362237
Kac-Ka=	0.000	0.80	0.17	0.1687522173	-0.0000000000	-0.1474362237
		0.90	0.30	0.1687522173	-0.0000000000	-0.1474362237
		1.00	0.47	0.1687522173	-0.0000000000	-0.1474362237

OPTION 2: SOIL VMC CALCULATION USING TOPP'S UNIVERSAL EQUATION

```
INPUT
Trace L=  2.200
 Ph Vel=  0.990
Probe L=  0.625

OUTPUT
     Ka= 12.642
  Theta= 0.2369
Theta %= 23.693
```

OPTION 3: SOIL VMC CALCULATION USING PETERSON'S SOIL EQUATION

		TRACE DATA		SOLUTION BY NEWTON'S METHOD		
INPUT		x	y	X=Theta	Y=f(X)	dY/dX
Trace L=	2.200	-0.50	-0.41	0.2000000000	0.0329279701	-0.7626597132
Ph Vel=	0.990	-0.40	-0.19	0.2431751796	-0.0023494043	-0.8696268631
Probe L=	0.625	-0.30	-0.04	0.2404735562	-0.0000085884	-0.8632615838
		-0.20	0.07	0.2404636074	-0.0000000001	-0.8632380625
OUTPUT		-0.10	0.12	0.2404636073	-0.0000000000	-0.8632380622
Ka=	12.642	0.00	0.13	0.2404636073	-0.0000000000	-0.8632380622
		0.10	0.10	0.2404636073	-0.0000000000	-0.8632380622
SOLUTION		0.20	0.03	0.2404636073	-0.0000000000	-0.8632380622
Theta=	0.240	0.30	-0.06	0.2404636073	-0.0000000000	-0.8632380622
Theta %=	24.046	0.40	-0.16	0.2404636073	-0.0000000000	-0.8632380622
		0.50	-0.29	0.2404636073	-0.0000000000	-0.8632380622
CHECK		0.60	-0.42	0.2404636073	-0.0000000000	-0.8632380622
Kac=	12.642	0.70	-0.55	0.2404636073	-0.0000000000	-0.8632380622
Kac-Ka=	0.000	0.80	-0.68	0.2404636073	-0.0000000000	-0.8632380622
		0.90	-0.80	0.2404636073	-0.0000000000	-0.8632380622
		1.00	-0.90	0.2404636073	-0.0000000000	-0.8632380622

Time Conti...

Soil Materials

(2) $K_a = 3.03 + 9.3\,\theta_v + 146\,\theta_v^2 - 76.7\,\theta_v^3$ from Stott, Par

Ka	Theta	Ka	Theta	Ka	Theta	Ka	Theta	Ka	T
3.030	0.000	3.049	0.002	3.070	0.004	3.091	0.006	3.114	0
3.138	0.010	3.162	0.012	3.189	0.014	3.216	0.016	3.244	0
3.274	0.020	3.304	0.022	3.336	0.024	3.369	0.026	3.403	0
3.438	0.030	3.475	0.032	3.512	0.034	3.550	0.036	3.590	0
3.631	0.040	3.672	0.042	3.715	0.044	3.759	0.046	3.804	0
3.850	0.050	3.898	0.052	3.946	0.054	3.995	0.056	4.046	0
4.097	0.060	4.156	0.062	4.203	0.064	4.258	0.066	4.313	0
4.370	0.070	4.428	0.072	4.487	0.074	4.546	0.076	4.607	0
4.669	0.080	4.732	0.082	4.796	0.084	4.861	0.086	4.927	0
4.994	0.090	5.062	0.092	5.131	0.094	5.200	0.096	5.271	0
5.343	0.100	5.416	0.102	5.490	0.104	5.565	0.106	5.641	0
5.718	0.110	5.795	0.112	5.874	0.114	5.954	0.116	6.034	0
6.116	0.120	6.198	0.122	6.282	0.124	6.366	0.126	6.452	0
6.538	0.130	6.625	0.132	6.713	0.134	6.802	0.136	6.892	0
6.983	0.140	7.075	0.142	7.168	0.144	7.261	0.146	7.356	0
7.451	0.150	7.547	0.152	7.645	0.154	7.743	0.156	7.842	0
7.941	0.160	8.042	0.162	8.144	0.164	8.246	0.166	8.349	0
8.454	0.170	8.559	0.172	8.664	0.174	8.771	0.176	8.879	0
8.987	0.180	9.096	0.182	9.206	0.184	9.317	0.186	9.429	0
9.542	0.190	9.655	0.192	9.769	0.194	9.884	0.196	10.000	0
10.116	0.200	10.234	0.202	10.352	0.204	10.471	0.206	10.591	0
10.711	0.210	10.833	0.212	10.955	0.214	11.078	0.216	11.201	0
11.326	0.220	11.451	0.222	11.577	0.224	11.704	0.226	11.831	0
11.959	0.230	12.088	0.232	12.218	0.234	12.348	0.236	12.479	0
12.611	0.240	12.744	0.242	12.877	0.244	13.011	0.246	13.146	0
13.282	0.250	13.418	0.252	13.555	0.254	13.692	0.256	13.831	0
13.970	0.260	14.109	0.262	14.250	0.264	14.391	0.266	14.532	0
14.675	0.270	14.818	0.272	14.962	0.274	15.106	0.276	15.251	0
15.397	0.280	15.543	0.282	15.690	0.284	15.838	0.286	15.985	0
16.135	0.290	16.285	0.292	16.435	0.294	16.586	0.296	16.737	0
16.889	0.300	17.042	0.302	17.195	0.304	17.349	0.306	17.504	0
17.659	0.310	17.814	0.312	17.971	0.314	18.128	0.316	18.285	0
18.443	0.320	18.602	0.322	18.761	0.324	18.921	0.326	19.081	0
19.242	0.330	19.404	0.332	19.566	0.334	19.728	0.336	19.891	0
20.055	0.340	20.219	0.342	20.384	0.344	20.549	0.346	20.715	0
20.881	0.350	21.048	0.352	21.216	0.354	21.384	0.356	21.552	0
21.721	0.360	21.891	0.362	22.060	0.364	22.231	0.366	22.402	0
22.573	0.370	22.745	0.372	22.918	0.374	23.091	0.376	23.264	0
23.439	0.380	23.612	0.382	23.787	0.384	23.962	0.386	24.138	0
24.314	0.390	24.490	0.392	24.667	0.394	24.845	0.396	25.023	0
25.201	0.400	25.380	0.402	25.559	0.404	25.739	0.406	25.919	0
26.099	0.410	26.280	0.412	26.462	0.414	26.643	0.416	26.825	0
27.008	0.420	27.191	0.422	27.374	0.424	27.558	0.426	27.742	0
27.926	0.430	28.111	0.432	28.296	0.434	28.482	0.436	28.668	0
28.854	0.440	29.041	0.442	29.228	0.444	29.415	0.446	29.603	0
29.791	0.450	29.979	0.452	30.168	0.454	30.357	0.456	30.546	0
30.736	0.460	30.926	0.462	31.116	0.464	31.307	0.466	31.498	0
31.689	0.470	31.881	0.472	32.073	0.474	32.265	0.476	32.457	0
32.650	0.480	32.843	0.482	33.036	0.484	33.230	0.486	33.424	0
33.618	0.490	33.812	0.492	34.007	0.494	34.202	0.496	34.397	0
34.593	0.500	34.788	0.502	34.984	0.504	35.180	0.506	35.377	0
35.575	0.510	35.770	0.512	35.967	0.514	36.165	0.516	36.362	0
36.566	0.520	36.758	0.522	36.956	0.524	37.154	0.526	37.353	0
37.552	0.530	37.750	0.532	37.950	0.534	38.149	0.536	38.348	0
38.548	0.540	38.748	0.542	38.949	0.544	39.148	0.546	39.349	0
39.549	0.550	39.750	0.552	39.951	0.554	40.151	0.556	40.353	0
40.554	0.560	40.755	0.562	40.957	0.564	41.158	0.566	41.366	0
41.562	0.570	41.764	0.572	41.966	0.574	42.168	0.576	42.371	0
42.573	0.580	42.776	0.582	42.979	0.584	43.181	0.586	43.384	0
43.587	0.590	43.790	0.592	43.993	0.594	44.196	0.596	44.400	0

Granular Materials

$K_a \doteq 3.91 + 30.08\theta_v + 198.8\theta_v^2 - 417.3\theta_v^3$

Ka	Theta	Ka	Theta	Ka	Theta	Ka	Theta	Ka	Theta
3.910	0.000	3.971	0.002	4.033	0.004	4.098	0.006	4.163	0.008
4.230	0.010	4.299	0.012	4.369	0.014	4.440	0.016	4.513	0.018
4.588	0.020	4.664	0.022	4.741	0.024	4.819	0.026	4.899	0.028
4.980	0.030	5.062	0.032	5.146	0.034	5.231	0.036	5.317	0.038
5.405	0.040	5.493	0.042	5.583	0.044	5.674	0.046	5.766	0.048
5.859	0.050	5.953	0.052	6.048	0.054	6.145	0.056	6.242	0.058
6.340	0.060	6.440	0.062	6.540	0.064	6.641	0.066	6.743	0.068
6.847	0.070	6.951	0.072	7.055	0.074	7.161	0.076	7.268	0.078
7.375	0.080	7.483	0.082	7.592	0.084	7.702	0.086	7.812	0.088
7.923	0.090	8.035	0.092	8.148	0.094	8.261	0.096	8.374	0.098
8.489	0.100	8.604	0.102	8.719	0.104	8.835	0.106	8.952	0.108
9.069	0.110	9.186	0.112	9.304	0.114	9.423	0.116	9.542	0.118
9.661	0.120	9.781	0.122	9.901	0.124	10.021	0.126	10.142	0.128
10.263	0.130	10.385	0.132	10.506	0.134	10.628	0.136	10.750	0.138
10.873	0.140	10.995	0.142	11.118	0.144	11.241	0.146	11.364	0.148
11.487	0.150	11.610	0.152	11.733	0.154	11.856	0.156	11.980	0.158
12.103	0.160	12.226	0.162	12.349	0.164	12.473	0.166	12.596	0.168
12.719	0.170	12.842	0.172	12.964	0.174	13.087	0.176	13.210	0.178
13.332	0.180	13.454	0.182	13.576	0.184	13.697	0.186	13.819	0.188
13.940	0.190	14.060	0.192	14.181	0.194	14.301	0.196	14.420	0.198
14.540	0.200	14.658	0.202	14.777	0.204	14.895	0.206	15.012	0.208
15.129	0.210	15.246	0.212	15.362	0.214	15.477	0.216	15.592	0.218
15.706	0.220	15.820	0.222	15.933	0.224	16.045	0.226	16.157	0.228
16.268	0.230	16.378	0.232	16.487	0.234	16.596	0.236	16.704	0.238
16.811	0.240	16.918	0.242	17.023	0.244	17.128	0.246	17.232	0.248
17.335	0.250	17.437	0.252	17.538	0.254	17.638	0.256	17.737	0.258
17.835	0.260	17.932	0.262	18.028	0.264	18.124	0.266	18.218	0.268
18.310	0.270	18.402	0.272	18.493	0.274	18.582	0.276	18.671	0.278
18.758	0.280	18.844	0.282	18.928	0.284	19.012	0.286	19.094	0.288
19.175	0.290	19.254	0.292	19.332	0.294	19.409	0.296	19.485	0.298
19.559	0.300	19.632	0.302	19.703	0.304	19.773	0.306	19.841	0.308
19.908	0.310	19.973	0.312	20.037	0.314	20.099	0.316	20.160	0.318
20.219	0.320	20.276	0.322	20.332	0.324	20.386	0.326	20.438	0.328
20.489	0.330	20.538	0.332	20.586	0.334	20.631	0.336	20.675	0.338
20.717	0.340	20.757	0.342	20.795	0.344	20.832	0.346	20.867	0.348
20.899	0.350	20.930	0.352	20.959	0.354	20.986	0.356	21.011	0.358