

Rutting Prediction in Asphalt Concrete Pavements

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This state-of-the-art paper summarizes a number of procedures to either limit rutting to some specific amount or to estimate the expected quantity from repetitive traffic loading. Two methods are suggested for estimating permanent deformation from long-term loading. From a design standpoint, the procedure for limiting the rutting to some prescribed amount that is based on a limiting subgrade strain criterion is the procedure that could be used with more confidence since these criteria have been developed from analyses of existing design procedures and field trials. On the other hand, the methodology described for the estimation of the actual amount of rutting from laboratory repeated load or creep tests and either elastic or viscoelastic layered theory will require field documentation before it can be used with confidence. Nevertheless, such procedures can be used to compare alternatives in design by making rutting estimates for pavements made of different materials.

The distortion (or permanent deformation) mode of distress in asphalt pavements that results from both traffic- and non-traffic-associated causes is summarized below.

General Cause	Specific Causative Factor	Example of Distress
Traffic-associated	Single or comparatively few excessive loads	Plastic flow (shear distortion)
	Long-term (or static) load	Creep (time dependent) deformation
Non-traffic-associated	Repetitive traffic loading (generally a large number of repetitions)	Rutting (resulting from accumulation of the small permanent deformations associated with passage of wheel loads)
	Expansive subgrade soil	Swell or shrinkage
	Compressible material underlying pavement structure	Consolidation settlement
	Frost-susceptible material	Heave (particularly differential amounts)

(In this paper only the distortion from traffic-associated

causes is discussed.) This mode of distress can result from a single or a comparatively few excessive loads.

The available methodology permits the prediction of rutting resulting either from a specific number of repeated loads or from long-term loading. Few attempts have been made to predict the rutting resulting from the plastic flow or shear distortion associated with a single or comparatively few excessive loads since the concern of the engineer has been to design the material to resist such loads by using materials designs predicated on shear strength characteristics (1, 2). This approach has also been applied to the design of pavements to resist rutting from repetitive traffic loading. Tests such as the CBR (3), the R-value (4), or triaxial compression are widely used to determine the Mohr-Coulomb parameters ϕ and C under specific conditions (e.g., the specific time of loading and the temperature for asphalt concrete) (1, 5). Such methodology has been summarized by Monismith and Salam (6).

The following sections present a summary of existing procedures that permit the estimation of rutting from repetitive and creep loading, as well as guidelines for limiting strain values at the subgrade level. The general framework in which such estimates can be made is shown in Figure 1. This discussion will be concentrated primarily on blocks 7 through 11 since it is assumed that a pavement section will be available for analysis that has been designed by an existing procedure (e.g., the California R-value or the Corps of Engineers CBR).

RUTTING FROM REPETITIVE TRAFFIC LOADING

Two approaches are available for the study of rutting from repeated traffic loading. In one method the vertical compressive strain at the subgrade surface is limited to some tolerable amount associated with a specific number of load repetitions [e.g., Dorman and Metcalf (7)]. By controlling the characteristics of the materials in the pavement section through materials design and proper construction procedures and using materials of adequate stiffness and sufficient thickness so that this strain level is not exceeded, permanent deformation equal to or less than some prescribed amount is ensured.

The second procedure involves an estimation of the actual amount of rutting that might occur by using appropriate materials characterization information and an analysis procedure that assumes the pavement structure to be represented as a layered elastic or viscoelastic system.

Limiting Subgrade Strain Criteria

Three criteria for limiting subgrade strain in highway pavements that can be used to ensure that permanent deformation in the subgrade does not lead to excessive rutting at the pavement surface are discussed in this section and listed in Table 1. The first of these (7) was developed by elastic analyses of pavements designed according to the CBR procedure and for the AASHTO Road Test. The criteria listed in Table 2 are associated with ultimate rut depths of about 19 mm ($\frac{3}{4}$ in).

The second criterion suggested, also shown in Table 1 (8), was developed by examining pavement structures designed according to the California design procedure. These strain values are less than those suggested in the first; it is possible, however, that smaller limiting values of permanent deformation occur in pavements designed by the California procedure.

Hicks and Finn (9) have analyzed the various sections of the San Diego test road in the same manner. The results of their analyses are shown in Figure 2. These values are more conservative than those (also shown in Figure 2) obtained from an analysis of pavements designed according to the California procedure. In this test road, however, relatively small values, generally less than 7.5 mm (0.3 in), of permanent deformation were being obtained at the time that the criteria were developed.

The criteria developed by Witczak (10) for airfield pavements, based in part on the analysis of field trials conducted by the U.S. Army Corps of Engineers Waterways Experiment Station, are appropriate (11). These criteria are also summarized in Table 1. These values are substantially higher than those associated with highway pavements.

In using these criteria, the pavement is analyzed as a layered elastic structure. Appropriate values for the properties of each of the layers are used, and the environmental influences resulting from temperature and moisture conditions associated with specific locales are considered.

Rutting Estimation From Repeated Traffic Loading

A number of procedures are available for the estimation of the amount of rutting from repeated traffic loading. They may be categorized as

1. The use of elastic layered systems to represent the pavement structure with materials characterization by repeated load triaxial compression tests or creep tests (particularly for asphalt-bound layers), and
2. The use of viscoelastic layered systems to represent the pavement structure with materials characterization by means of creep tests.

Representation of Pavement as an Elastic Layered System

A number of investigators (12, 13, 14) have suggested that a pavement may be represented as a layered elastic system in the determination of the state of stress or strain resulting from a surface loading. The amount of rutting can then be estimated for some specified number of load

repetitions with the use of an appropriate constitutive relationship.

The use of this type of analysis requires the relations between plastic strain and applied stress for each of the pavement components, i.e.,

$$\epsilon^p = f(\sigma_{ij}) \quad (1)$$

where

$$\begin{aligned} \epsilon^p &= \text{plastic or permanent strain, and} \\ \sigma_{i,j} &= \text{stress state.} \end{aligned}$$

It is then possible to estimate the permanent deformation occurring in each particular layer by computing the permanent strain at a sufficient number of points within the layer to reasonably define the strain variation with depth. The permanent deformation is the sum of the products of the average permanent strains and the corresponding differences in depths between the locations at which the strains were determined (Figure 3), i.e.,

$$\delta_i^p(x, y) = \sum_{i=1}^n (\epsilon_i^p \Delta z_i) \quad (2)$$

where

$$\begin{aligned} \delta_i^p(x, y) &= \text{rut depth in the } i\text{th position at point } (x, y) \text{ in} \\ &\text{the horizontal plane,} \\ \epsilon_i^p &= \text{average permanent strain at depth } [z_i + \\ &\quad (\Delta z_i/2)], \text{ and} \\ \Delta z_i &= \text{difference in depth.} \end{aligned}$$

The total rut depth can be estimated by summing the contributions from each layer. From the knowledge of the plastic strain at various numbers of load repetitions, the development of rutting with traffic can be estimated.

This approach has been used to predict permanent deformation in either a portion of or the total pavement structure by Barksdale (13), Morris and others (15), McLean and Monismith (16), Freeme and Monismith (17), Snaith (18), Brown and Snaith (19), Hills and others (20), van de Loo (21), and Chomton and Valayer (22).

The method of using repeated load triaxial compression tests has been used by Brown and others (23) and Monismith and others (24) on subgrade type materials. The water content, dry density, stress state, and number of load applications all influence the development of permanent strain. The results of repeated load testing can be expressed in equations of the form

$$\epsilon^p = AN^b \quad (3)$$

$$\Delta\sigma = \epsilon^p / (\ell + m\epsilon^p) \quad (4)$$

where

$$\begin{aligned} \epsilon^p &= \text{permanent strain,} \\ \Delta\sigma &= \text{applied stress,} \\ N &= \text{number of stress applications, and} \\ A, b, \ell, \text{ and } m &= \text{experimentally determined coefficients.} \end{aligned}$$

For granular materials, Barksdale (13) has developed considerable data, which can be represented by an equation of the following form:

$$\begin{aligned} \bar{\epsilon}^p / \bar{\sigma} &= (1/K\sigma_3^n) \left\{ 1 - [\bar{\sigma}R_r(1 - \sin\phi)] / [2(C\cos\phi + \sigma_3 \sin\phi)] \right\} \\ &\quad \times (N/N_0)^m \end{aligned} \quad (5)$$

where

Figure 1. Block diagram of a distortion subsystem.

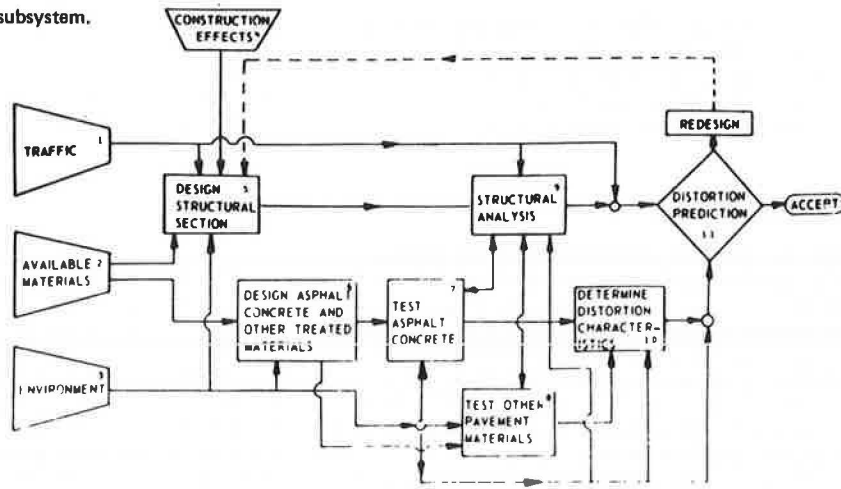


Figure 2. ϵ -N relation derived from panel ratings for all base types.

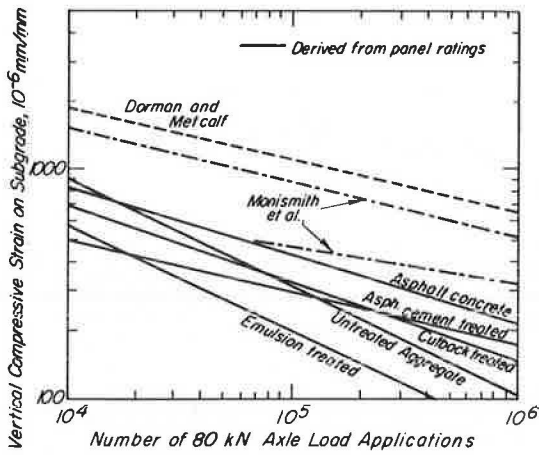


Figure 3. Schematic representation of pavement system used to estimate permanent deformation.

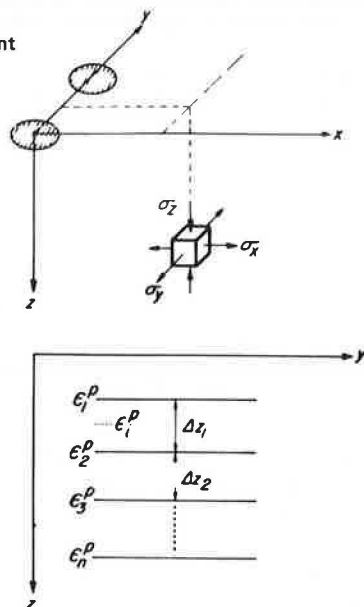


Table 1. Allowable subgrade compressive strain values corresponding to different load applications.

Load Applications (MPa)	Compressive Strain on Subgrade (mm/mm)		
	Highway Pavements		Airfield Pavements
	Dorman and Metcalf (7)	Monismith and McLean (8)	Witezak (10)
6.9			19.2×10^{-4}
69			16.8×10^{-4}
690	1.05×10^{-3}	8.0×10^{-4}	15.2×10^{-4}
6 900	6.5×10^{-4}	4.8×10^{-4}	14.6×10^{-4}
69 000	4.2×10^{-4}	2.9×10^{-4}	
690 000	2.6×10^{-4}	1.7×10^{-4}	

Note: 1 mm = 0.039 in; 1 Pa = 0.000 145 lb/in².

- $K\sigma_3^a$ = relationship defining the initial tangent modulus as a function of confining pressure,
- C = cohesion,
- ϕ = angle of internal friction,
- R_r = constant relating compressive strength to an asymptotic stress difference; $0.75 \leq R_r \leq 1$,

and these parameters are determined at a specific number of stress repetitions, N_0 .

For asphalt concrete, a number of approaches have been developed by Morris and others (15), McLean and Monismith (16), Snaith (18), and Freeme and Monismith (17).

Morris and others (15) have developed regression equations for the laboratory test data over a range of temperatures and for a range of both tensile and compressive stresses. The functional form of their expressions is

$$e^p = f(\sigma_1, \sigma_3, T, N) \pm E \tag{6}$$

where E = estimate of error.

Similarly, McLean and Monismith (16) have fitted to their test data a third order polynomial of the form

$$\log e^p = C_0 + C_1 (\log N) + C_2 (\log N)^2 + C_3 (\log N)^3 \tag{7}$$

in which the influences of stress state, time of loading, and temperature are described by the coefficients C_0 , C_1 , C_2 , and C_3 .

Snaith (18) has expressed his data by an equation of the form

$$\log e^p = (a + b \log t) \tag{8}$$

where a and b are functions of temperature and applied stress and are estimated directly from the laboratory test results.

Freeme and Monismith (17) have suggested an expression in which the relationship for the vertical strain at a point in the pavement structure (Figure 3) is estimated from

$$\epsilon_z^p = \delta(T)N^\alpha \bar{\sigma}^{n-1} t[\sigma_z - \frac{1}{2}(\sigma_x + \sigma_y)] \quad (9)$$

where $\bar{\sigma}$ = equivalent stress, and is defined as $\frac{1}{2}[(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$. The coefficients $\delta(T)$, α , and n must be determined experimentally.

Some attempts have been made to predict the accumulation of rutting in various pavement structures by using these constitutive relationships together with the process illustrated in Figure 3.

Morris and others (15) have prepared an estimate, shown in Figure 4, of the rutting occurring in a 290-cm asphalt concrete section of the Brampton Test Road as a function of time. The overall results are encouraging, as shown by the reasonable correspondence of the measured and predicted values, but suggest that much of the permanent deformation results from accumulations in the lower part of the asphalt-bound layer because of the comparatively high tensile stresses that occur near the underside of the section.

McLean and Monismith have compared the form of accumulation of permanent deformation with that observed by Hofstra and Klomp (25) in a laboratory test track (16). Although the mixtures are different, the shapes of the curves are similar, indicating the potential applicability of such a procedure. They have also examined the influence of certain designer-controlled variables on the rutting that might occur in thick asphalt-bound layers. Snaith (18) has used the same approach in his equations to predict the accumulation of rutting with load applications of the same form.

Barksdale (13) has also used this methodology to provide a relative indication of the rutting potential of different granular materials used in a particular pavement structure. His results are illustrated in Figure 5 in which the rutting potential is shown in terms of a rut index rather than in terms of the actual rutting associated with the different materials.

These analyses included studies of the influence of the asphalt concrete stiffness, the subgrade stiffness, and the layer thickness. Reducing the stiffness of the asphalt concrete by a factor of two increased the pavement deformation more than proportionally, indicating that the influence of the asphalt concrete stiffness may be substantial. The subgrade stiffness appeared to have almost no effect on the accumulation of permanent deformation in the asphalt concrete layer although it will influence the total permanent deformation at the pavement surface. For a given thickness of asphalt concrete, the total deformation will increase as the stiffness is reduced. The influence of layer thickness on permanent deformation, within the range investigated (300 to 500 mm or 12 to 20 in), was minimal.

This comparison, together with those illustrated above, indicates that the use of elastic theory for stress and strain distribution, together with a constitutive relationship determined from laboratory repeated load tests, has the potential to assist in the estimate of permanent deformation accumulation in thick asphalt-bound layers.

Freeme and Monismith (17) have estimated the amount of rutting in the layered system shown in Figure 6. The shape of the relationship with load repetition is reasonable.

An alternative approach proposed by Hills and others (20) and Chomton and Valayer (22) is the use of creep

tests on asphalt concrete, together with elastic layer theory, to represent the response of the pavement structure to load.

Observations of the development of rut depth with load repetitions in laboratory test tracks (two-layer pavements consisting of asphalt concrete resting directly on subgrade) provide data that, when suitably transformed, have the same shape as the test results for laboratory creep tests in uniaxial compression (Figure 7). The quantities in this figure are estimated as follows.

$$S_{mix} \text{ (laboratory creep)} = \sigma/\epsilon_{mix} \quad (10)$$

where

σ = applied creep stress at temperature T = constant,
 ϵ_{mix} = axial strain at particular time t , and
 S_{mix} = corresponding mix stiffness at temperature T and time t .

$$S_{bit} \text{ (laboratory creep)} = \sigma/\epsilon_{bit} \quad (11)$$

where S_{bit} = asphalt stiffness [estimated by using the procedure of Hills and others (20)].

$$S_{mix} \text{ (rutting test on field pavement)} = Z\sigma_0/[B(r/H_0)] \quad (12)$$

where

Z = f (radius of loaded area/thickness of asphalt-bound layer (H_0), $E_{subgrade}/E_{asphalt \text{ concrete}}$),
 σ_0 = tire contact pressure,
 r = total rut depth at pavement surface, and
 B = proportion of total rut depth in asphalt-bound layer.

For the S_{bit} (rutting test), only the viscous component, $(S_{bit})_v$, of S_{bit} is estimated.

$$1/(S_{bit})_v = t/3\eta \quad (13)$$

and

$$3\eta = \lim_{t \rightarrow \infty} (tS_{bit}) \quad (14)$$

for the time of loading in the rutting test $t = nt_w$, where n is number of wheel bases and t_w is time of loading for one wheel passage. At different temperature conditions this becomes

$$(S_{bit})_v = 3/[t_w \Sigma(n/\eta)_T] \quad (15)$$

The use of this methodology requires the measurement of S_{mix} in the laboratory, and the estimation of S_{bit} by the procedure of Hills and others and $(S_{bit})_v$ from knowledge of the traffic and temperature conditions and the nomographic procedure to give the rut depth, r , from $Z\sigma_0/[B(r/H_0)]$ for the specific pavement conditions. Comparisons between estimated and computed values are given in a paper by van de Loo in this Record. While the estimation procedure appears applicable to all materials, the computational one has been used only for asphalt-bound materials. This should not be considered a limitation, however, since as suggested by van de Loo (21), the creep test may become a useful mix design test to differentiate between mixtures.

Representation of Pavement as a Viscoelastic Layered System

Barksdale and Leonards (26) and Elliott and Moavenzadeh (27, 28) have suggested that rutting can be estimated by assuming that the pavement can be represented as a vis-

Figure 4. Permanent deformation versus time relationship of section 3 of Brampton Test Road.

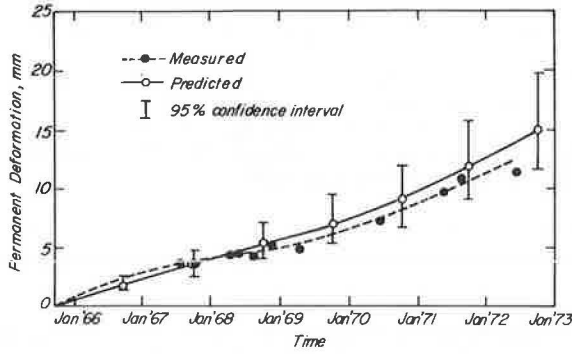


Figure 6. Estimated rutting in experimental test road: (a) total rut depth as a function of load application and (b) total rut depth at various layer interfaces.

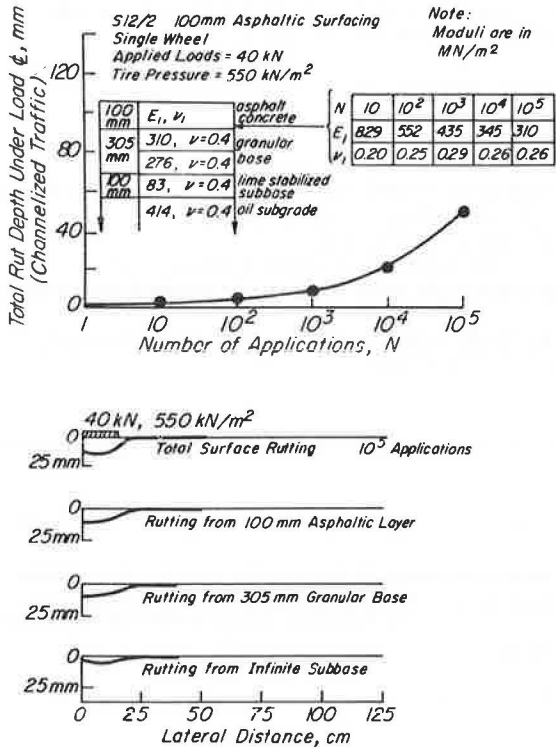
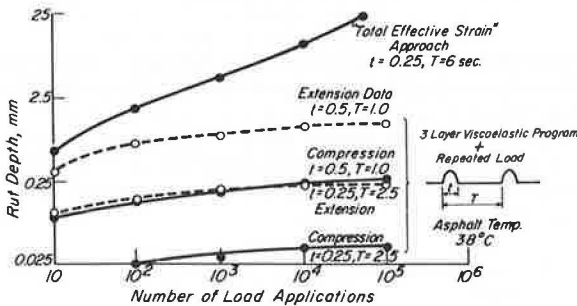


Figure 8. Comparison of rut depth predictions using the total effective strain and three-layer viscoelastic approaches.



coelastic layered system and then using creep tests to represent the response characteristics of the various materials in the pavement structure.

The accumulation of pavement deformation in the structure, for which the results of the elastic analysis were reported above (10), was examined using the FHWA Structural Subsystem (29). Compliances were determined in tension and compression and variations in the time of loading, *t*, and the total length of time before the next

Figure 5. Variation of rut index with percent fines for crushed granite gneiss bases.

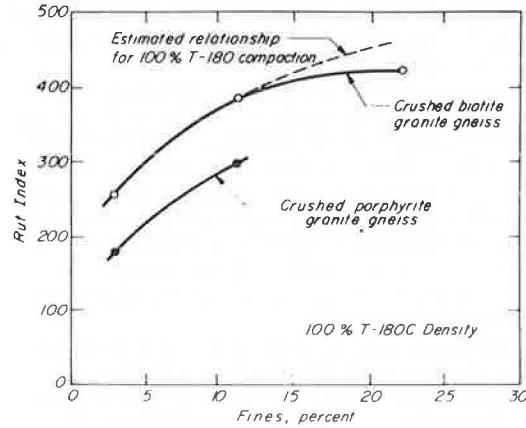


Figure 7. Creep and rutting tests on A9 test track.

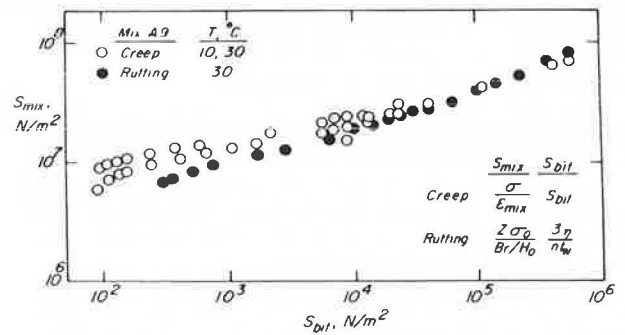
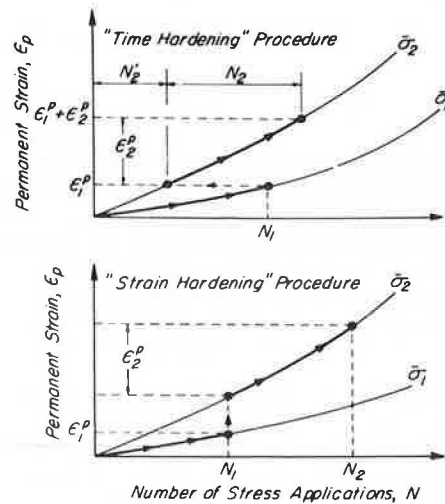


Figure 9. Procedures for the prediction of cumulative loading from simple tests.



load is applied, T (17). Figure 8 shows a summary of these determinations. The permanent deformations that were estimated by using this procedure are substantially less than those computed using the elastic analysis for essentially the same conditions. Also, the accumulation of permanent deformation with the number of load applications determined by this procedure does not show the same shape as that observed by Hofstra and Klomp (25).

Cumulative Loading Considerations

In the laboratory it is convenient to apply stresses of a single magnitude to the specific material under investigation.

Since the actual stress sequence in the field is not known, it is desirable to be able to predict the results of cumulative loading from the results of simple loading tests. At present, at least two methods are available for this: a time-hardening procedure and a strain-hardening procedure, both illustrated schematically in Figure 9.

In the time-hardening method, if the specimen is loaded for N_1 repetitions of stress state $\bar{\sigma}_1$, the resulting permanent strain will be $\epsilon_1^p(N)$. The equivalent number of repetitions, N_2 , at stress $\bar{\sigma}_2$ that would have given the same permanent strain is obtained as shown in Figure 9, and if further N_2 applications of σ_2 are applied, the total strain will continue to follow the path.

The strain-hardening procedure requires the determination of ϵ_1^p after N_1 repetitions of stress σ_1 . The number of repetitions at stress $\bar{\sigma}_2$ is then taken equal to N_1 , and a further N_2 application is applied. The total permanent strain is the sum of ϵ_1^p plus ϵ_2^p .

Both approaches have been used to predict the measured responses in cumulative loading from test data at single stress levels and compared with the experimental results by Monismith and others. Neither method gives a solution that agrees quantitatively with the experimental results. However, the predicted results are in qualitative agreement and bracket the actual data. The time-hardening procedure gives better agreement if the stress levels are successively increased, whereas the strain-hardening method gives closer agreement if the loads are successively decreased.

CREEP DEFORMATION

As with rutting from repeated traffic loading, creep deformation from standing loads may be estimated by using either elastic or viscoelastic theory. The use of viscoelastic theory together with creep compliances for the components of the pavement structure that are likely to creep is one feasible way to examine the problem. Alternatively, the use of elastic theory together with stiffness moduli whose dependence on time and temperature are known is another. This requires solutions for a series of stiffness moduli corresponding to different times and gives curves of the same form as above.

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