Theoretical Basis for Frost-Action Research

A. R. JUMIKIS, Associate Professor of Civil Engineering,
Rutgers University

The purposes of this article are to (1) direct the attention of the highway designer, builder, owner and user to the adverse effect of the freeze-thaw phenomena on the performance of street, highway and runway pavements; (2) indicate the need for effective frost action research; and (3) stimulate interest among research workers, highway engineers and others in the importance of frost action research in highway and airport engineering, with a view to improving technical service of the nation's transportation system.

For these purposes, a theoretical basis as an imperative and indispensable tool to work with is here briefly reviewed, the physical concepts involved are mentioned and the principal factors to be studied outlined.

The evaluation of the geotechnical and thermal properties of the materials in question, and their application to a practical problem on a scientific basis, with or without modifications, may, it is believed, help to suggest to the investigator an approach to a satisfactory solution of almost any frost action research problem.

The theories displayed (which are based on the discipline of heat transfer) are by no means the ne plus ultra for this field of research. However, based on the present state of knowledge, they can be considered as representing a theoretical basis for frost action research in soil mechanics (géotechnique) and highway engineering. As our knowledge increases, theories can then be modified and corrected.

● THIS article is a summary of thoughts resulting from the writer's reading of the pertinent literature, from his observations in practice through many years, from experienc gained in research in the fields of soil mechanics, foundation engineering, highway engineering and the building of airports, and from teaching undergraduate and graduate courses in these and other related fields, as well as from practical consulting work.

For several years now highway owners and engineers have become increasingly conscious of the kind and amount of damage to highways caused by frost action. Both contir uous (seasonal) freezing followed by thawing (in northern regions), and periodic (cyclic) freezing and thawing (in more temperate regions) of the pavement-supporting soil, parti ularly in areas where roads are kept free of snow in winter, create difficulties for traff and maintenance. These difficulties consist of differential frost heaves, development of boils, and damage or destruction of pavements, making driving conditions dangerous. Repairs of such damage usually cost huge sums.

Observations made of existing roads in the Atlantic states showed that many miles of roads of all types, which under normal climatic conditions have served well for many years, were damaged seriously in the severe winter of 1947-48. From experience with road performance in this particular winter, one concludes that engineers are really wor ried as to how to combat effectively frost on highways and airports.

FACTORS CONTRIBUTING TO DAMAGE

Some of the main factors contributing to the damage to roads by frost action include: (1) climatic conditions, for example, freezing temperatures and their duration; (2) the soil itself, particularly silts; (3) precipitation; (4) drainage conditions; (5) soil moisture; (6) position of ground-water table; and (7) vehicular loads.

Great damage occurs particularly in thawing periods, when improper soil below a pavement under unfavorable conditions becomes soft from saturation by melting ice water without proper drainage. The consequence is the loss of bearing capacity of the soil, resulting in damage to pavements and causing so-called spring breakup. The inte relationship of the various factors involved in damaging roads is complex. A variation any one of the factors involved influences to a greater or lesser extent the others, the properties of the soil, as well as the whole thermal system, soil-moisture-temperature.

These facts revive and focus more sharply the two well-known fundamental requirements for a good road: (1) a solid foundation and (2) good drainage. These requirements were understood by the early Roman and Incan road builders, and still are to be honored. The idea could be somewhat extended by saying, "A road must be dry even in wet weather."

By drainage is here understood taking care not merely of the water on the road surface, from floods and groundwater, but also of the melting water from the ice in the thawed pavement-supporting soil. In practice, drainage is often overlooked. One can appreciate its importance simply by remembering that excess moisture in the pavement-supporting soil reduces the bearing capacity of the soil.

NEED FOR FROST ACTION RESEARCH

Although the development in highway transportation in the past 30 years has made considerable progress, relatively little progress was made in arriving at more effective methods for evaluating the thermal system of soil-moisture-temperature in connection with the suitability and performance of highways under freeze-thaw conditions. Because of the complex nature of this thermal system, engineers in their efforts to solve a problem involving temperature differences, heat transfer and moisture migration in soils, relied for the most part upon a practical approach or depended upon mechanics alone. Relatively little attention in this matter has been given to a theoretical basis or approach.

Now, we often realize that a satisfactory explanation of a thermal problem cannot be found merely by experimentation or by way of mechanics alone. We must admit the fact that we have not arrived at a point where so-called practical methods in such researches have developed to a sufficiently refined point to give satisfactory solutions pertaining to pavement performance under freeze-thaw conditions.

If we hope to get from research all the answers we desire and to understand soil and pavement performance under freeze-thaw conditions, any approach or method of research must be based on a theoretical foundation. In addition to geology, mechanics, soil mechanics, hydraulics, and hydrology, we should also consult with increasing frequency the disciplines of mathematics and heat transfer. This is because none of the complex frost problems in soils can be undertaken without some knowledge of the thermal conductivity and diffusivity of the soil-moisture medium and the basic laws of heat transfer. In other words, we should utilize every theoretical tool available to our present state of knowledge and technology; because: (1) the system "soil-moisture-heat" is a complex thermal system requiring appreciation; (2) there is a need for better understanding of the physical factors and their mutual relationship in this thermal system and the laws governing it; and (3) large sums and great effort usually are spent in investigations pertaining to frost action in highway soils.

All these points, without reservation, manifest the great need for frost action research in highway engineering on a solid theoretical basis. The value of frost action research within the scope of highway research is now generally recognized, and therefore it is indisputable. Like all other phases of highway research, frost action research is vital, and is of national importance.

Experience indicates that conclusions drawn from observations on the effects of geophysical, climatic and soil conditions on highway behavior and performance in one area are not, in general, valid for other areas, however similar those areas may be. Therefore, it is believed that frost action research should be carried out under conditions as they prevail in each particular locality.

PURPOSE OF RESEARCH

The purpose of frost action research is to: (1) study the various factors interacting within the thermal system of soil-moisture-temperature and to try to find the fundamental relationships between factors entering into highway design and construction (of particular interest are thermal and thermoösmotic processes in soils in winter and summer, the climatic influence on the physical properties of soils, loss of bearing capacity of pavement-supporting soils under thawing conditions, spring breakup, study of materials to be used in lieu of rapidly diminishing supplies of gravel material in certain areas, and how to us substitute materials to advantage in highway construction); (2) investigate soils in orde to know which ones are particularly susceptible to heaving under freezing conditions; and (3) provide observation and test data to establish a method or criteria index for the evaluation of frost danger to subgrade soils and to base and subbase courses where condition are conducive to frost action. All this, in turn, serves to obtain quantitative and qualitative knowledge for design purposes.

The ultimate goal of frost-action research is to provide the engineer with knowledge concerning design and construction of better roads, airports, and other earthworks and thus to contribute to the improvement in the service of our nation's transportation system It is necessary because highways may be said to be the backbone of the nation's life and are considered to be the most-important factor in civilization, for progress and for national defense. Of course, the magnitude of our efforts in research should be great enou to match the magnitude of the important highway performance problem under freeze-thaw conditions.

PLACE OF FROST-ACTION RESEARCH

Having familiarized ourselves with the important frost action problem in highway engineering, accepting the thought that frost action research of the thermal system of soil-moisture-temperature must be based on a theoretical basis and that among other discipli heat transfer should also be consulted and having formulated the need and purpose of suc research, one might ask what is the place of frost action research in engineering?

The answer, fortunately, is not too difficult to be found. The place is géotechnique (s mechanics). This discipline permits us to subdivide it in phases paralleling the disciplin of general mechanics (see Table 1). Hence, it seems that the place of frost action resea

TABLE 1

SOIL MECHANICS (GÉOTECHNIQUE)

General Mechanics	Statics
Statics	Statics (all static soil tests, bearing tests, stability of slopes and foundations, static stabilization of soils, earth pressures)
Dynamics	Dynamics (all dynamic soil tests, seismic soil investigation, dynamic compaction of soils, pile driving, earthquakes and vibrations in soils induced by vibrating machinery)
Hydrodynamics	Hydrodynamics (consolidation, suction force, moisture migration in soils, permeability, lowering of groundwater table, injections [grouting])
Heat Transfer with Thermodynamics	Thermal Soil Mechanics (formation of ice, freezing and thawing, frost penetration, heaving, moisture migration in soils in cold and hot regions, heat transfer in soils, thermoösmosis, artificial freezing operations, permafrost)
	Geo-Electric Soil Investigation
	Electro-Ösmosis (de-watering of fine grained soils)
	Chemical Stabilization of Soils
	Nuclear Soil Mechanics (non-destructive tests for the determination of soil density and moisture content by means of radioactive isotropes)

in soils is in thermal soil mechanics under the aegis of geotechnique.

SCOPE OF RESEARCH

Having assigned the frost action research a place, we can now proceed with a brief discussion of the general scope of research. To start anything in research, there must be a theoretical basis for it. Using a theoretical basis, it is possible to treat the frost penetration problem analytically, provided the necessary quantities and soil constants, characterizing soil geotechnical and thermal properties, are readily furnished by tests. It is believed that the results of frost-action research based on sound theoretical considerations can be made available, with modification and adjustment, for practical application to highway and airport engineering.

The general scope of a frost research program should comprehend: (1) formulation of the problem; (2) library studies; (3) mathematical analyses; (4) building and construction of research facilities; (5) laboratory and field research; (6) studies of research material and testing of the hypotheses and theories; (7) scientific travel for gathering and exchanging information; and (8) correlation work and adjustment of findings to practice.

By mathematical analyses is here understood the treatment of the thermal system of soil-moisture-temperature by means of heat transfer theories. These theories have the advantage of (1) having a broad bearing and (2) the generality of their methods of analysis, which permits applying them in the theoretical and practical approach to the solution of frost problems in soils.

Of course, in any domain of scientific knowledge, some basic principles are treated by means of mathematics. Engineering has always been founded on a base of mathematics, and the trend today is toward more extensive use of it. As the Committee on Adequacy and Standards of Engineering Education of the American Society for Engineering Education recently concluded, "The greatest potential for future development in science and technology is to be found in mathematics."

In an article entitled "Have We Lost Control of Our Profession?" in "American Engineer" for May, 1954, Major General Samuel D. Sturgis, Jr., says:

"What is of real concern to me, however, is the gradual acceptance on the part of the average engineer in the average firm of an attitude geared to just getting by and no more. A continuance of such an attitude in any large segment of engineering firms will lead inevitably to a deterioration of the over-all quality of American engineering and a decrease in the confidence and esteem of those we serve, which it has always been our good fortune to enjoy . . . . One insidious influence on both the quality and quantity of engineers, the full impact of which is scarcely yet being felt, is the general drift in public schools away from mathematics and science towards the so-called social studies. The mental discipline of the three R's has been thrown overboard in favor of happy, well-adjusted children."

Hence, we conclude that the solutions pertaining to the problems of our thermal system are governed to a great extent by mathematics.

THEORETICAL BASIS

Having established the need for a theoretical basis in frost action research, it is pertinent to review briefly the following principal theories: (1) heat flow in the steady state, (2) heat flow in the unsteady state, (3) temperature oscillation theory, (4) sudden changes in temperature, (5) cold snaps, (6) Neumann's theory, (7) Stefan's theory, and (8) Ruckli's suction force theory.

Frost penetration in soils and thawing is simultaneously a geotechnical and a heat transfer problem. As a theoretical basis for analysis, the following laws apply: (1) the amount of heat in a differential soil element is proportional to its mass and to its temperature; (2) heat flows from a higher to a lower temperature; (3) the rate of heat flow across an area is proportional to it and to the temperature gradient at a point of that area; and (4) upon freezing, the migration of soil moisture takes place from a warmer medium toward the cold front (thermoösmosis).

1. Depending upon field conditions, heat conduction in the steady state or unsteady state is to be considered.[1] Where the temperature gradient is linear and constant, and the soil can be considered homogeneous, isotropic material, with constant geotechnical and thermal properties, the law of steady-state flow of heat can be applied:

$$q = K \cdot A \cdot \frac{dT}{dx}, \tag{1}$$

where

q = rate of heat flow;

K = thermal conductivity;

A = the area measured normal to the direction of flow, and

$\frac{dT}{dx} = i$ = temperature gradient.

2. (a) Analysis furnishes the following general partial differential equation for unsteady-state heat conduction in rectangular coordinates:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial^2 y} + \frac{\partial^2 T}{\partial^2 z} \right), \tag{2}$$

where

T = temperature in the soil;

t = time;

α = coefficient of diffusivity of a soil;

x, y, and z = space coordinates.

This equation expresses the conditions that govern the flow of heat in soil. The soil itself is considered to be homogeneous. (b) Assuming that on highways cold penetrates the soil in one direction only (vertically), i. e., normal to the upper road surface ($y = 0$, $z = 0$), we have a special case of Equation (2), which can be rewritten as follows:

$$\frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial x^2} \tag{3}$$

This type of equation has wide application in the theory of heat transfer as well as in the unidimensional consolidation theory of soils.

3. Observations show that the highway surface- and air-temperatures oscillate diurnally sinusoidally around a mean temperature, T_0. Hence, the solution of the partial differential equation (3) must also fit the surface boundary condition (at $x = 0$)

$$T = T_0 \cdot \sin(w \cdot t), \tag{4}$$

where

$w = \frac{2 \cdot \pi}{P}$, P being the period, and

x = depth below the ground (road) surface.

The solution of equation (3) is:

$$T = T_0 \cdot \left(e^{-x \cdot \sqrt{\frac{w}{2 \cdot \alpha}}} \right) \cdot \sin\left(w \cdot t - x \cdot \sqrt{\frac{w}{2 \cdot \alpha}} \right), \tag{5}$$

where

e = 2.71 ... = the base of natural logarithms.

---

[1] A full treatment of the various theories can be found in A. R. Jumikis, "The Frost Penetration Problem in Highway Engineering," 1955, Rutgers University Press, (in preparation).

Respective maximum and minimum points on the temperature oscillation wave are delayed progressively by $x\cdot\sqrt{\frac{w}{2\cdot\alpha}}$ as depth x increases. The amplitude of the oscillating temperature wave is damped rapidly by

$$\left(e^{-x\cdot\sqrt{\frac{w}{2\cdot\alpha}}}\right)$$

as depth, x, increases.

This solution is valid for a homogeneous and uniform soil. No consideration to formation of ice lenses in the subgrade is taken into account. Nevertheless, it gives a fairly good idea as to how the temperature fluctuations in the soil take place.

The quantities α (thermal diffusivity) and x (depth) are to be furnished by tests and investigations, respectively. Besides, this theory allows determining the thermal diffusivity, α, of a soil or pavement in place by means of temperature measurements at different depths. Tests in the laboratory can also be performed.

4. Sudden Changes of Temperature. For the solution of the problem as to how deep will the freezing temperature (say 0 C. = 32 F.) penetrate a soil in a certain period of time (α must be furnished by tests), if the surface temperature is lowered to T_S^0, the following equation, which is also a particular solution of the general partial differential Equation 3, can be used;

$$\frac{T - T_S}{T_0 - T_S} = \frac{2}{\sqrt{\pi}}\cdot\int_0^{x\cdot\eta}\left(e^{-\beta^2}\right)\cdot d\beta = G\left(x\cdot\eta\right), \tag{6}$$

where

T = temperature at depth x (for example, T = 0 C. = 32 F.),

T_0 = average initial temperature of the soil,

T_S = surface temperature,

t = time,

$\eta = \frac{1}{2\cdot\sqrt{\alpha\cdot t}}$,

$\beta = \frac{x}{2\cdot\sqrt{\alpha\cdot\left(t - \tau\right)}}$

τ = time variable (limits 0 and t),

$G\left(x\cdot\eta\right)$ = Gauss's probability integral (or error function).

For example, assuming T = 0 C. = 32 F., T_0 = 5 C. = 41 F., T_S = 20 C. = -4 F., and with α = 0.005 ft.²/hr., and t = 24 hours, x = 1.25 ft.

5. The following expression

$$T = \frac{2}{\sqrt{\pi}}\cdot\int_{x\cdot\eta}^{\infty} f\left(t - \frac{x^2}{4\cdot\alpha\cdot\beta^2}\right)\cdot\left(e^{-\beta^2}\right)\cdot d\beta\ , \tag{7}$$

which is also a particular solution of the general partial differential Equation 3, enables a more accurate evaluation of the effect of surface temperature fluctuation (so-called cold snaps) to be made than is possible if one assumes that they are simple periodic sine variations.

This expression is particularly well suited for problems where it is necessary to find soil temperatures at a certain depth, when a period of uniform soil temperature, say 0 C. = 32 F., is broken by a cold snap lasting several days.

6. Neumann's Solution. Taking into consideration ice and water, Neumann gives two equations for a moist, isotropic, semiinfinite body whose temperature to begin with is positive and constant. By suddenly lowering the surface temperature to a new constant but freezing value, the thermal process is started. The two equations are:

$$\frac{\partial T_1}{\partial t} = \alpha_1 \cdot \frac{\partial^2 T_1}{\partial x^2} \quad \text{for ice,} \tag{8}$$

and

$$\frac{\partial T_2}{\partial t} = \alpha_2 \cdot \frac{\partial^2 T_2}{\partial x^2} \quad \text{for water,} \tag{9}$$

where ξ is the thickness of the advancing ice layer, which is a function of time, thermal and other physical properties of the soil.

Considering latent heat of fusion, and setting up boundary conditions, the solutions of Equations (8) and (9) are:

$$T_1 = B_1 + D_1 \cdot G(x \cdot n_1), \tag{10}$$

and

$$T_2 = B_2 + D_2 \cdot G(x \cdot n_2), \tag{11}$$

where

$$G(x \cdot n) = \frac{2}{\sqrt{\pi}} \cdot \int_0^{x \cdot n} \left(e^{-\beta^2} \right) \cdot d\beta$$

is the Gauss's probability integral. One of the boundary conditions is:

$$\xi = b \cdot \sqrt{t}\ , \tag{12}$$

i.e., the frost penetration depth ξ is proportional to $\sqrt{t}$. The coefficient m is to be determined from the following transcendental function:

$$b_1 \cdot \left(T_f - T_s \right) \cdot \frac{e^{-\frac{m^2}{4 \cdot \alpha_1}}}{G\left(\frac{m}{2 \cdot \alpha_1} \right)} - b_2 \cdot \left(T_o - T_f \right) \cdot \frac{e^{-\frac{m^2}{4 \cdot \alpha_2}}}{I - G\left(\frac{m}{2 \cdot \sqrt{\alpha_2}} \right)} = \frac{Q_L \cdot \delta_s \cdot \sqrt{\pi} \cdot W \cdot m}{2}, \tag{13}$$

where

$$b_1 = \frac{K_1}{\sqrt{a_1}} = \sqrt{K_1 \cdot c_1 \cdot \delta_1}\ ,$$

$$b_2 = \frac{K_2}{a_2} = K_2 \cdot c_2 \cdot \delta_2\ ,$$

a_1 and a_2 are coefficients of thermal diffusivity,

Q_L = latent heat (when water is converted into ice),

δ_S = density of ice, viz., frozen soil,

δ_1 = density of ice,

δ_2 = density of water,

c_1 and c_2 are specific heats,

T_s = surface temperature,

T_o = initial temperature,

w = moisture content in soil,

and T_f = 32 F. = freezing temperature.

7. Stefan's Solution - Stefan simplified the foregoing theory assuming that the temperature gradient in the ice layer is linear, and the temperature of the water is 0 C. = 32 F. The frost penetration depth, ξ, according to Stefan, is expressed:

$$\xi = \sqrt{\frac{2 \cdot K_1}{Q_L \cdot \gamma_1} \cdot T_s \cdot t} \qquad (14)$$

The first derivative of ξ with respect to time t gives the rate of frost penetration.

8. Suction Force. Neumann's and Stefan's theories are valid only for soils where the moisture present is motionless. However, on formation of ice lenses, moisture in soils is subject to migration or flow towards a cold boundary (ice lenses). The upward flow of moisture in soils can be considered as being caused by the so-called suction force, P_S, the magnitude of which, according to Ruckli, can be expressed as follows:

$$P_S = \frac{\left(\gamma_w\right) \cdot \left(\Delta h\right) \cdot \left(H - \xi\right)}{\left(1.09\right) \cdot \left(t\right) \cdot \left(k_s\right)}, \qquad (15)$$

where

P_s = subpressure (real or fictive),

γw = unit weight of water,

Δh = amount of permissible frost heave,

H = depth of the ground-water table below ground surface,

δ = frost penetration depth,

t = time required to obtain an upward flow (in laboratory or actual freezing period),

k_s = coefficient (experimental) of proportionality of vertical flow of moisture through the soil concerned.

It seems that the suction force, inaugurated on freezing, would be an important soil constant in predicting frost penetration depths and calculating the amount of lowering the ground water table in order to interrupt moisture supply to the growing ice lenses.

9. Ruckli's Theory. The theory by R. Ruckli furnished a differential equation for frost penetration depth, ξ, taking into consideration the vertical flow of soil moisture toward a cold front upon freezing, suction force, porosity of soil, moisture content in soil, unit weight of the soil, depth of the ground-water table, diffusivity of the soil, duration of the cold period and its intensity, and other thermal properties and soil mechanical characteristics of the soil:

$$\frac{d\xi}{dt} = \frac{A}{\xi} - B \cdot v - \frac{C}{\sqrt{\xi}} \qquad (16)$$

in which A, B and C are constants to be determined for any given case, and v = velocity of the upward flow of soil moisture toward the ice lenses. The general solution of this equation can be found by plotting the integral curve of Equation (16), or analytically.

It seems to be obvious that basic theories contribute to the understanding of natural phenomena and provide an objective guide in theoretical as well as applied research. Particularly are they helpful when we have to face new and unfamiliar situations. In such cases they facilitate handling the problem in question with real competence. Therefore, heat transfer theories can be considered as great untapped sources of knowledge for the solution of the complex frost problems.

OFFICE, LABORATORY, AND FIELD RESEARCH

From the foregoing discussion it can be seen that for the evaluation of frost susceptibility of soils used in highway engineering, the following general studies, characterizing the properties of the particular soil, should be made; their results, as well as other information relative to the particular problem, should be studied and correlated: (1) climatologic data (duration and intensities of frost periods); (2) soil temperatures (in the field and laboratory); (3) ground-water table fluctuations and temperatures; (4) frost heaves on highways; (5) frost penetration depths; (6) physical and mechanical properties of soils entering into the evaluation of their frost susceptibility; (7) moisture content variation in unfrozen and frozen soils; (8) permeability of soils; (9) suction force studie in soils; (10) thermal properties of soils; (11) thermal properties of concrete pavements (12) thermal properties of bituminous pavements; and (13) correlation of findings.

Whatever our frost action research program should be, and whatever the definition of the practical problem might be, two kinds of research and studies should be done anyway: (1) research on thermal diffusivity and conductivity of soils and highway pavements and (2) research on suction force in soils upon freezing. If we have this information, we may apply any one of the aforementioned theories, with or without modifications, to a practical problem to be investigated.

Of course, important elements in any research are research-minded men, time, space to do research, basic equipment, a good library and financial support. Needless to menti that, above all, enthusiasm for research is a contributory factor of research workers, sp sors and those who direct research, as well as good cooperation and interest of all parties involved. Besides, as in any other field of scientific and technical endeavor, new facts, their better understanding and their better application can be expected from frost action re search. This contributes to the advancement of knowledge.

What has been learned by some research workers today might result in new, practical applications tomorrow which would be directly useful in highway engineering.

ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to the College of Engineering of Rutgers University and to the Joint Highway Research Committee, representation on which is divided between Rutgers University and the New Jersey State Highway Department, for sponsoring the presentation of this article.