Foundations to Resist Tilting Moments Imposed on Upright Cantilevers Supporting Highway Signs

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To have an upright cantilever structurally strong enough and at the same time as economical as possible, requires careful analysis of the supporting foundation. However, neither time nor money permit foundations to be put in that way. It is usually much cheaper to make a foundation enough larger to include expected factors of ignorance and safety. Nevertheless, some engineering must be used and the solution contained in this paper is a good beginning point.

Certain observations about horizontal soil resistances seemed to be established enough to use as a basis to start. The first is that for a vertical cantilever, a slim deep foundation is the most economical for a given load with limits of the strength of the foundation as an efficient structure. The soil has horizontal resistance to movement depending on its cohesive or granular makeup, or both. The resistance varies with the depth or the amount of overlying earth above. There is some relation between movement and resistance. In this analysis it is assumed that (a) within the limits used the soil is an elastic body, (b) the strength of a foundation varies with its projected area, (c) no part of the soil shall be stressed above its ability to withstand the load, and (d) in empirical solution errors shall be kept on the side of over-design.

In the solution itself, several empirical methods were used. It was impractical to design a foundation in a given soil for a given load at a given height, but if one took a foundation of given dimensions in a given soil and chose a neutral axis, then by integrating the soil resistance one finds both shear and moments imposed. A family of curves can then be drawn, from which generalizations and empirical solutions follow.

This type of analysis seems to be justified from model studies and tests run in 1945, then on some 12 years of usage covering many thousands of poles from 20 to 200 ft high supporting any type of load, and latest on the tests run by the Research Department of the Ohio State Highway Department.

THE RESISTANCE of the soil to the tilting forces on a deep, slim foundation cannot be exactly evaluated. There is, however, much need to determine approximate solutions and keep errors in assumption or solution on the side of safety and also to try to suggest factors of safety as great as factors of ignorance.

Using the best references available, it seems that certain approximations can be made. As in many other empirical solutions, it is sometimes necessary to find the limiting relationships and stay within those limits in order to keep the solution from being too cumbersome. Acknowledgment is made to Terzaghi (1) and Hogentogler (2).

It will develop that the most economical foundations for this purpose are slender and of some depth, and that their strength against tilting is such that such forces as bearing and uplift should be set aside as negligible.

DEFINITION OF TERMS

Units are feet and pounds on unit (1-ft) width of foundation.

R_p = passive horizontal soil resistance
R_a = active horizontal soil pressure
R_t = net resistance of soil to horizontal movement at depth "Z" as a maximum allowable pressure
R_d = net resistance of soil to horizontal movement due to its deflection
R = actual net resistance of soil to horizontal movement at depth "Z"
M = moment of horizontal load P at L distance from neutral axis of the foundation; this moment is solved for a foundation of unit width
M_z = moment imposed on foundation of depth D, by the soil, integrated around neutral axis
M_f = bending moment in foundation
V_f = horizontal shear in foundation
D = depth of foundation
Z = any depth
Z_1 = depth of neutral axis of foundation
dZ = increment depth
C = coefficient of soil cohesion
ϕ = angle of internal friction
G = weight of soil
L = vertical distance from load "P" to neutral axis of foundation
h = distance of load above groundline (not used except to find degree of error, use "L" instead in calculations)
A = Anderson's constant of cohesive resistance for soil in question
B = Anderson's constant of internal friction resistance of soil in question

DEVELOPMENT OF THEORY

Under the elastic theory it is accepted that resistance to motion is directly proportional to deflection (Fig. 1). Again in the study of soils it is generally assumed that resistance to unit deflection varies with depth. This solution is based on the assumption that these two relationships are straight line functions:

The net resistance of a soil to horizontal movement of something in it is the difference of the pressures on its two sides or passive resistance less active pressure. Thus $R_t = R_p - R_a$. Terzaghi (1) gives

$$R_p = 2C \tan (45^\circ + \phi/2) + GZ \tan^2 (45^\circ + \phi/2)$$

$$R_a = -2C \cot (45^\circ + \phi/2) + GZ \cot^2 (45^\circ + \phi/2)$$

or

$$R_t = 2C \left[\tan (45^\circ + \phi/2) + \cot (45^\circ + \phi/2)\right] + GZ \left[\tan^2 (45^\circ + \phi/2) - \cot^2 (45^\circ + \phi/2)\right]$$

For simplicity let $a = 2C \tan (45^\circ + \phi/2) + \cot (45^\circ + \phi/2)$ and $b = G \tan^2 (45^\circ + \phi/2) - \cot^2 (45^\circ + \phi/2)$

Thus $R_t = a + bZ$

From theory thus far:

R is proportional to R_t and to R_d.

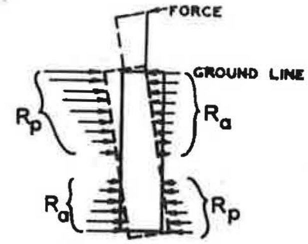

Figure 1.

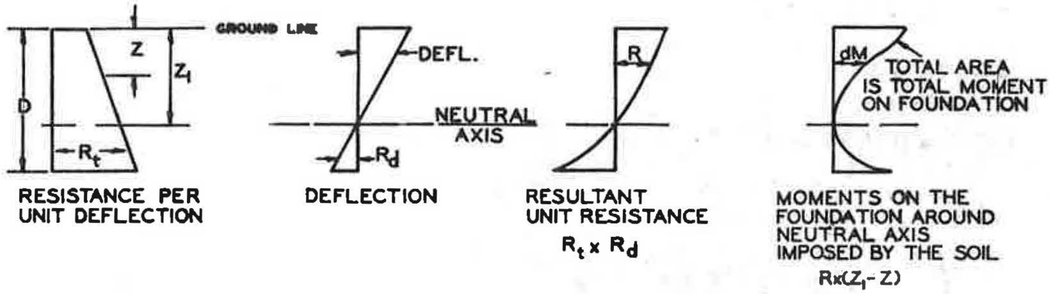

Figure 2. Development of forces and moments.

However, a deflection cannot give a resultant resistance greater than R_t, which is the capability of the soil to resist.

Thus $R = R_t \cdot R_d \cdot$ constant

Now R_d is proportional to the distance from the neutral axis; that is, $R_d = K (Z_1 - Z)$ (Fig. 2).

or, substituting, $R = (a + bZ) K (Z_1 - Z)$ (including both constants in "K")

But R must not exceed a + bZ.

That is, $R = a + bZ = (a + bZ) K (Z_1 - Z)$.

From inspection, the groundline will be the weak point for a cylindrical foundation.

Thus, at 0 depth, $Z = 0$, $R = a = K a Z_1$

or $K = 1/Z_1$ and,

$$R = (a + bZ) (1 - Z/Z_1) = a - \left(\frac{a}{Z_1} - b\right) Z - \frac{b}{Z_1} Z^2 \text{ (Fig. 2)}$$

Integrating the moments from the bottom of the foundation by means of integrating shear gives answers too critical of height of load for any use. But integrating the resisting turning moments imposed on the foundation by the soil and around the neutral axis, gives the first useful approach.

$dM = R \cdot$ lever arm, or $R (Z_1 - Z)$

or $dM = (a - aZ/Z_1 + bZ - bZ^2/Z_1)\ (Z_1 - Z)$

$$= a\,Z_1 - (2a - bZ_1)\,Z + (a/Z_1 - 2b)\,Z^2 + \frac{bZ^3}{Z_1}$$

$$M = \int dM_z d_z = a\,Z_1 Z - \left(\frac{2a - bZ_1}{2}\right) Z^2 + \left(\frac{a}{3Z_1} - \frac{2b}{3}\right) Z^3 + \frac{b}{4Z_1} Z^4 + C_1$$

but if $Z = D$, $M = 0$; then at $Z = 0$, $M = C_1$

$$\text{Thus } -C_1 = aZ_1D - \left(\frac{2a-bZ_1}{2}\right) D^2 + \left(\frac{a}{3Z_1} - \frac{2b}{3}\right) D^3 + \frac{b}{4Z_1} D^4$$

Since all moments total zero, the moment imposed on the foundation from the force above the ground = C_1 (all about neutral axis) then external moment

$$M = -aZ_1D + \left(\frac{2a - bZ_1}{2}\right) D^2 - \left(\frac{a}{3Z_1} - \frac{2b}{3}\right) D^3 - \frac{b}{4Z_1} D^4$$

(about the neutral axis)

$$\text{and } V_f = az - \frac{1}{2}\left(\frac{a}{Z_1} - b\right) Z^2 - \frac{b}{3Z_1} Z^3 - a\,D + \frac{1}{2}\left(\frac{a}{Z_1} - b\right) D^2 + \frac{b}{3Z_1} D^3$$

If $Z = 0$ $V_f = -\,a\,D + \frac{1}{2}\left(\frac{a}{Z_1} - b\right) D^2 + \frac{b}{3Z_1} D^3$ which represents the shear imposed from the structure above the groundline. Integrating V_f, the moment in the foundation is

$$M_f = \int V_f\,dZ = \frac{a}{2} Z^2 - \frac{1}{6}\left(\frac{a}{Z_1} - b\right) Z^3 - \frac{b}{12Z_1} Z^4$$
$$- aDZ + \frac{1}{2}\left(\frac{a}{Z_1} - b\right) D^2Z + \frac{b}{3Z_1} D^3Z + C_2$$

$Z = D$, $M = 0$

$$-C_2 = \frac{a}{2} D^2 - \frac{1}{6}\left(\frac{a}{Z_1} - b\right) D^3 - \frac{b}{12Z_1} D^4 - aD^2 + \frac{1}{2}\left(\frac{a}{Z_1} - b\right) D^3 + \frac{b}{3Z_1} D^4$$

$$C_2 = \frac{a}{2} D^2 - \frac{1}{3}\left(\frac{a}{Z_1} - b\right) D^3 - \frac{b}{4} D^4$$

$$M_f = \frac{a}{2} Z^2 - \frac{1}{6}\left(\frac{a}{Z_1} - b\right) Z^3 - \frac{b}{12Z_1} Z^4 - aDZ + \frac{1}{2}\left(\frac{a}{Z_1} - b\right) D^2Z + \frac{b}{3Z_1} D^3Z$$
$$+ \frac{a}{2} D^2 - \frac{1}{3}\left(\frac{a}{Z_1} - b\right) D^3 - \frac{b}{4} D^4$$

To study stresses in the foundation itself, the net soil forces are integrated on the foundation, from the bottom to the top, to give the shear V_f.

$$V_f = R\,dz = a - \left(\frac{a}{Z_1} - b\right) Z - \frac{b}{Z_1} Z^2\,dZ$$
$$= aZ - \frac{1}{2}\left(\frac{a}{Z_1} - b\right) Z^2 - \frac{b}{3Z_1} Z^3 + C_2$$

$Z = D$, $V_f = 0$, and $V_f = C_2$, or when $Z = 0$,

$$V_f = -aD + 1/2\left(\frac{a}{Z_1} - b\right) D^2 + \frac{b}{3Z_1} D^3$$

Taking various values of Z_1/D, a, and b, the moment and shear at D = o can be found, and from these two values height of load, in this case from the groundline up, can be determined. However, these values are very critical and complex and set aside as unwieldy, in favor of the following, which is a simple, adequate development for resisting moment of the foundation. However, internal stresses in the foundation are developed further in a later section.

It was found that M_z (about the neutral axis) varies only three percent to five percent over the range of heights of loads from one times foundation depth to 20 times foundation depth. Consequently, the analysis will use M_z (around the neutral axis). It was also found that if a value of $Z_1 = 2/3$ D was assumed, the errors would be on the safe side. Incidentally, this depth of neutral axis $Z_1 = 2/3$ D was observed in model studies of both granular and plastic soils.

Substituting then, the 2/3 value for Z_1/D, we get:

$M = 1/6\ aD^2 + 1/24\ bD^3$ (basic formula). Remember again that M is ft-lb allowable, around the neutral axis per unit width of foundation. As developed later, the practical formula will be $M = AD^2 + BD^3$ with values given for A and B for various soils.

CERTAIN VALUES

Hogentogler (2) gives basic values of certain soils, which can be used with the basic formulas of Terzaghi (1) to arrive at values of a and b in the basic formula. The arbitrary weight of soil is 100 pcf, for certain soils. The terms are fairly broad but certain additional factors of safety which are later mentioned are beneficial.

TABLE 1

Types of Soil	Coef. of Cohesion	Angle of Internal Friction (deg)	a	b
Silt, wet	0	10	0	73
Sand, wet or dry	0	34	0	326
Clay, very soft	200	2	800	27
Clay, medium	1000	6	4000	42
Clay, very stiff	2000	12	8120	87
Cemented sand and gravel	500 to 1000 say 750	34	3600	326
Sandy clay	1000	34	4800	326
Silty clay	200	14	800	102

FACTORS OF SAFETY

Certain additional strengths of the foundation may well be mentioned. The preceding theory takes into account only the cylindrical projection of the foundation. It is most certain that there is some conical effect, particularly in sandy soils. Again there is no consideration given for skin friction, which has considerable effect, particularly if the foundation has rough vertical and bottom surfaces. The soil can withstand deflections at certain depths beyond the assumed solution and continue to exert full pressure, and hence allow other depths to take greater loads, after the plastic theory. Soil also has certain ability to withstand short-time loads greater than the limits assumed. Housel (3) suggests an overload factor of 2.5 where some small movement is not dangerous.

It should be remembered that foundations should be extended below the frost line to resist "heaving." Also, freshly disturbed soil such as trenches, etc., may subtract from the above-mentioned factors of safety.

SPECIAL SHAPES

Since in all soils seen thus far, the weakest point was at the groundline, the design should be balanced by increasing the diameter of the top 1/3, letting the bottom diameter of the foundation remain at unity.

The top diameter is increased to where R_t at the bottom is as great as allowable. The top diameter is increased to N, keeping the bottom diameter as unity. At bottom R was $\frac{a+bD}{2}$

Bottom R is increased to a + bD or increased by $\frac{a + bD}{2}$.

Assuming again a neutral axis 2/3 down, the top or groundline resistance must be increased by the same amount, times the respective lever arms of the top and bottom. Top increase is (N-1)a at the groundline and since there is twice the lever arm as at the bottom, $2\,(N-1)a = \frac{a + bD}{2}$, from which $N = \frac{bD}{4a} + \frac{5}{4}$, which is the multiplier of the top diameter for most efficient value for a bottom diameter of unity. A much more involved solution gives the ideal increase in top diameter, but values remain nearly the same. This is an efficient shape, volume-wise, and it normally takes the top third below the frost line.

Increasing the top 1/3 by the ratio N, or to N times unity,

$$M = \int_0^D M\,dZ + (N-1) \int_0^{D/3} M_Z dZ$$

$$= (.1296N + .037)\ a\ D^2 + (.017N + .0247)\ bD^3$$

which is abbreviated by substitution to $AD^2 + BD^3$ (Table 2).

NON-HOMOGENEOUS SOILS

Where soils are in layers of different types, and this is more usual than not, there is a different type of problem. Usually the top layer is of one type and the subsoil of an entirely different composition. These cases are solved by assuming the top third to be of one type soil and the bottom 2/3 of another. Actually the portion of the foundation around the neutral axis has little value and therefore the last assumption does not introduce practical errors. More than two kinds of soil may affect the solution but main interest is in the soil at the top 1/3 and bottom 1/6 of the foundation.

The solution for the most efficient foundation will include the top width greater than the bottom width by the ratio "N"

SOLUTION FOR TWO TYPES OF SOIL

The most practical solution for this problem is to assume a straight line characteristic for the two soils. Soil net resistance values may be as shown in Figure 3.

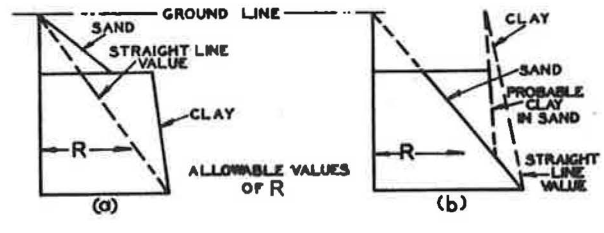

Figure 3.

Where there is a sandy type soil on top and a clay soil below, a straight line curve is less than the actual values. Where the clay type overlies the sand, the weakest part might be at the top of the sand, but this usually has sufficient penetration of clay to give it cohesive characteristics. Again, this point is near enough the neutral axis that there is not enough deflection to run up the force to a dangerous point.

Therefore, there is a new substitute or empirical soil whose "a^1" value is "a_1", and whose "b^1" value is

$$b^1 = \frac{a_2 + b_2 D - a_1}{D}$$ (where a_1 and b_1 are values for soil at top and a_2 and b_2 are values for soil at bottom)

This might possibly give negative values for "b," where a shallow foundation lies in a good hard clay top and a sandy bottom, but the equations and formulas still hold true within practical limits. Rather than use negative values the "a" value is reduced to give zero value of "b".

The "N" values are solved the same way as before. However, "N" is limited by practical dimensions to about five. Values for specific soils are given in Table 2 in which "A" and "B" values are developed.

SPECIAL SHAPES

If the direction of maximum moment is known, and it generally is, it is most efficient to increase the width of the top third at right angles, to the direction of force. Certain variables of this general shape are shown in Figure 4.

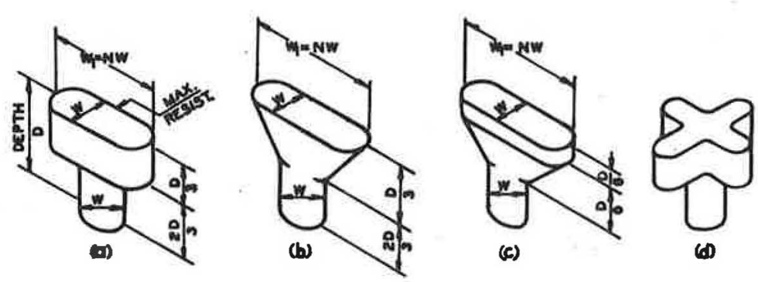

Figure 4.

Shape (a) (Fig. 4) is that used in the general formula, and shapes (b) and (c), which might be easier to dig, have about 90 percent of the strength of the formula. It might be noted that shape (c) is the most efficient foundation on a tilting resistance per unit volume for a given over-all depth that the author has investigated. Shape (d) is used where the direction of maximum force is not known. A pinching at the middle or

TABLE 2

Values for various soil combinations - contemplated depths between 5' and 1

Where $M = AD^2 + BD^2$, wide upper portions by ratio of "N"

M = safe tilting moment around neutral axis, per foot bottom width of foundation.

Soil Symbols

S Cl - Sandy clay
S - Loose sand or loose sand and gravel
Cem S - Cemented sand and gravel
Sf Cl - Very soft clay
M Cl - Medium clay
H Cl - Hard clay
St - Silt
St Cl - Silty clay - loamy clay

Upper Soil	Lower Soil	a_1	b_1	a_2	b_2	a or a^1	b or b^1	N	.1296N +.037	.017N +.0247	A	B
S Cl	S Cl					4800	325	1.4	.218	.049	1040	15.9
S Cl	S	4800	325	0	320	3200	0	1.25	.199	.046	1470	0
S Cl	Cem S	4800	325	3600	325	4800	205	1.25	.199	.046	950	9.4
S Cl	H Cl	4800	325	8100	85	4800	305	1.4	.218	.049	1040	14.9
S	S					0	320	Say 5	.683	.110	0	35.2
S	S Cl	0	320	4800	325	0	645	Say 5	.683	.110	0	71.0
S	Cem S	0	320	3600	325	0	500	Say 5	.683	.110	0	55.0
Sf Cl	S Cl	800	27	4800	325	800	590	3	.425	.076	340	44.8
Sf Cl	M Cl	800	27	4000	40	800	250	2	.296	.059	230	14.7
Sf Cl	H Cl	800	27	8100	85	800	570	3	.425	.076	340	43.3
M Cl	S	4000	40	0	325	3250	0	1.25	.199	.046	645	0
M Cl	Cem S	4000	40	3600	325	4000	285	1.4	.218	.049	870	14.0
M Cl	M Cl					4000	40	1.25	.199	.046	800	1.8
M Cl	H Cl	4000	40	8100	85	4000	350	1.4	.218	.049	870	17.1
H Cl	H Cl					8100	85	1.25	.199	.046	1610	3.9
St Cl	St Cl					800	100	1.5	.231	.051	180	5.1
St Cl	M Cl	800	100	4000	40	800	250	2	.296	.059	235	14.7
St Cl	H Cl	800	100	8100	85	800	570	3	.425	.076	340	43.3
St Cl	S Cl	800	100	4800	325	800	590	3	.425	.076	340	44.8
St Cl	Cem S	800	100	3600	325	800	510	3	.425	.076	340	38.8

increasing bottom and top diameters is theoretically more efficient, but not practical due to small diameters used at bottom.

INTERNAL STRESSES IN THE FOUNDATION

Integrating R_t from the bottom of the foundation upward, gives the internal moment in the foundation. Effective "a" and "b" values are from Table 2. Thus a general equation for the stresses in the foundation can be derived:

$$R = a + bZ - \frac{aZ}{Z_1} - \frac{bZ^2}{Z_1}$$

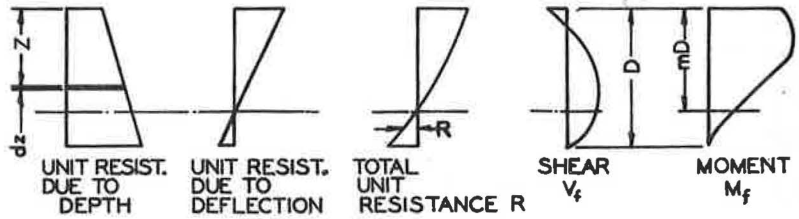

Figure 5.

Integrating, where V_p = shear at base of pole

$$V_f = aZ - \frac{a}{2Z_1} Z^2 + \frac{b}{2} Z^2 - \frac{b}{3Z_1} Z^3 + C \text{ (since at } Z = 0,\ V_f = V_p = C)$$

$$Z = D,\ V_f = 0, \text{ thus } C = -aD + \frac{1}{2}\left(\frac{a}{Z_1} - b\right) D^2 + \frac{b}{3Z_1} D^3$$

$$V_f = aZ - \frac{a}{2Z_1} Z^2 + \frac{b}{2} Z^2 - \frac{b}{3Z_1} Z^3 - aD + \frac{1}{2}\left(\frac{a}{Z_1} - b\right) D^2 + \frac{b}{3Z_1} D^3$$

Now it is found that the distribution of the shear in the foundation varies with the ratio of a to b (that is, cohesiveness versus granular nature of the soil) and that the a-to-b ratio effect is modified by total depth. To simplify the solution, certain ratios are taken as follows:

$$X = \frac{Z}{D} \text{ or } Z = XD;\ m = \frac{Z_1}{D} \text{ or } Z_1 = mD; \text{ and } b = \frac{na}{D} \text{ or } n = \frac{bD}{a}$$

Substituting, since $C = V_p$

$$V_p = aD\left(-1 + \frac{1}{2m} - \frac{n}{2} + \frac{n}{3m}\right)$$

$$V_f = aD\left(X - \frac{x^2}{2m} + \frac{nx^2}{2} - \frac{nx^3}{3m} - 1 + \frac{1}{2m} - \frac{n}{2} + \frac{n}{3m}\right)$$

Similarly, where M_p = moment at base of pole

$$M_f = \int V_f dZ = D\int V_f dx$$

$$-M_p = aD^2\left(-\frac{1}{2} + \frac{1}{3m} - \frac{n}{3} + \frac{n}{4m}\right)$$

$$M_f = aD^2\left(\frac{x^2}{2} - \frac{x^3}{6m} + \frac{nx^3}{6} - \frac{nx^4}{12m} + \frac{x}{2m} - \frac{nx}{2} + \frac{nx}{3m} + \frac{1}{2} - \frac{1}{3m} + \frac{n}{3} - \frac{n}{4m}\right)$$

Therefore,

$$\frac{h}{D} = \frac{4 - 6m + 3n - 4mn}{12m - 6 + 6mn - 4n}$$

The most encountered values are $\frac{h}{D} = 4$, and since they represent slightly higher values of foundation stresses, they are used for the following proportionate values of shear, moment and depth in the foundation.

It is suggested that the foundation be designed in size for the poorest soil in the class studied, but that the stresses in the foundation be studied as well for the strongest of the soils considered, since shears

TABLE 3

Values for m h/D	n = 0	n = .1	n = 1	n = 10	n = ∞
2	.533	.540	.587	.663	.688
4	.519	.526	.573	.653	.678
6	.514	.521	.568	.649	.675
10	.508	.516	.563	.645	.672

and moments may be greater in the foundation for the later condition, even for the same groundline moments.

TABLE 4

$\left(\frac{h}{D} = 4\right)$

	n = 0		n = .1		n = 1.0		n = 10		n = ∞	
x	V_f/aD	M_f/aD^2	V_f/aD	M_f/aD^2	V_f/aD	M_f/aD^2	V_f/aD	M_f/aD^2	V_f/bD^2	M_f/bD^2
.0	-.035	-.143	-.036	-.148	-.0431	-.186	-.130	-.505	-.0084	-.0354
.1	.055	-.142	.055	-.147	.0326	-.186	.007	-.512	-.0034	-.0360
.2	.126	-.133	.126	-.137	.137	-.173	.198	-.502	.0077	-.0360
.4	.211	-.098	.211	-.099	.256	-.135	.620	-.420	.0401	-.0312
.6	.218	-.054	.219	-.057	.296	-.078	.891	-.263	.0654	-.0203
.8	.147	-.016	.152	-.017	.219	-.024	.766	-.090	.0599	-.0070
.9	.085	-.004	.089	-.003	.129	-.007	.478	-.025	.0382	-.0021
1.0	.0	0	0	0	0	0	0	0	0	0

TABLE 5

$\left(\frac{h}{D} = 2\right)$

	n = 0		n = .1		n = 1.0		n = 10		n = ∞	
x	V_f/aD	M_f/aD^2	V_f/aD	M_f/aD^2	V_f/aD	M_f/aD^2	V_f/aD	M_f/aD^2	V_f/bD^2	M_f/bD^3
.0	-.062	-.125	-.066	-.127	-.080	-.160	-.218	-.440	-.015	-.0301
.1	+.037	-.126	+.025	-.129	+.016	-.164	-.018	-.456	-.011	-.0315
.2	+.098	-.119	+.099	-.123	+.101	-.158	+.111	-.444	+.001	-.0321
.4	+.188	-.090	+.190	-.093	+.227	-.124	+.539	-.389	+.036	-.0287
.5	+.204	-.073	+.210	-.073	+.261	-.099	+.715	-.326	+.049	-.0247
.6	+.200	-.049	+.206	-.056	+.270	-.072	+.824	-.248	+.060	-.0191
.8	+.138	-.015	+.141	-.016	+.204	-.023	+.752	-.089	+.056	-.0067
.9	+.078	-.003	+.081	-.004	+.121	-.006	+.456	-.025	+.036	-.0021
1.0	+ 0	0	0	0	0	0	0	0	0	0

Relative moment curves for various n values are shown in Figures 6 and 7.

Where a foundation is the shape of an inverted frustrum of a cone, it is seen that the relative value of "b" diminishes. This geometry is as follows:

$$R = N_a + \left[b - \frac{(N-1)}{D} a\right] Z$$

Thus where the top of a foundation has been increased to N times the bottom diameter, one may assume

$$R = Na + b^1 Z,$$

$$\text{Where } b^1 = b - \frac{(N-1)a}{D}$$

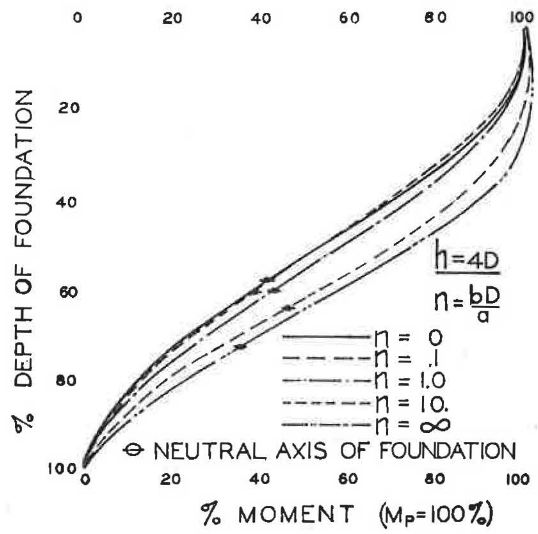

Figure 6.

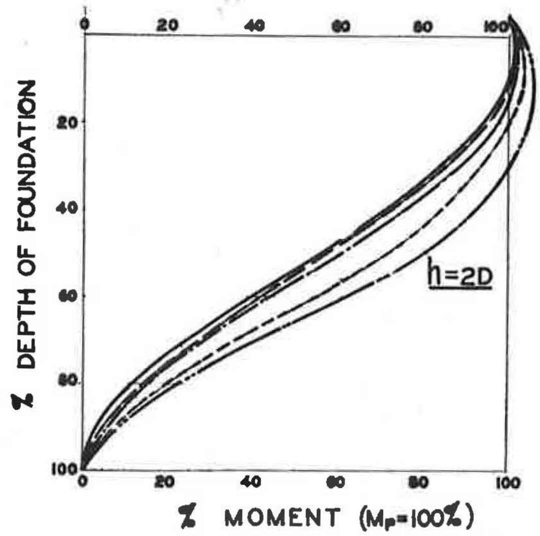

Figure 7.

Example - Say a = 900, b = 400, d = 12', N = 3, a^1 = 2700, b^1 =

$$400 - \frac{2.900}{12} = 250$$

and R = 2700 + 250Z, and the foundation can be computed by using a^1 and b^1 in place of a and b in the formulas. It should be noted the width of the foundation is the bottom width.

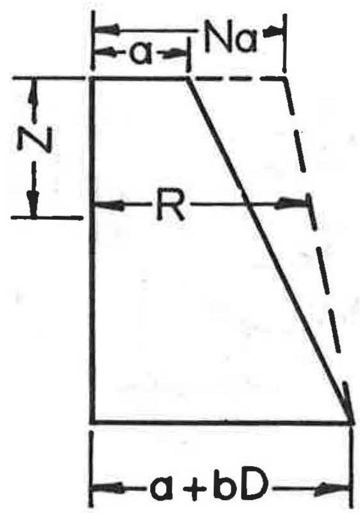

Figure 8.

SUMMARY

1. In general the most efficient foundation to resist tilting moment is slim and deep; its slimness only limited by practical limitations such as internal strength and means of digging.

2. If moments are taken around an assumed neutral axis, general equations can be used whose errors are on the safe side and whose accuracies are within about 5 percent, assuming soil values to be absolute.

3. Special shapes are so much more efficient that general formulas incorporate the basic general shape (Fig. 4a).

4. The allowable tilting resistance of a foundation in ft-lb per unit bottom width is $M = AD^2 + BD^3$ (see Table 2 for values of "A" and "B").

5. Foundation strengths (for reinforcing) may be checked according to formula in that section, but if foundation bolts go near the bottom of a foundation, further reinforcing is probably unnecessary.

EXAMPLES

1. Find practical dimensions for a foundation for a dead end pole with the following:

Five thousand-lb design load at 30-ft height. Constant load about $\frac{1}{2}$ this value.

Soil - 8 in. of top soil, 4 ft sandy clay, then hard clay bottom soil.

From Table 2, $M = 1040D^2 + 14.9D^3$ and $N = 1.4$; at the surface moment = 150,000 ft-lb (not used).

Ignore the top .67 ft as being too liable to be disturbed.

Assume a depth of not over 9 ft which gives a neutral axis 6 ft deep.

Then M = 5000 x 36.67 = 183,333 ft-lb.

Contemplate a bottom width of 24 in.

Then M per unit width = 91,667 or $91{,}667 = 1040D^2 + 14.9D^3$.

D	D^2	D^3	$1040D^2$	$14.9D^3$	Allow. M
8'	64	512	66,500	7,600	74,100
9'	81	729	84,300	10,800	95,100

(which is sufficient)

The total depth is 9 ft 8 in. after adding the top soil.

Thus a foundation (Fig. 9).

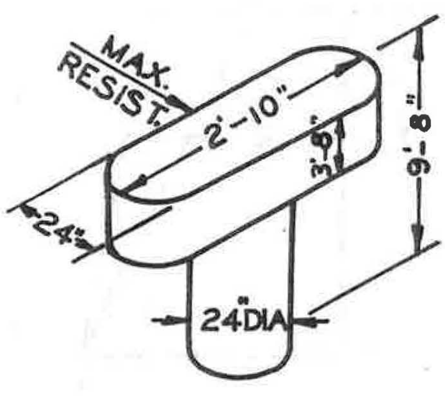

Figure 9.

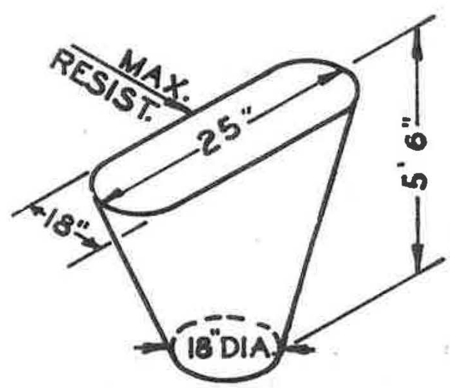

Figure 10.

2. Foundation for 1200-lb occasional load, direction known at 26 ft.

Soil - medium clay overlying cemented sand and gravel.

From Table 2 A = 870, B = 14.0, N = 1.4

Assume a depth of 6 ft, neutral axis = 4 ft.

Moment around neutral axis = 1200 x 30 = 36,000. If bottom = 18 in. dia., $M = 24{,}000 = 870D^2 + 14D^3$

D	D^2	D^3	$870D^2$	$14D^3$	Allow. M
5'	25	125	21,700	1750	23,450
6'	36	216	31,300	3020	34,320
5.1'	26	132	22,600	1850	24,450

Use 5 ft 6 in. for even half feet. In order to get away from heaving a sloped foundation was used (Fig. 10).

REFERENCES

1. Terzaghi, "Theoretical Soil Mechanics." Wiley and Sons, Inc., Arts. 12 and 127.
2. Hogentogler, "Engineering Properties of Soil." McGraw-Hill, Table 22, p. 216 and Table 23, p. 220.
3. Housel. Proceedings, ASCE, Part 2, p. 1056 (October 1943).