Hawaii's Experience with Vertical Sand Drains

K. B. HIRASHIMA, Testing Engineer
Territorial Highway Department, Honolulu

Settlement observations taken on two sand-drain projects were plotted and compared with the estimated time rate of settlement. Settlement calculations were made by combining the separate percent consolidations due to the vertical drainage and the radial drainage of the pore water. The coefficient of consolidation for the vertical drainage of the pore water was determined by means of a laboratory consolidation test. The coefficient for the radial drainage of water was computed from the results of field permeability tests and from observations on the rate of settlement shortly after the installation of sand drains.

● VERTICAL sand drains to accelerate the settlement of highway embankments have been used on two recent projects in the Territory of Hawaii: (1) Federal-Aid Project No. F 15(3) at the crossing of Kalanianaole Highway over Kahanaiki Swamp, and (2) Federal-Aid Secondary Project No. S 223(2), Haleiwa Cutoff Road, across the Kiikii Swamps.

The author is indebted to the pioneer work of O. J. Porter (1) and others in regard to design and construction methods involving vertical sand drains. Since these aspects of the subject have been competently and adequately covered by others and data are readily available, no extended discussion concerning them will be attempted here. Instead, this paper will attempt to deal with the more-technical phases of the subject about which there appears to be a paucity of information in the literature.

Both swamps referred to above were formed of alluvial sediments washed in from higher ground mixed with some organic material and occur over areas which, in the geologic past, were probably under sea level or, at least, close to the shore line, so that they are underlain by sandy material of marine origin. By marine origin is meant that the sand was either part of an ocean beach, or a sand dune in case the shore line was some distance out. In the case of Kahanaiki Swamp on Project F 15(3) the entire material over the sand bottom was soft, so there was a drainage face at the bottom of the swamp. In the case of the Kiikii Swamps on Project S 223(2), part of the alluvial deposits over the sand had consolidated or hardened sufficiently so that, apparently, it was uncompressible and relatively impervious to the flow of moisture. Vertical drainage was considered to be only in an upward direction. The existence of this hard layer was discovered during the soil-profile investigations prior to construction. A cylindrical steel mandrel was used to place the sand drains, and during actual construction it was possible, by means of the heavy pile driving hammer used, to drive the steel mandrel through this hard layer, although with considerable effort.[1] The subsequent settlement data appeared to show that this hard layer is not consolidating under the weight of the embankment. Thus, the driving of the sand piles through this hard layer, although it has done no harm, has not affected the settlement of the embankment, which is due entirely to the consolidation of the upper, softer layer.

WORKING TABLE, CONSTRUCTION METHODS, SETTLEMENT DATA

A working table measuring approximately 2 feet 6 inches in thickness was first laid over the surface of the swamp so as to provide a relatively stable surface over which equipment could be operated. Immediately after the working table was leveled off, holes were dug through it, settlement platforms installed over the top of the soft swamp layer, and the holes backfilled.

The settlement platforms consisted of a base 3 feet by 3 feet built of heavy 2-inch planks with a length of ½-inch galvanized pipe attached at the center. As the embankment was built up, additional lengths of ½-inch pipe were added as needed. Elevations were taken on the pipes as the work progressed and the results plotted to show the settlement with time. This will be discussed later (see Figs. 5 and 6).

A paper on the subject was presented by the author at the June 1952 convention of the

[1] That is relative to the effort required to penetrate the upper softer layer.

Western Association of State Highway Officials at Seattle, Washington, at which time one of the projects here discussed was still under construction. As of April 6, 1953,[2] Project F 15(3) has been completed and in service for 19 months. Project S 223(2) has been completed and in service for 7⅓ months. Levels were taken on the finished pavement recently and compared to levels at the time of completion. The additional settlements under service thus observed are shown on the time-settlement curves (Figs. 5 and 6).

Placing of sand drains was begun as soon as the working table was ready to receive the necessary equipment.

Using a power auger, holes were dug through the working table to the top of the swamp. A steel mandrel was then driven through the soft, compressible layer to stable (noncompressible) material below. The mandrel was driven, without leads, by means of a regular pile driving hammer. It was a simple matter to stand up the mandrel by letting it fall of its own weight into the soft layer. Guy wires tied onto truck winches then kept the mandrel plumb. For the lengths of mandrels used on the two projects here reported, up to approximately 40 feet, the above method of driving without leads proved practicable. (Driving with leads was tried and found to be much slower.)

The mandrels on the two projects were both fabricated by the contractors from heavy 18-inch-diameter steel pipe. An orange-peel arrangement, which resulted in a conical point when closed by means of an inside catch, was used for the driving end on Project F 15(3). After driving to the necessary depth, the mandrel was filled with sand. A "fish line" extending down the inside of the mandrel to the point was pulled to open the orange peel. This allowed the sand to be deposited in the hole as the mandrel was slowly withdrawn. On the other project, the contractor used a solid, conical shoe hinged on one side for his driving point. In driving, the resistance of the ground kept the shoe in the closed position. In raising the mandrel, the shoe dropped of its own weight to the open position, thus allowing the sand to run out.

One phenomenon that has to be guarded against is that of arching of the sand in the mandrel. To overcome this and to insure proper density of the sand in the hole, the mandrel was provided with a tight cover and compressed air was applied to the top of the sand when withdrawing the mandrel from the hole. The pressure of the air and the rate of withdrawal of the mandrel have to be carefully regulated, otherwise there is danger of blowing and dispersing the sand into the swamp muck or of having a partially empty hole.

Cost data for the two Hawaii projects have been analyzed in Appendix A.

SOIL TESTS

Undisturbed samples were taken and consolidation tests were run. Due to a lack of deep sampling tools, the samples were obtained only from relatively near the surface.

From the consolidation tests the following data were obtained: (1) the voids-ratio versus applied pressure relationship and (2) time-settlement relationship for the sample. From the time-settlement relationship for the sample, the coefficient of consolidation was calculated by the square root of time (Gilboy's) method.

Since the laboratory procedures in carrying out the above tests and the various empirical rules for adjusting the laboratory data are well known, the subject will not be considered further here.

The pressures on the soft swamp material at various stations along the centerline profile, due to the highway embankment, were calculated and the corresponding voids-ratios estimated from the voids-ratio versus pressure relationship developed by the consolidation tests. The total settlement was then estimated by means of the following formula:

$$S = \frac{e_1 - e_0}{1 - e_0} D \qquad (1)$$

where S = total settlement

e_1 = ultimate voids-ratio due to applied loads

e_0 = voids-ratio prior to application of loads

D = thickness of compressible layer

The voids-ratios e_0 and e_1 were taken as the average of the voids-ratios at the top and at the bottom of the compressible layer. The pressure at the bottom of the compressible layer due to the embankment

[2] Date of this paper.

load was calculated by means of the following formula:

$$P = \frac{q}{\Pi} (B + \sin B) \qquad (2)$$

where P = pressure at bottom of compressible layer at a point directly below the center of the embankment section
q = intensity of surface loading
Π = 3.1416
B = angle subtended by the width of the embankment at the point where P is estimated.

METHOD OF COMPUTING TIME-SETTLEMENT

In ordinary consolidation, the drainage of the pore water is in the vertical direction and takes place according to the following differential equation, due to Terzaghi (2).

$$\frac{\partial P}{\partial t} = \frac{c_v \partial^2 P}{\partial z^2} \qquad (3)$$

$$c_v = \frac{k_v (1 + e)}{a\gamma} \qquad (4)$$

where P = hydrodynamic excess pressure of the pore water at any time, subsequent to t = 0, due to the embankment load
z = depth below surface at which the pressure P is measured.
t = time
c_v = coefficient of consolidation for vertical drainage of the pore water
k_v = coefficient of permeability of the swamp material for vertical flow of the pore water
a = coefficient of compressibility of the swamp material
e = voids ratio
γ = density of the pore water

The boundary conditions appropriate to the present problem were taken as,

$$\left.\begin{aligned} P &= P_1 \text{ when } t = 0 \\ P &= 0 \text{ for } z = 0 \\ \frac{\partial P}{\partial z} &= 0 \text{ for } z = H \end{aligned}\right\} \qquad (5)$$

where P_1 = pressure due to the weight of the embankment, assumed to be uniform throughout the whole depth of the swamp
H = half-depth of soft-compressible layer for Project F 15(3) and the whole depth of soft material in the case of Project S 223 (2)

The basic conception of the theory of consolidation is that when the hydrodynamic excess pressure P in the pore water reduces to zero, the consolidation is complete and grain-to-grain contact of the soil results. Hence, $(P_1 - P)/P_1$ is a measure of the state of consolidation and

$$\text{Percent Consolidation} = U\% = \frac{P_1 - P}{P_1} \times 100 \qquad (6)$$

The differential equation (Equation 3) has been completely solved for various boundary conditions and various tables and curves are available in the literature. For the boundary conditions here specified, the curve (Fig. 1) shows the relationship between the percent of consolidation and the nondimensional time factor T_v,

$$T_v = \frac{c_v}{H^2} t \qquad (7)$$

If we know the values of c_v and H for the swamp, we can compute by Equation 7 the time factor T_v corresponding to any time t subsequent to the application of the embankment load onto the swamp, and thus find the corresponding percent of consolidation (U%) from the curve of Figure 1. This percent of consolidation applied to the total ultimate settlement computed by Equation 1 is then the estimated settlement up to the time t.

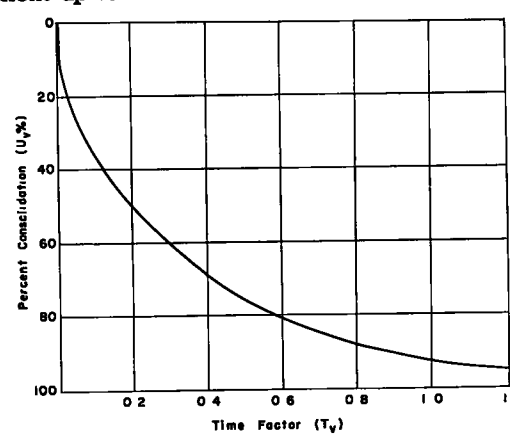

Figure 1. Chart for determining percent of consolidation due to vertical drainage.

With the installation of sand drains the drainage of the pore water can take place

in horizontal radial directions as well as vertical. For this three-dimensional consolidation, the appropriate differential equation is

$$\frac{\partial P}{\partial t} = c\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial y^2}\right) \qquad (8)$$

Transforming to cylindrical coordinates, and making use of axial symmetry, Equation 8 becomes

$$\frac{\partial P}{\partial t} = c\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial z^2}\right) \qquad (9)$$

Equation 9 is based on the assumption that the coefficients of consolidation in the vertical and radial (horizontal) directions are the same. In general this is not the case so that instead of Equation 9 we will have,

$$\frac{\partial P}{\partial t} = c_v \frac{\partial^2 P}{\partial z^2} + c_r\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right] \qquad (10)$$

where $c_v = \frac{k_v(1+e)}{a\gamma}$ = coefficient of consolidation for the vertical drainage of pore-water

k_v = coefficient of permeability for the vertical flow of pore-water

$c_r = \frac{k_r(1+e)}{a\gamma}$ = coefficient of consolidation for the radial (horizontal) drainage of pore-water (10a)

k_r = coefficient of permeability for the radial (horizontal) flow of pore-water

Equation 10 can be solved as a whole or, more conveniently, it can be split into two parts,

$$\frac{\partial P}{\partial t} = c_v \frac{\partial^2 P}{\partial z^2} \qquad (3)$$

$$\frac{\partial P}{\partial t} = c_r\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right] \qquad (11)$$

Each part with appropriate boundary conditions is then solved separately and the corresponding percent of consolidation at a given time t computed for each. It can be shown that these partial percent consolidations can be combined according to the following formula[3] to give the combined percent of consolidation applicable to equation 10.

$$100 - U\% = (100 - U_v\%)(100 - U_r\%)(1/100) \qquad (12)$$

[3]See Reference 3.

where $U\%$ = combined percent consolidation

$U_v\%$ = percent consolidation due to the vertical drainage of water

$U_r\%$ = percent consolidation due to the radial (horizontal) drainage of water

The part $U_v\%$ can be estimated easily by means of the curve in Figure 1. To calculate $U_r\%$ is not so easy, since published data on the subject are comparatively meager. The method of computing the percent of consolidation due to radial drainage, corresponding to Equation 11 is given in Appendix B. Numerous curves suitable for engineering use in connection with consolidation problems is given in a work by Barron (4).

Unlike the vertical percent of consolidation, the radial percent of consolidation depends not only on a time factor T_r but also on a parameter b/a, where b is the radius of the area, assumed to be circular, drained by each sand drain and a is the radius of the sand drain itself. The sand drains were arranged in a hexagonal pattern as shown in Figure 2 and the values of b/a were 14.2 and 11.9 for Project F 15(3) and S 223(2), respectively.

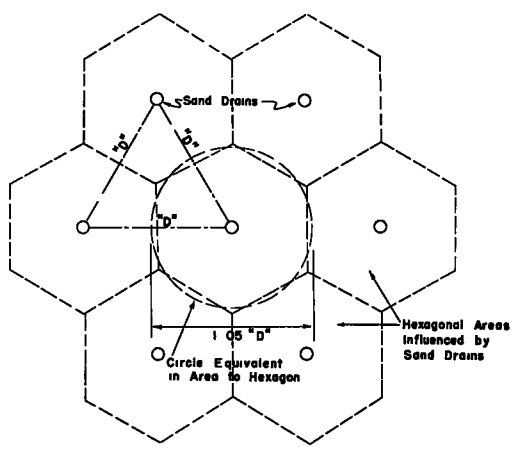

Figure 2.

Figure 3 shows the relationship, computed as shown in Appendix B, of $U_r\%$ to the time factor T_r where

$$T_r = \frac{c_r t}{4b^2} \qquad (13)$$

where T_r = time factor for radial drainage of pore-water

t = time

c_r = coefficient of consolidation for radial drainage of pore-water
b = radius of circular area influenced by each sand drain

If curves such as Figures 1 and 3 or corresponding tabular data are available, the problem reduces to the following steps: (1) For any time t, subsequent to the instant when the embankment load is applied, compute the corresponding time factors by Equations 7 and 13; (2) From the curves of Figures 1 and 3 find the corresponding percents of consolidation U_v% and U_r%; (3) Combine the above according to Equation 12; and (4) the combined percent of consolidation applied to the ultimate settlement as calculated by means of Equation 1 will give the settlement up to the time t.

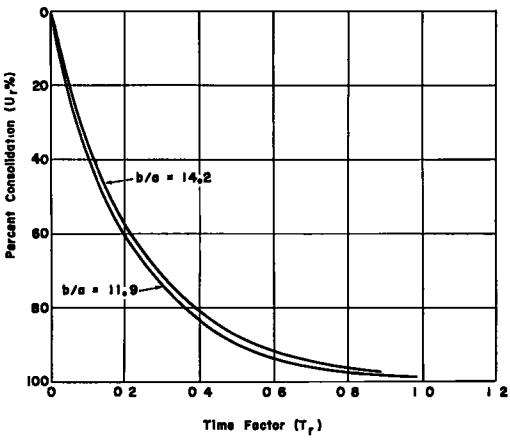

Figure 3. Percent of consolidation for radial (horizontal) drainage.

In the theory, it is assumed that the total consolidating pressure is applied suddenly at time t = 0. Physically this is impossible. The working table was laid in one lift, but the embankment was built up in layers and the work took many days to complete. Hence, the embankment load was actually applied gradually and not suddenly as assumed in the theory. Because of this fact, the settlement was computed by the method of superposition, which is valid because the differential equations are linear. The time when each convenient increment of load was applied was taken as a new zero of time in computing the settlement due to that particular increment. These partial settlements for corresponding times on the time scales of Figures 5 and 6 were added to give the total calculated settlements. The results were plotted as calculated time-settlement curves.

The consolidation due to vertical drainage begins as soon as work on the working table starts, but consolidation due to radial drainage does not begin until some time later when work on the sand drains begins. This introduces an error. However, the error is significant for small values of t only. For large values of t, that is for long time effects, the error is negligible.

In the determination of the time factors the only unknown quantities are the two coefficients of consolidation. The first, c_v, can be computed from the laboratory consolidation test. Various techniques have been devised with this object in view and have been thoroughly discussed in the literature so that additional comment at this time seems unnecessary.

The direct laboratory determination of c_r, the coefficient of consolidation for radial drainage of pore water, is probably not possible. In the case of alluvial sediments, as in swamps, the deposit is stratified. For use in the consolidation problem, the value of the coefficient of consolidation that is required is not that of the individual layers but of the deposit as a whole.

Referring to Equations 4 and 10(a), we see that c_v and c_r are proportional to the respective coefficients of permeability. The ratio of the permeabilities is therefore also the ratio of the coefficients of consolidation.

DETERMINATION OF THE PERMEABILITY RATIO

In theory we can obtain samples from various strata and determine the coefficients of permeability of the various strata. For vertical flow, the permeabilities are in series and for horizontal flow they are in parallel. Then following the electrical analogy of conductances in series and in parallel, it can be seen that the permeability of the deposit as a whole in the horizontal direction is always greater than the permeability in the vertical direction. In practice, however, the method is cumbersome and uncertain due to the presence of poorly defined strata and lenses of different materials.

In the analysis of the settlement for Project F 15(3), the coefficient of consolidation for horizontal drainage was estimated from the difference in rate of settlement before and shortly after the installation of the sand drains. In the case of Project S 223(2), an independent check of the ratios of the coefficients of consolidation was made by conducting field permeability tests.

The field permeability tests were made using the tube method and the auger-hole method.

In the tube method, a tube known diameter is placed tightly in a hole of the same size to a known depth below a water table as shown in Figure 4. The water is then pumped out of the tube down to some known elevation below the water table and above the bottom of the tube. Water from the surrounding area is allowed to flow into the tube from the bottom. The time it takes for the water to rise in the tube a given distance is measured. The permeability is computed by means of the following formula:

$$k = \frac{\pi R^2 \ln (h_1/h_2)}{A\,t} \qquad (14)$$

where k = coefficient of permeability
R = radius of tube
$\ln$ = natural logarithms
h_1, h_2 = initial and final water levels in tube (see Fig. 4)
t = time required for water to rise in tube from h_1 to h_2
A = a coefficient determined by use of an electric analogue

Since the flow of water is upward into the tube, the coefficient of permeability thus measured is that in the vertical direction.

In the auger-hole method suggested by Kirkham and Van Bavel (6, 7), an auger hole of known diameter is dug down to a known depth. The water in the hole is pumped out and its rate of rise noted. If the auger hole is dug all the way down to an impermeable layer, the problem is subject to exact mathematical analysis. If the hole is not carried down to an impermeable layer, the problem can be solved by means of an electrical analogy. In both cases, the equipotential and stream lines can be drawn and from a study of these curves it is seen that the flow into the auger hole is predominantly horizontal. Hence, the

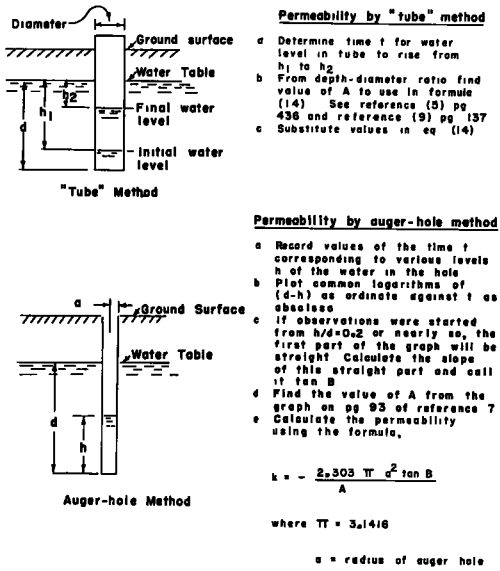

Figure 4.

permeability as computed by this auger hole method is predominantly horizontal.

Spangler in his "Soil Engineering" describes this auger-hole method (9) and gives what appears to be a simplified formula for computing the coefficient of permeability. In our experience the formula gave values of the permeability that was much too high. Hence, in our work we followed strictly the method of Van Bavel and Kirkham (7).

The auger-hole method requires several hours to perform. Since the very act of boring an auger hole puddles the soil, it is necessary to fill and empty the hole several times in order to de-puddle the soil pores. Hence, a single determination takes at least a day. The tube method takes much longer. With the thought that it might be of interest, a brief description of both methods is given in connection with Figure 4.

Having thus determined the permeability in the vertical and radial (horizontal) directions, we can use their ratio to compute the value of c_r/c_v. In this way, for Project S 223(2), the value of c_r was estimated to be 0.0146 or 0.015 sq. ft. per hour based on a value of c_v of 0.002 sq. ft. per hour, as determined by the laboratory consolidation test.

The above values of c_v and c_r were used to calculate the time-settlement curves for Stations 25+10 and 25+75 on Project S 223(2). The actual long-term

rate of settlement appeared to point to a value of c_r somewhat lower than given in the above and so a ratio of c_r/c_v of 6 was used to calculate the settlements at Stations 47+43 and 48+20.[4]

settlement is shown in heavy lines and the calculated settlement in light lines. The observed settlement curves in both cases show a sharp drop at the time of installation of the sand drains. On both projects

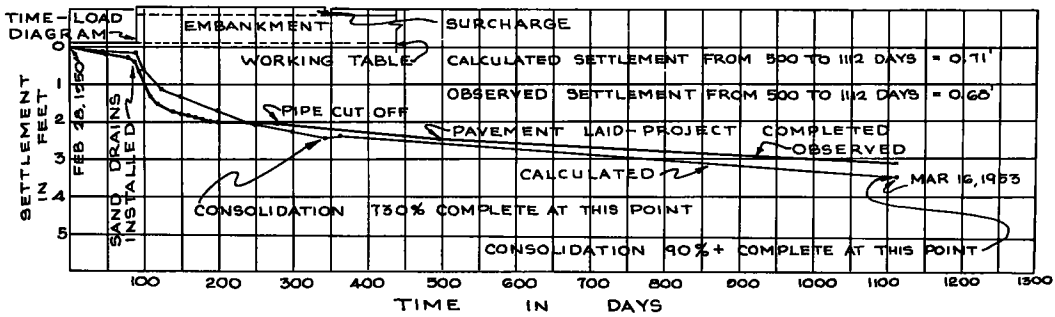

Figure 5. Comparison of calculated versus observed settlements for vertical sand drains over the Kahanaiki swamp, Project F 15 (3), Station 269 + 25.

It is interesting to compare the estimated combined percent of consolidation as of a recent date, April 6, 1953, with the hypothetical percent of consolidation as of the same date calculated on the assumption of vertical drainage only (no sand drains).

TABLE 1
COMBINED PERCENT CONSOLIDATION COMPARED WITH PERCENT CONSOLIDATION DUE TO VERTICAL DRAINAGE ONLY
PROJECT S 223(2)

Station	Date Started	Date Sand Drains Installed	Date Last Data Obtained	Ratio c_r/c_v	Calculated: Consolidation without Sand Drains	Calculated: Consolidation with Sand Drains
					%	%
25+10	10-27-51	1-23-52	4-6-53	7½	28 4	91 4
25+75	10-27-51	1-30-52	4-6-53	7½	39 6	92 1
47+43	10-25-51	12-11-51	4-6-53	6	25 5	88 8
48+20	10-25-51	12-11-51	4-6-53	6	22 9	88 4

A glance at the last two columns shows that the sand drains were effective in speeding up the settlement. It seems safe to assert that future settlement, if any, will be slight; whereas if sand drains had not been installed, further progressive settlement could be expected resulting in high maintenance costs. The depths at the four stations listed in Table 1 are comparable and from the last column of the table, the combined percent of consolidation does not appear to be greatly affected by a change in the ratio of c_r/c_v from 7½ to 6.

SETTLEMENT CURVES

The observed settlement and the calculated combined settlement were plotted for a number of stations on both projects. An example from each project is presented (see Figs. 5 and 6). The observed

some lateral plastic displacement was observed, indicating too fast a rate of loading, which probably accounts for the greater rate of observed settlement at the beginning. It can be seen that, if a constant displacement of about ½ foot be added in Figure 6, beginning shortly after the time the sand drains were installed, the calculated and observed settlement curves will agree closely and the above-mentioned plastic displacement would be accounted for. On Project F 15(3) a longitudinal plastic displacement under a bridge abutment appears to have taken place, although actually the problem is more complicated because erosion damage took place during a rainstorm that occurred shortly after the project was completed.

During March and April of 1953, after both projects had been completed and in service for some time, levels were taken and the actual settlements at various points since completion were compared with the previously computed values. Such a comparison is given in Table 2.

Considering the fact that c_r and c_v vary from station to station and in view of the many other uncertainties involved, the observed settlement curves are considered to be in good agreement with the calculated.

CONCLUSION

Based on our experience on the two projects mentioned, vertical sand drains are an effective and satisfactory method of accelerating the consolidation of soft, com-

[4] A recheck of our field-permeability test data gave a value of c_r/c_v of slightly less than 6, or a value of c_v of 0.012 sq. ft. per hr.

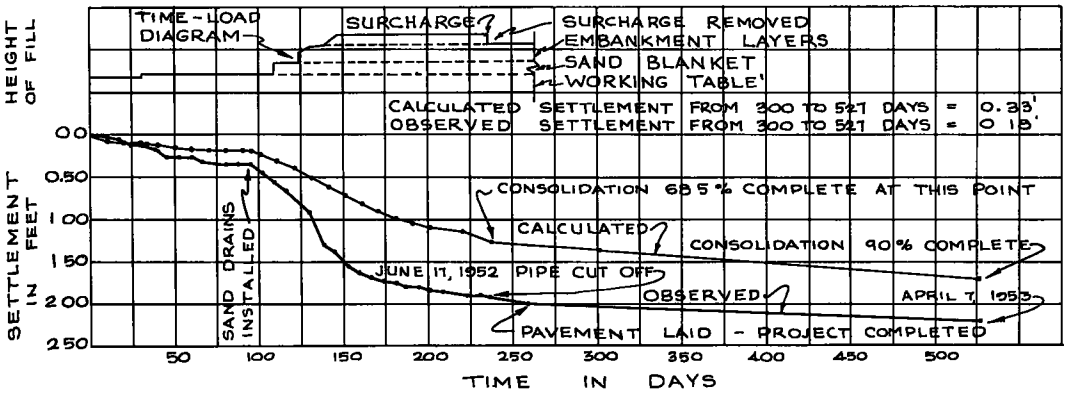

Figure 6. Comparison of calculated versus observed settlements for vertical sand drains over Kiikii swamps, Project S 223 (2) Station 25+75.

TABLE 2
OBSERVED AND CALCULATED SETTLEMENTS SINCE COMPLETION OF SAND DRAIN PROJECTS

Project	Station	Observed Settlement	Calculated Settlement
		ft.	ft.
F 15(3)	268+00	0.05	0.81
F 15(3)	269+25	.68	.71
F 15(3)	271+50	.47	.99
F 15(3)	273+50	.44	.37
S 223(2)	25+10	.17	.49
S 223(2)	25+75	.18	.33
S 223(2)	47+43	.09	.54
S 223(2)	48+20	.22	.57

pressible foundations. Too small a diameter of sand drains is probably not advisable, because of possible difficulties in filling the mandrel with sand and of the arching of the latter due to friction of the sides. The 18-inch-diameter sand drains used on the two projects were of satisfactory size in these respects. In general, compressed air must be used to force the sand out of the mandrel, and both the pressure of the air and rate of withdrawal of the mandrel must be carefully regulated. The spacing of the sand drains can be varied so that reasonably complete consolidation will have taken place by the time the project is completed.

The coefficient of consolidation due to radial drainage of the pore water can be estimated by means of field permeability tests.

In computing the settlement due to vertical drainage of the pore water, it was assumed that the pressure distribution throughout the compressible layer was rectangular, whereas actually it was somewhat trapezoidal. However, because of the relatively small part it plays in the total combined consolidation, it is not believed the error is serious.

The discrepancy between the estimated settlements and the observed settlements for the periods since the completion of the two projects could be due to errors in the values of the coefficients of consolidation, to errors in the estimated ultimate settlements, or to other indeterminate factors.

In the field permeability tests, comparatively large "samples" are involved compared to the usual laboratory samples. Moreover, what is measured is the rate of seepage of the swamp water itself and not distilled water, which may have different seepage characteristics (8). Hence, the field permeability tests lead to more-representative values for the swamp material as a whole, at least theoretically.

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APPENDIX A

Cost Data on Vertical Sand Drains

Kalanianaole Highway, Federal Aid Project F 15 (3)

Number of sand drains	249
Total length	8521 lin. ft.
Average length of drain	34 ft.
Diameter of sand drains	18 in.
Volume of sand required	646 cu. yd.

Labor Cost

Foreman	296 hr.	at $1.75	$518.00	
Riggers	684 "	" 1.40	957.60	
Crane Operator	296 "	" 1.80	532.80	
Pitman	296 "	" 1.00	296.00	
Laborers	780 "	" 1.10	858.00	
Truck Drivers*	86 "	" 1.25	107.50	
				$3,269.90

Equipment Costs

Crane, leads, and hammer	296 hr.	at $12.00	$3,552.00
Compressors	592 "	" 3.50	2,072.00
Air tanks	592 "	" .50	296.00
Post hole digger	110 "	" 3.50	385.00
6 c. y. Trucks	86 "	" 2.00	172.00
Cost of mandrel (estimated)			1,000.00
			$ 7,477.00

Total cost - labor and material	$10,746.90
Cost per lin. ft. exclusive of sand, taxes, overhead, and profit	1.26

NOTE: On this project, the contractor had free access to a sand pit, so that the cost of sand is not included in the above costs. Haul from sand pit to project approximately 4 miles.

* Hauling Sand from Sand Pit to Project.

Haleiwa Road Cut-off, Federal Aid Project S223(2)

No. of sand drains	390
Total length	17,023 lin. ft.
Average length of drain	43 lin. ft.
Diameter of sand drains	18 inches
Volume of sand required (including spillage)	1,400 c. y.
No. of working days required to complete work	39 days

Equipment Cost (Average Cost Per Day)

1 - P & H Crane	8 hrs. at	$9.80 =	$ 78.40
1 - Trencher	8 " "	6.15 =	49.20
1 - Hough Loader	8 " "	2.80 =	22.40
1 - 5000# Hammer	8 " "	2.50 =	20.00
1 - 500 cfm Compressor	8 " "	7.00 =	56.00
Total Cost Equipment			$226.00

Labor Cost (Average Cost Per Day)

1 - Superintendent			$25.00
2 - Operators	16 hr. at	$2.00 =	32.00
3 - Operators	24 " "	1.85 =	44.40
1 - Mechanic	8 " "	1.80 =	14.40
1 - Laborer	8 " "	1.45 =	11.60
2 - Laborers	16 " "	1.25 =	20.00
1 - Laborer	8 " "	1.10 =	8.80
Total Cost Equipment			$156.20
W.C. & P.L. Insurance - 2.906%			4.53
			$160.73

Material Cost

1.400 c. y. at $1.25 = $1,750.00

Following is Cost Per Lin. Ft.

Mandrel $1,000.00 / 17,023 = $0.0587 per lin. ft.

Equipment Cost 39 days at $226.00 = $8,814.00
$8,814.00 / 17.023 = $0.5178 per lin. ft.

Labor Cost 39 days at $160.73 = $6,268.47
$6,268.47 / 17.023 = $0.3682 per lin. ft.

Material Cost
$1,750.00 / 17,023 = $0.1028 per lin. ft.
TOTAL $1.0475 per lin. ft.

SUMMARY:
Cost per lin. ft. of driving sand drains - including vertical holes and sand, backfill, but exclusive of taxes, overhead, and profit $1.05.

APPENDIX B

Consolidation Due to Horizontal Flow

The differential equation of consolidation due to radial, horizontal flow is (see eq. (4), Page 291 of "Theoretical Soil Mechanic" by Terzaghi).

$$\frac{\partial P}{\partial t} = c\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right) \qquad (1)$$

Where P = hydrodynamic pressure within the pores of the compressible material at any time t

t = time

r = radial distance from the center of a vertical sand drain to the point where the pressure P is measured

c = coefficient of consolidation for flow in the horizontal direction

Boundary Conditions: Let the radii of a sand drain and that of the equivalent circular area drained by it be a and b respectively. Then if the applied load is P_1 per unit area the boundary conditions are

$$P = 0 \text{ for } r = a \qquad (2)$$

$$\frac{\partial P}{\partial r} = 0 \text{ for } r = b \qquad (3)$$

$$P = P_1 \text{ for } t = 0 \qquad (4)$$

The boundary condition (Equation 2) requires that the pressure drop abruptly to zero at the boundary between the sand drain and the compressible material. This condition can be met if the sand drains are free-draining. The boundary condition (Equation 3) requires that there be no flow across the outer radius b of the area drained. This means that the flow at any point must be radially toward the nearest sand drain. Thus the requirement is a reasonable one.

Solution of Equation (1). The solution of Equation (1) subject to the given boundary conditions is

$$P = P_1 \sum_{k=1}^{k=\infty} e^{-u_k^2 ct} A_k B_0 (u_k r) \qquad (5)$$

Where

$$B_0 (u_k r) = J_0 (u_k r) - \frac{J_0 (u_k a)}{N_0 (u_k a)} N_0 (u_k r) \qquad (6)$$

e = base of Naperian logarithms

$J_0 (u_k r)$ = Bessel function of first kind of zero order

$N_0 (u_k r)$ = Bessel function of the second kind of zero order (Neumann function)

A_k and u_k are constant; to be determined in the manner set forth below.

Determination of u_k: In differentiating Equation (5) with respect to r as called for by the boundary (Equation 3), we note that $B_0 (u_k r)$ is the only factor containing r. Hence the u's must be determined in such a way that

$$\frac{\partial B(ur)}{\partial r} = 0 \quad \text{for } r = b \qquad (7)$$

Making use of the relations, -

$$\frac{\partial J_0(ur)}{\partial r} = -uJ_1(ur) \qquad (8)$$

and

$$\frac{\partial N_0(ur)}{\partial r} = -uN_1(ur) \qquad (9)$$

we find that Equation (7) is equivalent to

$$J_1(ub) - \frac{J_0(ua)}{N_0(ua)} N_1(ub) = 0 \qquad (10)$$

Where $J_1(ur)$ = a Bessel function of the first kind of the first order

$N_1(ur)$ = a Bessel function of the second kind of the first order

Equation (10) has an infinite number of roots $u_1 b$, $u_2 b$, $u_3 b$, $u_4 b$, etc. These may be found by a process of successive approximations and the values of u_1, u_2, u_3, u_4, etc., determined.

Determination of A_k: The coefficients A_k are determined by the following equation:

$$A_k = \frac{\frac{2a}{u_k} B_1(u_k a)}{b^2\left[B_0(u_k b)\right]^2 - a^2\left[B_1(u_k a)\right]^2} \qquad (11)$$

Where $B_1(u_k a) = \frac{J_0(u_k a)}{N_0(u_k a)} N_1(u_k a) - J_1(u_k a)$ (12)

For t = 0, Equation (5) reduces to

$$P = P_1 \sum_{r=1}^{r=\infty} A_k B_0(u_k r) \qquad (13)$$

and the purpose of Equation (11) is to determine the coefficients A_k in such a way so that

$$A_1 B_0(u_1 r) + A_2 B_0(u_2 r) + A_3 B_0(u_3 r) + \text{etc.} = 1 \qquad (14)$$

for all values of r. Then

$P = P_1$ for t = 0 for all values of r and the boundary condition Equation (4) will hold true.

Percent Consolidation: We can compute the hydrodynamic pressure for any given point subsequent to t = 0 by means of Equation (5). The difference $P_1 - P$ is the loss of pressure and is a measure of the amount of consolidation. Thus the percentage consolidation at a point is,

$$U_P\% = \frac{P_1 - P}{P_1} \times 100 \qquad (15)$$

Where $U_P\%$ = percent consolidation at a given point

To find the average percentage consolidation over the entire area influenced by a sand drain, we need to find the average loss of pressure over the area. Thus,

$$U\% = 100 \times \frac{\Pi(b^2-a^2)P_1 - 2\Pi\int_a^b Prdr}{\Pi(b^2-a^2)P_1} \qquad (16)$$

Where U% = average percent consolidation over the entire area influenced by a sand drain.

Evaluating the integral in Equation (16) after first substituting for P from Equation (5), we find

$$U\% = 100 \left[1 - \frac{\Sigma^2 \frac{a}{u_k} A_k B_1 (u_k a) e^{-u_k^2 ct}}{b^2 - a^2} \right] \quad (17)$$

(The summation in the above is with respect to the successive values of k)

By a suitable transformation, it can be easily shown that the value of U% in Equation (17) depends on a parameter b/a.

By means of Equation (17), the percentage consolidation corresponding to various values of time can be computed and a time-consolidation curve plotted. In general, if we are interested mainly in long-time effects, say consolidation greater than 30%, one term of the series under the summation sign in Equation (17) will give sufficiently accurate results. Many times the coefficient of consolidation c is unknown and it is desirable to plot a general curve from which a time-consolidation curve can be plotted later when the value of c is determined. We can do this by introducing a zero-dimensioned time factor,

$$T = \frac{ct}{4b^2} \quad (18)$$

The exponential factor in Equation (17) then becomes,

$$e^{-u_k^2 ct} = e^{-4b^2 u_k^2 T} \quad (19)$$

By plotting T against U%, we obtain a time-factor, consolidation curve such as Figure 3.

SUMMARY

The steps in computing a time-consolidation curve are as follows:

1. From the spacing of the sand drains determine the radius of the equivalent circular area drained by each. Call this radius b. Also decide on the radius a of the sand drains.
2. By means of tables such as Jahnke and Emde's "Tables of Functions," solve the Equation (10) for values of u. Call these roots (more properly eigen values) in order of magnitude u_1, u_2, u_3, u_4, etc.
3. Having determined the u's, we find from Equation (11) the coefficients A_1, A_2, A_3, A_4, etc., corresponding to the u's.
4. Finally by means of Equation (17), we can compute the percentage consolidation corresponding to various assumed values of t (or T) and thus obtain a time-consolidation curve (or time factor, consolidation curve).

EXAMPLE

Given sand drains 18 inches in diameter (a = 0.75 feet) spaced 17 feet center to center in a geometrical pattern as shown in Figure 6. ($b = \frac{1.05 \times 17}{2} = 8.93$ ft.).

The parameter b/a = 11.9. Compute the percent consolidation due to radial, horizontal flow for a time-factor T = .0542

Solution

1. Values of b and a are as given above.
2. Second step. Compute the values of the u's satisfying Equation (10). This is equivalent to finding the values of u for which the graph of the function crosses the

horizontal zero axis. As a preliminary, assume various values of u, compute the values of the function and note their signs, whether plus or minus. In this way we find that the function changes signs as follows:

from - 0.1072 for u = 0.1 to + 0.3996 for u = 0.2
from + 0.2382 for u = 0.5 to - 0.3013 for u = 0.6
from - 0.8391 for u = 0.8 to + 0.8037 for u = 1.0
from + 0.0637 for u = 1.7 to - 0.0588 for u = 1.73

Therefore the first four roots u_1, u_2, u_3, and u_4, lie between the above values. The first root u_1 lies between 0.1 and 0.2. The algebraic difference between the two values of the function is the change in value of the function as u varies from 0.1 to 0.2. Dividing the change in the function by the change in the value of u gives the derivative or slope of the secant line.

$$\text{Slope} = \frac{0.3996 - (-0.1072)}{0.2 - 0.1} = 5.068$$

Thus the function changes 5.068 units for a unit change in u. But for u = 0.1, the function is -0.1072 and is increasing algebraically as u increases. In order for the function to increase from 0.1072 to zero, the value of u must therefore increase by

$$\frac{0.1072}{0.5068} = 0.0211$$

Hence u = 0.1 + 0.0211=0.1211 to a first approximation. Substituting this value of u in Equation (10), we find that the function does not become zero but has a value of +0.0192.

We next repeat the above approximation for the shorter interval u = 0.1 to u = 0.1211. This second approximation gives us a value of u = 0.1178. For this value of u the function on the left of Equation (10) has the value + 0.0015. By a third approximation, we find u_1 = 0.1175 and for this value of u, the value of the function in Equation (10) is zero for practical purposes. In this way the first four roots of Equation (10) are found to be

$$u_1 = 0.1175$$
$$u_2 = 0.5455$$
$$u_3 = 0.9390$$
$$u_4 = 1.7151$$

3. Third step. Having found the values u_1, u_2, u_3, etc., the next step is to find the values of the coefficients A_k by means of Equation (11). To compute A_1, we need the values of the following:

$$B_1(u_1a) = B_1(0.088)$$
$$B_0(u_1b) = B_0(1.049)$$

Looking up the tabular values of the various functions involved we find,

$$A_1 = \frac{\dfrac{2x.75}{0.1175}\left[(-0.6163)(-7.2332) - 0.0440\right]}{8.93^2\left[0.7432 - (-0.6163)(0.1255)\right]^2 - 0.75^2\left[-0.6163)(-7.2332) - (0.0440)\right]^2}$$
$$= 1.3191$$

In this way the coefficients are found to be

$$A_1 = 1.3191$$
$$A_2 = 0.2168$$
$$A_3 = 0.0612$$
$$A_4 = -0.0849$$

4. Fourth step. In computing the percent consolidation by means of Equation (17), we split the part under the summation sign into two parts as follows:

u_k	$\frac{a}{u_k} B_1(u_k a)$	2A	$2A_k \frac{a}{u_k} B_1(u_k a)$
u_1	28.1731	2.6382	74.3263
u_2	3.6272	0.4336	1.5728
u_3	3.8625	0.1224	0.4728
u_4	-0.7765	-0.1698	+0.1318

and

u_k	$e^{-4b^2 u_k^2 T}$
u_1 =0.1175	0.799
u_2 =0.5455	0.006
u_3 =0.9390	0.0000(-)

The third term is less than the fourth place of decimals. Hence we need to take into account only the first two terms at the most.

Thus U for T = 0.0542 we find is

$$U\% = 100 \times \left[\frac{1 - 74.33 \times 0.799 + 1.573 \times .006}{8.93^2 - .75^2}\right] = 25.01\%$$

Repeating the calculations of Step 4 for various values of T, enough points can be found to enable one to plot a curve such as those of Figure 5 for the particular value of b/a involved.

The above example is not an hypothetical one. It and the curves of Figure 5 were computed for actual projects.

APPENDIX C

Figures A and B are centerline profiles through the sand-drain portions of the two projects. The profiles show: (a) original ground surface prior to construction; (b) location of the water table; (c) variation in thickness along the centerline of the compressible swamp material; (d) finished centerline profile of the pavement as of the respective dates of completion; (e) settlement of the pavement surface since completion and up to the respective dates given; (f) bottom of fill, as of the respective dates of completion, as determined by settlement observations; and (g) bottom of fill, as of the respective dates of completion, as calculated by means of theoretical formulas.

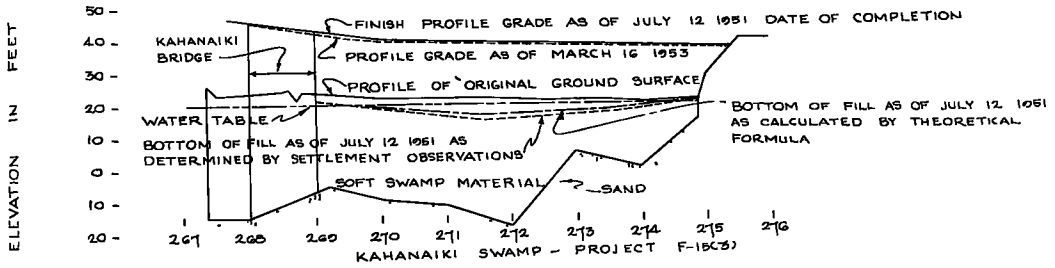

Figure A.

It is to be noted that, as of the respective dates of completion, the calculated percent consolidation over the deeper portions was as low as 70 percent. Additional settlement was expected for at least the following 2 years. But as shown in Table 2 the actual additional settlement proved in most cases to be much less than the calculated. The expected additional settlements are all fractions of a foot and, in view of the relatively crude nature of soil testing and the many uncertainties inherent in the problem, it is not reasonable to expect very close agreement between the calculated and observed values.

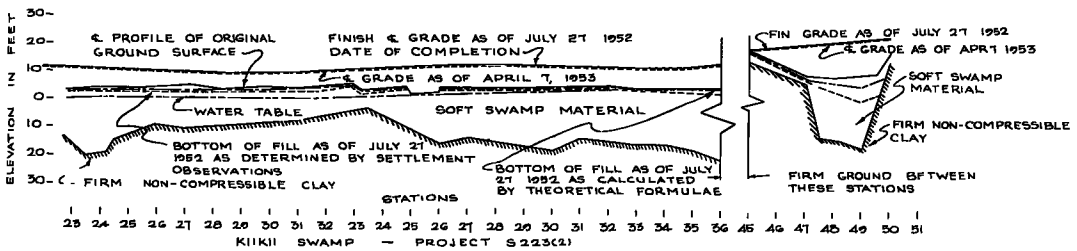

Figure B.

From a practical point of view, what one wishes to know is whether additional large settlements should be expected in the future. Both theory and actual observations up to the present time indicate that, in the case of these two projects, consolidation is now over 90 percent complete, so the chances are that little, if any, additional settlements will take place in the future.

Both swamps are located in low, flat areas and are composed of alluvial materials with a considerable admixture of organic matter. The latter ranges from 12.9 percent to 14.6 percent in the case of the Kahanaiki swamp and from 17.5 percent to 19.0 percent in the case of the Kiikii swamp.