# Stress Relaxation in Soil Compaction

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A discussion of time effects in soil compaction is based on a study of the stress relaxation phenomenon in specimens of clay, loess, and a sand-clay mixture tested in a state of uniaxial compression. The data are analyzed with the aid of rheologic models. The work reported by Sowers and Kennedy (HRB Bull. 93, 1953) is examined in terms of the stress relaxation phenomenon. The variables included in the study are the soil type, moisture content, total strain, number of compacting load applications per unit time, and the compactive energy per unit volume of soil compacted. The significance of the results is also discussed with regard to the standard laboratory compaction techniques and the existing methods of field compaction.

• RESEARCH in soil compaction was conducted by Sowers and Kennedy (*6*) to determine the relationship between the number of compacting load applications and the soil densities produced, keeping the total compacting energy per unit volume constant for each compaction test. The material tested was a medium plasticity clay. In a phase of these investigations identical soil samples were compacted using first the Standard AASHO method and, second, single applications of static pressure to each of three layers of soil in a standard compaction mold. The second method was adjusted by trial and error to utilize the same amount of total work as the first. The density obtained by the second method was greater than that obtained by the first in this and similar experiments. Through these results the authors concluded that for a given amount of work, the greatest compaction results when the work is exerted in a single application. Sowers and Kennedy further concluded that each successive application of the same pressure to the soil results in less and less work per application. This implies that impact loading is less effective than work equivalent static loading. They present the following physical reasoning (*6*):

> The cause can be inferred from our knowledge of soil structure. The compaction of soil under pressure is the result of elastic deflection of the soil structure and plastic movement of the soil grains into a more dense arrangement. The elastic deflection absorbs work in the form of strain energy while the plastic deformation absorbs work and transforms it to heat. When the pressure is removed, the elastic part of the deflection is largely recovered. The strain energy is dissipated in the viscous resistance of the soil to swelling and in some plastic deformation and rearrangement. During a second load application the deflection is largely elastic with only a small amount of plastic deformation because the

pulse loading was found to be about 0.020 sec. This time interval is the time of loading of a soil sample during the laboratory impact compaction test.

It remains now to establish an order of magnitude of the time of loading of a soil sample during the static compaction test. Because of the nature of this compaction, this order may be determined qualitatively as follows.

In the application of the static load to the soil sample, the pressure is built up to a maximum value in increments. Since the load piston is in contact with the soil during this pressure build-up and since the pressure load is not released until the maximum predetermined value of pressure is reached, the soil is subject to work loading during this entire build-up time. It is not unreasonable to say that this build-up would require an order of many seconds.

To determine an order of magnitude of this time in the static compaction test, a test procedure following the outline of the testing by Kennedy was performed in the laboratory. To insure the shortest possible time of loading, the static pressure was built up to its maximum value in just two increments, the displacement of the piston being measured after each increment to determine the pressure-displacement relationship. The time required for the application of this static load was about 20 sec. This time value may be considered a lower bound of the time of loading during a static compaction test. Its order is about a thousand times that of the time of loading for an impact compaction test.

## STRESS RELAXATION STUDY

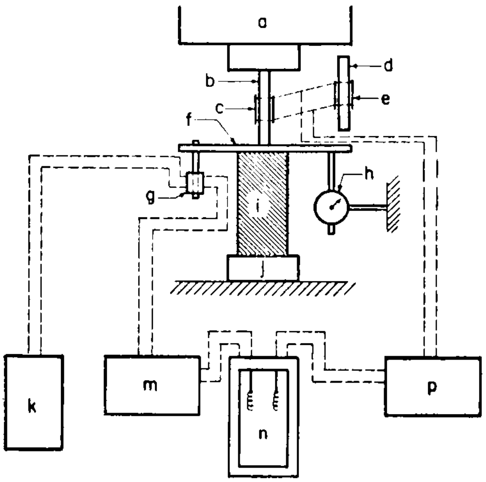
### *Experimental Apparatus*

To obtain the time of relaxation of soils of varying types and moisture contents, uniaxial compression stress relaxation tests were conducted. In studying the stress relaxation response of a material, a strain is suddenly imposed on a specimen and held constant while the stress $S$ required to maintain it is measured as a function of time. Because of the physical limitations of the experimental apparatus available, cylindrical test specimens 2.5 in. in diameter and 5.5 in. in height with a height-diameter ratio of 2.2 were used. The specimens were prepared by a compaction process. To insure that the size of the preparation mold had no influence on the density produced by a compaction test with an expenditure of compactive energy per unit value equal to that of the Standard AASHO, check compaction tests using the same type material were performed with both the standard mold and the smaller preparation mold.

A schematic representation of the experimental apparatus is given in Figure 1. The soil specimen rests on the fixed base of the compression machine and the load is transmitted to the specimen through a dynamometer which is fixed between the top of the specimen and the movable head of the machine. The dynamometer consists of wire-resistance strain gages mounted on a thin-walled aluminum-alloy tube. The dynamometer response was amplified and recorded on an oscillograph. An indicator dial and a linear differential transformer were used to measure the travel of the movable head.

### *Rheologic Models*

The workers in the field of rheology frequently represent the response of materials with mechanical models composed of springs and dashpots connected in series or parallel as an aid to the understanding of the material response to loading. By the addition of a sufficient number of Voigt or Maxwell elements, it is pos-



a. Movable head of compression machine
b. Active dynamometer
c. Active strain gages
d. Inactive dynamometer (for temperature compensation)
e. Inactive strain gages
f. Dynamometer base
g. Linear differential transformer
h. Indicator dial for deformation measurements
i. Soil sample
j. Fixed base of compression machine
k. Audio oscillator (input for transformer)
m. Amplifier for transformer
n. Oscillograph
p. Amplifier for dynamometer

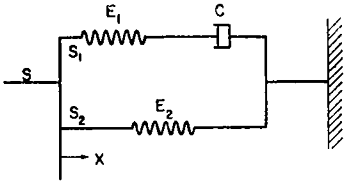
Figure 1. Schematic diagram of testing apparatus.

sible to fit the creep and stress relaxation response of a material to a high degree of accuracy. For the study of stress relaxation response in cohesive soils the model element given in Figure 2 seems to be a reasonable compromise between the response of soil as a real material and the simplicity of a mathematical model. Its use is limited to the stress relaxation problem and any other soil phenomenon may have an entirely different mathematical model representation. The model (Fig. 2) displays complete time-dependent elastic reversibility which cohesive soils do not exhibit. Thus the model seems reasonable for stress relaxation studies but is not reasonable for studying the deformation of soils under both loading and unloading. It is important to remember that such rheologic models are not intended to be true representations of the material, but are only schematic aids to understanding the response.

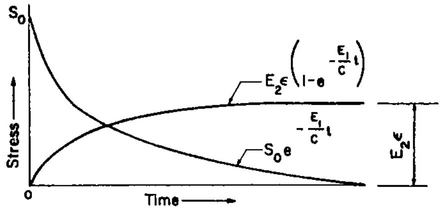
The following is the development of analytic stress-strain-time relations based on the response mechanism represented by the three parameter rheological model (Fig. 2).

The rheological soil model is subjected to a stress $S$, which will cause the model to deform in the direction indicated by $x$ of Figure 2 and stress will thereby be introduced into both branches of the model. The entire stress $S$ will be equal to the sum of the stresses in each branch and can be expressed as

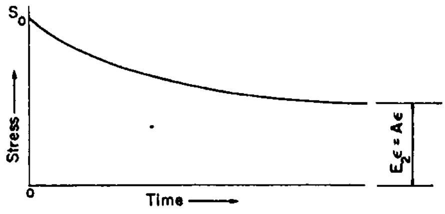
$$S = S_1 + S_2 \tag{1}$$



(a) Assumed Rheological Soil Model



(b) Plot Of The Two Terms Of Equation (8)



(c) Superposition Of Two Terms Of Equation (8) To Give Stress Relaxation Curve

Figure 2. Rheologic soil model and response.

In the branch containing the spring only, the stress is

$$S_2 = E_2 x, \quad \text{or} \quad \dot{x} = \frac{\dot{S}_2}{E_2} \tag{2}$$

where the dot indicates the time rate of change.

In the branch containing the spring and dashpot, the deformation is the sum of the deformations of the spring $x_s$ and the dashpot $x_d$.

$$x = x_s + x_d \quad \text{and} \quad \dot{x} = \dot{x}_s + \dot{x}_d \tag{3}$$

But inasmuch as

$$\dot{x}_s = \frac{\dot{S}_1}{E_1} \quad \text{and} \quad \dot{x}_d = \frac{S_1}{C},$$

$$S_1 = C\dot{x} - \frac{C\dot{S}_1}{E_1} \tag{4}$$

But $S_1 = S - S_2$ and $\dot{S}_1 = \dot{S} - \dot{S}_2 = \dot{S} - E_2\dot{x}$.

Therefore, substituting in Eq. 4 gives

$$S_1 = \frac{C}{E_1}(E_1 + E_2)\dot{x} - \frac{C\dot{S}}{E_1} \tag{5}$$

From Eq. 1

$$S + \frac{C}{E_1}\dot{S} = E_2\left(x + \frac{C(E_1+E_2)}{E_1E_2}\dot{x}\right) \tag{6}$$

In studying stress relaxation, a strain is suddenly imposed and then held constant, while the stress required to maintain it is measured as a function of time. Thus for stress relaxation:

$$\epsilon = \text{strain} = \text{constant and } \dot{\epsilon} = 0 = \dot{x}.$$

Therefore, Eq. 6 becomes

$$\frac{C}{E_1}\dot{S} + S = E_2\epsilon,$$

which has the solution

$$S = E_2\epsilon + D \exp\left(-\frac{E_1}{C}t\right). \tag{7}$$

To evaluate the constant of integration, $D$, the condition used is

$$S = S_0 \quad \text{at} \quad t = 0.$$

Thus

$$D = S_0 - E_2\epsilon.$$

Therefore, Eq. 7 becomes

$$S = E_2\epsilon\left[1 - \exp\left(-\frac{E_1}{C}t\right)\right] + S_0 \exp\left(-\frac{E_1}{C}t\right) \tag{8}$$

which may be considered to be the stress relaxation for the rheological model of Figure 2.

Thus the stress $S$ as a function of time is the sum of two time-dependent relations as shown in Figure 2b and 2c. Figure 2c also indicates those values ($S_0$ and $A\epsilon$) which must be obtained experimentally to verify or refute the soil model assumed.

Eq. 8 can also be presented in the following form:

$$S - A\epsilon = (S_0 - A\epsilon)e^{-Bt} \tag{9}$$

where

$$A = E_2 \quad \text{and} \quad B = \frac{E_1}{C}$$

Therefore

$$\frac{S - A\epsilon}{S_0 - A\epsilon} = e^{-Bt}$$

and

$$Bt = 2.3 \log\left(\frac{S_0 - A\epsilon}{S - A\epsilon}\right) \tag{10}$$

The value $B$ here actually represents the slope of the plot of $t$ versus 2.3 $\log[(S_0 - A\epsilon)/(S - A\epsilon)]$. It should be noted that $1/B = C/E_1$ has the dimensions of time and is called the time of relaxation.

The results of the stress relaxation tests are summarized in Tables 2 through 4.

Figure 3 is a typical graph of time versus the logarithm of

$$\left(\frac{S_0 - A\epsilon}{S - A\epsilon}\right)^{2.3}$$

TABLE 2
SUMMARY OF TEST RESULTS—CLAY SAMPLES

| Sample No. | % Strain | Time Relaxation (sec) | Moisture Content (%) |
|---|---|---|---|
| C-1-A | 1 | 0.365 | 21.7 |
| C-1-D | 1 | 0.389 | |
| C-1 | 1 | 0.274 | |
| C-1 | 2 | 0.509 | |
| C-2 | 1 | 0.368 | 19.4 |
| C-2 | 2 | 0.435 | |
| C-2 | 3 | 0.323 | |
| C-3 | 2 | 0.372 | 25.9 |
| C-3 | 4 | 0.304 | |
| C-4 | 1 | 0.527 | 23.6 |
| C-4 | 2 | 0.497 | |
| C-5 | 1 | 0.419 | 20.6 |
| C-5 | 2 | 0.337 | |
| C-5 | 3 | 0.365 | |
| C-5 | 4 | 0.298 | |
| C-5 | 5 | 0.376 | |
| C-6 | 1 | 0.399 | 22.4 |
| C-6 | 2 | 0.527 | |
| C-6 | 3 | 0.415 | |
| C-6 | 4 | 0.333 | |
| C-7 | 1 | 0.375 | 17.3 |
| C-7 | 2 | 0.285 | |
| C-7 | 3 | 0.218 | |
| Range | | 0.218 to 0.527 | |
| Average | | 0.390 | |

TABLE 3
SUMMARY OF TEST RESULTS—CLAYEY SAND SAMPLES

| Sample No. | % Strain | Time of Relaxation (sec) | Moisture Content (%) |
|---|---|---|---|
| S-1 | 1 | 0.288 | 13.1 |
| S-1 | 2 | 0.268 | |
| S-1 | 3 | 0.325 | |
| S-1-A | 3 | 0.348 | |
| S-2 | 1 | 0.315 | 11.2 |
| S-2 | 3 | 0.265 | |
| S-3 | 1 | 0.382 | 13.9 |
| S-3 | 2 | 0.343 | |
| S-3 | 3 | 0.381 | |
| S-4 | 1 | 0.189 | 12.4 |
| S-4 | 2.1 | 0.213 | |
| S-5 | 1 | 0.218 | 8.5 |
| S-6 | 1 | 0.267 | 10.1 |
| S-6 | 2 | 0.226 | |
| Range | | 0.189 to 0.382 | |
| Average | | 0.272 | |

for a stress relaxation test. If the rheologic model of Figure 2a actually represented the physical response of the soil used in the study, just one slope and hence only one relaxation time would have been obtained. The presence of two distinct slopes indicates two relaxation times and hence that the assumed model of Figure 2a is not general enough to represent the relaxation mechanism. By the addition

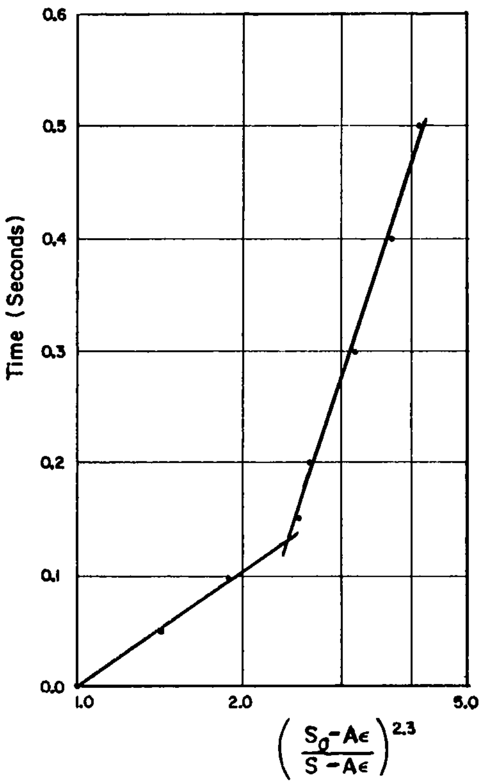


Figure 3. Typical relaxation curve.

TABLE 4
SUMMARY OF TEST RESULTS—LOESS SAMPLES

| Sample No. | % Strain | Time of Relaxation (sec) | Moisture Content (%) |
|---|---|---|---|
| L-1 | 1 | 0.153 | 19.7 |
| L-1 | 2 | 0.250 | |
| L-1 | 2.54 | 0.273 | |
| L-1 | 4 | 0.353 | |
| L-1 | 5 | 0.355 | |
| L-2 | 1 | 0.345 | 26.6 |
| L-2 | 2 | 0.192 | |
| L-2 | 3 | 0.139 | |
| L-3 | 1 | 0.131 | 17.4 |
| L-3 | 2 | 0.270 | |
| L-3 | 3 | 0.271 | |
| L-4 | 1 | 0.257 | 21.6 |
| L-4 | 3 | 0.266 | |
| Range | | 0.139 to 0.355 | |
| Average | | 0.254 | |

of a sufficient number of Maxwell elements in parallel, it is possible to fit the relaxation response of a real material to almost any degree of accuracy. Thus, the material may be represented by a spectrum of relaxation times.

For the problem under consideration, a better rheologic model than that of Figure 2a would be that given in Figure 4.

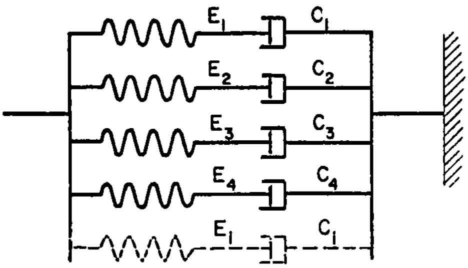


Figure 4. General rheologic model representing a spectrum of stress relaxation times.

Depending upon the values of the ratios $C_i/E_i$, that is the values of the relaxation times, and the duration of interest for the problem under consideration, the model of Figure 4 may degenerate into the form given in Figure 2a.

The soil response (Fig. 3) shows two distinct relaxation times. For times of interest less than 0.8 sec, Figure 4 degenerates into Figure 2a where $C_1/E_1 = 1/B_1$ and the remaining Maxwell elements in parallel act as if they were elastic elements represented by $E_2$. For times of interest less than 3.0 sec but greater than 0.8 sec, Figure 4 becomes $C_1/E_1 = 1/B_2$ and the remaining elements act as a single elastic element $E_2$.

Since the magnitude of the time of loading used by Sowers and Kennedy was very large for the static compaction test and the duration of loading in the standard compaction test was very small, the relaxation times obtained from the experiments are those of interest in this study. There are indications that relaxation times greater than the two obtained also exist and, because of the limitations of the experimental apparatus, some smaller than those obtained. Typical values for the second times of relaxation are shown in Table 5.

TABLE 5
TYPICAL SECOND TIMES OF RELAXATION

| Sample No. | Strain (%) | $B_2$(sec) |
|---|---|---|
| C-1-A | 1 | 1.29 |
| C-1-D | 1 | 1.52 |
| C-2 | 1 | 0.74 |
| C-2 | 3 | 2.42 |
| C-4 | 2 | 2.20 |
| C-5 | 1 | 1.11 |
| C-5 | 5 | 1.61 |
| C-6 | 4 | 1.15 |
| S-1 | 1 | 0.84 |
| S-1 | 2 | 1.24 |
| S-1 | 3 | 0.51 |
| S-3 | 3 | 1.88 |
| S-4 | 1 | 2.35 |
| S-4 | 2.1 | 2.71 |
| L-1 | 4 | 2.74 |
| L-1 | 5 | 1.53 |

Thus the conclusions regarding the effects of time on compaction presented by Sowers and Kennedy can definitely be explained in terms of the stress relaxation phenomenon. For the large load durations, in their work, a large number of relaxation mechanisms (Maxwell elements) respond, hence the greater permanent strains and greater densities. The standard compaction test, on the other hand, applies an impact loading of very short duration and after a few blows of the hammer most of the response is elastic hence smaller permanent strains and smaller densities.

In summary, the range of the first relaxation times for all samples was from 0.139 sec to 0.527 sec and the average was 0.311 sec. For the clay samples the first relaxation times were the largest and ranged from 0.218 sec to 0.527 sec, with 0.390 sec as an average value. The clayey sand produced an intermediate range of first times of relaxation varying from 0.189 sec to 0.382 sec with 0.272 sec as an average. The values are in good agreement with Whitman and Taylor who determined the time of relaxation of a pure sand by means of a triaxial compression test to be 0.2 sec during their research on the relationship between load-rise time and shearing strength (*9*). The loess samples produced the smallest first relaxation times and ranged from 0.139 sec to 0.355 sec with 0.254 sec as an average value.

The time of loading for the impact compaction test was of the order of magnitude of 0.020 sec, a level well below the level of even the smallest time of relaxation obtained.

The times of relaxation determined by the laboratory testing varied somewhat with soil type. The longest times of relaxation prevailing for the clay soil, the shortest for the loess (or silt), with the clayey sand values being intermediate. There was a relatively small difference between the averages for the loess and the clayey sand for the entire range of testing, but there was a more significant difference in the average values when compared with the clay.

Figure 5 indicates the relationship of soil density and relaxation time. Other attempts to plot the relaxation time against various parameters were inconclusive. The graph of the dry density vs the average time of relaxation for a given moisture content shows perhaps the most consistent results of all. For all three soil types, the average times of relaxation distribute themselves around a maximum value of dry density.

The time of relaxation seems to be related to the moisture content of the soil samples and hence to the density. The trend is by no means conclusive (Fig. 6), because the graph of average time of relaxation vs moisture content shows a different trend for the sandy soil than for the

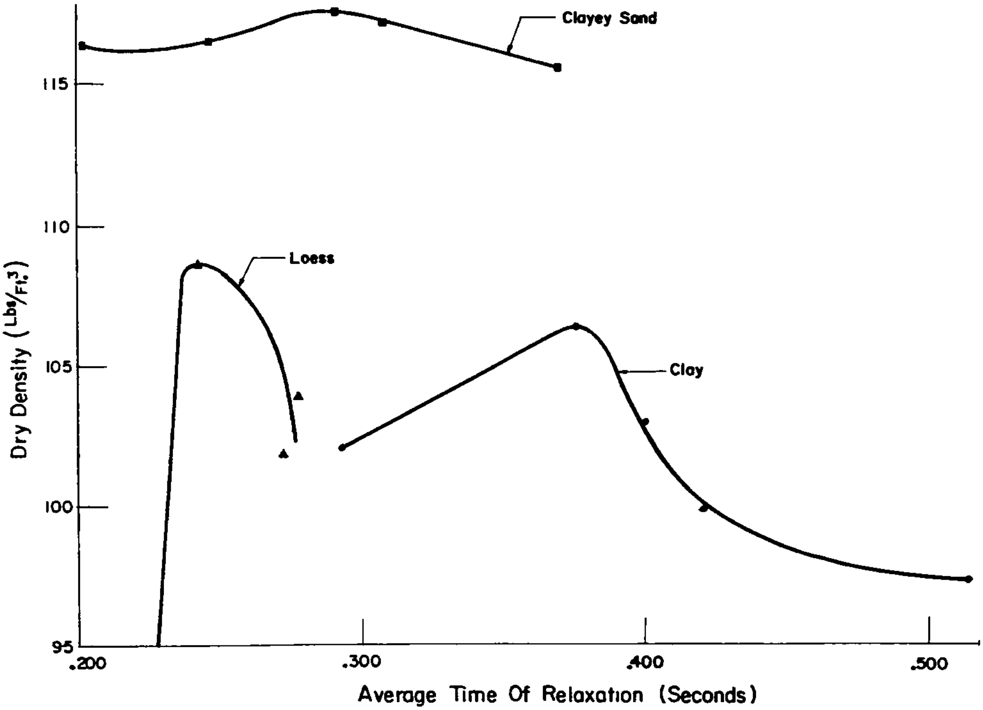


Figure 5. Dry density *vs* average relaxation time.

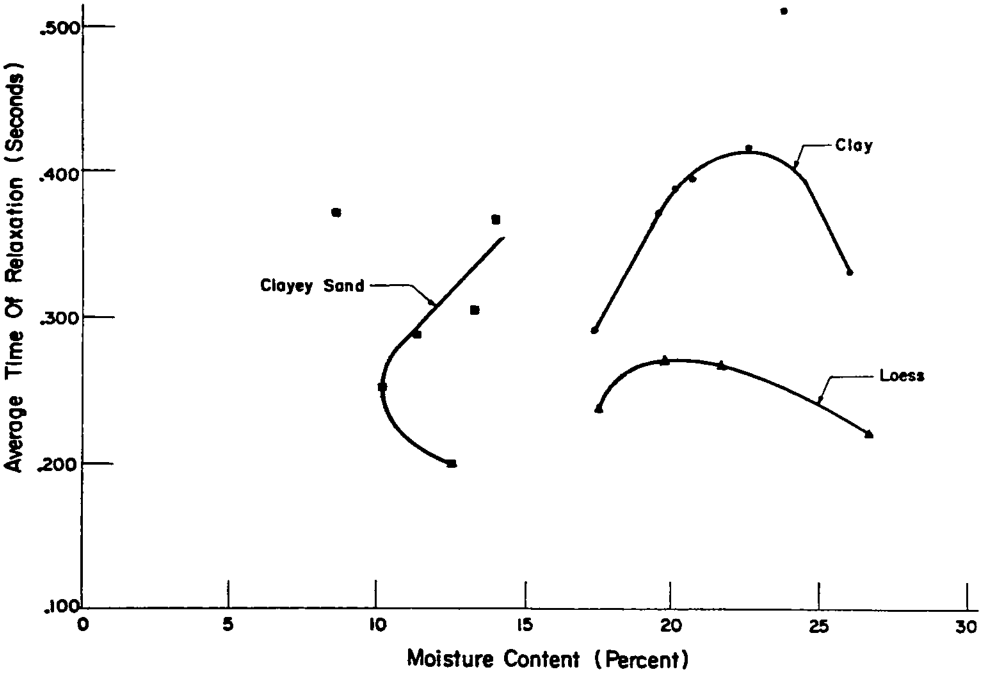


Figure 6. Moisture content *vs* average relaxation time.

clay and loess. Although the shape of the curves in Figure 6 are similar to those of the standard compaction test, the maximum value of the relaxation time does not correspond to the optimum moisture content.

A graph of time of relaxation vs unit strain fails to establish a clear-cut relationship except perhaps in the case of the loess samples (Fig. 7). The indicated trend is for the time of relaxation to increase with

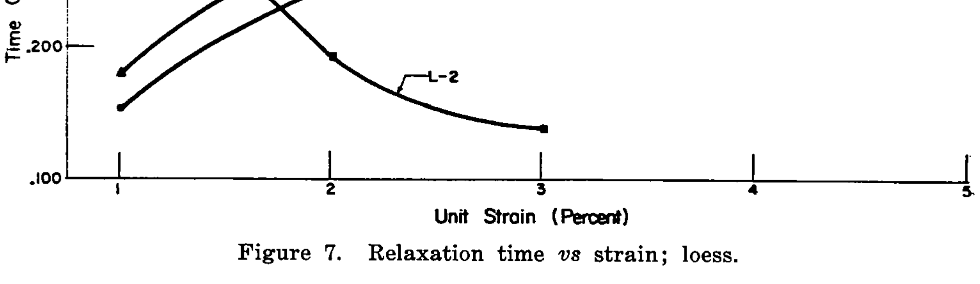


Figure 7. Relaxation time *vs* strain; loess.

the unit strain up to a maximum value and then to decrease with increasing strain. When correlated with Figure 6, the results obtained for the moisture content of 27 percent seem to indicate that the data obtained on the side where the time of relaxation decreases with increasing strain. Thus it would be expected that for strains less than 1 percent the loess at a moisture content of 27 percent would exhibit decreasing values of relaxation time for decreasing values of percent strain. Although the data obtained are not extensive enough to draw any conclusions, it seems that such relationships may hold for the clay and sand samples (Figs. 8 and 9).

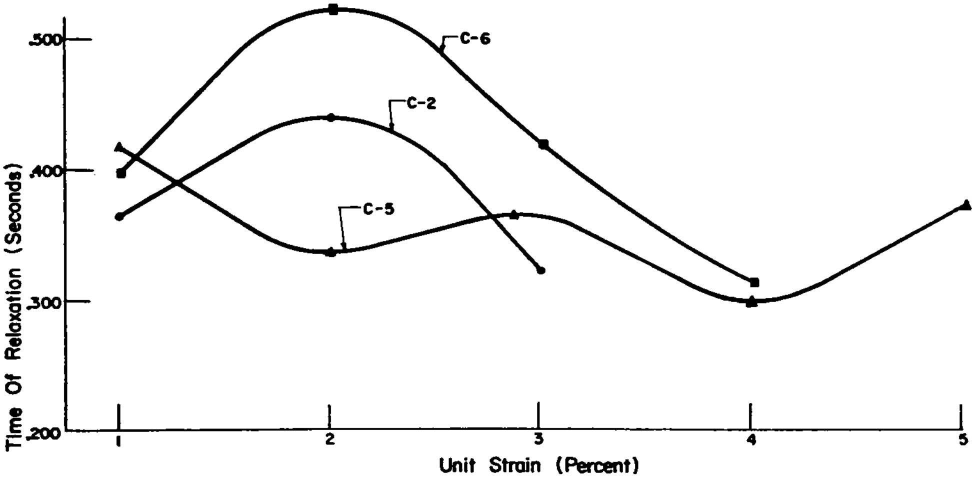


Figure 8. Relaxation time *vs* strain; clay.

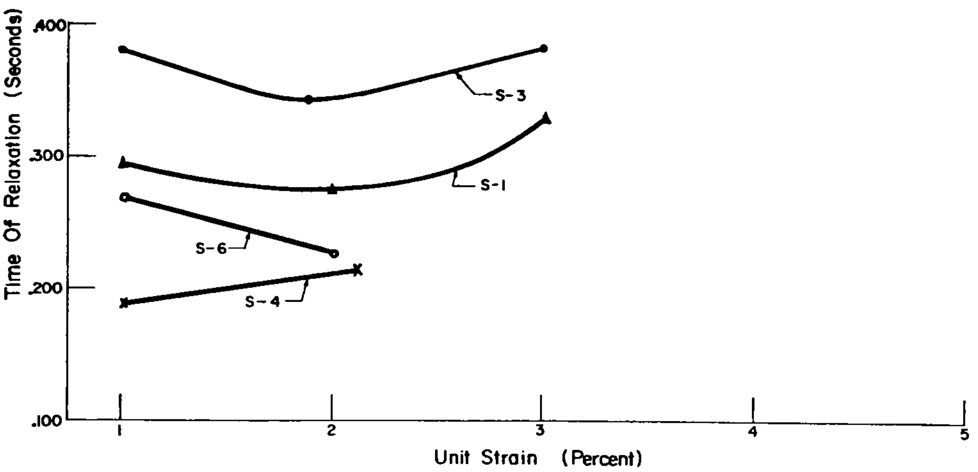


Figure 9. Relaxation time *vs* strain; clayey sand.

## CONCLUSIONS

### *Time Effect of Stress Relaxation on Compaction*

The laboratory testing described herein demonstrated:

1. That there is a spectrum of stress relaxation times for soils and that the smaller times measured, for a wide range of soil types, moisture contents, and applied strains, are of the order of magnitude of from 100 to 600 milliseconds.
2. That the time of loading during a standard laboratory compaction test was approximately 20 milliseconds.
3. That the time of loading during a static application of equal compactive energy was greater than 20 sec.

These magnitudes clearly demonstrate that for a standard laboratory compaction test the time of loading is of much shorter duration than the times of relaxation for clayey, silty, and sandy soils and further that the time of loading during a static application of energy is of much longer duration than the times of relaxation.

The establishment of these orders of magnitude of times and the observation of Sowers and Kennedy that less efficient compaction is achieved in the standard laboratory compaction test than in a test where the equivalent compactive energy per unit volume is expended in a single application allows the conclusion to be drawn that the interpretation of the authors relating the relative magnitudes of load duration and relaxation times to the differences in density achieved is correct. This interpretation, as stated previously, is that in order to achieve more efficient compaction the time of loading should be greater than the times of relaxation.

Sowers and Kennedy observed that densities achieved with a single application of static load (where the time of loading was greater than the times of relaxation) was about 11 percent greater than the density achieved with a standard laboratory compaction test (where the time of loading was less than the time relaxation) using equal compactive energies per unit volume.

These observations suggest very strongly that compaction, like consolidation, is at least in part a time phenomenon and that superior compaction will occur when the time of loading is of a longer duration than the times of relaxation.

### *Extrapolation of Results to Field Compaction*

The extrapolation of these laboratory findings to actual field compaction is considered in a qualitative fashion. It is difficult to correlate maximum laboratory densities achieved in compaction tests with densities achieved in the field because of the following reasons:

1. The soil sample tested may not be typical of the fill being placed.
2. The moisture content of the field fill is hard to maintain uniformly near the optimum level.
3. The compactive energy expended per unit volume of field fill is difficult to determine.
4. Compaction methods and procedures vary in different localities.

In terms of this research the only attempt that will be made to relate laboratory findings with field compaction is to compare reported load duration times to which a soil is subjected during field compaction with the times of relaxation determined herein. This comparison will allow some conclusions to be drawn.

A series of field tests conducted by the Roads Research Laboratory of Great Britain (*8*) established that the duration of the loading pulse generated by a smooth wheel roller and a track-laying tractor was from 2 to 5 sec. These durations were determined by inserting pressure gages at different depths in a layer with the aforementioned equipment. The pressure impulse generated was recorded by the gages as the vehicle passed over them. As may be expected, the duration of the loading impulse increased with increasing depth. The shortest impulses were recorded on gages inserted 4 to 5 in. below the surface of the loose soil layer. These pulse durations were of the order of 2 sec. In a general sense, this value may be selected as a lower bound of impulse duration and hence time of loading. Whiffin (*8*) does not mention the velocity of the compacting equipment, but it was thought not to vary much from the speed of compaction equipment usually used in the field.

A comparison of this time of loading with the times of relaxation determined herein lead to the conclusion that field compaction is being performed efficiently in terms of the time phenomenon reported in this paper since the time of loading is greater than the times of relaxation. The duration of the pulse deeper in the loose soil layer indicates that, in a time-efficiency sense, rather thick lifts of soil could be compacted. It would be anticipated that the same densities would be achieved using either thick lifts of soil with many passes or shallow lifts with fewer passes since the pulse duration is sufficiently long.

A visual determination of the duration of impulse from a graph of time versus pressure from Whiffin (*8*) would show that the actual compacting impulse recorded is of a much shorter duration than that of the entire pulse as reported by him. In the case of a sandy soil the duration of compacting impulse was about 0.64 sec for a gage 3.6 in. below the surface of the loose soil. Since this duration is still in excess of the relaxation times determined herein for all of the sand samples and since other compacting impulses recorded by Whiffin are larger than the impulse in sand, no revision of the above conclusions is necessary.

It should be emphasized that the results and conclusions in this paper are merely qualitative, exploratory studies of stress relaxation effects on soil response. Much additional research is needed before any definite quantitative conclusions can be expressed. Additional research is also needed to determine the time of loading during actual field compaction in order that comparisons can be made with the times of relaxation.

## ACKNOWLEDGMENTS

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## REFERENCES

1. Cowin, S. C., Kondner, R. L., and Ayre, R. S. "Bibliography Relating to Vibratory Cutting, Penetration and Compaction of Soils." Johns Hopkins Univ. (1958).
2. Cowin, S. C., Kondner, R. L., and Ayre, R. S., "A Critical Review of Selected Literature Relating to the Vibratory Cutting, Penetration and Compaction of Soils." Johns Hopkins Univ. (1958).
3. Freudenthal, A. M., "The Inelastic Behavior of Engineering Materials and Structures, John Wiley and Sons, New York (1950).
4. Kennedy, C. M., "A Laboratory Investigation of the Efficiency of Different Methods of Soil Compaction." M. S. Thesis, Georgia Inst. of Tech., (1953).

5. KONDNER, R. L., "A Non-Dimensional Approach to the Vibratory Cutting, Compaction and Penetration of Soils." Dept. of Mechanics. Johns Hopkins Univ. (1960).
6. SOWERS, G. F., AND KENNEDY, C. M., "Effect of Repeated Load Application on Soil Compaction Efficiency." HRB Bull. 93, pp. 61-64 (1954).
7. TAYLOR, D. W., AND WHITMAN, R. V., "The Behavior of Soils Under Dynamic Loading." Parts 1, 2 and 3, Mass. Inst. of Tech. (1952, 1953, 1954).
8. WHIFFIN, A. C., "The Pressures Generated in Soil by Compaction Equipment." Symposium on Dynamic Testing of Soils, ASTM Spec. Tech. Publ. No. 156 (1954).
9. WHITMAN, R. V., "The Behavior of Soils Under Transient Loading." Proc. Third International Conference on Soil Mechanics and Foundation Engineering, Vol. 1 (1957).