DEPARTMENT OF DESIGN

Influence of Vehicle Speed on Pavement Deflections

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• AS A PART of the series of tests performed on the AASHO Road Test at Ottawa, Ill., measurements were obtained of the actual pavement deflections for a range of vehicle types and vehicular speeds. In all cases it was found that the deflections decreased as the speed of the vehicles increased. Similar results have been reported on tests at Road Test One-MD (*1*) and in a study of the subgrade support characteristics on US 52 in Indiana (*2*). Of interest in the present paper is the explanation of the mechanism responsible for this phenomenon.

FORMULATION OF PROBLEM

Regardless of the simplifications made in the analysis of a pavement system subjected to transient loads, at least the following factors must be considered: (a) the nature of the forcing function, (b) the inertia of the pavement and its support, (c) elastic deformations (restoring force), and (d) time-dependent deformations. As the last three factors represent basic parameters of viscoelastic theory (mass m, spring function k, and damping function c, respectively), it is natural that a solution be sought within the framework of this theory. The particular model chosen for the pavement-support system is the simple Voigt element shown in Figure 1. Although more complicated models could be postulated (*3*, *4*, and *5*) the resulting solutions tend to obscure the controlling mechanism of the phenomenon under consideration.

The loading function will be taken as a rectangular pulse of intensity P

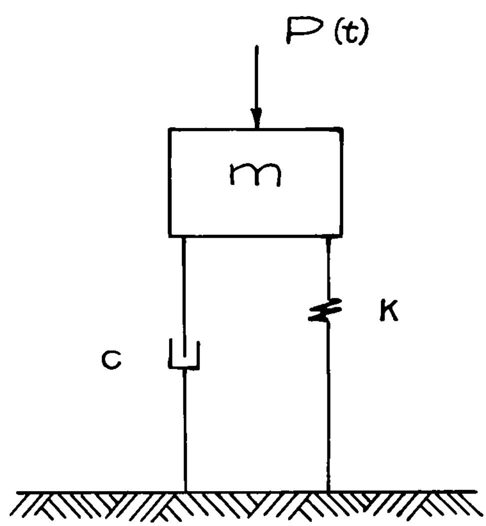

Figure 1. Simple Voigt element.

and of duration t_1 as shown in Figure 2. Here it is assumed that a point in the pavement system is subjected to a load of uniform intensity P during the t_1 seconds required for the passage of the vehicle.

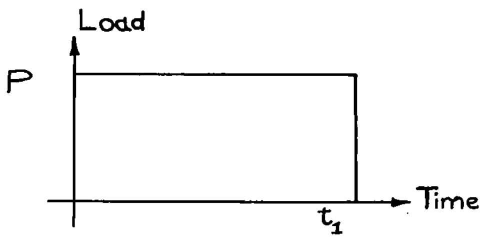

Figure 2. Loading function.

SOLUTION

Taking $y(t)$ as the dynamic deflection, the differential equation of motion is

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = P\,[u(t) - u(t - t_1)] \qquad (1)$$

which can be rewritten as

$$\frac{d^2y}{dt^2} + 2aw\frac{dy}{dt} + w^2y = \frac{p}{m}\,[u(t) - u(t - t_1)] \qquad (2)$$

in which

$w = \sqrt{k/m}$ = undamped natural frequency;

$a = c/2mw = c/c_{cr}$ = damping factor;

$u(t - t_1)$ = unit step function = 0 when $t < t_1$ and 1 when $t > t_1$.

By Laplace transforms the solution of Eq. 2 is found to be

$$y(t) = \Delta_{st}\left[1 + \frac{e^{-awt}}{\sqrt{1-a^2}}\sin\left(wt\sqrt{1-a^2} - \psi\right)\right] \qquad (3a)$$

for $t < t_1$

$$y(t) = \Delta_{st}\left\{\frac{e^{-awt}}{\sqrt{1-a^2}}\sin\left(wt\sqrt{1-a^2} - \psi\right) - \frac{e^{-aw(t-t_1)}}{\sqrt{1-a^2}}\sin\left[w\sqrt{1-a^2}\,(t - t_1) - \psi\right]\right\}$$

for $t > t_1$ (3b)

in which

$\Delta_{st} = P/k$ = static deflection

$$\psi = \tan^{-1}\frac{\sqrt{1-a^2}}{-a}$$

Deflection curves of actual highway pavements under loads show that the motion is not oscillatory but rather a creeping back to the equilibrium position. Such systems are said to be "overdamped"; that is, the damping factor a is greater than unity. Taking $a > 1$ in Eqs. 3a and 3b, it follows that

$$D = \frac{y(t)}{\Delta_{st}} = 1 - \frac{e^{-awt}}{\sqrt{a^2-1}}\sinh\left(wt\sqrt{a^2-1} + \phi\right)$$

for $t < t_1$ (4a)

$$D = \frac{e^{-aw(t-t_1)}}{\sqrt{a^2-1}} \sinh [w\sqrt{a^2-1}\,(t-t_1)+\phi] - \frac{e^{-awt}}{\sqrt{a^2-1}} \sinh (wt\sqrt{a^2-1}+\phi)$$

for $t > t_1$ (4*b*)

in which

$D = y(t)/\Delta_{st}$ = dynamic deflection factor

$\phi = \tanh^{-1}\sqrt{a^2-1}/a$

CONCLUSION

A plot of the dynamic deflection factor as a function of time ($w = 20$ cycles per sec) for loading durations of $t_1 = 0.078$ sec and 0.314 sec with $a = 2$ is shown in Figure 3.

Noting that the time t_1 for the passage of the vehicle varies inversely with the velocity, as say

$$v = \frac{L}{t_1} \qquad (5)$$

in which L is the effective length of the loading pulse, Eqs. 4*a* and 4*b* can be solved to yield the ratio of the maximum deflection factor, $D_{max} = y(t)_{max}/\Delta_{st}$ as a function of v/wL for any damping factor a. A plot of this relationship is given in Figure 4 for a range of a-values. It is immediately apparent from this plot that deflections decrease as vehicular speeds increase.

A search of the literature was made to obtain a measure of L and w for highway pavements. Deflection patterns in the Road Test One-MD report (*1*, Figs. 99 and 103) indicate that an effective length of 20 ft is not unreasonable. The reduction in length (24 to 28 ft in the report) to 20 ft is to account for the difference between the assumed rectangular pulse and the actual sinusoidal type of loading of real vehicles. Estimates of w were more difficult to obtain. The DEGEBO studies (*6*, Table 18-1) of the natural frequency of various soils (from peat to sandstone) indicated a range of frequencies of 12.5 to 34.0 cycles per sec. Later studies by Nijboer and Van der Poel (*7*) on

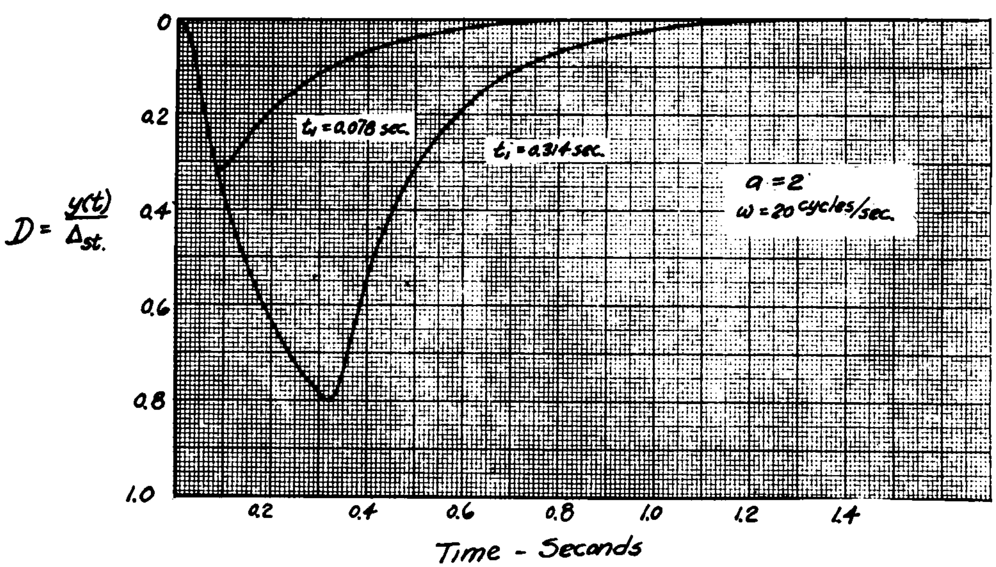

Figure 3. Dynamic deflection factor as function of time.

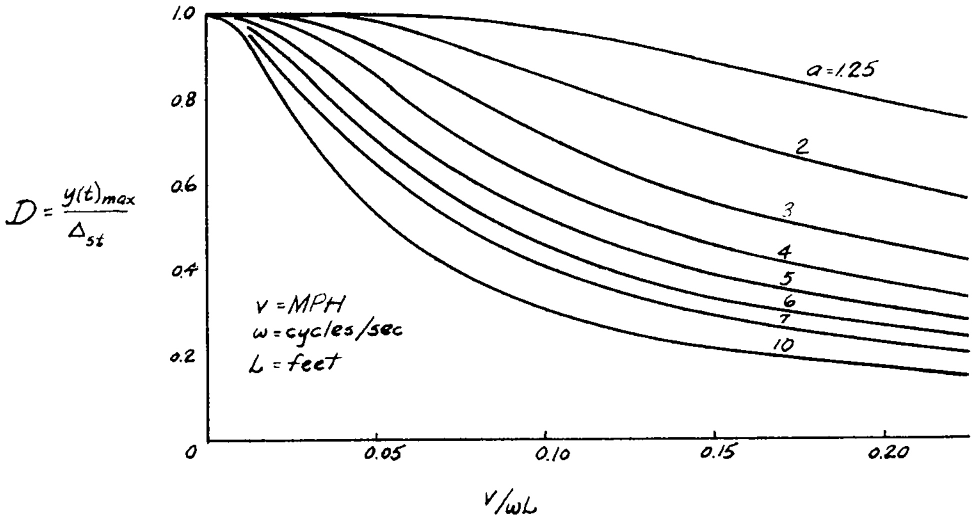

Figure 4. Maximum deflection factor as function of $v/w\ L$.

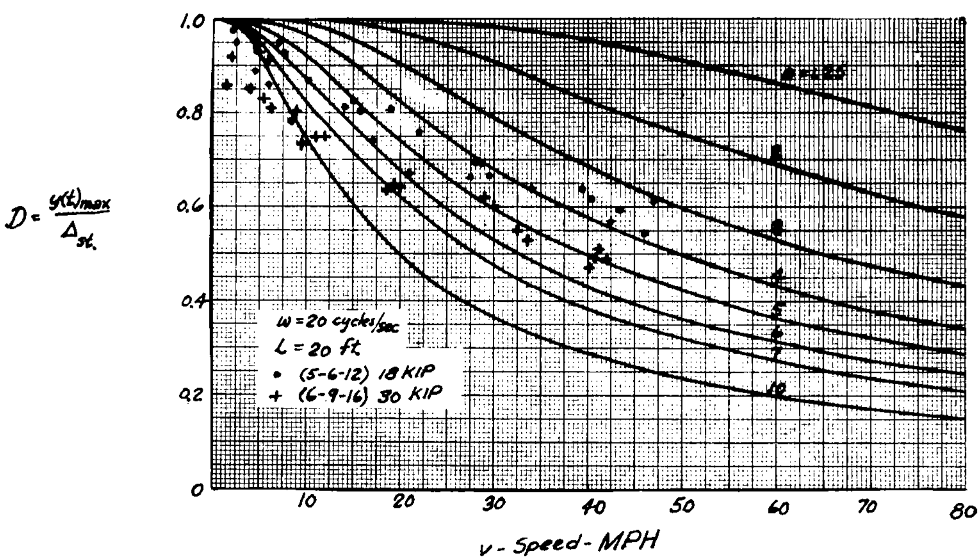

Figure 5. Maximum deflection factor as function of velocity.

the dynamic characteristics of asphaltic pavements gave approximately the same range in frequencies for these pavements. Hence, as an average value the frequency was taken as 20 cycles per sec. Making these assumptions, the abscissa scale in Figure 5 was established.

The crosses and dots in Figure 5 represent some deflection ratios obtained from the AASHO Road Test data at Ottawa, Ill. (Fig. 6). Por-

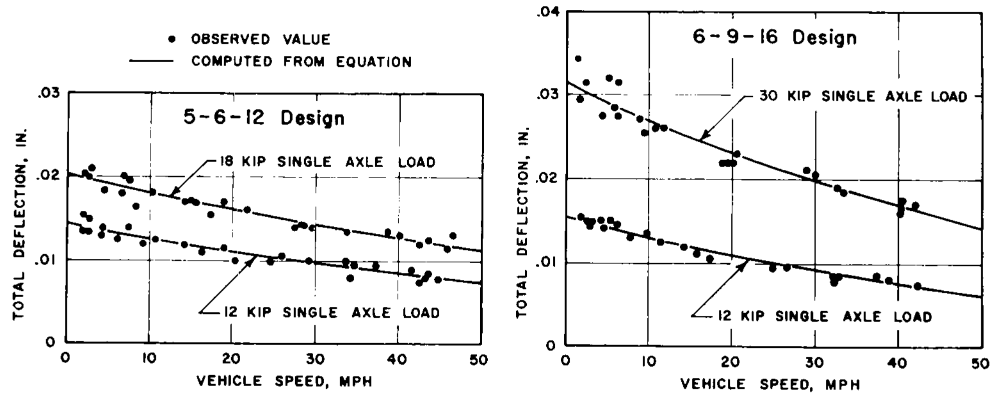

Figure 6. Variable speed study, effect of vehicle speed on total deflection (courtesy of AASHO Road Test).

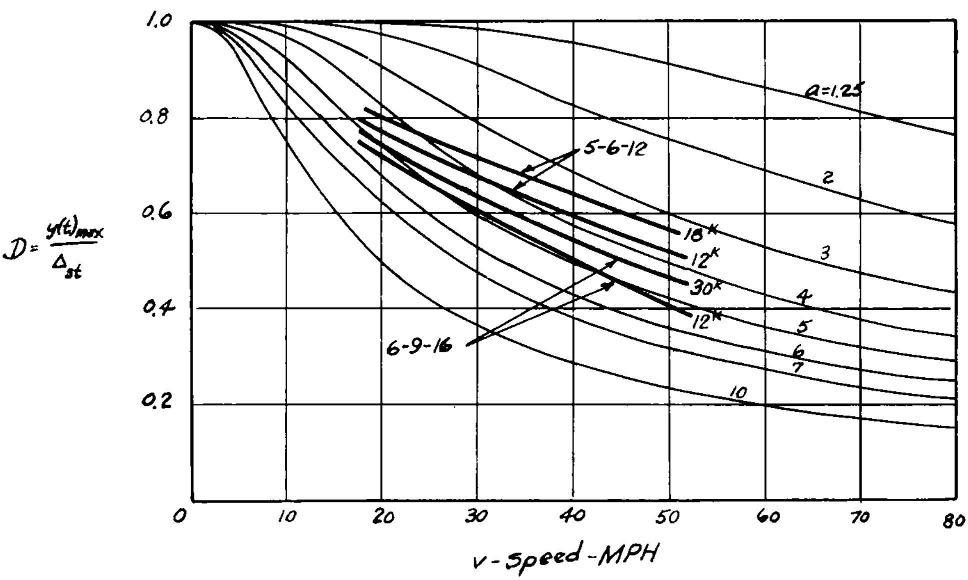

Figure 7. Portions of best-fit curves.

tions of the best-fit curves, computed from the Road Test equation (solid curves in Fig. 6), are reproduced in Figure 7.

It is difficult to conclude on the basis of the above that a reliable procedure has been developed whereby an engineer can predict the effect of vehicle speed on the deflection of highway pavements. This can only be verified by carefully conducted tests. However, the general aspects of these results do demonstrate the ability of viscoelastic theory to supply a mechanistic model of the pavement system.

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