Viscoelastic Properties of Asphalt Concrete

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A continuing research program is currently under way at the University of California for the purpose of establishing the usefulness of viscoelastic analysis as applied to an understanding of the rheologic characteristics and structural responses of asphalt concrete. As a preliminary step, the asphalt concrete mixture to be used in preparing the test slabs was subjected to an extensive program of triaxial compression testing, covering a range of load types including creep, stress relaxation, constant rate-of-strain, and repeated load lateral pressures from 0 to 250 psi and temperatures from 40 to 140 F. From the results of triaxial tests at a specific temperature and lateral pressure, viscoelastic constants for a four-element rheologic model were derived; values for these constants were obtained for the entire range of ambient conditions studied. This phase of the research thus produced a basic system of data relating the general stress-strain-time characteristics of the test mixture.

A comparison of the deflections predicted by viscoelastic theory with those measured during the slab tests indicated that the slab profiles from both sources had the same general shape and time dependence, but that the measured values had magnitudes considerably greater than those given by theory. The deviations between the two sets of values increased with increased temperature.

Also included is a discussion of the probable source of the differences found between the slab test data and theoretical predictions.

• A CONTINUING RESEARCH program is currently under way at the University of California for the purpose of establishing the usefulness of viscoelastic analysis as applied to an understanding of the rheologic characteristics and structural behavior of asphalt concrete. This avenue of approach may aid in the understanding of the behavior of asphalt concrete pavements inasmuch as (a) the load conditions that may be imposed on such pavements cover a wide range in time—from the essentially static condition associated with vehicle parking areas to the rapidly applied repeated loads occurring on heavy-duty highways and airfield

taxiways—and (b) the stress vs strain characteristics of asphalt concrete are time-dependent.

Although elastic theory may prove useful in the design of the pavements under fast-moving wheel loads, the effect of very slowly applied or static wheel loads cannot be considered by this approach. Moreover, the accumulation of small deformations under repeated load application and the subsequent rutting of asphalt concrete also cannot be accounted for by elastic theory. Thus, an approach wherein the time dependence of the stress vs strain characteristics of asphalt concrete can be considered would seem appropriate for analysis of certain aspects of pavement behavior.

A number of papers have already been presented dealing either with the basic viscoelastic properties of asphalt concrete or with the development of possible theoretical relationships for the behavior of flexible pavements (*1, 2, 3, 4, 5*). Recently attention has been directed to an attempt at joining these two avenues of research through a study of the reactions of actual paving slabs subjected to static loading on an idealized elastic foundation.

This paper is concerned with a presentation and discussion of two types of test data: (a) the results of four types of triaxial compression tests used to evaluate the viscoelastic properties of an asphalt concrete mixture over a range in temperatures and lateral pressures, and (b) the results of deflection measurements performed on test slabs of the same mixture under static loading on a large spring base. Included as an essential part of the discussion is a presentation of the mathematical relationships necessary to define the rheologic properties of the mixture in terms of a simple linear viscoelastic model both for the triaxial compression tests and for the slab tests. An important feature of this paper is a comparison of actual test data with similar results predicted from theory.

MATERIALS

The scope of the investigations made it necessary to restrict the laboratory investigation to tests on a single asphalt concrete mixture. The physical properties of the asphalt and aggregate used in this mixture have been described in detail elsewhere (*1*); therefore, only a brief description is included here.

One 85- to 100-penetration asphalt cement was used in the investigation. Results of standard tests on this asphalt are given in Table 1. Based on absolute viscosity measurements by sliding plate microviscometer (*6*) and capillary viscometer over a range in temperatures, the asphalt would be classed as a material approaching Newtonian behavior. These results are presented elsewhere (*1*).

The aggregate used in the mixtures was a crushed granite from Watsonville, Calif. (*7, 8*), with a uniform specific gravity of 2.92. It was originally intended that one dense gradation of the material (a ⅜-in. maximum size gradation conforming to the 1954 State of California standard specifications) be used in both the triaxial compression and the slab tests. For the triaxial compression test specimens, this was readily ac-

TABLE 1
IDENTIFICATION TESTS ON ASPHALT

Test	Result
Pen. at 77 F., 100 g, 5 sec	96
Pen. at 39.2 F., 200 g, 60 sec	24
Penetration ratio	25
Flash point, Pensky-Martens (°F)	445
Viscosity at 275 F (SSF)	138
Heptane-xylene equivalent	20/25
Softening point, ring and ball (°F)	110
Thin film oven test, 325 F, 5 hr:	
Percent weight loss	0.51
Percent pen. retained	53
Duct. of residue, cm at 77 F	100+

complished by screening the material then recombining each of the fractions in the amount required to prointo individual-size fractions and duce single specimens. The grading curve for these specimens is shown in Figure 1, together with the specification limits.

As described later, the slabs were prepared in the Richmond, Calif., laboratories of the California Research Corporation, and the aggregate was supplied from the stocks of that organization. Due to the large quantities of aggregate required, these stocks could not be combined to duplicate exactly the grading curve used for the aggregate in the triaxial compression specimens. The grading finally accepted as the best compromise is also shown in Figure 1. By comparing the two materials, this gradation appears deficient in sizes near the No. 8 sieve. The effect of this disparity was evaluated on additional specimens prepared using the second gradation.

PREPARATION OF TEST SPECIMENS

Two different types of specimens were required for the laboratory work associated with this investigation. For the triaxial compression tests, cylindrical specimens 2.8 in. in diameter and 6.5 in. in height (nominal dimensions) were used. Details of the method of preparation using kneading compaction (*9*) are also included elsewhere (*1*). For these specimens, one asphalt content of 6 percent (by dry weight of aggregate) was selected, this amount approximating the design value for the type and gradation of aggregate used, based on the State of California mix design procedure. The ingredients for the specimens were mixed at 230 F. The density of the specimens as determined by water displacement was approximately 150 pcf with only minor variations between individual samples.

The slab tests used thin "plates" of asphalt concrete approximately 40 by

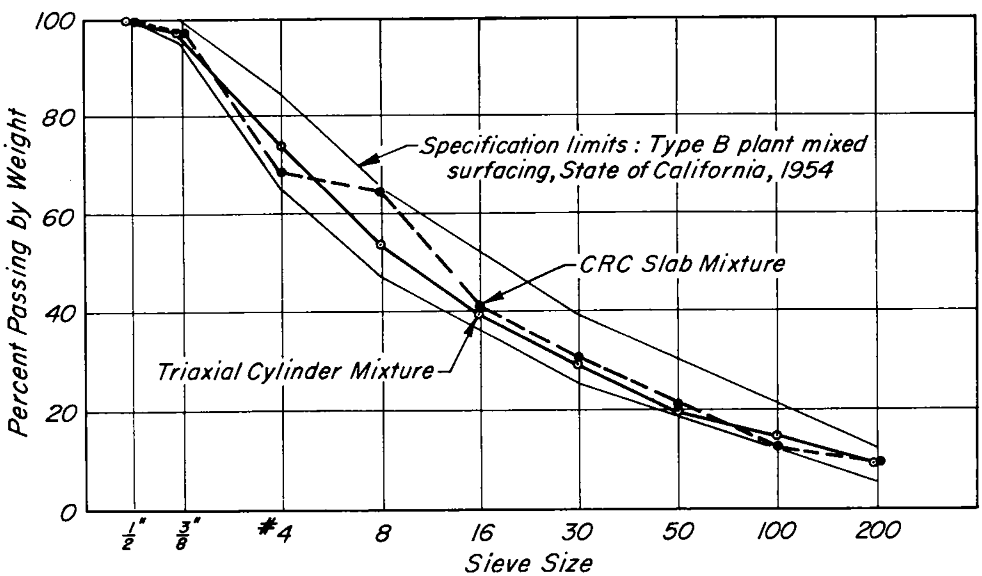

Figure 1. Grading curves for Watsonville aggregate.

40 in. square and 1.5 in. deep. The slab specimens were prepared at the Richmond laboratories, where equipment of the size necessary for such an operation was available. This process involved the production of all the required slabs simultaneously by the construction of a small (approximately 4 ft wide by 25 ft long) paving strip with the desired thickness (about 1.5 in.) Upon cooling, the pavement was broken into segments approximating the size of the test slabs and transported to the University of California laboratories for final trimming and testing. Each segment was supported during this entire process by a heavy plywood form to prevent possible damage from accidental bending.

Inasmuch as a complete description of the functions of the California Research Corporation paving laboratory has been published by that organization (*10*), no attempt made to detail the methods used to produce the test strip from which the slabs were taken. However, the mixing temperature employed was 300 F (slightly above the 230 F used for the triaxial cylinders, due to the heat losses involved in handling the paving mixture), and considerable compactive effort was applied to the strip by alternate applications of steel and pneumatic rollers.

Also, the results of the procedure described were gratifying, as the test slabs were quite uniform in texture and thickness. Samples cut from the slabs after all test measurements had been completed gave a unit weight by water displacement of 152.5 pcf.

Unfortunately, extraction tests on the slab mixture (performed as a routine check by the California Research Corp.) disclosed that the actual asphalt content of the compacted slabs was only 5.1 percent, instead of the desired 6.0 percent, increasing the necessity for checking the comparative properties of the original triaxial test mixture and of the modified slab mixture.

INSTRUMENTATION AND TEST TECHNIQUES

Four types of triaxial tests were performed to determine the rheologic behavior of the asphalt concrete mixture: (a) creep tests, (b) stress relaxation tests, (c) constant rate-of-strain determinations, and (d) repeated axial load tests. The tests were conducted at three temperatures, 40, 77, and 140 F, with three lateral pressures being used at each temperature, 0 (unconfined), 43.8, and 250 psi. These values of temperature and pressure were selected to cover the range of practical interest. The apparatus and procedures used in these tests have been described elsewhere (*1*).

For the creep tests, three levels of stress were employed for each temperature and lateral pressure combination. In the repeated load tests (essentially, repeated creep tests), only one level of stress was employed due to the relatively long time required for a particular test. The cycle selected for the repeated load determinations involved application of the desired stress for 1 sec and at the rate of 20 applications per min. For the stress relaxation tests, three levels of initial strain were tested for each set of ambient conditions. The constant rate-of-strain determinations were performed at load rates of 0.01, 0.10, and 1.0 in. per min for the various conditions.

The satisfactory completion of the triaxial test program required the preparation and testing of over 250 specimens. Thus, it is believed that these test data can be accepted as an accurate portrayal of the stress-strain-time characteristics of this test mixture over the wide range of test conditions.

The slab tests required the design and construction of considerable

special equipment. Essentially, the proposed structural analog required apparatus by which the slab could be placed under static load on a foundation of constant elastic properties. In addition, this apparatus had to include some means for controlling the environmental temperature within fairly rigid limits. Provision also had to be made for measuring the deflected shape of a test slab and for observing changes in that shape with time.

A major portion of the device constructed to satisfy these requirements is shown in Figure 2. The flexible foundation was provided by approximately 1,600 ⅞-in. diameter coil springs, arranged in a pattern of equilateral triangles in such a way that each spring would account for 1 sq in. of the surface area. Because each spring was required by specification to have a constant of 200 lb per in., such a configuration thus provided a foundation with a vertical resistance k of 200 psi per in. A $\frac{1}{32}$-in. rubber membrane was placed on the surface of the springs to protect them from the entrance of foreign material and to provide an improved bearing surface for the asphalt concrete slabs. Plate bearing tests performed on the spring base justified the use of a theoretical foundation constant of 200 psi per in.

Loads for the various slab tests were provided by the use of pneumatic cells and are similar to those used in the creep and repeated load tests (*1*). Inasmuch as a rather large range of loads was involved, it was necessary to employ several sizes of these cells to obtain sufficient sensitivity. A typical intermediate size is shown in Figure 2, attached to the frame used to provide a suitable reaction surface.

The entire spring base (as well as the reaction frame) was securely

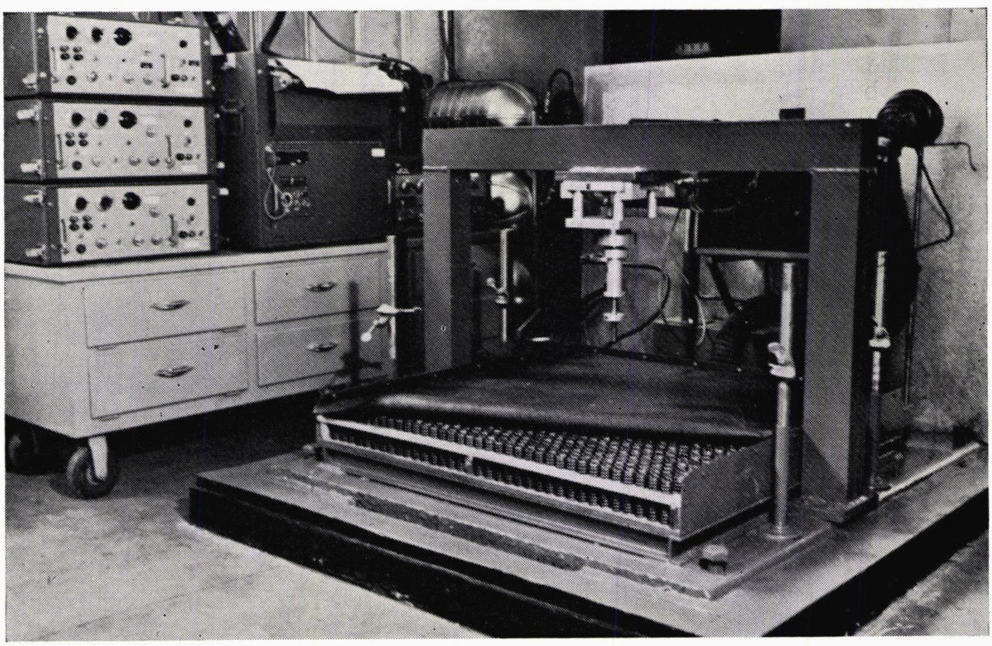

Figure 2. Slab-testing device with rubber membrane stripped back to show spring base (note pneumatic load cell hanging from the center of the reaction frame; electronic recorder for use with differential transformers appears in background).

fixed to a large block of cast iron. This provided an extremely level and rigid support for the unit, as well as an excellent heat-stabilizing sink for temperature control.

The test unit (Fig. 2) was designed to fit under a heavily insulated plywood cover, which was set on wheels to facilitate its movement. When locked in place over the spring base, the cover completed a nearly airtight enclosure with very low thermal conduction. Temperature control within the enclosure was obtained by the use of a large refrigeration unit in combination with an electric heater; both were attached to a master thermostat. A battery of electric blowers was used to provide a high degree of air circulation. These devices combined to permit temperature control within ± 1 F, over a range from $+30$ to $+150$ F.

Records of deflected shapes of loaded slabs were provided through the use of a system of linear variable differential transformers. A single small transformer was rigidly mounted in the center of the coil spring, directly below the load center, with its movable core attached to the rubber membrane. This device was used to record the center deflection of a given slab. Deflections at locations away from this point were indicated by several larger transformers above the slab, spaced along a radial line from the center. A typical test setup of this type is shown in Fig. 3. The control wires from these gages were run through the insulated cover to an electronic chart recording unit (background Fig. 2).

In performing a given slab test, the slab was first carefully slipped from its plywood form onto the spring base. The deflection gages were then positioned as shown in Fig. 3 and calibrated. Next, the cover was rolled over the slab and the temperature controls were actuated. As soon as the temperature of the slab reached equilibrium at the desired level (usually about 24 hr) the test could be performed. Load was applied by means of the pneumatic cell, using a rubber-surfaced plate to simulate a flexible bearing surface. The load was maintained either until the deflections as (indicated on the recorder) reached equilibrium or until the capacity of the transformers was exceeded.

This test procedure was carried out at the same three temperatures employed in the triaxial testing, 40, 77, and 140 F. Two slabs were loaded at each temperature, with differing stress levels. A 1-in. diameter rubber loading foot was used at 40 F and 77 F; and a 2-in. diameter foot was used at 140 F. These limitations were imposed by the necessity for maintaining an approximation of infinite boundary conditions for the slab. The resultant data from the slab tests are discussed in detail and compared with that predicted by viscoelastic theory in a later section.

BASIC MODEL THEORY

The use of model systems composed of purely elastic springs and purely viscous dashpots to aid in derivations associated with the use of linear viscoelastic concepts is by this time a fairly familiar feature of such analyses (*1, 2, 11, 12*) and no attempt is made to amplify this subject here.

The viscoelastic model (Fig. 4) was selected for study in connection with the investigations discussed in this paper. It should be noted that this is the same four-element model suggested by Kühn and Rigden (*13*) for use in research with asphalt cements. The authors (*1*) have also suggested its possible value for describing the rheologic behavior of asphalt concrete. This model is capable of accounting for instantaneous elastic deformation, retarded elastic deformation, and viscous flow. The first two types of deformation are

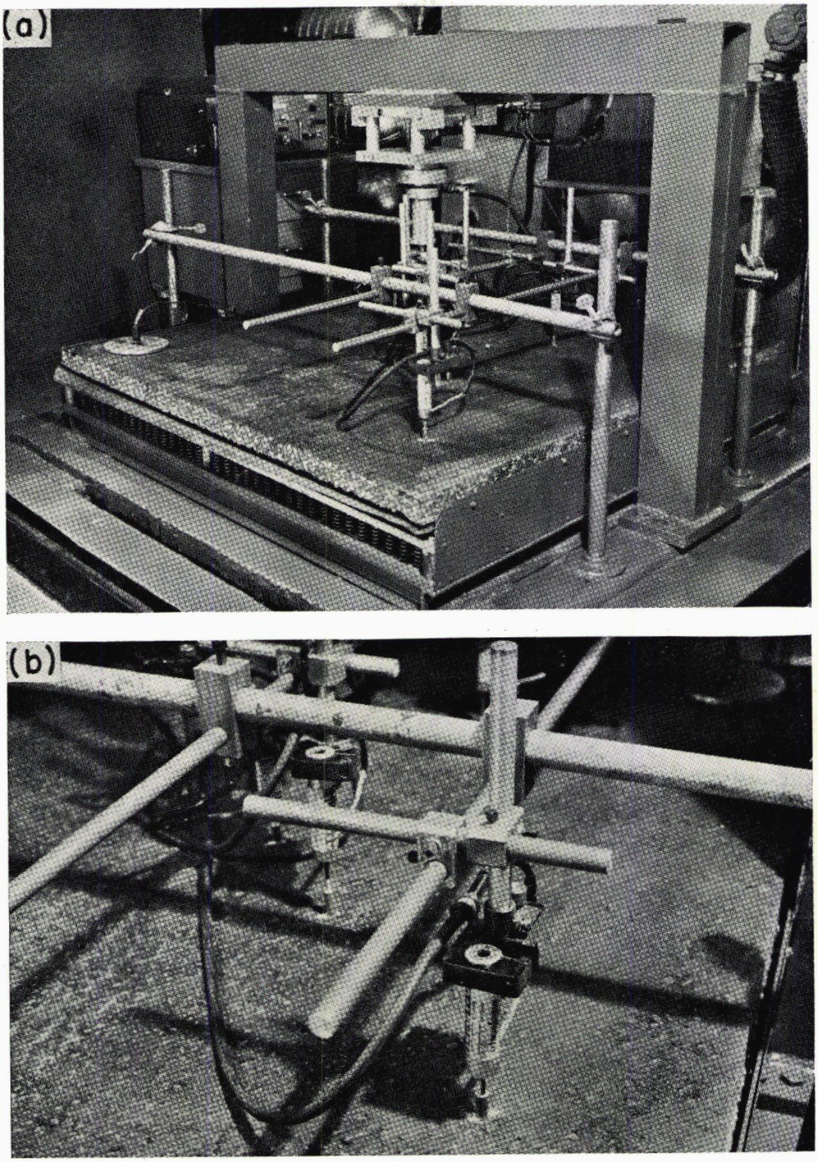

Figure 3. Slab test apparatus ready for operation, showing (a) instrumented slab ready for testing and (b) closeup of differential transformer installation.

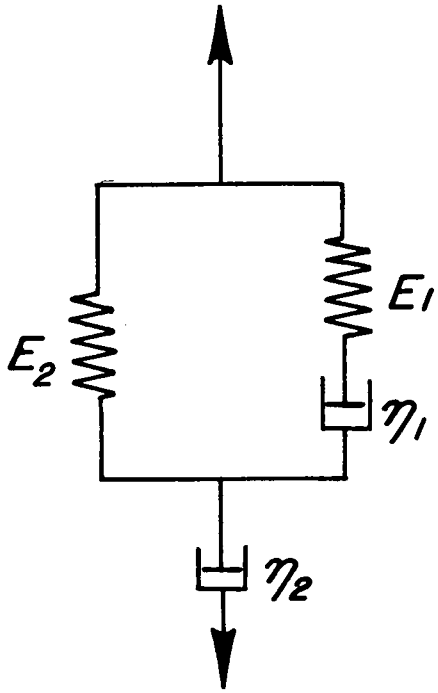

Figure 4. Four-element model.

recoverable, whereas the viscous flow is, of course, irrecoverable. It can be seen that these are precisely the properties often suggested as being characteristic of asphalt paving mixtures (*14, 15*).

Mathematically, the four-element model shown in Figure 4 is equivalent to the Burgers model, a system often mentioned in discussions of viscoelastic analysis; the Burgers model also has the three types of response previously mentioned. The basic differential equation governing the stress-strain-time characteristics of the four-element configuration can be stated as

$$\left[\frac{d^2}{dt^2}+\left(\frac{E_1}{\eta_1}+\frac{E_1+E_2}{\eta_2}\right)\frac{d}{dt}+\frac{E_1\,E_2}{\eta_1\,\eta_2}\right]\sigma\ (t)=$$

$$\left[\left(E_1+E_2\right)\frac{d^2}{dt^2}+\left(\frac{E_1\,E_2}{\eta_1}\right)\frac{d}{dt}\right]\varepsilon\ (t)\quad (1)$$

This equation can be simplified:

$$\left[\frac{d^2}{dt^2}+C_1\frac{d}{dt}+C_2\right]\sigma\ (t)=$$

$$\left[C_3\frac{d^2}{dt^2}+C_4\frac{d}{dt}\right]\varepsilon\ (t)\qquad (2)$$

in which

$$C_1=\frac{E_1}{\eta_1}+\frac{E_1+E_2}{\eta_2}$$

$$C_2=\frac{E_1\,E_2}{\eta_1\,\eta_2}$$

$$C_3=E_1+E_2$$

$$C_4=\frac{E_1\,E_2}{\eta_1}$$

Eqs. 1 and 2 can be written in terms of differential operators $Q(t)$ and $P(t)$ as

$$Q(t)\,\sigma=P(t)\,\varepsilon\qquad (3)$$

For the creep test, where $\sigma_1(t)=\sigma_0$ (a constant) and $\varepsilon_1(0)=\dfrac{\sigma_0}{E_1+E_2}$ and σ_1 and ε_1 refer to axial stress and strain, the solution of Eq. 1 becomes

$$\varepsilon_1(t)=\sigma_0\left[-\frac{E_1}{E_2(E_1+E_2)}\exp\left(-\frac{t}{\tau^*}\right)+\frac{1}{E_2}+\frac{t}{\eta_2}\right]\qquad (4)$$

in which

$$\tau^* = \eta_1 \left[\frac{E_1 + E_2}{E_1 E_2} \right]$$

For creep recovery on removal of the stress σ_0 at $t = t_0$, the solution of Eq. 1 for $t > t_0$ is

$$\varepsilon_1(t) = \sigma_0 \left\{ \frac{E_1}{E_2(E_1+E_2)} \left[\exp\left(-\frac{t-t_0}{\tau^*}\right) - \exp\left(-\frac{t}{\tau^*}\right) \right] + \frac{t_0}{\eta_2} \right\} \quad (5)$$

For the relaxation test, where $\varepsilon_1(t) = \varepsilon_0$ (a constant) and $\sigma_1(0) = \varepsilon_0\ (E_1+E_2)$, the solution of Eq. 1 is

$$\sigma_1(t) = \frac{\varepsilon_0}{R_2 - R_1} [(C_3 R_2 - C_4) \exp(-R_2 t) + (C_4 - C_3 R_1) \exp(-R_1 t)] \quad (6)$$

in which C_1, C_2, C_3, and C_4 are the same as for Eq. 2,

$$R_1 = \frac{C_1}{2} + \frac{1}{2}\sqrt{C_1^2 - 4C_2}$$

$$R_2 = \frac{C_1}{2} - \frac{1}{2}\sqrt{C_1^2 - 4C_2}$$

For the constant-rate-of-strain test, where $\varepsilon_1(t) = C\,t$ and $\varepsilon_1(0) = 0$, the solution of Eq. 1 is

$$\sigma_1(t) = \frac{C}{(R_2 - R_1) R_1 R_2} [R_2\ (C_3 R_1 - C_4) \exp(-R_1 t) + R_1\ (C_4 - C_3 R_2) \exp(-R_2 t) + C_4(R_2 - R_1)] \quad (7)$$

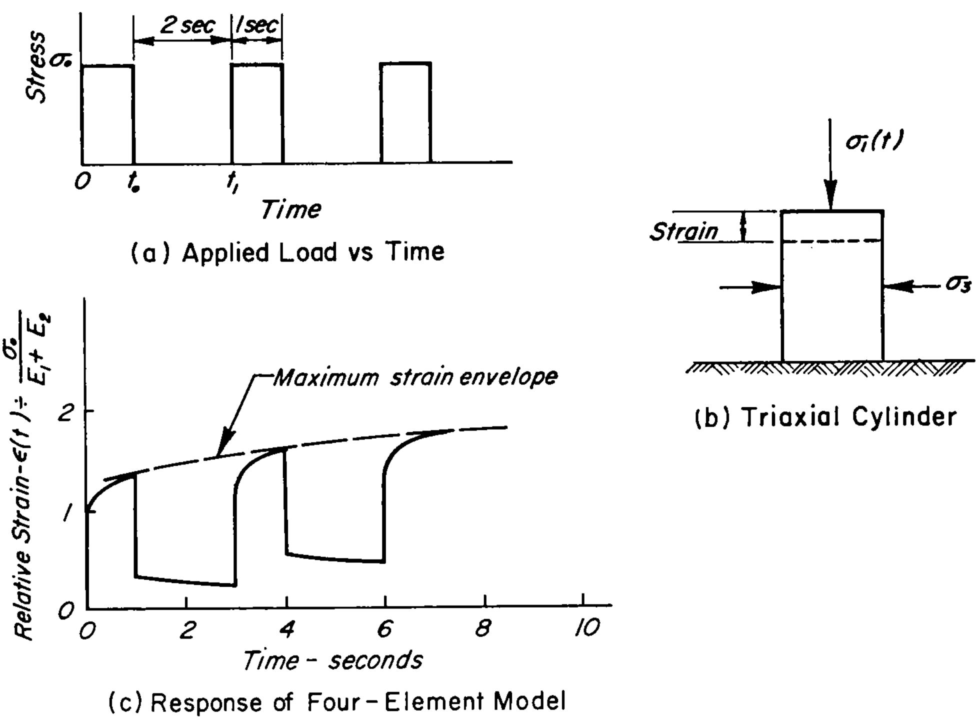

Figure 5. Creep and repeated-load relationships for the four-element model.

in which all constants are the same as for Eq. 6.

Eqs. 4 and 5 can be used to deal with repeated-load applications by the simple expedient of shifting the time scale so that these two relationships fit each successive cycle of loading. This process may be visualized by inspection of Figure 5. For the first load cycle, Eqs. 4 and 5 can be applied directly, because this load condition is identical to creep loading. The four-element model, if undisturbed during creep recovery, will rebound until only the irrecoverable viscous flow due to the free dashpot remains. If, however, a second cycle of load is applied before rebound is complete, cumulative buildup of strain in addition to viscous flow may occur. Eqs. 4 and 5 may be used to study the second cycle of load in Figure 5, because the value of strain at $t=t_1$ can be taken as the new initial value. By this same approach, load cycling may be examined indefinitely.

APPLICATION OF MODEL THEORY TO ANALYSIS OF TRIAXIAL TEST RESULTS

Eqs. 4 through 7 completely establish the mathematical relationships necessary for employing the four-element model in connection with the various types of triaxial tests discussed herein. These relationships were used in conjunction with test data to compute values for the four constants, E_1, E_2, η_1, and η_2 over the range of ambient conditions selected for study. Figures 6 through 9 show plots of these numerical values versus temperature and lateral pressure.

The viscoelastic constants for the four-element model are most easily approximated from the results of a creep test, although they can be estimated, with some difficulty, from relaxation test data. Figure 10 shows how such processes may be carried out. In general, the values plotted in Figures 6 through 9 were estimated,

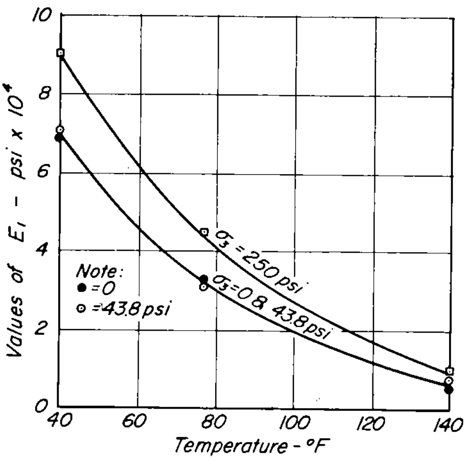

Figure 6. Relation of E_1 to temperature and lateral pressure.

for a particular set of ambient conditions, by first considering creep data from tests under those conditions. Four constants providing a satisfactory agreement between theory and data were selected and then applied to the equations governing the constant-rate-of-strain and relaxation tests, for further comparisons between theory and test results. The constants were then adjusted, if necessary, to provide the best possible over-all agreement for all three test types. To reduce the work involved in performing these numerical manipulations, Eqs. 4 through 7 were

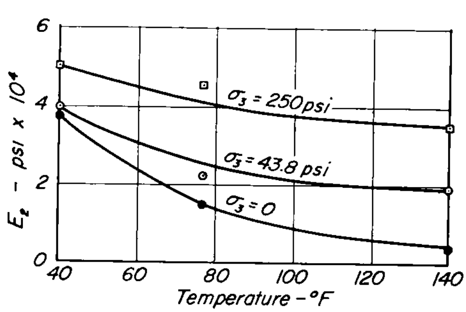

Figure 7. Relation of E_2 to temperature and lateral pressure.

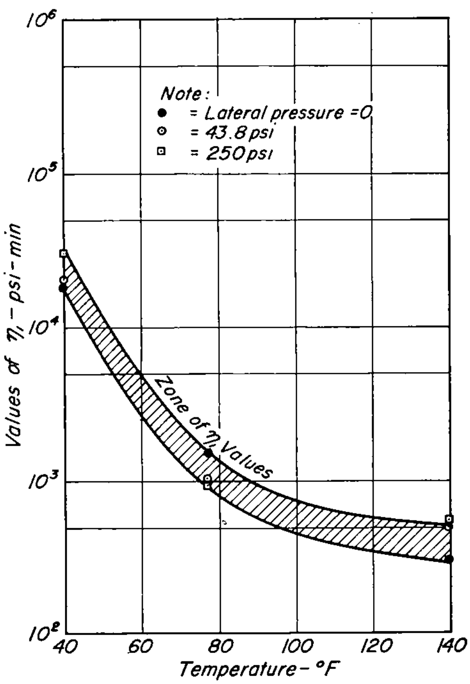

Figure 8. Relation of η_1 to temperature and lateral pressure.

programed for solution on a digital computer.

The values plotted in Figures 6 through 9 were obtained by considering each set of ambient conditions independently; because tests at three temperatures (40, 77, and 104 F) were involved, with three lateral pressures at each temperature (0, 43.8, and 250 psi), nine individual determinations were made. However, when plotted in final form the constants seemed to indicate reasonably clear trends, which may be summarized as follows:

1. E_1 (Fig. 6). Values of E_1 indicated a marked decrease with increases in temperature. Lateral pressure increases produced a corresponding increase in E_1, although a considerable increase in lateral pressure was required to produce an appreciable effect (note that values for $\sigma_3=0$ and $\sigma_3=43.8$ psi lie almost on the same curve).

2. E_2 (Fig. 7). This plot shows the same general characteristics as Figure 6. However, values of E_2 seemed less susceptible to changes in temperature than those for E_1, and more susceptible to changes in lateral pressure.

3. η_1 (Fig. 8). Values of η_1 displayed a marked decrease with increases in temperature. The effect of lateral pressure, however, was not clear cut.

4. η_2 (Fig. 9). This constant showed considerable increases in value with increases in either temperature or lateral pressure.

It should be emphasized that these values of E_1, E_2, η_1, and η_2 are simply arbitrary constants selected for use in connection with Eq. 1. It would probably be a mistake to attempt any

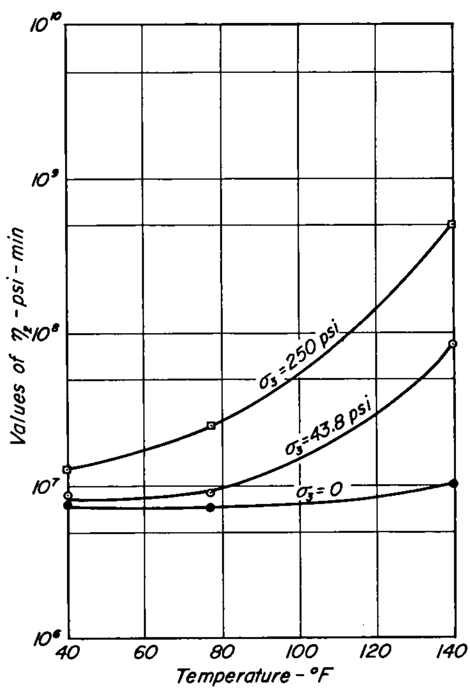

Figure 9. Relation of η_2 to temperature and lateral pressure.

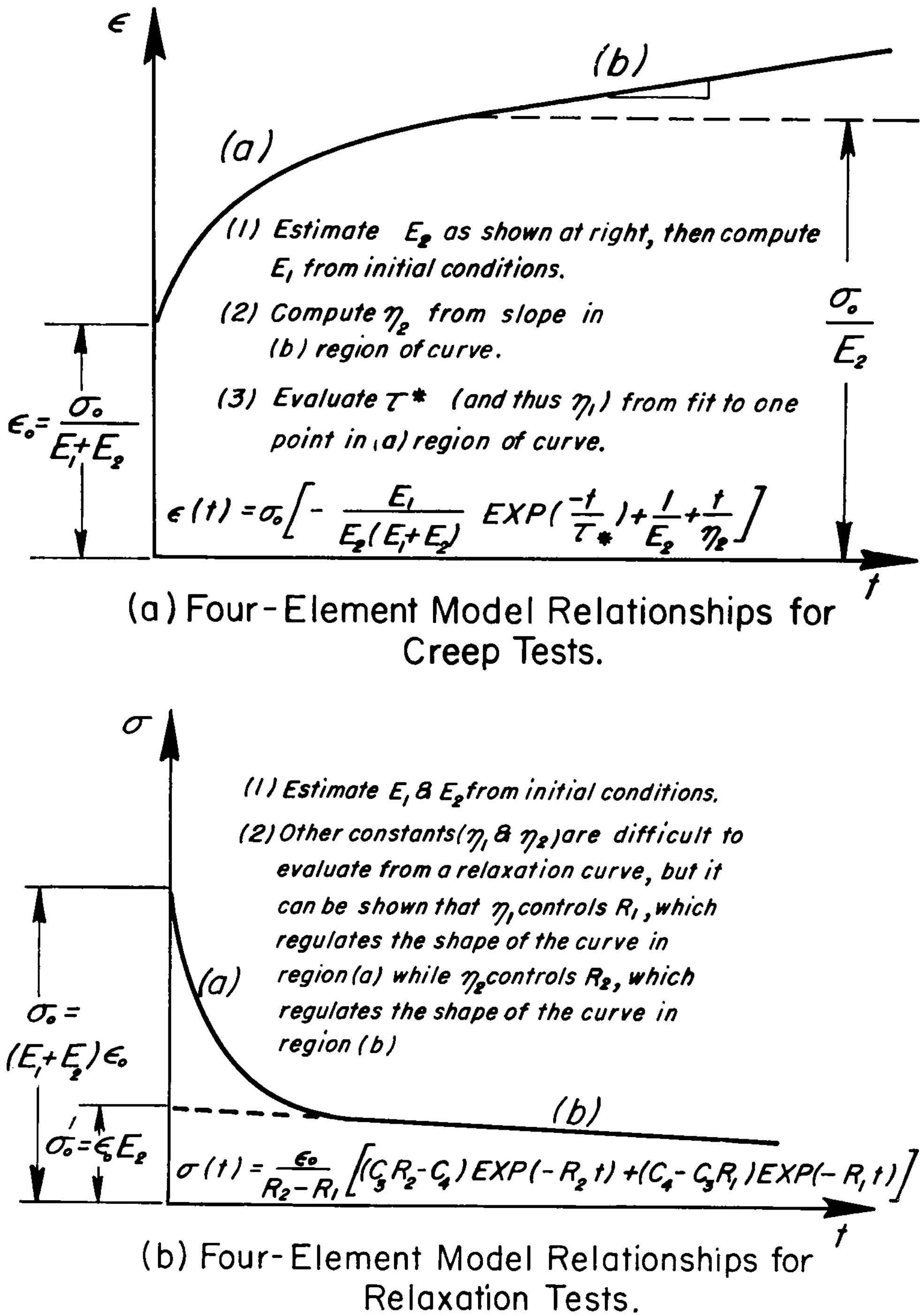

Figure 10. Determination of four-element model constants from triaxial compression test results.

significant connection of the suggested trends with recognized rheologic criteria.

To demonstrate the ability of the four-element model to reflect the characteristics of the paving mixture selected for study, Figures 11 through 14 are presented. These figures show comparisons of unconfined test data at 40 F with similar data as predicted by the four-element model relationships. Although the agreement between actual and predicted data was not perfect, the model appeared to reflect the characteristics of the material to a marked degree over a wide range of load situations. The comparisons given in Figures 11 through 14 are fairly typical of those obtained for the other ambient conditions studied; the disagreement between test data and theoretical predictions seldom exceeded 30 percent and was usually much less. Only for the case of repeated loading at high temperature and low lateral pressures, where some unexplained deviations occurred, did the four-element model fail to achieve satisfactory expression of the rheologic characteristics of the paving mixture.

It should be noted in Figure 14 that no attempt has been made to correct the deformation under the first stress application for the results presented. (This initial load increment can vary due, for example, to seating and apparatus continuity problems.) The shapes of the theoretical and actual curves however, are quite similar up to about 10^4 stress applications.

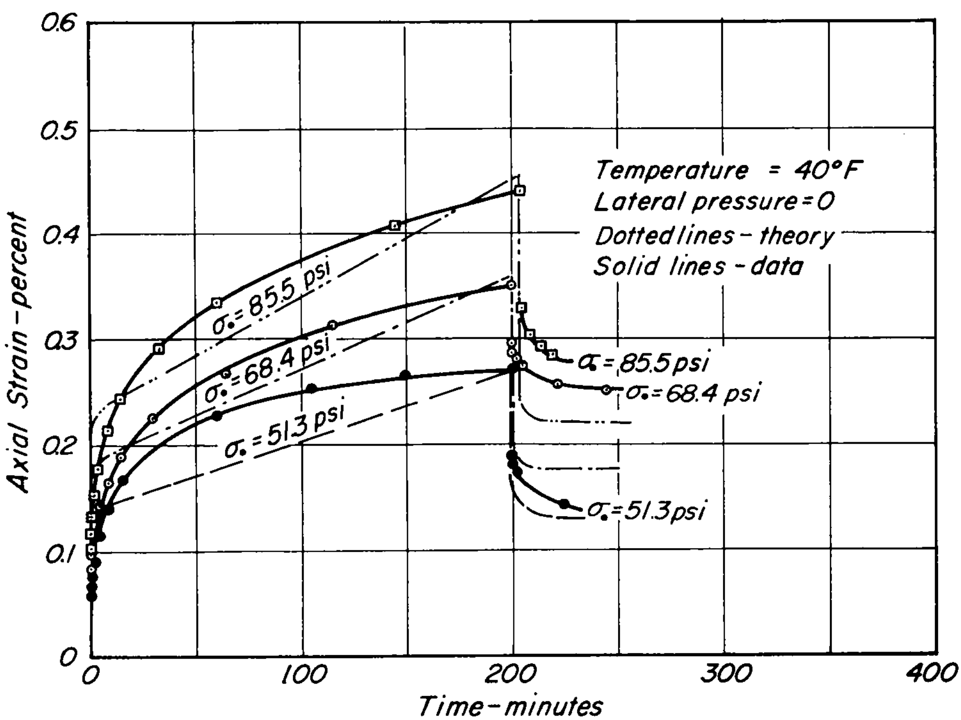

Figure 11. Comparison of creep test data with data predicted by four-element model.

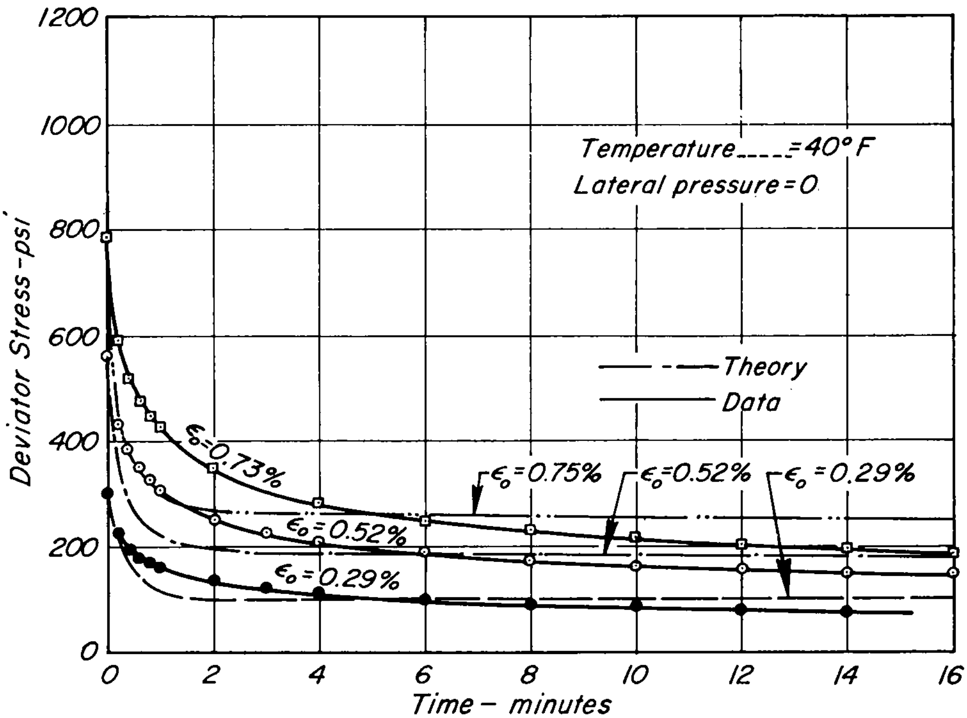

Figure 12. Comparison of relaxation test data with data predicted by four-element model.

VISCOELASTIC SLAB THEORY

The viscoelastic equations and coefficients presented in the preceding discussion were used to produce theoretical predictions of the deflections measured during slab tests on the spring base. For the purpose of this paper, it was assumed that a paving slab resting on the spring base could be closely approximated by the structural analogy of a thin viscoelastic plate on a Winkler foundation (approximated by a set of independent springs).

Hertz (*16*) has presented a solution for the structural responses of a thin elastic plate on a Winkler foundation. He showed that the deflections of such a plate could be obtained, in the case of an infinitely large plate in a state of axial symmetry, from

$$D\nabla^4 w(r) = q(r) - p(r) \tag{8}$$

in which

$$D = \frac{E\,h^3}{12(1-\nu^2)};$$

E = elastic modulus;

h = thickness;

ν = Poisson ratio of plate,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\,\frac{\partial}{\partial r};$$

$w(r)$ = vertical deflection of plate, as a function of radius, r, from center of load;

$q(r)$ = surface loading; and

$p(r)$ = foundation reaction.

It has already been mentioned that the basic differential equation for the

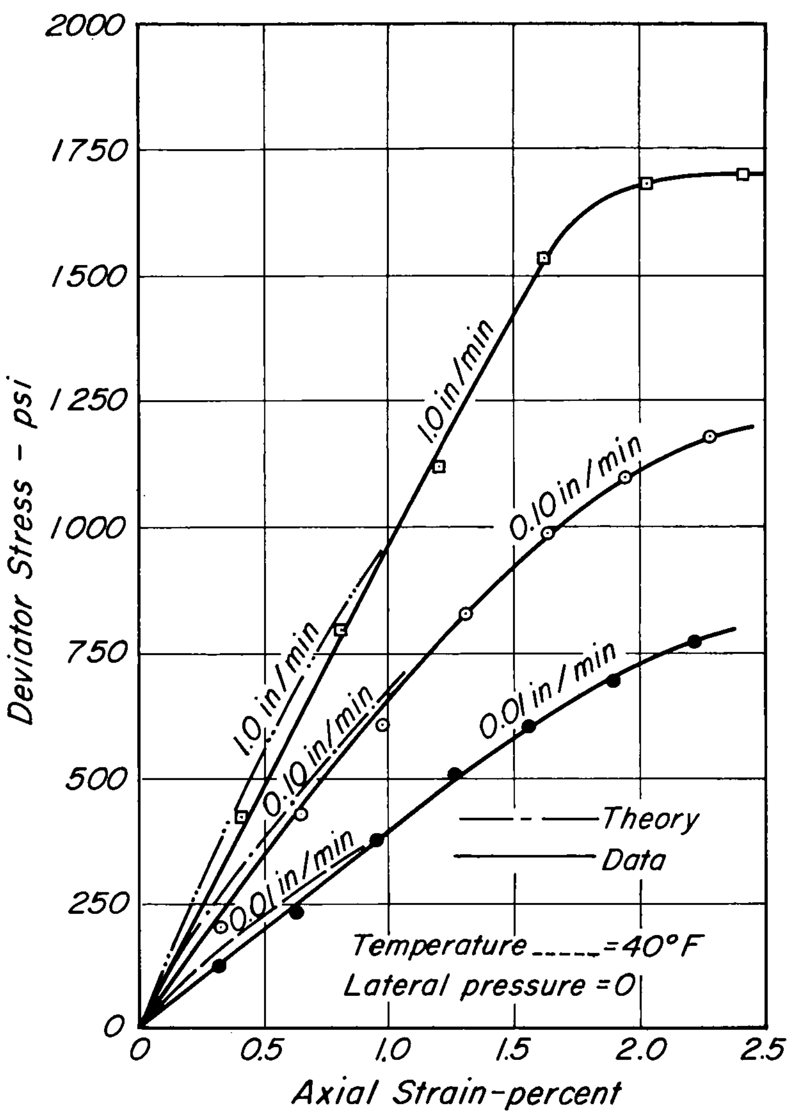

Figure 13. Comparison of constant-rate-of-strain test data with data predicted by four-element model.

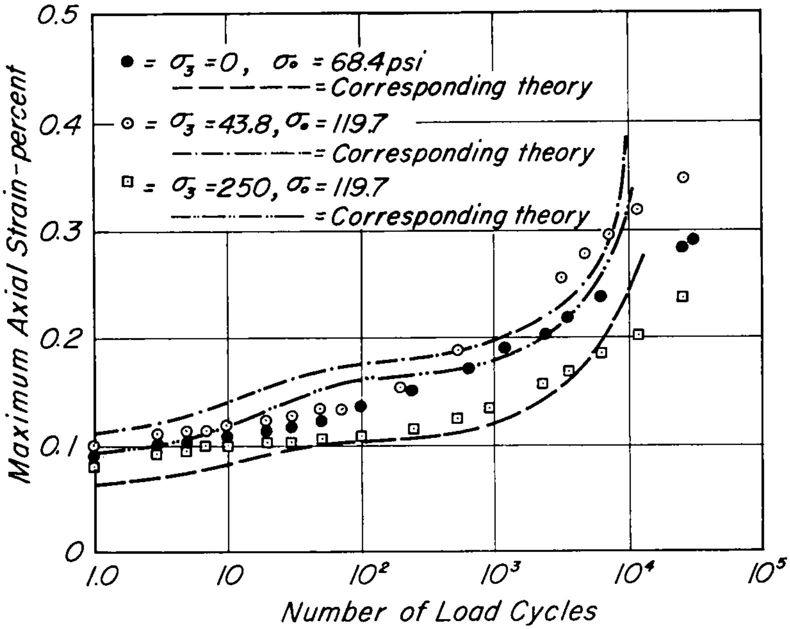

Figure 14. Comparison of repeated-load test data with those predicted by four-element model—40 F.

four-element model can be expressed in terms of the differential operators $P(t)$ and $Q(t)$. By replacing the modulus of elasticity in the D-term of Eq. 8 with the ratio of these operators, $P(t)/Q(t)$, an expression for the viscoelastic behavior of the slab can be obtained:

$$D(t)\nabla^4 w(r,t) = q(r,t) - p(r,t) \tag{9}$$

It can be seen that the time-dependence of w, q, and p is also implied by this process. The solution of this differential equation, utilizing an application of the correspondence principle relating elastic and viscoelastic boundary value problems, was carried out through the use of integral transform methods (*4*). The result for the deflections due to a uniform load of radius r_1 applied at $t=0$ and held constant is:

$$w(r,t) = \frac{q_0}{k}\int_0^\infty \frac{f(a,b)}{\left[\left(\frac{l_0}{r_1}\right)^4 \lambda^4 + 1\right]\left[a\,b(a-b)\right]} J_1(\lambda) J_0\left(\frac{\lambda}{r_1}r\right) d\lambda \tag{10}$$

in which

λ = an integration variable;

q_0 = unit load on area of radius r_1;

k = foundation modulus;

C_1, C_2, C_3, *and* C_4 = basic viscoelastic constants, Eq. 2;

$$l_0^4 = \frac{D^*}{k};$$

$$D^* = C_3\left[\frac{h^3}{12(1-\nu^2)}\right];$$

$$a=\frac{-\left[A\left(\frac{l_0}{r_1}\right)^4\lambda^4+C_1\right]+\left[A^2\left(\frac{l_0}{r_1}\right)^8\lambda^8+B\left(\frac{l_0}{r_1}\right)^4\lambda^4+F\right]^{\frac{1}{2}}}{2\left[\left(\frac{l_0}{r_1}\right)^4\lambda^4+1\right]};$$

$$b=\frac{-\left[A\left(\frac{l_0}{r_1}\right)^4\lambda^4+C_1\right]+\left[A^2\left(\frac{l_0}{r_1}\right)^8\lambda^8+B\left(\frac{l_0}{r_1}\right)^4\lambda^4+F\right]^{\frac{1}{2}}}{2\left[\left(\frac{l_0}{r_1}\right)^4\lambda^4+1\right]};$$

$$A=C_4/C_3;$$

$$B=2\frac{C_1C_4}{C_3}-4C_2;$$

$$F=C_1^2-4C_2;\text{ and}$$

$$f(a,b)=\{\exp(at)[a^2b+c_1ab+C_2b] -\exp(bt)[ab^2+C_1ab+C_2a] +C_2(a-b)\}$$

The foregoing equation is tedious to evaluate by desk computational methods; therefore, it was programed for the digital computer.

The derivation of Eq. 10 for use in the analysis of the deflections of the asphalt paving slabs tested for this paper involved certain important assumptions, as follows:

1. The thin-plate theory defined by Eq. 8 was derived by neglecting the effects of transverse normal and shearing stresses.
2. The basic theory also required the assumption that the plate (slab) material had equal properties in tension and compression.
3. Poisson's ratio for the plate (slab) material was considered to be independent of time, in order to achieve an important (although not absolutely essential) mathematical simplification.
4. The lateral dimensions of the plate were taken as infinite, in relation to the size of the area affected by the imposed load.

The importance of these assumptions in influencing the ability of the theory, as exemplified by Eq. 10, to describe the deflections of the test slabs is discussed later.

COMPARISON OF VISCOELASTIC SLAB THEORY WITH TEST DATA

The results of deflection measurements on a typical test slab at 40 F are shown in Figures 15 and 16. Figure 15 shows deflection measured under the center of load versus time; Figure 16 shows the profiles of the slab for various times after load application. Similar data were obtained for each of the slabs tested. The loads at each temperature were selected arbitrarily from theory as being those which would produce deflections roughly corresponding to values obtained from field measurements on typical pavement installations.

Also shown in Figures 15 and 16 are the theoretically predicted curves produced by the application of Eq. 10. These curves were computed for each of the six test slabs by applying the viscoelastic coefficients (C_1, C_2, C_3, C_4) obtained from the unconfined triaxial compression tests at the same temperatures. The constants derived from unconfined tests were used because, except for the region directly below the loaded area, lateral pressures in the slabs were estimated as

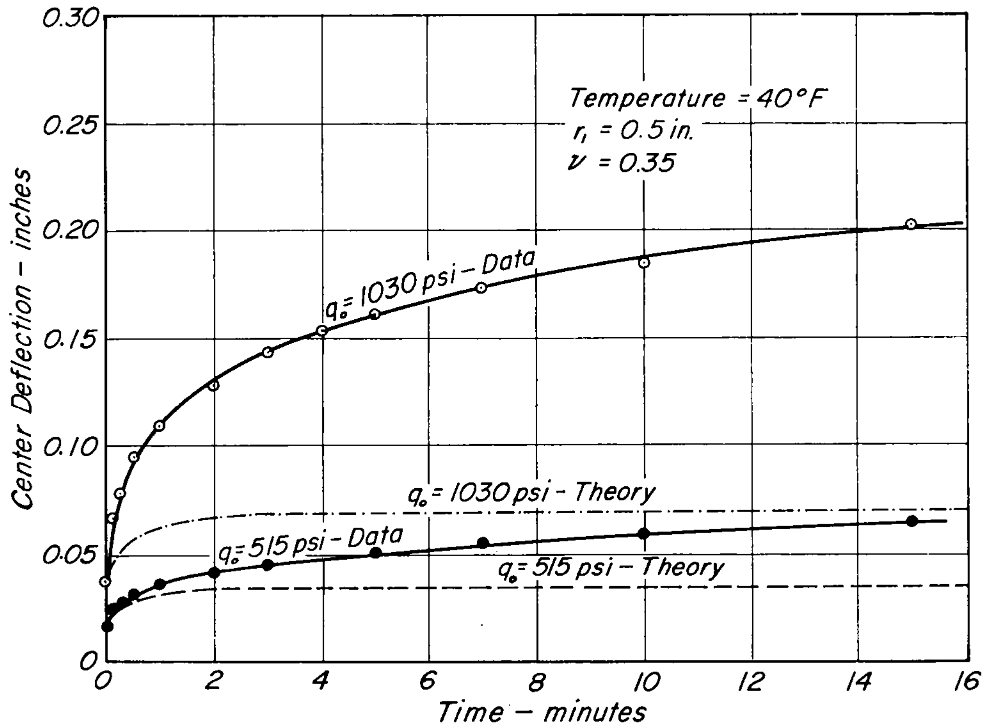

Figure 15. Comparison of measured deflections under centers of loaded slabs with deflections predicted by viscoelastic theory.

being negligible. Poisson's ratio for each temperature was estimated from the results of volume change measurements previously reported (*1*).

A large disparity between theory and test data is apparent from an inspection of Figures 15 and 16; similar disagreement was found for all of the comparisons made. It is interesting to note, however, that the disagreement was affected to some extent by temperature. The test results at 40 F, for example, show a ratio of measured deflections to those predicted by theory of almost 2:1 after 15 min of load application (Figures 15 and 16); at 77 F the ratio was more on the order of 3:1.

Although data such as those shown in Figures 15 and 16 might raise a question as to the value of viscoelastic analysis as applied to problems such as the one under consideration, there are a number of important factors which should be cited in argument against such an early, and possibly mistaken, conclusion. These factors are considered in the following.

It was mentioned earlier that slight differences in asphalt content and aggregate gradation existed between the triaxial test cylinders and the slab specimens. A comparison between the results of triaxial compression tests in creep, stress relaxation, and constant-rate-of-strain loading for the two mixtures failed to disclose any significant differences in their rheologic properties. Thus it was believed that the explanation of

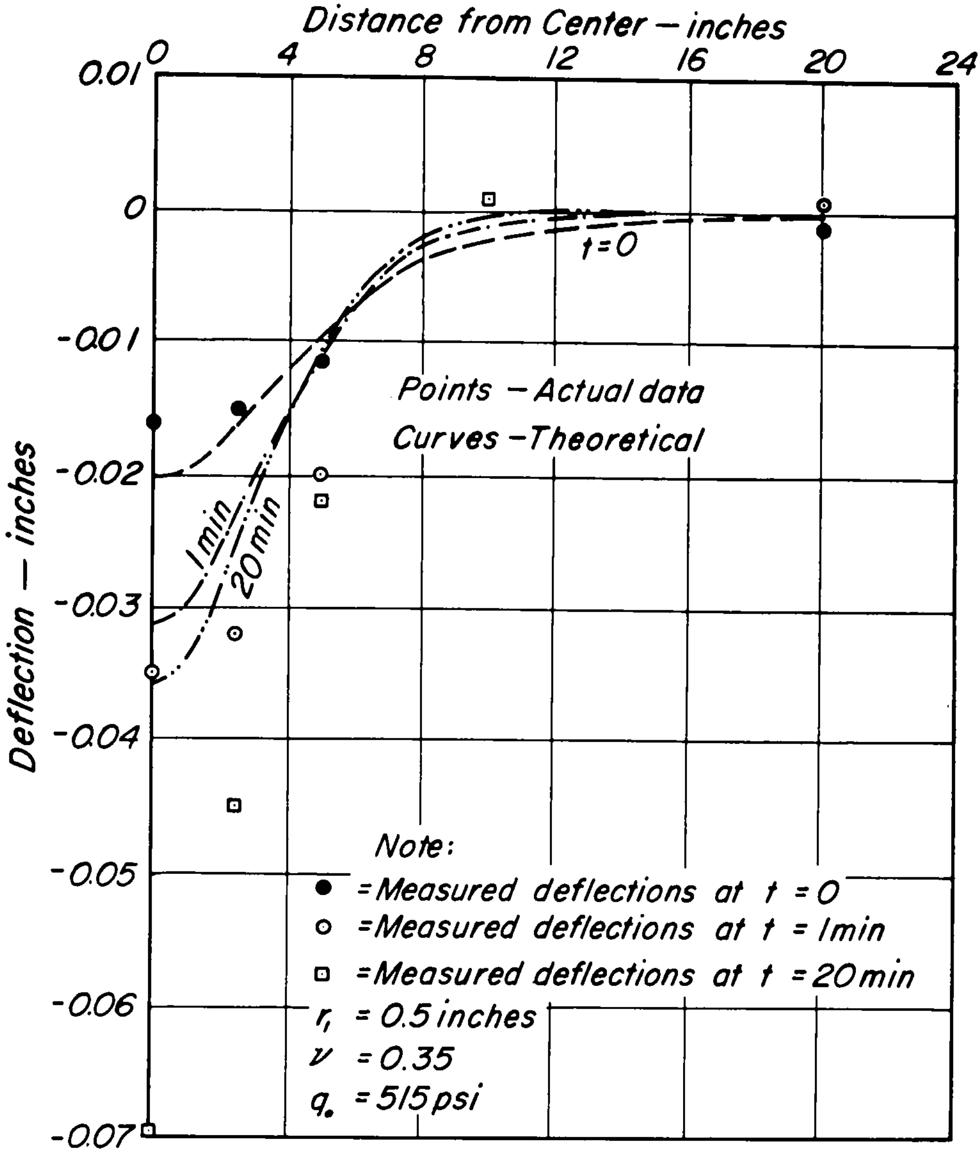

Figure 16. Comparison of actual and theoretical deflection profiles of a test slab for various times after load application—40 F.

the disagreement between test results and the viscoelastic thin-plate theory found in the slab studies might be afforded by a critical examination of the assumptions made in deriving and applying that theory, as follows:

1. The basic assumption that transverse normal and shearing stresses could be neglected is open to question; however, it is unlikely that the errors introduced by this simplification would be of the order of magnitude discussed here.

2. The assumption that asphalt paving mixtures have the same properties in tension and compression is almost certainly in error. Unfortunately, few data are available on this subject at present. Quite probably, the stiffness of asphalt mixtures in tension is less than in compression; this difference might also be time-dependent. These factors could easily cause errors in this theory, as given by Eq. 10, of the magnitude shown in Figures 15 and 16. Research is currently under way to evaluate this possibility.

3. The assumption of a constant Poisson's ratio for a viscoelastic material can also be questioned. However, for the purpose of this paper it was found expedient to employ this assumption, after careful consideration of the possible resultant effect on theoretical calculations. Values for the Poisson's ratio of the slab material at the desired temperatures were estimated from the results of volume change measurements during unconfined constant-rate-of-strain tests, as has already been mentioned. The results are given in Table 2. As can be seen, the assumption of a constant ratio at each temperature is nearly correct.

4. The infinite boundary conditions assumed for Eq. 10 were apparently satisfied at all temperatures used during slab testing. In all cases the deflections measured at a radius of 20 in. from the center of load (*i.e.*, the edge of the slab) were relatively negligible, as suggested by theory.

TABLE 2

POISSON'S RATIOS COMPUTED FROM VOLUME CHANGE MEASUREMENTS (CONSTANT-RATE-OF-STRAIN TESTS)

Temp. (°F)	Load Rate (in./min)	Poisson's Ratio
40	0.01	0.371
40	0.10	0.358
40	1.00	0.305[1] (approx.)
77	0.01	0.492
77	0.10	0.484
77	1.00	—[1]
140	0.01	0.495
140	0.10	0.498
140	1.00	—[1]

[1] Volume changes difficult to record at this load rate.

5. In addition to the formal assumptions previously listed, use of a linear viscoelastic model in deriving Eq. 10 implied that the levels of stress and strain imposed on the test slabs be kept in the linear range. This range was not well defined in the triaxial compression test phase of this research, due to the time demands required for such definition. If the loads imposed on the test slabs were such that the linear material range was exceeded, the disagreement shown in Figures 15 and 16 might easily be explained in part. Further investigation of this problem is also under way.

SUMMARY AND CONCLUSIONS

The ability of an analytical system founded on a simple, linear, viscoelastic model to express the rheologic characteristics of an asphalt concrete was demonstrated by the data presented herein. The comparisons made here to triaxial compression test results through the use of the four-element model clearly showed that it could provide a general approximation of the properties of the

material under study over a wide range of load types and ambient conditions. The errors obtained during such comparisons in most cases were less than 30 percent.

The success obtained in the application of viscoelastic analysis to the prediction of the deflections of the slabs of the test mixture placed under static loading on an elastic foundation was less marked. However, further research into areas such as the comparison of relative properties of asphalt mixtures in tension and compression and the study of the linear limits of these properties holds promise that such analysis can be vastly improved. It must be remembered that elastic techniques, as they might be applied to such a problem, would only be capable of giving one set of deflections under a specific load; the necessary variations in those deflections with time would not be available. Thus, with further attention to some of the considerations mentioned in the foregoing discussion, it would seem that viscoelastic theory might yet serve to provide a better analytic framework for application to specific loading conditions for asphalt concrete pavements.

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