Serviceability Criteria for Prestressed Concrete Bridge Girders

Andrzej S. Nowak and Hassan H. El-Hor, *University of Michigan*

Serviceability limit states often govern in the design of prestressed concrete bridge girders, yet the corresponding acceptability criteria are not clearly justified. The paper deals with allowable stresses and ultimate moment. The requirements of the AASHTO standard specifications are demonstrated on pretensioned bridge girders AASHTO Types III through VI. Moments are calculated for the considered limit states and for various spans. It is observed that the ratio of these moments varies with regard to span length. In most cases, the final tension stress (after the final loss of prestress) determines the minimum required number of prestressing strands. On the other hand, the important limit state is compression stress in concrete, as overloading may lead to unacceptable permanent deformations. Revision of the serviceability limit states, consistent in format with the new AASHTO load and resistance factor design specifications, is suggested. The formulation of a compression limit state in concrete that is based on the elastic limit and tension limit states in concrete and the modulus of rupture is proposed.

Design of prestressed concrete bridges specified by the AASHTO (*1*) is a combination of working stress design and ultimate strength design. Prestressed concrete girders are designed to satisfy the allowable stress requirements at service load conditions. Then, the ultimate flexural capacity of the section is also checked. In most cases the allowable tension stress governs. However, most of the code calibration effort was directed to the development of the ultimate load criteria (*2*). Therefore, there is a need to consider the allowable stress design.

The objective of this paper is to review and compare the design criteria for prestressed concrete bridge girders, with regard to the ultimate limit states (ULS) and serviceability limit states (SLS), as specified in AASHTO standard specifications and AASHTO Load and Resistant Factor Design (LRFD). In particular, the analysis was focused on ultimate moment and allowable stresses in tension and compression.

Limit States

A structural component can be in a safe state or a failure state. The limit state is defined as the boundary condition separating these two states. In general, failure is considered as the inability to perform a function(s), such as, carrying the loads, providing a shelter, or satisfying certain deformation criteria (deflection or vibration). The limit states can be put into categories, depending on the following functions:

- ULS,
- SLS, and
- Fatigue limit states.

ULS is related to load-carrying capacity. A structure that reaches a ULS is on the brink of failure, in the form of a collapse, overturning, or rupture. ULS for a beam is defined as reaching the ultimate moment carrying capacity, or shear capacity, but also loss of stability. The limit state can be considered for a component or for the whole structure (bridge). In general, a bridge reaches a ULS only after several components (girders) have reached their ULSs. If moment in a girder is equal to the ultimate moment, the girder cannot take any more loading, but it does not necessarily mean a failure. In most cases, the bridge loading still can be increased until several girders reach their ULSs.

SLS's are related to bridge performance under load levels lower than those used at ULS. Examples of SLS include cracking, deflection, vibration, and excessive permanent deformation. In general, the consequences of reaching an SLS are much less severe than that for ULS. Cracking is undesirable; it may lead to corrosion of reinforcement or prestressing steel or both. However, opening of a crack once in a while may be acceptable. Deflection and vibration are two limit states that are difficult to define. The acceptability criteria are not clear and appear to be subjective. Compression stress in concrete may exceed the elastic limit, and this may result in a permanent deformation. Therefore, there is a need to control stress at the top fibers of the girder at transfer (after the wires are cut) and bottom part of the composite girder after the final loss of prestress.

FLSs are related to load-carrying capacity under repeated loads. Multiple application of load, even at a level that is lower than the ultimate load, can lead to rupture. Bridge structures often carry millions of trucks, and each passage can be considered as a load cycle. Welded steel components in tension are vulnerable to fatigue failure.

In practice, the limit states are formulated using various load and resistance parameters, $x_1, \ldots x_n$, in form of the so-called limit state functions, $f(x_1, \ldots x_n)$. Limit state function is an equation:

$$f(x_1, \ldots x_n) = 0 \tag{1}$$

so that if $f(x_1, \ldots x_n) > 0$, the structure (or component) is in a safe state, and if $f(x_1, \ldots x_n) < 0$, the structure (or component) is in a failure state.

Safe realization of structures requires that the probability of reaching a limit state be kept at an acceptable low level. That probability is often called the probability of failure (P_F) and it depends on cost of investment (C_I), and consequences of failure (C_F). The cost (C_I) includes the costs of design, construction, and operation (use). The optimum probability of failure corresponds to the minimum total expected cost:

$$\min C_T = C_I + \Sigma(P_{Fi}\ C_{Fi}) \tag{2}$$

where

P_{Fi} = probability of failure for limit state i and
C_{Fi} = cost of failure for limit state i.

The consequences of failure vary depending on limit state; therefore, the products of P_{Fi} and C_{Fi} are calculated for all limit states involved in the design.

In the design codes, the acceptability criteria are also formulated in terms of limit state functions. Safety reserve is ensured by specifying conservative values of load and resistance parameters. The probability-based approach has been used to derive the optimum load and resistance factors for the ULS (2). However, for the SLS, the consequences of failure (reaching the limit state) are usually an order of magnitude (or more) lower than for ULS. Therefore, there is a need for quantification of the load and resistance parameters for SLS and the development of a basis for calibration.

Design Criteria

The design of prestressed concrete bridge girders on the basis of AASHTO (*1*) is based on the calculation of stresses under the so-called service loads and their comparison with specified allowable stresses. Stresses are calculated at midspan. Service loads are determined as unfactored effects of dead load, live load, and impact. The prestressing force is also considered and its effect is reduced by estimated prestress losses. The ultimate moment carrying capacity is calculated and compared with the total factored load. The design requirements are reviewed by considering AASHTO girders Types III through VI.

The calculation of dead load (DL) does not involve much uncertainty. The statistical parameters of DL are available (2). On average, DL exceeds the design values by about 3 to 5 percent. The coefficient of variation is 0.08 to 0.10.

Design live load (LL) is calculated using HS20 truck or lane loading. It was found that the actual truck traffic can produce much higher load effects (*3*). The expected maximum 75-year lane moments can be as large as 2.10 of HS20 moment for about a 150-ft (45-m) span or about 1.60 of HS20 moment for a 20-ft (6-m) span. Design dynamic load (impact) (IL) is specified as a function of span length:

$$\mathrm{IL} = [50/(125 + L)]\ \mathrm{LL} \tag{3}$$

where L = span length (1 ft = 0.3 m). The actual dynamic load is a function of bridge span, roughness of the surface and vehicle dynamics (*4*). It has been observed that IL decreases with increasing truck weight, and for very heavy vehicles IL = 0.15 LL. The coefficient of variation for (LL + IL) is about 0.18.

The girder distribution factor (GDF) specified by AASHTO (*1*) for prestressed concrete girders is *s*/5.5, where *s* = girder spacing in feet (1 ft = 0.3 m). The resulting GDFs are conservative in most cases. A recent study by Zokaie et al. (*5*) indicated that the current AASHTO specifications are overly conservative for longer spans and girder spacings (by about 50 percent) but are too liberal for shorter spans and girder spacings.

Prestressing force is the major design consideration. The stress level is controlled by the number of strands and initial prestressing force. Prestress loss is estimated at two stages: immediately after the wires are cut (initial loss of prestressing force) and at the end of economic life (final loss of prestressing force).

The design stresses under service load are calculated for unfactored dead load and HS20 truck plus impact (Equation 3), with GDF equal to *s*/5.5. The calculations are carried out to determine the maximum stresses in compression and tension. Tension stress at the top fibers of the girder is considered after the wires are cut (after initial loss of prestressing force). It is calculated using the following formula:

$$\sigma_{ti} = F_i/A_c - F_i e_o/Z_t + M_{\min}/Z_t \qquad (4)$$

and corresponding compression stress at the bottom is calculated as follows:

$$\sigma_{ci} = F_i/A_c + F_i e_o/Z_b - M_{\min}/Z_b \qquad (5)$$

where

A_c = area of concrete,
F_i = initial prestressing force,
e_o = eccentricity of strands,
Z_t = section modulus with respect to top fibers for noncomposite section,
Z_b = section modulus with respect to bottom fibers for noncomposite section, and
$M_{\min}$ = moment caused by self-weight of the girder.

Other stresses are calculated for a composite section. Maximum compression in the top fibers is checked under live load and after the final loss of prestressing force. Maximum tension stress (if any) is calculated at the bottom fibers, also after the final loss of prestressing force. The compression stress is calculated as follows:

$$\sigma_{cs} = F_s/F_c(1 - e_o/K_b) + M_p/Z_t + M_c/Z'_{tc} \qquad (6)$$

and tension stress:

$$\sigma_{ts} = F_s/A_c(1 - e_o/K_t) - M_p/Z_b - M_c/Z_{bc} \qquad (7)$$

where

F_s = final prestressing force,
Z_{bc} = section modulus with respect to bottom fibers for composite section,
Z'_{tc} = section modulus with respect to extreme top fibers for composite section,
$K_t = -Z_b/A_c$,
$K_b = Z_t/A_c$,
$M_c = M_{sD} + M_{LL}$,
$M_p = M_g + M_s$,
M_s = moment caused by slab weight,
M_{sD} = superimposed moment,
M_g = moment caused by girder weight, and
M_{LL} = moment caused by LL and impact.

The specified allowable stresses are listed in Table 1 (*1*). Prestressed concrete bridge girders designed by AASHTO are required to satisfy the initial and final concrete stresses shown in Table 1 at any section along the girder. It is assumed that the considered stresses are exposed to a corrosive environment; therefore, the allowable tension stress is $3\sqrt{f'_c}$. The specifications allow a maximum of 75 percent of ultimate prestressing steel stress, f'_s, to be applied initially at transfer for low relaxation strands. The resistance reduction factor for prestressed concrete sections in flexure is $\phi = 1.0$.

The allowable tension stress is specified to control the occurrence of cracking. Tension may occur at the top of the beam immediately after the wires are cut. It may also be present at the bottom, as the result of LL. Then, the maximum tension can be expected after the final loss of prestressing force (at the end of economical life). A cracked girder requires a different analytical model than an uncracked section. Cracking may cause an accelerated corrosion of reinforcement or prestressing steel. However, the problem can be controlled by ordinary reinforcement. The physical limit is the tensile strength of concrete (moment corresponding to the modulus of rupture in concrete), or, after the initial

TABLE 1 Allowable Stresses Specified by AASHTO (*1*)

Types of Stress		Stress (psi)
Initial stress in concrete at transfer	Tension	$6\sqrt{f'_{ci}}$
	Compression	$0.6f'_{ci}$
Final stress in concrete	Tension *	$3\sqrt{f'_c}$
	Compression	$0.4f'_c$

* severe corrosive environment

(1 psi = 6.894 kPa)

crack occurred, the decompression moment. Therefore, the limit state function can be formulated as

$$M_r - M_{DL} - M_{LL} - M_{IL} = 0 \tag{8}$$

where

M_r = moment corresponding to the tensile strength limit in concrete and includes the prestressing effect and loss of prestressing force, if any;
M_{DL} = moment caused by dead load;
M_{LL} = moment caused by live load; and
M_{IL} = moment caused by dynamic load (impact).

The cracking moment M_{cr} can be determined as a function of the tensile strength of concrete (f_r). The mean modulus of rupture is about 700 psi (4.8 MPa) for concrete with $f_c' = 5{,}000$ psi (34.5 MPa); this compares with allowable tension stress of 530 psi (3.6 MPa) (*6*).

The allowable compression stress is specified to avoid excessive permanent deformation. As in the case of tension stress, two cases are considered. Immediately after transfer, maximum compression occurs at the bottom of the girder. After the final prestress loss, the maximum compression stress must be checked at the top. The physical limit for permanent deformation is elastic limit, which is assumed to correspond to about 0.6 f_c'. Therefore, the limit state function for compression can be formulated as follows:

$$M_{el} - M_{DL} - M_{LL} - M_{IL} = 0 \tag{9}$$

where M_{el} = elastic moment (moment corresponding to elastic stress limit in concrete) and includes the prestressing effect and loss of prestressing force, if any.

The ultimate moment for a prestressed concrete girder, M_n, is calculated from the following formula:

$$M_n = A_{ps} f_{ps} d_p (1 - 0.6\rho) \tag{10}$$

where

A_{ps} = area of prestressing steel;
f_{ps} = yield strength;
d_p = effective depth;
$\rho = A_{ps}f_{ps}/(bd_pf_c')$ (reinforcement ratio);
b = width of the section (effective slab width); and
f_c' = strength of concrete.

The actual moment carrying capacity is a random variable. The mean value is about 5 percent larger than the design value calculated using Equation 10, and the coefficient of variation is 0.075 (*2*).

The ultimate moment (M_n) is compared with the factored load effect (M_u) calculated as follows:

$$M_u = 1.3\, M_{DL} + 2.17\ (0.5)\ (GDF)\ (M_{LL} + M_{IL}) \tag{11}$$

where

M_{DL} = moment caused by dead load;
M_{LL} = moment caused by live load (per lane);
GDF = girder distribution factor; and
M_{IL} = moment caused by dynamic load (impact).

In addition to the ultimate moment, AASHTO (*1*) requires that the cracking moment (M_{cr}) be checked and defined as

$$M_{cr} = (Z_{bc}/Z_b)[F_{pe}A_{ps}(e_o - K_t)] - f_rZ_{bc} \tag{12}$$

where f_r = modulus of rupture.

Analysis of AASHTO Girders

The calculations are performed for prestressed concrete AASHTO-type Girders III through VI. The cross sections are shown in Figure 1. Simple spans are considered from 40 through 120 ft (12 through 36 m). It is assumed that all the considered bridges carry at least two traffic lanes, that girders are composite with concrete deck slab, and that strands are draped at the third points.

Further it is assumed that dead load, in addition to the girder weight, includes two normal-size parapets, a 1-in. (25-mm) haunch, diaphragms 1 ft (0.3 m) wide, a wearing surface of 30 psf (1.44 kN/m^2) and a stay-in-place form work of 15 psf (0.72 kN/m^2). The thickness of the cast-in-place concrete deck varies with the girder spacing. It is assumed that the nominal final concrete strength in the pretensioned girder is 6,500 psi (45 MPa) and in the deck it is 4,500 psi (31 MPa). Concrete strength at transfer is considered to be 5,500 psi (38 MPa). The prestressing steel is composed of 0.5-in. (12-mm) low relaxation strands with an ultimate strength of 270 ksi (1860 MPa).

The number of prestressing strands is the single most important parameter that determines the resistance for ULS and SLS. For each limit state, i, the number of required strands (n_i) is determined. The calculations are carried out for the following limit states considered in the design: allowable initial tension stress for concrete, allowable initial compression stress for concrete, allowable final tension stress for concrete, allowable final compression stress for concrete, and ultimate moment. The number of strands required for the ultimate load-carrying capacity is denoted by n_u. For tension stress, the number of strands required is denoted by n_{ti} for the initial stage (after transfer), and n_{ts} for the final stage (after final prestress loss). Similarly, for compression stress, the number of strands required is denoted by n_{ci} for the initial stage (after transfer), and n_{cs} for the final stage (after final prestress loss).

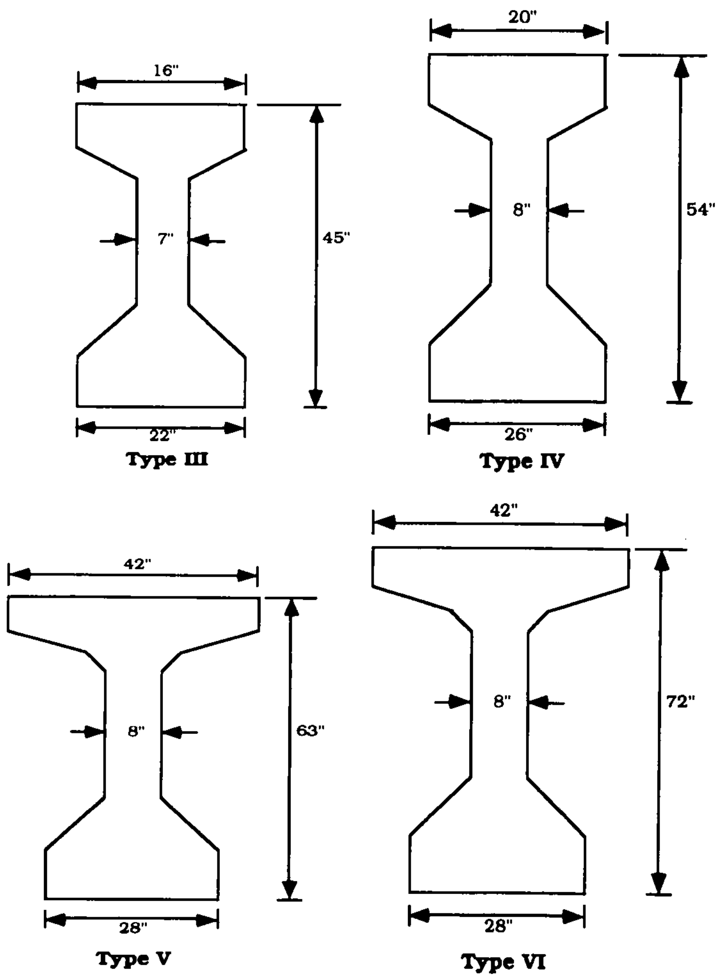

FIGURE 1 AASHTO girders Types III through VI. All dimensions are in inches (1 in. = 25 mm).

Only one limit state is considered at a time. If, for example, the ultimate moment is considered, then the number of strands is determined only with regard to the required moment carrying capacity. The results are shown in Figure 2. The presented numbers of strands are calculated for girder spacing of 8 ft (2.4 m) and slab thickness of 8 in. (200 mm). The size of the AASHTO-type girder is selected depending on the span length: for spans 40 to 60 ft (12 to 18 m) AASHTO Type III, for spans 60 to 80 ft (18 to 24 m) AASHTO Type IV, for spans 80 to 100 ft (24 to 30 m) AASHTO Type V, and for spans 100 to 120 ft (30 to 36 m) AASHTO Type VI.

The sign of expected stress at the initial stage and final stage are opposite. Immediately after transfer, prestressing force is the major factor that increases the critical tension and compression stresses. Therefore, an upper bound is imposed on the required number of strands. For spans up to about 70 ft (21 m), tension in the top fibers of the girder govern, and for longer spans, compression at the bottom governs. In the final stage, after the final loss of prestress, prestressing force decreases the critical stresses. Therefore, a lower bound is determined for the required number of strands. The feasible domain is shown as the shaded area in Figure 2. There is a considerable variation of the required numbers of strands. However, it is clear that the design is governed by the allowable tension stress (in the final stage).

The effect of the allowable tension stress on the required number of strands is shown in Figure 3. Various limits are considered from 0 (no tension allowed) to 10 times the square root of f'_c.

For the considered AASHTO girders, moments are calculated for various limit states, in particular:

- Moment corresponding to the allowable tensile stress in noncomposite girder,
- Moment corresponding to the allowable compression stress in noncomposite girder,

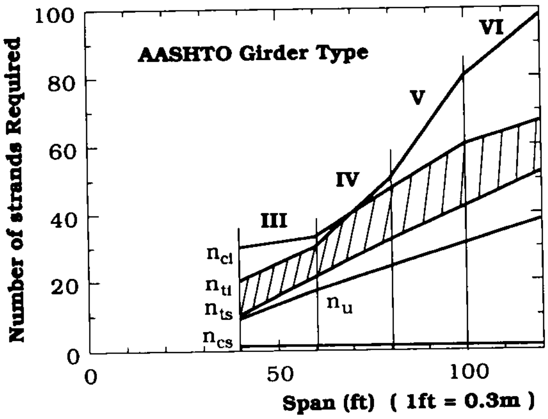

FIGURE 2 Required number of strands: n_u = ultimate moment, n_{ti} = tension at the initial stage (after transfer), n_{ci} = compression at the initial stage, n_{ts} = tension at the final stage (after final prestress loss), and n_{cs} = compression at the final stage.

- Moment corresponding to the allowable tensile stress in composite girder,
- Moment corresponding to the allowable tensile stress in composite girder, and
- Moment corresponding to the ultimate load-carrying capacity in composite girder.

Each moment is determined with regard to only one limit state (other limit states are disregarded). The ratios of these moments and the ultimate moment are plotted in Figure 4 for AASHTO girder Types III through VI.

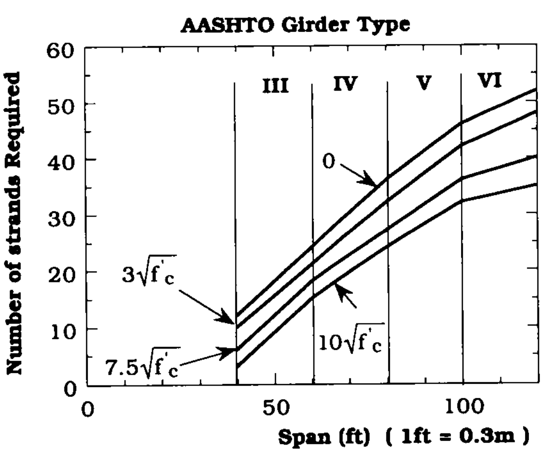

FIGURE 3 Required number of strands for various values of the allowable tension stress at the final stage.

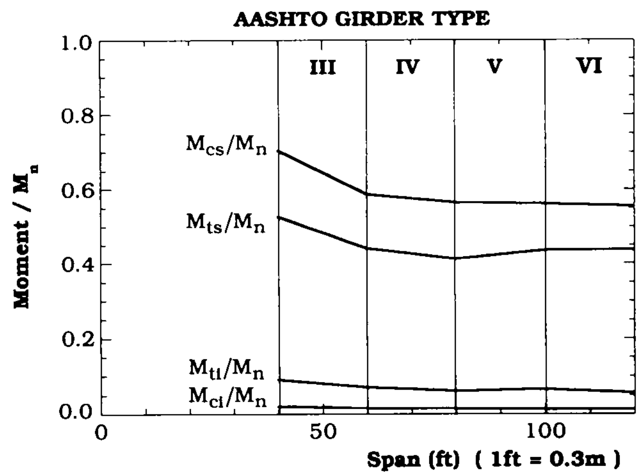

FIGURE 4 Moment ratios calculated for allowable tension and compression stresses.

The nominal moment, M_n, is calculated for the composite section using Equation 10. The ratios of moments M_{cs}/M_n, M_{ts}/M_n, M_{ti}/M_n, and M_{ci}/M_n vary with span length. Moments M_{ti} and M_{ci} are applied to a noncomposite section and, therefore, they appear to be small compared with M_n.

Conclusions

Ultimate limit states and serviceability limit states are considered for prestressed concrete girders. The minimum required number of strands is calculated for various limit states, including allowable initial tension stress, initial compression stress, final tension stress, final compression stress, and ultimate moment. The results confirm that the final tension stress governs the design.

Serviceability limit states based on allowable stress in concrete require further consideration. Design resistance and loads are not realistic and, therefore, the calculated stress have no physical meaning. The actual concern is an excessive permanent deformation of the girder. Therefore, it is suggested that the elastic limit stress in compression be checked. Furthermore, because the live loads are unrealistic, the use of factored loads specified in the new LRFD AASHTO (*1*) is suggested. Tension stress can be controlled by additional reinforcement. The girder distribution factors are overly conservative in most cases.

It is suggested that the design be based on the following limit states:

- Tension stress in concrete (initial and final),
- Elastic limit for compression stress in concrete, and
- Ultimate moment.

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