

# Structural Safety of Prestressed Concrete and Composite Steel Highway Bridges

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Traditionally, the safety evaluation of existing bridges and code calibration of newly developed structural design specifications for the ultimate limit states are usually based on the maximum factored design loads. The advantage of this approach is that it does not require detailed design computations. Past experience with prestressed concrete girder bridges indicates that the design of such bridges is governed by the allowable stresses requirement at release or under service load effects. Similarly, the design of composite steel beam bridges is generally controlled by overloading for compact sections and by the maximum stress criterion for noncompact sections. The reliability of bridges designed according to AASHTO's Load Factor Design code is evaluated on the basis of actual designs. Reliability is measured in terms of the reliability index for the ultimate flexural capacity limit state. The statistical data on strength are generated starting from statistics on material properties and using simulation methods. Statistical data on load components are compiled from the available literature. The scope of the study covers a wide range of precast sections and rolled beams, span lengths, and beam spacings.

**H**ighway bridges traditionally have been designed on the basis of deterministic values of load and resistance. The use of minimum material properties, specified load intensities, and prescribed computational procedures serves the important role of ensuring uniformity in the nominal performance

of bridges. The deterministic approach has been reinforced by the large extent to which structural engineering design is codified and by the lack of feedback about actual performance of structures. The lack of information about actual behavior of bridges combined with the use of codes embodying relatively high safety factors can lead to the view that absolute safety can be achieved. Absolute safety is unattainable; in addition, the use of very high levels of safety can be undesirable because it may require the deployment of almost infinite resources.

Conventional methods of computing the safety of existing bridges and calibration of newly developed structural design codes for an ultimate limit state usually assume that the maximum factored design load effect governs the design. The advantage of this approach is that it does not require detailed design computations, such as the selection of the precast or rolled section and determination of the thickness of the concrete deck. The nominal capacity in this case is assumed to be equal to the applied factored load and divided by the capacity reduction factor. This approach neglects the effect of other design requirements, such as serviceability and overloading, on the final capacity of the structure.

Past experience with prestressed concrete I-girder and box beam bridges indicates that the design of such bridges is governed by the allowable stresses requirement at release of the prestress and under service load effects. The magnitude of the allowable stresses under

service load effects is a function of the exposure condition (corrosive versus noncorrosive environments). Similarly, the design of composite steel beam bridges is generally controlled by overloading for compact sections and by the maximum stress criterion for noncompact sections.

In 1994, AASHTO published its Load Resistance Factor Design (LRFD) specifications (1) for the design of highway bridges. The first edition of the specifications was intended to provide a framework for future versions. The limit states that could be calibrated for prestressed concrete, structural steel, and composite structures were mainly related to strength. Other limit states, such as serviceability and overloading, were made equivalent to the standard Load Factor Design (LFD) specifications until enough data are available to be able to calibrate them reliably.

In this paper, the reliability of prestressed concrete girders and composite steel beams is evaluated on the basis of actual designs. The bridges are designed in accordance with the current AASHTO's LFD specifications (2). Simply supported prestressed concrete I-girders, as well as compact and noncompact composite steel beams, are considered in this study. The limit state function considered is the flexural capacity at ultimate. The structural reliability is measured in terms of the reliability index. The statistical data on strength are generated starting from statistics on material properties and using simulation methods. Statistical data on load components are compiled from the available literature. The scope of the study covers a wide range of precast

sections and rolled beams, span lengths, and beam spacings.

### STATEMENT OF THE PROBLEM

Traditionally, the safety of prestressed concrete and composite steel bridges has been generally based on considering the mean value of the resistance ( $\mu_R$ ) to be to the following (3):

$$\mu_R = \lambda[\alpha_D(D) + \alpha_{L+I}(L + I)] \left( \frac{1}{\phi} \right) \quad (1)$$

where

- $\alpha_D$  and  $\alpha_{L+I}$  = dead load and live load (including impact) factors, respectively;
- $D$  and  $L + I$  = nominal dead load and live load plus impact effects, respectively;
- $\lambda$  = mean-to-nominal ratio; and
- $\phi$  = resistance reduction factor.

The use of Equation 1 to compute the safety of prestressed concrete girders at ultimate can be erroneous because this equation assumes that the factored moment capacity governs the design of the section over the allowable stress requirement. For illustration, Figure 1 shows the required number of strands versus the simple span length for a 28/63 I-girder (the first and second numbers represent respectively the bottom flange width and the total beam depth, in inches). The girders are

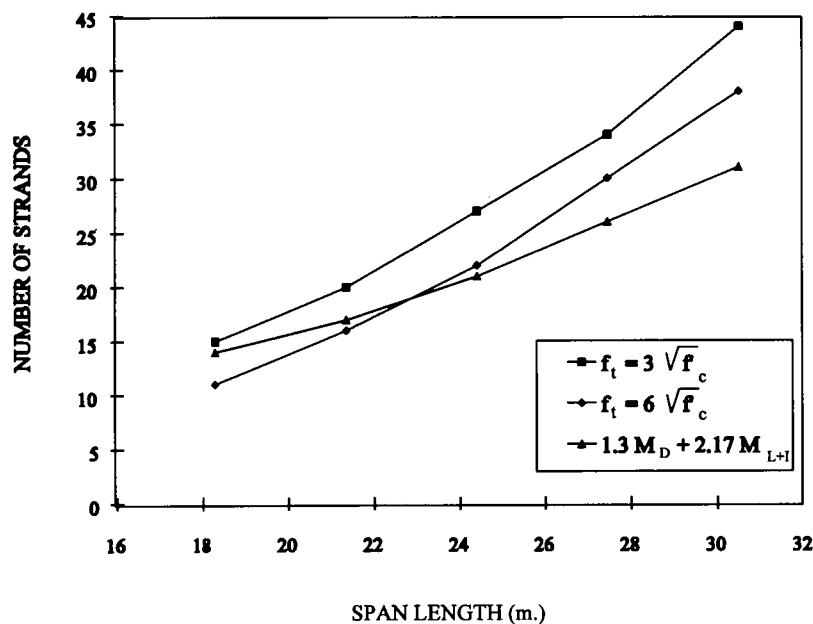


FIGURE 1 Number of strands versus simple span length for 28/63 P/S concrete I-girder.

designed according to the AASHTO code and have a spacing of 2.44 m (8 ft.). Two separate limit states are considered in the design: (a) serviceability (with a maximum allowable final tensile concrete stress,  $f_t$ , limited to  $3\sqrt{f'_c}$  or  $6\sqrt{f'_c}$ , where  $f'_c$  is the nominal strength of concrete at 28 days), and (b) ultimate strength. In the useful range of application, the curves representing the allowable stress condition are consistently above the one representing ultimate strength requirement, thus indicating that the serviceability limit state governs the design of the girder.

Similarly, code calibration based on Equation 1 alone can be inadequate when applied to steel bridges in flexure because it assumes that the steel beam is compact and that the capacity is based on the plastic stress distribution. In general, some wide-flange rolled steel beams do not satisfy AASHTO's ductility requirement for compactness and, therefore, the capacity should be based on the moment at first yield. Further, the maximum stress and overloading conditions usually control the design of noncompact sections for the practical range of application. For example, Figure 2 shows the maximum simple span length versus the girder spacing for a W36  $\times$  210 composite beam designed in accordance with the AASHTO code. Analysis of the section in plastic bending for beam spacings in the range of 1.83 to 3.66 m (6 to 12 ft) showed that it does not satisfy AASHTO's ductility requirement (Equation 10-128a of the specifications). Therefore, the section is classified as noncompact according to the specifications. The design of the composite beams in Figure 2 is based

on three different and separate conditions. These conditions include the (a) plastic moment capacity, (b) yield moment capacity, and (c) overloading. The analysis shows that the design of the composite beams for the considered beam spacings is governed by the yield moment requirement (i.e., maximum stress in the bottom flange limited to the specified yield stress). The maximum simple span lengths based on the maximum stress condition for the considered beam spacings are about 25 percent less than the corresponding designs that are based on the plastic moment capacity. For this reason, the use of Equation 1 can underestimate the actual safety of steel bridges designed by AASHTO.

### AASHTO'S GENERAL DESIGN REQUIREMENTS

AASHTO's specifications are used for the design of typical simple span I-girder and spread box beam bridges. The AASHTO code requires interior girders to have an ultimate capacity in flexure ( $\phi M_u$ ) at least equal to the factored load effect ( $M_u$ ):

$$\phi M_u \geq M_u \quad (2)$$

in which  $M_u$  is computed according to AASHTO's Group I load combination:

$$M_u = 1.3[M_{DL1} + M_{DL2} + (5/3) M_{L+I}] \quad (3)$$

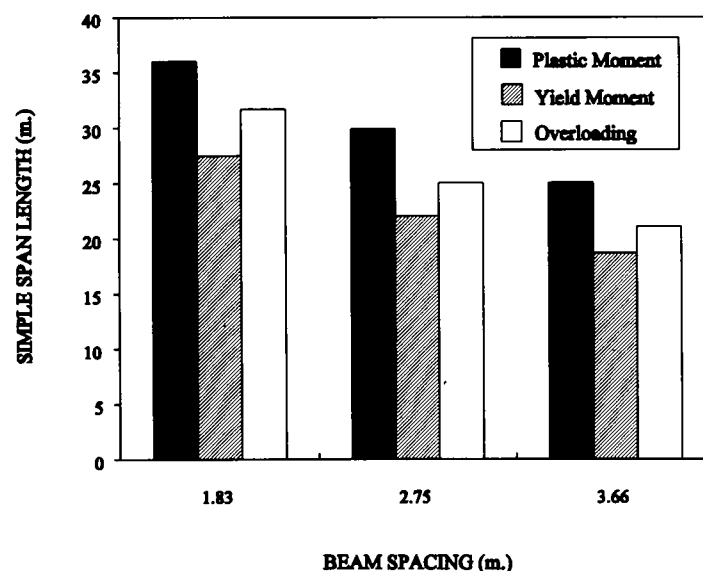


FIGURE 2 Span capability of W36  $\times$  210 composite steel beams.

TABLE 1 Allowable Initial and Final Concrete Stresses by AASHTO

TYPES OF PRESTRESS		ALLOWABLE STRESS
INITIAL STRESS AT TRANSFER OF P/S	TENSION	$3\sqrt{f'_{ci}}$
	COMPRESSION	$0.6 f'_{ci}$
FINAL STRESS UNDER DESIGN LOADS	TENSION	$3\sqrt{f'_c}$ OR $6\sqrt{f'_c}$
	COMPRESSION	$0.4 f'_c$

where

$M_{DL1}$  = dead load moment on the noncomposite beam

$M_{DL2}$  = superimposed dead load moment on the composite beam, and

$M_{L+I}$  = live load plus impact moment on the composite section.

$M_{DL1}$  is estimated by selecting deck dimensions, girder section, and stay-in-place forms;  $M_{DL2}$  is caused by the effect of the weight of concrete barriers, railing, and future wearing surface; and  $M_{L+I}$  is based on AASHTO's girder distribution factors, the HS20-44 (truck or lane) or alternate military loading, and AASHTO's impact coefficient. Additional specific requirements for prestressed concrete and composite steel bridges are listed below.

### Prestressed Concrete Girders

The ultimate flexural capacity of under-reinforced pre-tensioned concrete girders having a rectangular section behavior is based on the following expression:

$$\phi M_n = \phi A_s^* f_{su}^* d \left( 1 - \frac{0.6 \rho^* f_{su}^*}{f'_c} \right) \quad (4)$$

where

$\phi$  = capacity reduction factor of 1.0;

$A^*$  = area of prestressing steel strands;

$f_{su}^*$  = stress in the prestressing steel strands at ultimate;

$d$  = depth of prestressing steel strands;

$\rho^*$  = reinforcement ratio (equal to  $A^*/bd$ ); and

$b$  = effective width of section.

Prestressed concrete bridge girders are also required to satisfy the initial and final concrete stresses shown in Table 1 at any section along the girder, where  $f'_c$  and all stresses are in pounds per square inch. AASHTO allows a maximum stress of 70 percent of the ultimate prestressing steel stress ( $f'_s$ ) to be applied initially at transfer for stress relieved strands. The corresponding stress at transfer for low relaxation strands is 75 percent of  $f'_s$ . Slight overstressing up to 85 percent of  $f'_s$  for a short period is permitted to offset seating losses.

### Composite Steel Beams

The ultimate strength of compact composite steel beams designed by AASHTO is based on the fully plastic stress distribution shown in Figure 3. Composite beams in positive bending qualify as compact when their steel section meets two requirements. First, the depth of the

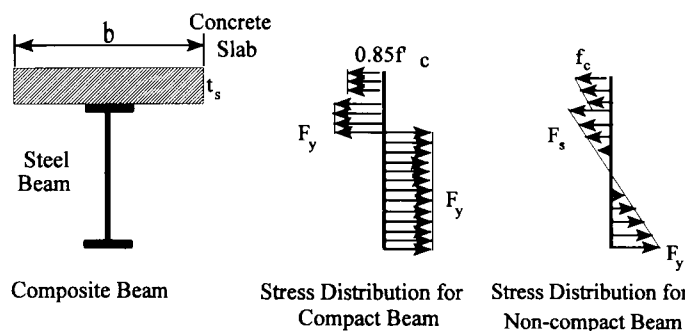


FIGURE 3 Plastic stress distribution for compact composite steel beams.

web in compression at the plastic moment ( $D_{cp}$ ) should satisfy the following inequality:

$$\frac{2D_{cp}}{t_w} \leq \frac{19230}{\sqrt{F_y}} \quad (5)$$

where  $t_w$  is the web thickness and  $F_y$  is the specified minimum yield strength in pounds per square inch. The second requirement limits the depth from the top of the concrete slab to the neutral axis in plastic bending ( $D_p$ ) to the following value:

$$D_p \leq \frac{(d + t_s + t_h)}{7.5} \quad (6)$$

where

- $d$  = depth of steel section,
- $t_s$  = thickness of concrete slab, and
- $t_h$  = average thickness of the concrete haunch above the top flange (in.).

Also, the AASHTO code requires the ratio of the projecting top compression flange width,  $b'$ , to its thickness ( $t$ ) not to exceed the value determined by

$$\frac{b'}{t} = \frac{2200}{\sqrt{1.3(f_{DL1})_{tf}}} \quad (7)$$

where  $(f_{DL1})_{tf}$  is the top flange compressive stress caused by noncomposite dead load (in pounds per square inch). This expression should be satisfied by both compact and noncompact composite beams.

When the steel section does not satisfy the compactness requirements of Equations 5 and 6, AASHTO requires that the maximum strength of the section be taken as the moment capacity at first yield,  $M_y$ . In this case, the maximum factored moment caused by the applied loading,  $M_u$ , as given by Equation 3, will be smaller than or equal to  $\phi M_y$ , where  $\phi$  is equal to 1.0. AASHTO also requires the sum of stresses produced by the applied loading on the noncompact beam to be below the yield stress at any point. For unshored construction, the total stress must satisfy the following expression:

$$1.3[f_{DL1} + f_{DL2} + (5/3)f_{L+I}] \leq F_y \quad (8)$$

where  $f_{DL1}$ ,  $f_{DL2}$ , and  $f_{L+I}$  were defined earlier. For shored construction, the stress  $f_{DL2}$  in Equation 8 is 0.

Finally, the design of composite steel beams should satisfy AASHTO's overloading requirement regarding the stress in the steel section:

$$f_{DL1} + f_{DL2} + (5/3)f_{L+I} \leq 0.95F_y \quad (9)$$

The above requirement applies to all composite sections, whether compact or noncompact.

## RELIABILITY MODELS

Reliability-based safety evaluation of structures starts with the formulation of the limit state functions. These limit states are conditions under which a structure can no longer serve its intended purpose during its life span. In general, the reliability of a structural member or system for the ultimate flexural capacity limit state can be expressed by the use of a failure function,  $G$ , as

$$G(R, Q) = R - Q \quad (10)$$

where  $R$  is the resistance and  $Q$  is the total load effect. Failure occurs if  $G$  is less than or equal to 0. Load components and resistance are random by nature because of the inherent variability in material and load, lack of statistical data, mathematical idealization, approximate design procedures, and human error. Therefore,  $G$  is a random variable because it is a combination of random variables, as indicated by Equation 10. Structural safety can be measured in terms of a reliability index ( $\beta$ ), as in the following:

$$\beta = \frac{\mu_G}{\sigma_G} \quad (11)$$

in which  $\mu_G$  and  $\sigma_G$  denote the mean and standard deviation of  $G$ , respectively. The relationship between the probability of failure ( $P_f$ ) and reliability index is expressed as the following:

$$P_f = 1 - \Phi(\beta) \quad (12)$$

where  $\Phi$  is the cumulative standard normal distribution function of  $G$  ( $\mu = 0$  and  $\sigma = 1$ ). Figure 4 shows a typical probability distribution of  $G$  and a graphical definition of the reliability index and probability of failure.

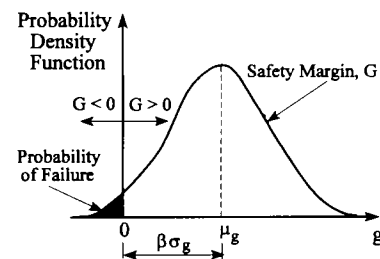


FIGURE 4 Random representation of the safety margin.

In this study the reliability index is computed using the Rackwitz-Fiessler method (5) because  $R$  and  $Q$  do not have the same probability distributions. This iterative procedure is based on approximating the true probability density functions of the random variables by normal distributions at the point of maximum probability (design point).

### BRIDGE LOAD MODELS

The statistical parameters of load and resistance are needed for evaluating the reliability index. The maximum load effects on a highway bridge are mainly caused by dead load, live load, dynamic load, environmental loads, and accidental loads (braking forces, vehicle collision, etc.). Environmental loads do not govern for short and medium span bridge superstructures and, hence, they are not treated in this study. Therefore, the total load effect ( $Q$ ) for a bridge girder can be represented by the following:

$$Q = [D + L + I] \quad (13)$$

where  $D$  is dead load,  $L$  is live load, and  $I$  is the dynamic load effect. The statistical parameters of the total load effect ( $Q$ ) are determined using Turkstra's rule (6), and the probability distribution is approximated by a normal distribution.

It is convenient to consider four components of dead load, according to quality control measures. These components are weight of precast members ( $D_1$ ), cast-in-place concrete elements ( $D_2$ ), asphalt ( $D_3$ ), and miscellaneous items ( $D_4$ ). The bias (mean-to-nominal) ratios and coefficients of variation (COV) of dead load components are shown in Table 2 (7).

Nowak's live load model (8) is used in this study. The live load model is based on truck surveys in North America. It was shown that the governing combination for multiple-lane short and medium span bridges is caused by two trucks traveling side-by-side on the bridge. Actual or "more accurate" girder distribution factors (GDFs) are needed for evaluating the live load

means per girder. The following expression for GDF for I-girders is considered (9):

$$\text{GDF} = 0.15 + \left(\frac{S}{3}\right)^{0.6} \left(\frac{S}{l}\right)^{0.2} \left(\frac{k_g}{lt_s^3}\right)^{0.1} \quad (14)$$

where

$S$  = girder spacing (ft),

$l$  = span length (ft),

$t_s$  = concrete slab thickness (ft), and

$k_g$  = ( $\text{ft}^3$ ) is evaluated from the following:

$$k_g = n(I + Ae_g^2) \quad (15)$$

where

$n$  = modular ratio

$I$  and  $A$  = moment of inertia and the area of the beam or girder, respectively, and

$e_g$  = eccentricity of the beam with respect to the slab. The statistics of the live load model include professional factors to account for uncertainties in the value of the GDF.

The dynamic load on bridges is generally caused by the dynamic properties of the structure, the suspension system of the vehicle, and surface roughness and bumps. The mean value of impact is considered to be 15 percent of live load with a high coefficient of variation of 0.80 (8).

### BRIDGE RESISTANCE MODELS

The component strength for most highway bridges depends on the bridge type, girder layout and geometry, material properties, and section dimensions. The random nature of the strength is mainly a result of the variability of material strength, accuracy of strength prediction theories, and fabrication. The statistical properties of the material strengths that are used in the study are compiled from the available literature (10) and presented in Table 3.

TABLE 2 Statistical Parameters of Dead Load

Class	Description	Bias	C.O.V
$D_1$	Factory-made Members	1.03	0.04
$D_2$	Cast-in-place Members	1.05	0.08
$D_3$	Wearing Surface	1.00	0.15
$D_4$	Miscellaneous Items	1.03	0.04

TABLE 3 Statistics of Materials Used in the Study

Material Variable	Mean-to-Nominal Ratio	Coefficient of Variation
Yield Stress for Flanges	1.05	0.10
Yield Stress for Webs	1.10	0.11
Modulus of Elasticity of Steel	1.00	0.06
Poisson's Ratio of Steel	1.00	0.03
Tensile Strength of Steel	1.00	0.11
Concrete Compressive Strength	0.99	0.18
Concrete Tensile Strength	1.04	0.10
Strength of Prestressing Strands	1.04	0.04

The flexural strength of concrete girders and composite steel beams is studied in terms of the moment-curvature relationship. The Monte Carlo simulation method is used to evaluate the statistical parameters of the ultimate moment capacity. Several pretensioned concrete girders with different reinforcement ratios are analyzed. A typical probabilistic moment-curvature relationship for an AASHTO Type III I-section at the mean and one standard deviation above and below the mean is shown in Figure 5. The analysis showed that the mean-to-nominal ratio and coefficient of variation of the moment capacity at ultimate are governed by the statistics of the prestressing strands and are equal to 1.04 and 0.08, respectively.

Similarly, several composite steel beams of various sizes are analyzed in flexure with consideration of material statistics. Typical probabilistic moment-curvature relationship for a composite beam having a W36  $\times$  210 rolled section is shown in Figure 6. The analysis showed that the ultimate moment carrying capacity has a mean-to-nominal ratio of 1.10 and a coefficient of variation equal to 0.12. These statistical properties include professional factors to account for uncertainties in the analysis, that is, the difference between theory and experiment.

The results of the analysis indicate that the probability distribution of the ultimate moment capacity of concrete and steel girders can be approximated by a lognormal model.

## FINDINGS

The AASHTO specifications are used for the design of simply supported prestressed concrete I-girders and composite steel beams. The live load is composed of the HS20-44 or alternate military loading, whichever governs. The girder spacing ranges between 1.82 and 3.65

m (6 and 12 ft). All bridges have two normal size parapets, an average haunch above the top flange 25.4 mm thick (1 in.), future wearing surface of 1.44 kN/m<sup>2</sup> (30 psf), and stay-in-place formwork of 0.72 kN/m<sup>2</sup> (15 psf). The specified 28-day concrete compressive strength in the cast-in-place deck is 27.6 MPa (4,000 psi). The thickness of the deck varies with the girder spacing.

## Prestressed Concrete Girders

Nominal final concrete strength of 44.8 MPa (6,500 psi) is specified for the pretensioned girder. Concrete strength in the girder at transfer is considered to be 37.9 MPa (5,500 psi). The prestressing steel is composed of 12.7-mm (0.5-in.) low relaxation strands with 1862 MPa (270 ksi) ultimate strength. The steel strands are draped at the third points.

Figure 7 shows the ratio of the number of strands to satisfy serviceability to the number of strands required for strength, denoted by  $h$ , versus the span length. The plots are generated for allowable concrete stresses on the basis of corrosive and noncorrosive environments. The shaded areas represent designs that have different girder spacings. This deterministic analysis shows that allowable stresses govern the design for the spans and girder spacings considered. Figure 7 indicates that the number of strands to satisfy serviceability is about 1.15 to 1.45 times larger than the number of strands needed for strength.

Figure 8 shows the range of reliability indexes for the flexural capacity limit state based on designs to satisfy strength or allowable stresses. The range of reliability indexes for AASHTO designs is 5 to 6 for noncorrosive environments and 6 to 8 for corrosive environments. The range of reliability indexes based on the limit state ( $1.3 M_D + 2.17 M_{L+I}$ ) lies between 3.2 and 4.3.

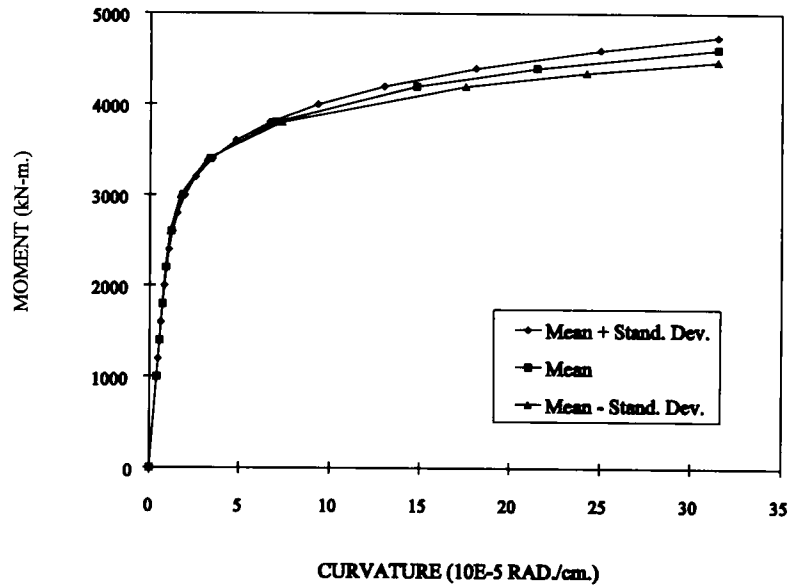


FIGURE 5 Probabilistic moment-curvature relationship for AASHTO type III P/S concrete I-girder.

### Composite Steel Beams

The design of the composite steel beams covers 10 wide-flange rolled sections. AASHTO M270 grade 36 (248 MPa) structural steel is specified for the wide-flange sections.

Figure 9 shows the maximum simple span length of composite beams with rolled sections for a profile of

girder spacings. Most of the considered steel beams are not compact because they do not satisfy AASHTO's ductility requirement (Equation 6). The study showed that the design of the noncompact beams was governed by the maximum stress requirement (Equation 8), whereas the overloading condition controlled the design of the compact beams (Equation 9). The reliability indexes for the composite steel beams in Figure 9 are pre-

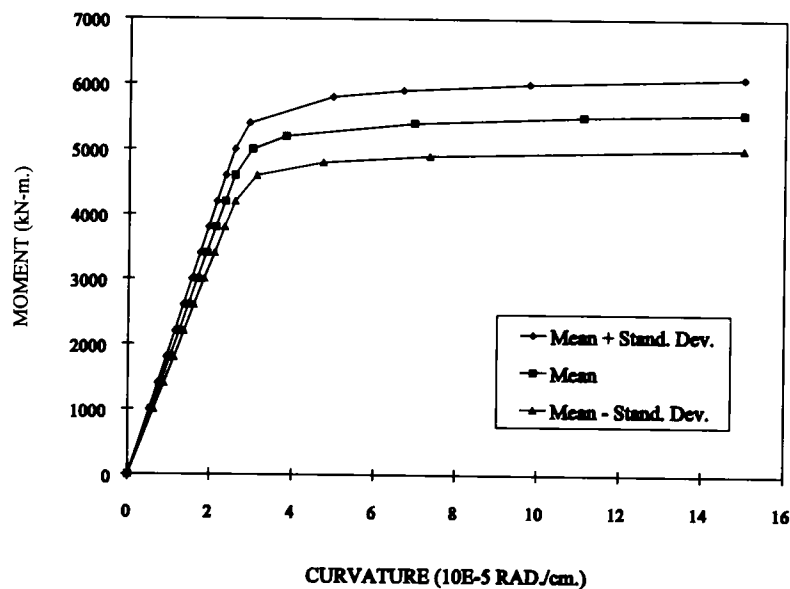


FIGURE 6 Probabilistic moment-curvature relationship for a W36  $\times$  210 composite steel beam.



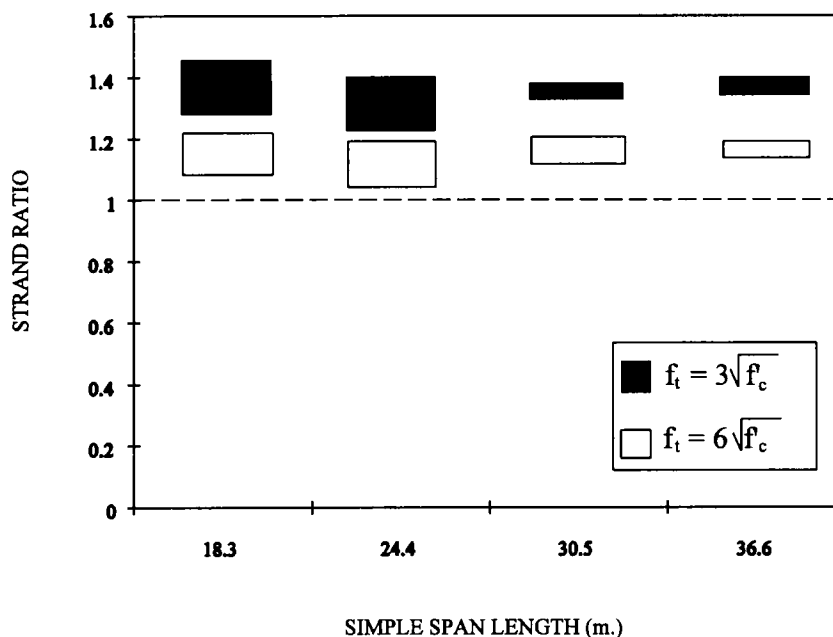


FIGURE 7 Strand ratio versus simple span length for P/S concrete I-girders.

sented in Figure 10. Also shown in Figure 10 is the range of the reliability indexes for bridges having the same beam spacings but assuming that the design is governed by  $(1.3 M_D + 2.17 M_{L+I})$ . The results of the reliability study indicate non-uniformity in the safety of steel bridges that are designed in accordance with the

current AASHTO code. In general, the reliability index varies between 5 and 7, depending on the beam spacing, span length, and section size. The corresponding reliability indexes for designs based on the factored moments are between 2.7 and 4. The main reasons for the non-uniformity in the reliability are because of

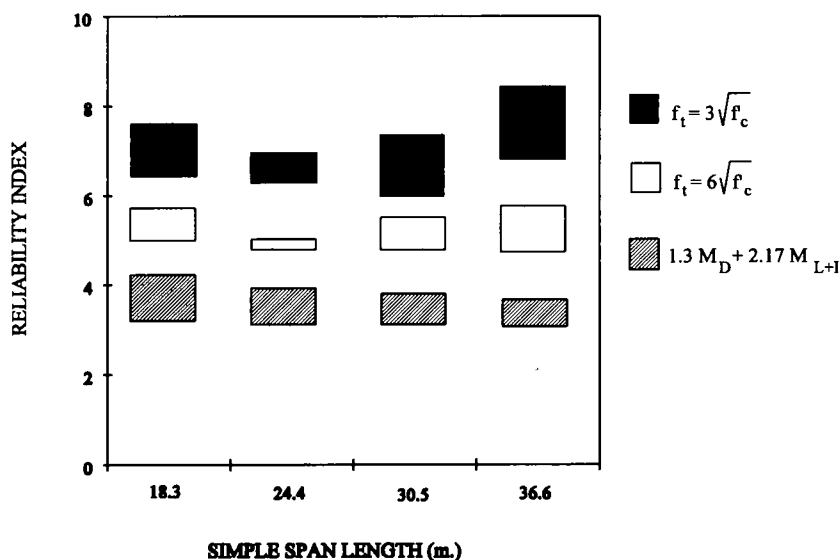


FIGURE 8 Reliability index versus simple span length for P/S concrete I-girders.

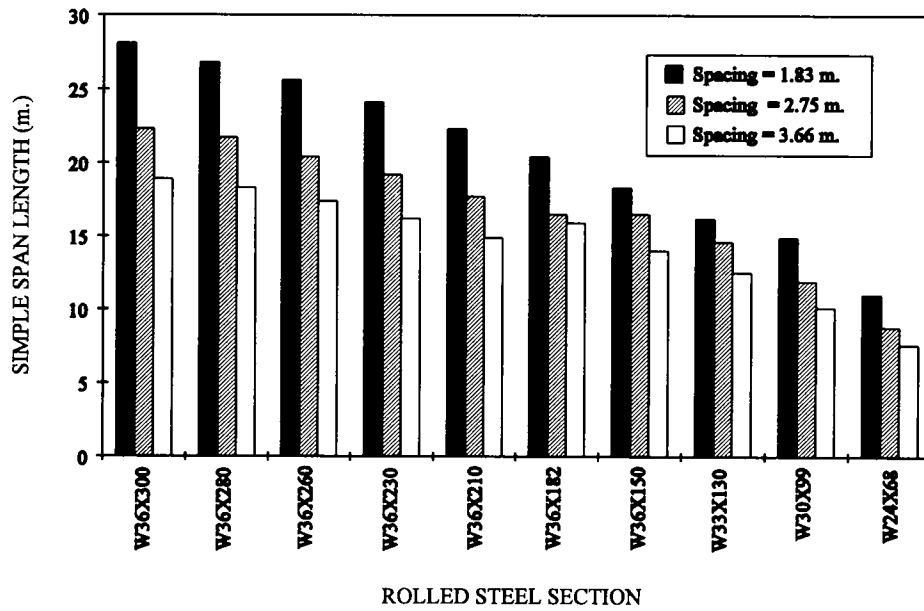


FIGURE 9 Maximum simple span length for composite steel beams.

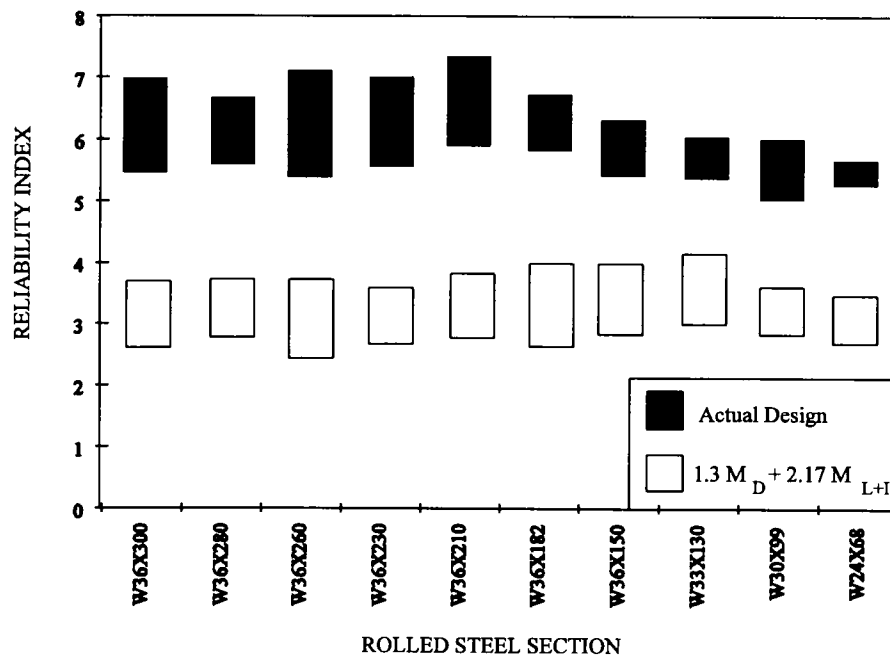


FIGURE 10 Reliability indexes for composite steel beams.

AASHTO's approximate live load girder distribution factor, which overestimates the live load for larger girder spacings, and the fact that the AASHTO is not a reliability-based code. The study also indicates that noncompact beams have higher reliability indexes than do compact beams because noncompact beams are more conservatively designed than compact ones using the current specifications.

## CONCLUSIONS

Simply supported pretensioned concrete girders and composite steel beams designed according to AASHTO specifications are investigated using reliability methods. Reliability indexes are computed on the basis of actual designs for a range of span lengths, girder spacings, and precast/rolled sections. The results of this study about bridge girders in flexure at ultimate lead to the following conclusions:

1. Current LFD-based specifications for concrete and steel bridges result in non-uniform safety for different spans and girder spacings.
2. Prestressed concrete girder design is usually governed by allowable service stress requirements. In most practical cases, the actual ultimate flexural capacity is 15 to 45 percent higher than the required strength because of the applied factored loads. Higher values are for designs that limit allowable concrete tensile stresses to  $3\sqrt{f'_c}$  instead of  $6\sqrt{f'_c}$ .
3. The design of noncompact composite steel beams that do not satisfy AASHTO's ductility requirement is governed by the maximum stress requirement. The design of compact beams is usually controlled by overloading.
4. The actual reliability prestressed concrete I-girders are in the range of 5 to 6 for noncorrosive environments and 6 to 8 for corrosive environments. The range of

reliability indexes based on the  $(1.3 M_D + 2.17 M_{L+I})$  limit state lies between 3.2 and 4.3.

5. The reliability index of composite steel beams varies between 5 and 7. Lower values of the reliability index are associated with compact beams and smaller girder spacings.

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