# Tests of Interactance Formulas Derived from O-D Data

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Several methods of forecasting future travel patterns in large urban areas are currently in wide use. All of these involve many complexities and require the use of modern high-speed data-processing equipment. The problems of trip projection involve two distinct areas of analysis: (1) estimates of the number of trips which the land uses in each subdivision or zone in the study area are likely to generate, and (2) the patterns of interzonal origins and destinations which are likely to result when trips are distributed between logical termini.

In practice, it is possible to make reasonable and satisfactorily accurate estimates of the number of trip ends that will be generated in each zone from consideration of various land use and population factors. The rates of trip production can be determined from analyses of the current origin-destination (O-D) data, or they can be estimated with a fair degree of accuracy from relationships found in other metropolitan areas with similar characteristics.

The projection of inter-zone travel patterns is a much more complex matter. Basically, all trip distribution methods may be placed in two classes, depending on the manner in which the current travel pattern is used in estimating future desires:

1. "Analogy" Methods—projections in which growth factors are applied to current interzonal movements.
2. "Synthetic" Methods—projections in which travel characteristics are derived from current O-D data, and applied to future land-use estimates to synthesize travel patterns.

●MANY THINGS must be considered when selection of a projection method is made. When current, accurate, home-interview O-D data are available, the relative ease of application would tend to recommend the "analogy" approach to traffic forecasting, especially if projections consist of simply up-dating a survey made a few years earlier. However, in most cases, highway planners need traffic forecasts of travel patterns as they will be at least 20 yr later. In this interim, it is expected that the limits of urbanization in most metropolitan areas will spread out to include areas which today are only partially developed, are open farm land or are totally undeveloped. Future traffic in these peripheral areas, which will be devoted to housing and industrial uses 20 yr hence, will be a wholly new development which cannot be anticipated by the application of growth factors to existing patterns.

For example, travel to and from a peripheral zone which today encompasses only sparse residential development may be oriented largely toward the central portion of the urban area; in any case, the number of movements reported in the current data will be few. If considerable housing is anticipated in the zone, and commercial and industrial development is expected in neighboring peripheral areas, it would not be logical to expect that future travel patterns would continue to be centrally oriented; new travel desires will develop between these outlying areas. Application of "growth factors" to current movements will only result in enormous expansion of the few reported interzonal interchanges, with a consequent failure to develop any traffic to the nearby shopping and employment opportunities.

Comparison of urban travel patterns at different years shows that travel desires are dynamic and respond readily to changes in social and economic conditions. For example, analogy with past conditions would fail to reveal the dramatic change in the peak-hour directional split that reportedly has occurred on the George Washington Bridge in recent years. Whereas in the past, peak-hour traffic was composed largely of middle- and upper-income New Jersey residents commuting to their places of employment in New York, recent industrial developments on the New Jersey side of the river have attracted employees from lower-income groups in the public and other low-rent housing developments in New York. Consequently, peak-hour traffic today on the bridge is almost equal in both directions. Other studies have revealed significant changes in travel habits due to the advent of television and widespread evening shopping (1).

Because of this inherent weakness in the "analogy" concept of traffic projection, considerable research has been done with the aim of establishing finite relationships which could be applied to a given set of land-use characteristics, so as to produce a synthetic travel pattern for any metropolitan area. Although a comparison of the validity of all projection methods would be desirable at this time, the scope of this paper is limited to a discussion of current travel patterns for three metropolitan areas which were developed by use of "interactance formulas" derived from existing O-D data. Inasmuch as the interactance formulas may be used in developing travel patterns for any series of zones and for any given year for which planning data are available, it has been possible to synthesize current travel patterns which are directly comparable with the home-interview data.

In these studies, the interactance formulas were applied to the actual numbers of trip ends reported in O-D surveys completed in three cities since 1957 (2, 3, 4). Synthetic travel patterns were developed for each area, and compared with the results of the expanded home-interview data. Various statistical tests have been applied to evaluate the reliability of the results. The procedures used in the study and the principal findings of the tests are summarized. The comparisons of the synthetic and reported travel patterns reveal that the interactance formulas produce results comparable in reliability to those obtained directly from the expanded home-interview data.

## ORIGIN-DESTINATION SURVEYS

Comprehensive O-D surveys have been completed during the past two years by this consultant in St. Louis, Mo., Kansas City, Kan.-Mo., and Charlotte, N.C. These cities present a wide range in population, area, density and travel habits, and have been selected to make the detailed analysis or test of the interactance formulas. Time and limited resources have precluded extensive tests of all projections heretofore. It is hoped that others will undertake further research in this area.

Characteristics of the survey areas are presented in Table 1. The populations range from slightly more than 202,000 in Charlotte, to 1,275,000 in St. Louis. Figure 1 shows the survey area, including the zones and analysis "rings" used in the St. Louis and Charlotte studies. Charlotte had 84 internal zones and Kansas City and St. Louis had 173 and 235 zones, respectively.

A comprehensive home-interview-type O-D study was conducted in each area. Home-interview data were obtained from 5 to 10 percent of the dwellings in each study area and expanded to represent the entire trip population. Consequently, individual zone-

TABLE 1

CHARACTERISTICS OF SURVEY AREAS

| Characteristics | St. Louis, Mo.[1] (1957) | Kansas City, Mo. (1957) | Charlotte, N.C. (1958) |
|---|---|---|---|
| Population[2] | 1,275,454 | 857,550 | 202,261 |
| Dwelling units[2] | 408,596 | 280,107 | 59,040 |
| Sample size (% dwelling units) | 5 | 5 | 10 |
| Survey area (sq mi) | 280 | 396 | 115 |
| Persons per sq mi | 4,550 | 2,160 | 1,760 |
| Trips reported[3] | 2,472,154 | 1,894,563 | 478,402 |
| Average number of trips per person | 1.94 | 2.22 | 2.36 |
| Cars owned[2] | 366,963 | 264,448 | 61,802 |
| Average number of cars per dwelling unit | 0.90 | 0.95 | 1.05 |
| Number of zones | 235 | 173 | 84 |

[1] The St. Louis study area consisted of the City of St. Louis and adjacent urbanized portions of St. Louis County, Missouri. The study did not include approximately 450,000 persons residing in Illinois portions of the Greater St. Louis area.
[2] Source: Home-interview data.
[3] Source: Table "B-1." Totals include all person trips, internal and external, reported in the dwelling unit interviews.

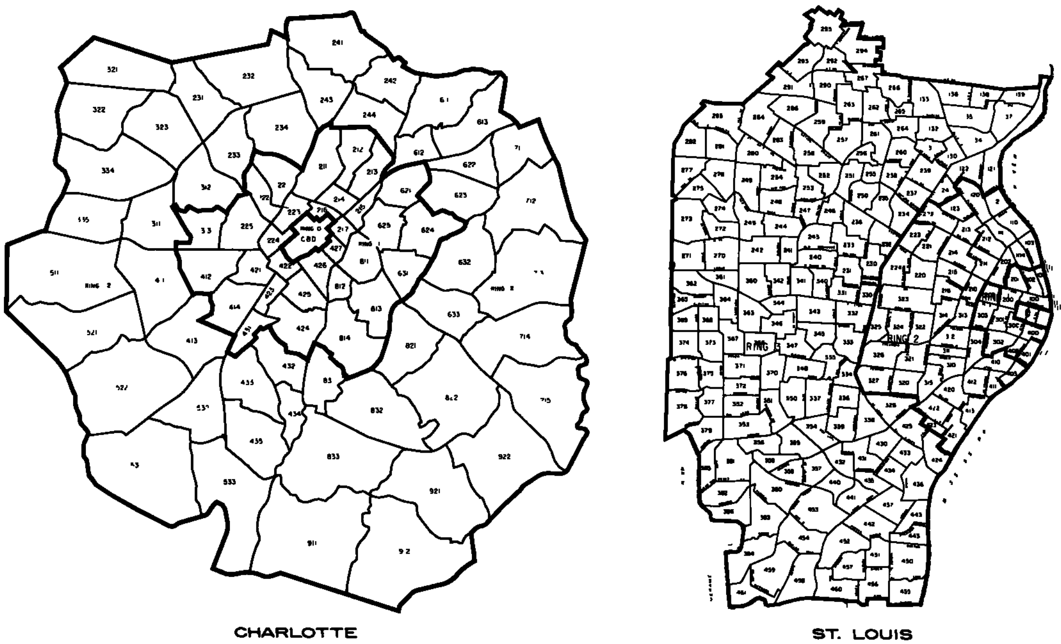



Figure 1. Traffic analysis rings— Charlotte, N.C. and St. Louis, Mo.

to-zone movements, most of which are small in magnitude, are subject to wide fluctuation due to sample size. However, the discrepancies caused by sample variability are minimized when a large directional movement is considered.

To determine the completeness of the data obtained in the home and external interviews, a "screenline check" was made. The results of the screenline checks for St. Louis, Kansas City and Charlotte are summarized in Table 2 with graphic comparisons given in Figure 2. The percent of reported trips, as compared to ground count, was 78.8 percent in Charlotte, which is slightly less than the usual minimum requirement. After considerable study it was ascertained that the screenline was subject to extensive "double crossing." Percentages of screenline crossings were 84.4 and 87.2 percent for St. Louis and Kansas City, respectively.

Another validity check consists of a comparison of the number of external cordon crossings made by residents of the area, as obtained in the home and roadside interviews. Because of the larger sample size of the latter, the expanded roadside interview

TABLE 2

TESTS OF RELIABILITY—SELECTED STUDIES

| Study | Percent of Ground Count Screenline—16 Hrs (All vehicles) | Percent of External Trips Reported in Internal Survey (All vehicles) |
|---|---|---|
| St. Louis, Mo., 1957 | 84.4 | 76.9 |
| Kansas City, Mo., 1957 | 87.2 | [1] |
| Charlotte, N.C., 1958 | 78.8[2] | 94.4 |

[1] Not reported, due to different methods of data collection in the Missouri and Kansas portions of survey area.

[2] It is estimated that a minimum of 11,882 cars and trucks crossed the screenline twice. Deducting these vehicles from the ground counts produces an over-all screenline comparison of about 84 percent. In addition, it is believed that a large number of vehicles registered outside the survey area made additional trips completely within the survey limits, although these were not reported in the survey data.

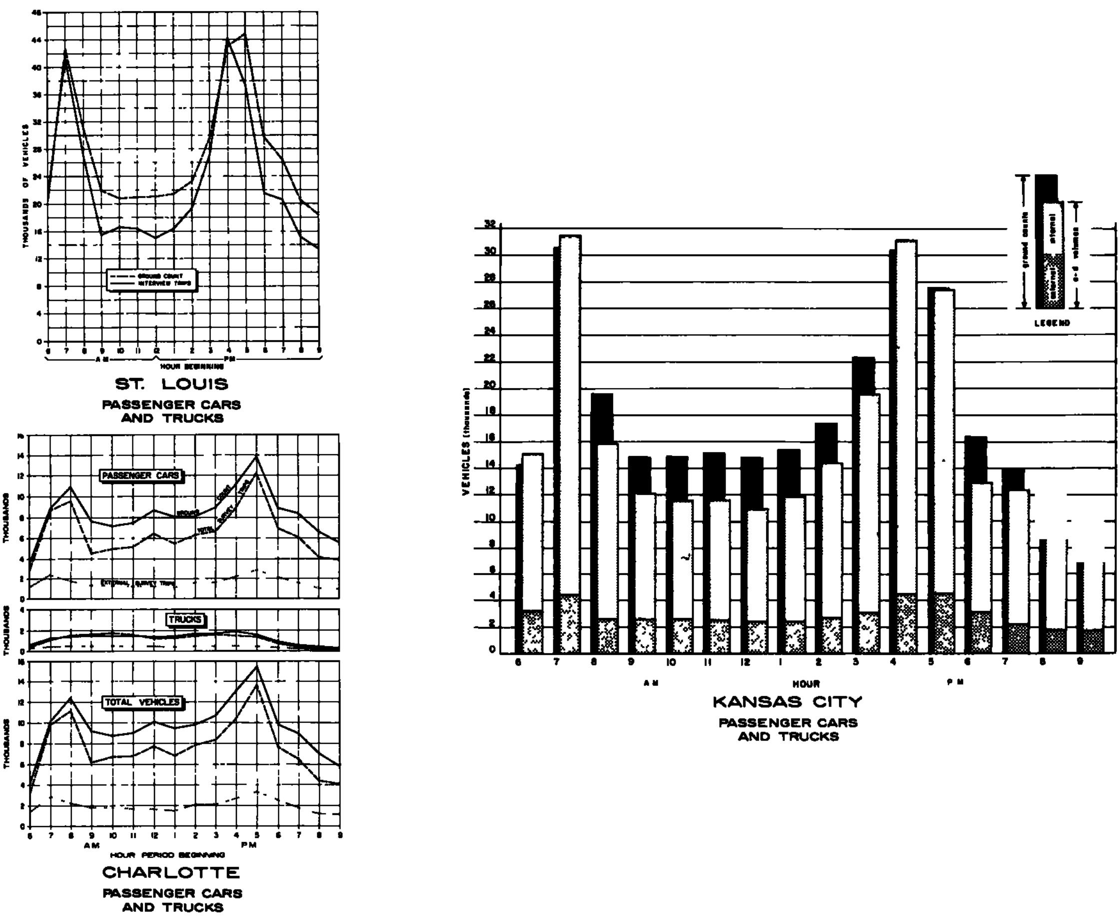



Figure 2. Comparison of traffic crossing screenlines—St. Louis, Mo.; Kansas City, Mo.; and Charlotte, N.C.

data are generally accepted as having greater reliability. The results of this comparison for the studies under consideration are given in Table 2. It is an accepted fact that in most home-interview O-D studies, this comparison reveals even greater variability in the data than is expected in the screenline check.

## Analyses of O-D Data

It has been found in these and other O-D studies that between 80 and 90 percent of all person trips reported in the dwelling-unit interviews either begin or end at home. Trips have, therefore, been reclassified into five basic purposes, depending on purpose at the non-home terminus. Trips between home and work were classified as "work" trips. "Commercial" trips were defined as those between home and "personal business," "medical-dental," "shopping," and "eat meal" purposes. "Social" and "school" trips were defined as trips between home and each of these purposes, respectively. Trips between work and commercial generators, with neither terminus at home, were classified as "miscellaneous" trips.

Several advantages are gained by analyzing and projecting person trips separately for each of these basic purposes. It is obvious that trip production for each purpose will be related to different land-use variables, thereby greatly systematizing the procedure of estimating trip ends.

Of even greater significance, however, is the distinct difference in distribution patterns for each of the various purpose categories. The importance of this finding cannot be overemphasized in highway planning work. In Table 3, cumulative distribution patterns are compared for four of the basic purposes. Although the results vary considerably among the three metropolitan areas studied, due mainly to differences in maximum driving radius, it is clear in each case that work trips are longer than trips

TABLE 3

COMPARISON OF TRIP LENGTHS BY PURPOSE—SELECTED STUDIES

| | Cumulative Percent of Trips by Residents, Internal Survey | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Trip Length | Work Trips | | | Commercial Trips | | | Social Trips | | | School Trips | | | Total Trips | | |
| (min) | St. Louis[2] | Kansas City[2] | Charlotte[2] | St. Louis | Kansas City | Charlotte | St. Louis | Kansas City | Charlotte | St. Louis | Kansas City | Charlotte | St. Louis | Kansas City | Charlotte |
| 0 | 3.6 | 2.2 | 5.8 | 15.2 | 10.8 | 18.0 | 11.5 | 14.3 | 15.1 | 21.7 | 30.5 | 23.4 | 10.1 | 9.8 | 12.9 |
| 0 - 3 | 15.3 | 13.9 | 11.5 | 45.4 | 35.1 | 27.4 | 34.8 | 39.4 | 22.5 | 59.2 | 51.7 | 35.8 | 31.7 | 29.9 | 21.5 |
| 3 - 6 | 22.6 | 17.2 | 25.8 | 55.9 | 43.7 | 48.1 | 45.6 | 46.8 | 40.6 | 68.3 | 52.5 | 57.9 | 40.8 | 36.8 | 40.0 |
| 6 - 9 | 35.0 | 26.4 | 46.5 | 68.6 | 56.7 | 68.0 | 58.7 | 55.8 | 60.3 | 76.8 | 65.2 | 78.2 | 53.2 | 47.9 | 60.2 |
| 9 - 12 | 47.6 | 40.6 | 66.4 | 78.2 | 68.0 | 82.0 | 67.9 | 65.0 | 75.2 | 83.4 | 70.6 | 87.3 | 63.7 | 59.5 | 76.4 |
| 12 - 15 | 59.3 | 54.9 | 80.5 | 84.8 | 77.3 | 90.3 | 76.7 | 75.8 | 85.2 | 87.9 | 75.5 | 93.1 | 72.7 | 70.4 | 86.6 |
| 15 - 18 | 69.1 | 66.8 | 90.0 | 89.4 | 84.3 | 96.2 | 84.0 | 82.4 | 92.5 | 91.9 | 78.5 | 96.5 | 80.0 | 78.8 | 93.5 |
| 18 - 21 | 77.1 | 76.5 | 95.7 | 92.8 | 90.1 | 98.8 | 88.8 | 87.5 | 96.7 | 94.4 | 85.7 | 99.3 | 85.5 | 85.4 | 97.3 |
| 21 - 24 | 84.3 | 83.2 | 98.5 | 95.2 | 94.0 | 99.5 | 92.6 | 91.8 | 98.6 | 96.7 | 88.0 | 99.9 | 90.1 | 89.9 | 99.0 |
| 24 - 27 | 89.5 | 88.7 | 99.5 | 97.2 | 95.8 | 100.0 | 94.7 | 94.3 | 99.6 | 97.8 | 97.6 | 100.0 | 93.3 | 93.4 | 99.7 |
| 27 - 30 | 93.7 | 93.4 | 99.9 | 98.6 | 97.9 | 100.0 | 96.6 | 96.3 | 99.9 | 98.4 | 99.9 | 100.0 | 95.9 | 96.1 | 100.0 |
| Over 30 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Avg Length (min) | 14.4 | 14.8 | 11.0 | 7.6 | 9.1 | 6.9 | 9.7 | 9.3 | 8.1 | 6.0 | 8.0 | 5.7 | 10.7 | 10.8 | 8.1 |

[1] Intra-zone trips.
[2] St. Louis and Kansas City data are for all modes; Charlotte, auto drivers only.

for the other purposes. In Kansas City, for example, although almost one-half of the total trips for all purposes were less than 9 min in length, only about one-fourth of the auto driver work trips were in this time category. The same phenomenon is observed in the data for St. Louis and Charlotte.

A series of special tabulations was prepared for analysis of the trip data. These included tabulations showing the number of trips for each purpose by zone residents, the number of "purpose" motivated trips made to and from each zone, and the distribution of each class of trip to other zones by successive 2- or 3-min time increments.

In the studies of generation of trips by zone residents, it was found that median family income, vehicle ownership, population density, and relative decentralization were the most significant factors. Although considerable variability was observed between populations of different characteristics in certain categories of trips (such as social and school trips), consideration of the four variables in conjunction with each other tends to give a balanced picture of the factors influencing trip production by residents. Similar studies of trip production at non-home termini revealed even greater variations between zones, but it was possible to develop general patterns relating trip production to land-use characteristics for zones in each of the "rings" into which the study areas had been subdivided.

The analysis of trip production rates tends to become complex, but once the basic relationships have been established, estimation of the number of trip ends in each zone at a given future year, by application of these relationships to anticipated land-use development, is a fairly straightforward process.

A means of estimating the number of trip ends in each zone must be developed to distribute trips between the respective home and non-home termini and thus formulate a future travel pattern. Obviously, this procedure involves many complexities, because trips originating in any one zone of the study area could conceivably have destinations in all of the other zones into which the area has been subdivided. The problem resolves itself into determining exactly what proportion of the trips (for each purpose) originating in Zone "A" shall have destinations in each of the other zones. Although this distribution could be accomplished by analogy with trips reported in the current O-D study, several basic factors discount the validity of this procedure:

1. The survey data represent only 5 to 10 percent samples, and many of the smaller inter-zonal movements were completely unreported.
2. Zones greatly increasing or decreasing in relative importance as trip generators will attract different proportions of trips from other zones than they do today.

Previous studies have demonstrated a definite inverse relationship between the number of trips a pair of zones will exchange and the driving time between the zones (1, 5, 6, 7, 8). Additional research in conjunction with the St. Louis, Kansas City, and Charlotte O-D studies has confirmed these findings.

Off-peak auto driving times were computed for all possible pairs of zones. Trips reported in the home interviews were summarized separately for home and non-home termini in each zone, including a further breakdown showing the number of trips for each purpose made to all zones within successive 2- or 3-min time increments.

The driving time between a pair of zones must be coupled with the trip generation potential of each zone and of all other zones in the study area to develop reasonable estimates of the rate of travel between the zones. For example, to estimate the number of shopping trips that would be made by the residents of Zone "A" to the retail outlets in Zone "B", consideration must be given to the total number of home-based shopping trips made by the residents of Zone "A"; the total number of trips made for shopping purposes to Zone "B" by residents of all zones; and the total number of shopping trips made to all other zones, as well as the travel time between Zones "A" and "B".

## Derivation of Interactance Formulas

The trip length studies were used in conjunction with the available population statistics to develop a series of curves relating the average off-peak driving time and the relative number of trip attractions to travel desires for each purpose between zones at successive increments of distance. Because trips tended to become shorter towards the periphery of each of the areas studied, separate analyses were made for each of several concentric "rings," based on relative decentralization (Fig. 1).

Although the slopes of the curves for each purpose differed considerably, they all exhibited the inverse relationship between trip production and travel time, to varying degrees, as illustrated in the curves for St. Louis (Figs. 3 and 4), Kansas City (Figs. 5 and 6), and Charlotte (Fig. 7).

Comparison of the curves for different purposes developed in each study revealed that attraction rates for work trips had the slowest rate of decrease as driving time

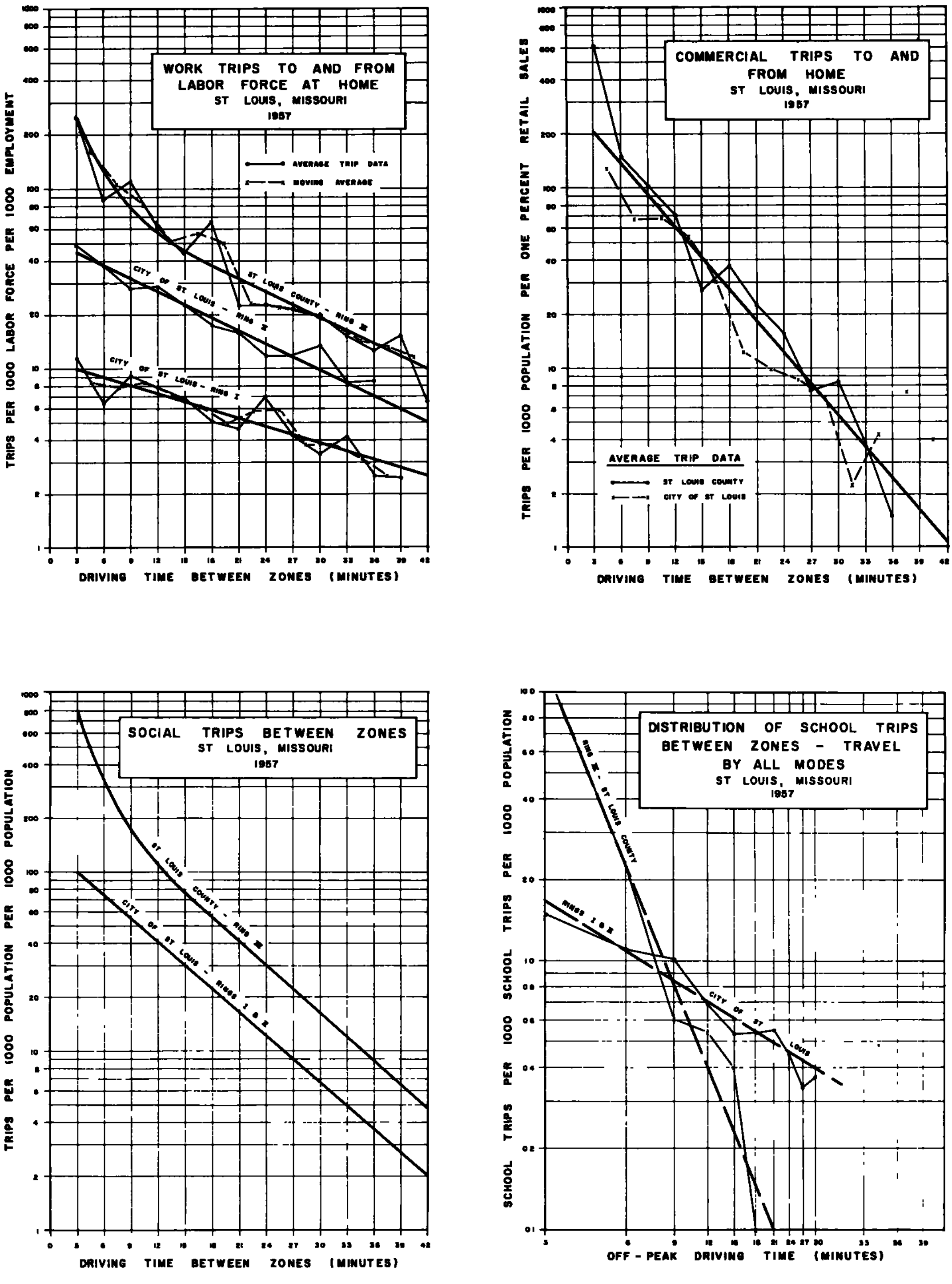
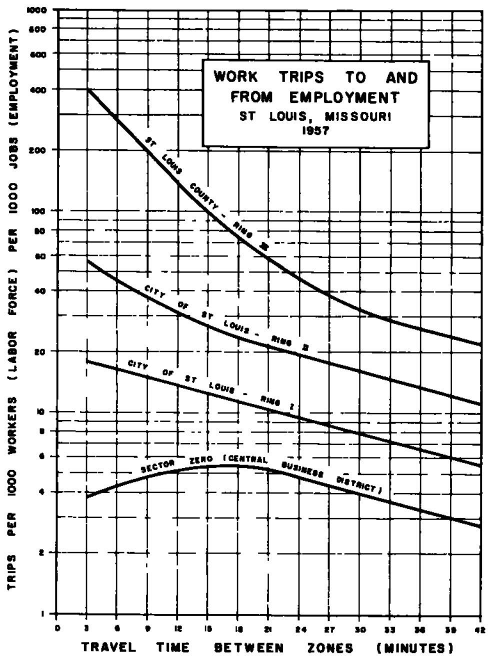
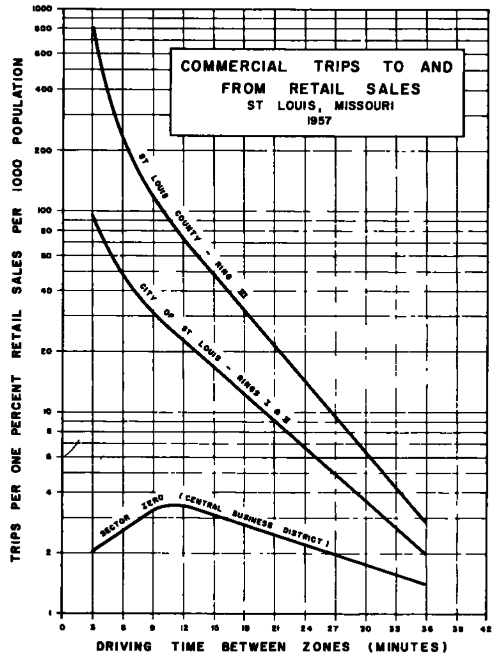



Figure 3. Inter-zonal distribution curves— St. Louis, Mo.



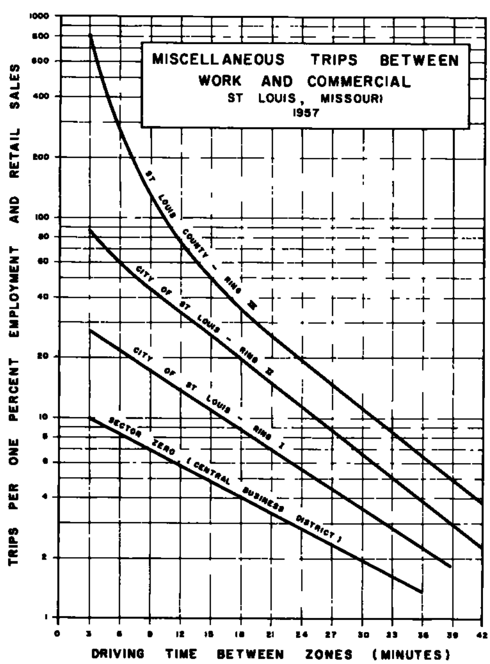




Figure 4. Inter-zonal distribution curves— St. Louis, Mo.

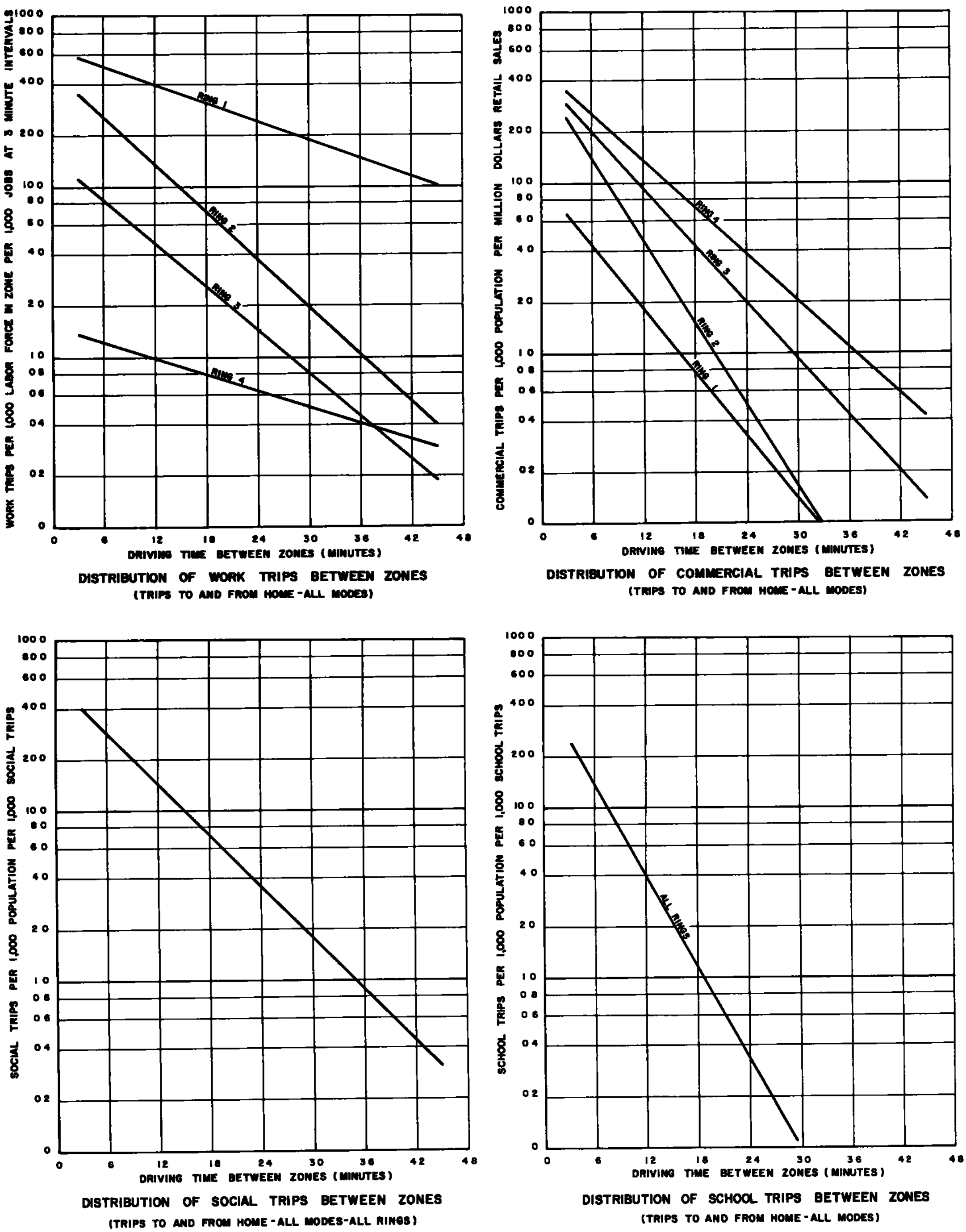



Figure 5. Inter-zonal distribution curves— Kansas City, Mo.

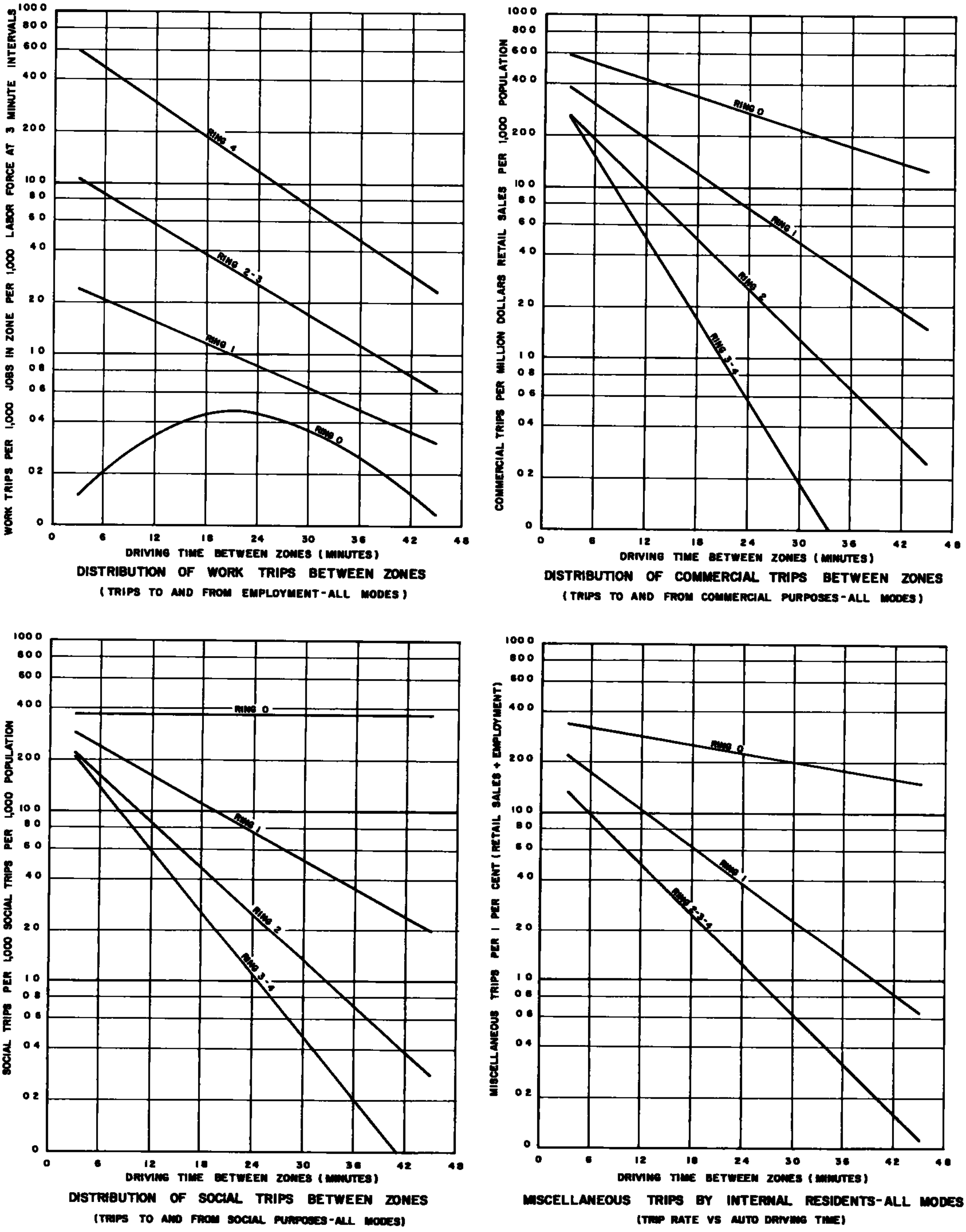



Figure 6. Inter-zonal distribution curves— Kansas City, Mo.

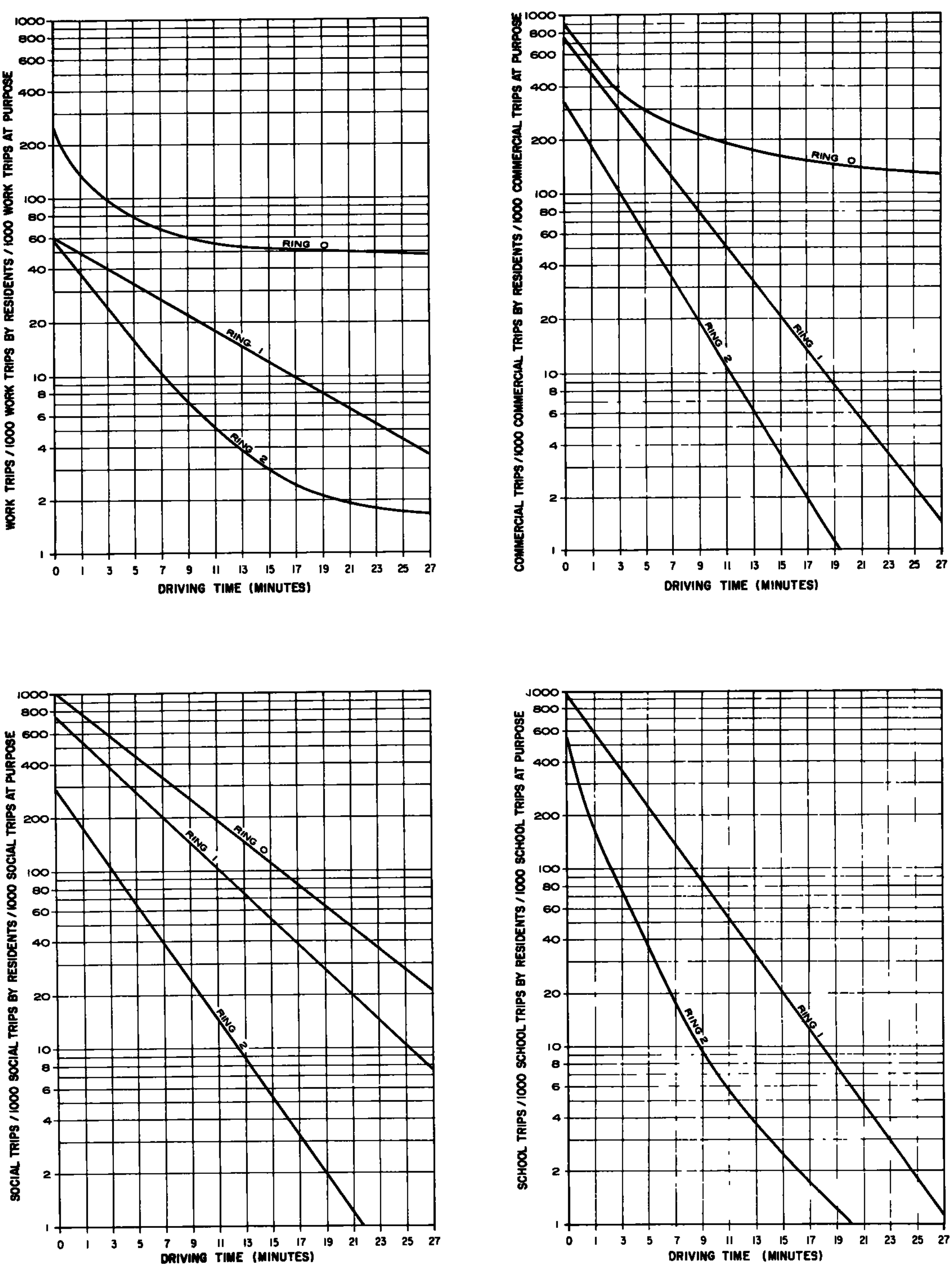



Figure 7. Inter-zonal distribution curves— Charlotte, N.C.

increases. The distributions of social trips also reflected a willingness on the part of area residents to make somewhat longer trips for this purpose. Trip attraction rates for commercial and miscellaneous trips decreased rapidly with increased travel time between zones, whereas the curves for school trips, most of which are local in nature, show the most rapid rate of decay.

## Synthesis of O-D Patterns

Relative "rates of attraction" between zones, as derived from the interactance formulas, are applied to estimated trip end totals in each zone to distribute the trips and produce a synthetic travel pattern. The same procedure is used in the synthesis of either current or future travel desires, the only difference being that the number of trip ends reported in the survey data is used in the first case, whereas future trip end estimates based on anticipated land-use development are used in the latter.

The computations involved in applying interactance formulas to distribute trip end estimates have been accomplished with an IBM 650 computer. Two input cards were prepared for each possible pair of zones, with each zone considered separately as the trip's origin. Distribution rates from the interactance formulas were applied on the basis of the appropriate curve for the origin zone in each case. (Therefore, in cases of travel between zones in different "rings," slightly varying estimates of trips were produced, if each of the "rings" exhibited distinctive distribution patterns.)

In making the first approximation of trips between zones, the attraction rate derived from the interactance curve was multiplied by the corresponding "attraction" in the destination zone, and the resulting products were totaled for each origin zone and balanced to the predetermined "control total." After the initial estimates of trips for each purpose had been made for each zone, these estimates were summated and divided into the respective control totals to establish balancing factors. The initial estimates were then multiplied by the balancing factors and punched out as the first approximation of inter-zonal travel for each purpose.

This procedure is repeated with each zone in the study area considered as origin, resulting in two separate estimates of travel for each purpose between each pair of zones. Averaging the values obtained in the first approximation does not necessarily yield values which will summate to the control total for the individual zones. It may be necessary to carry the averaging process through two or more cycles (iterations) to produce a well-balanced estimate of travel in which all of the trips attributed to each zone are approximately equal to the control total (9). It has been found in the application of the interactance formulas that one or two iterations will usually suffice to produce well-balanced inter-zonal trip distribution.

## Variability of Home-Interview O-D Data

Before a meaningful analysis can be made of the reliability of synthetic travel patterns, the variabilities inherent in the original expanded home-interview data must be considered. Due to limited sample size, movements reported in the survey data are subject to variations of different magnitudes from the true values that would be obtained from a 100 percent sample. An evaluation of the variations expected in the survey data is needed to give the proper perspective to the differences observed between the survey data and the projected data.

The travel patterns developed from the home-interview data and by the application of interactance formulas both consisted of very large numbers of relatively small inter-zonal movements. The smallest movement possible in the expanded home-interview data was limited by sample size, although inter-zonal movements as small as one or two trips per day undoubtedly do occur between many zone-pairs. A volume of this magnitude has no significance when considered separately, although it has been demonstrated that traffic flows consist largely of aggregations of such movements.

It is obvious that the magnitudes of individual movements, as reported in the expanded survey data, are subject to wide fluctuations due to sample variability. Inter-zonal volumes of 300 trips or less, which comprise 50 percent or more of the total traffic in the large urban areas considered in this study, are particularly susceptible to wide fluctuations.

The variability exhibited between different 5 percent samples has been demonstrated with data from a home-interview study made in Milwaukee in 1945 (10). A 50 percent sample of the dwelling units, consisting of 10 separate 5 percent samples, was taken in the census tract selected for study. The number of trip ends reported in the tract varied from about 3,900 to 5,300 among the various samples. Each of these extreme variations represents about 20 percent of the average value of 4,600 trip ends, which was obtained by expanding the 50 percent sample. The percentage of deviation between the two extreme values of expanded samples was even larger (about 36 percent). Although these actually observed variations appear larger than the expected variations discussed in theoretical treatments of sampling, Lynch concludes from the Milwaukee test results that "a five percent sample is in keeping with the over-all requirements as to cost and accuracy" for use in highway planning work.

The Milwaukee study was concerned with the sample variability of a relatively large number of trips. Even more pronounced deviations are expected when smaller volumes are considered. A discussion of the degree of reliability which could be placed in data secured by interviewing different sample sizes is given by Anderson (11). The findings of this investigation are partially summarized in Table 4, which gives the standard error for different trip magnitudes obtained from expanded 5 and 10 percent samples.

The assumptions made in this study were that the sample under consideration was complete; was selected by an approved random technique; and was derived from a normally distributed population. However, none of these conditions is fully met in the home-interview sampling technique. It is known from the screenline deficiencies that every one of the home-interview studies is incompletely reported to some degree; a stratified ordinal method of sample selection has been used in preference to random sampling techniques; normal distribution is doubtful if the sample is small. Therefore, although Table 4 indicates the relative reliability of 5 and 10 percent samples, the values obtained should be considered as favorable evaluations of the data.

It is evident from this study that individual inter-zonal movements of small magnitude, which constitute the bulk of urban traffic, are subject to wide sampling variations, ranging as high as 90 percent of the true value in the case of an expanded movement of 20 trips derived from a 5 percent sample. Somewhat greater confidence may be placed in the values for small movements derived from a 10 percent sample. The degree of error in movements of 1,000 trips or more is similar for both sample sizes given, because expected deviations decrease rapidly in the larger trip volume groups.

Although the variability associated with sampling has little significance to the highway planner, who is concerned with the relatively stable large volumes of trips assigned to proposed facilities, the results of the statistical tests described later are biased to the extent that the synthetic movements are being compared with survey data which in themselves are subject to varying degrees of error. For example, chances are about two out of three that the true value of a movement of 100 trips as derived from a 5 percent sample lies between 57 and 143 trips; if the latter value should happen to coincide with the true value, and the interactance curves develop an equal volume, an error of 43 trips would nevertheless be recorded between the survey and synthetic data. Be-

TABLE 4

EXPECTED ERROR FROM 5 AND 10 PERCENT RANDOM SAMPLES AND VARIOUS VOLUMES OF INTERCHANGE[1]

| Volume of Interchange From Expanded Sample | Standard Error of Estimate (Percentage) | |
|---|---|---|
| | 5% Sample | 10% Sample |
| 20 | 90 | 70 |
| 40 | 68 | 48 |
| 60 | 55 | 40 |
| 80 | 48 | 35 |
| 100 | 43 | 30 |
| 200 | 30 | 23 |
| 300 | 25 | 18 |
| 400 | 23 | 16 |
| 500 | 20 | 15 |
| 600 | 18 | 14 |
| 700 | 17 | 13 |
| 800 | 16 | 12 |
| 900 | 16 | 11 |
| 1,000 | 16 | 10 |
| 2,000 | 10 | 8 |
| 3,000 | 8 | 6 |
| 4,000 | 7 | 5 |
| 5,000 | 7 | 5 |
| 6,000 | 6 | 4 |
| 7,000 | 6 | 4 |
| 8,000 | 5 | 4 |
| 9,000 | 5 | 3 |
| 10,000 | 5 | 3 |

[1] Source: Anderson, O.K., "Statistical Evaluation of Origin-Destination Data." Unpublished Thesis, Bureau of Highway Traffic, Yale University, May 1951, pp. 33-37.

cause movements developed from only a few samples are not likely to show a normal distribution, it is doubtful that the distortion in the comparisons will average out even when many small movements are considered.

## RELIABILITY OF INTERACTANCE FORMULAS

The application of interactance curves to predict reasonable patterns of inter-zonal travel is a relatively new technique which has been developed by empirical methods. The procedures discussed herein are the outgrowths of direct investigations.

Interactance curves have been derived and used for trip distribution in a number of studies during the past few years. (First use of the interactance formulas by this firm was for a 25-yr projection of travel in Philadelphia, Pa., made in 1955. The technique has since been used in Omaha, Neb. (1956); Washington, D.C. (1956-58); Miami, Tampa and Tallahassee, Fla., (1957-58); St. Louis, Mo. and Kansas City, Mo., (1957-58); Charlotte, N.C. (1959); Pheonix, Ariz., (1959); and Philadelphia, Pa. (revised, 1959).) Techniques of the method have been improved with each new experience.

In the course of the trip projection work undertaken in St. Louis, Kansas City, and other recent studies, an effort has been made to develop some measure of the reliability of the techniques described previously. Due to limitations in both budget and time, these tests have not been as comprehensive as possible for detailed research, but the insight they provide is most encouraging. The results of each of the tests described herein have been found to be of direct value in improving the trip distribution technique.

To test the reliability of the interactance formulas for the purpose of predicting inter-zone trip distribution, trip attraction rates determined from the curves given in Figures 3 to 7 have been applied to the 1957 inter-zone trip ends generated by residents of each study area. The numbers of inter-zone trip ends generated in each zone, as reported in the home-interview survey, have been distributed to all other zones in the study area. Two independent estimates of travel were thus determined for each purpose of trips between every pair of zones. The two estimates were averaged by the method of successive approximations. Average vehicle occupancy rates for each trip purpose were applied to develop the number of auto drivers and auto passengers in each inter-zonal movement. Trip patterns were developed for combined travel by each mode.

In this manner a synthetic travel pattern was developed between all internal zones in each survey area. The projected movements are directly comparable with the expanded home-interview data. However, the complexity of the comparison is emphasized by the fact that in the St. Louis study area of 235 zones, there are 27,495 possible movements to be considered, exclusive of intra-zone movements.

### Comparison of Inter-Zonal Movements

The comparative analysis of expanded interview data with synthesized trip distribution patterns can be accomplished only by statistical methods. Direct comparison of the two sets of data reveals that the synthetic estimates of travel between most zone pairs differed by varying degrees from the values obtained in the home interviews. To evaluate the magnitude of these differences, the root-mean-square error has been computed for stratified volume classes of 100 trips, up to magnitudes of 1,000 trips, similar to procedures used in other recent studies of trip projection techniques (8, 12).

As a measure of variability, the root-mean-square error indicates the limits within which about two-thirds of the deviations between the observed and estimated values will fall. About 95 percent of all variations lie within twice the root-mean-square error, while almost all the differences between the theoretical and observed values are included by three times the root-mean-square error.

The root-mean-square error for each trip magnitude group was computed by summing the squares of the differences between the survey and synthesized volumes for each pair of zones, dividing the total squared differences by the number of zone pairs in the group, and finding the square root of the quotient. The procedures can be expressed by:

$$R = \sqrt{\frac{\left(T_{AB} - T^{1}_{AB}\right)}{n}} \qquad (1)$$

in which

$R$ = root-mean-square error of volume group

$T_{AB}$ = survey volume between Zones "A" and "B"

$T^1_{AB}$ = synthesized volume between Zones "A" and "B"

$n$ = number of zone-pairs in volume group (survey data)

## St. Louis Results

The results of this statistical test for St. Louis, for trips by all modes and by auto drivers, are given in Table 5, with graphic presentation in Figure 8. The number of zone pairs, the root-mean-square error of estimate, and the average percentage of error are given for each volume class. Similar characteristics are observed in the deviations exhibited by auto driver trips and trips by all modes, although the auto driver trips generally showed less variability. The root-mean-square error for all volumes of auto driver trips was 70 trips or about 61 percent of the average inter-zonal auto driver movement of 114 trips; the comparable error for trips by all modes was 120 trips, representing about 65 percent of the average inter-zonal movement of 186 person trips.

The magnitude of the root-mean-square error for each volume class increases as the average size of movement increases. However, the proportional error decreases somewhat in the larger volume classes, up to magnitudes of 1,000 trips. Although the proportional deviations may seem large at first consideration, they compare quite favorably with values obtained in other recent studies (8, 12). The effect of sample variability and other factors must also be considered to arrive at a realistic evaluation of the technique.

Although considerable variability is observed between the survey data and the theoretical movements when individual zone-to-zone interchanges were considered, a critical evaluation of the results of this test is difficult. Because the St. Louis study area was subdivided into a large number of relatively small zones, the volume of movement between zone pairs tended to be small. The average inter-zonal movement for all modes of travel between zones was only 186 trips. Zone pairs exchanging 1,000 or fewer trips by all modes accounted for over 75 percent of all travel in the study area. The expanded trip data prepared from the home-interview reports show only 298 zone pairs, or slightly more than 1 percent of the possible movements, with more than 1,000 trips between them. Trip movements of 2,000 or greater were reported for only 95 zone pairs, accounting for about 10 percent of all travel within the study area.

Most of these heavy trip movements occurred between adjacent zones. Of the 298 movements which exceeded 1,000 trips per day, 214 involved pairs of adjacent zones; only 84 movements were between non-adjacent zone pairs. Of the 95 zone pairs which generated travel exceeding 2,000 trips per day, 87 were adjacent to one another and

TABLE 5

ROOT-MEAN-SQUARE ERRORS OF ESTIMATE, SYNTHETIC TRIP DISTRIBUTION ST. LOUIS, 1957—AUTO DRIVERS AND ALL MODES

| Trip Volume | Auto Drivers | | | All Modes | | |
|---|---|---|---|---|---|---|
| | Number of Zone Pairs | Root-Mean-Square Error | Percent | Number of Zone Pairs | Root-Mean-Square Error | Percent |
| Less than 100 | 6,799 | 50 | 100 | 6,287 | 66 | 132 |
| 100 - 200 | 1,475 | 94 | 63 | 2,073 | 112 | 75 |
| 200 - 300 | 498 | 127 | 51 | 943 | 147 | 59 |
| 300 - 400 | 221 | 167 | 48 | 473 | 191 | 55 |
| 400 - 500 | 115 | 187 | 42 | 290 | 215 | 48 |
| 500 - 600 | 73 | 239 | 43 | 145 | 236 | 43 |
| 600 - 700 | 50 | 330 | 51 | 131 | 277 | 43 |
| 700 - 800 | 38 | 342 | 46 | 102 | 307 | 41 |
| 800 - 900 | 25 | 445 | 52 | 64 | 328 | 39 |
| 900 - 1,000 | 24 | 412 | 43 | 52 | 420 | 47 |
| Over 1,000 | 79 | 845 | | 298 | 863 | |
| All volumes | | 70 | | | 120 | |

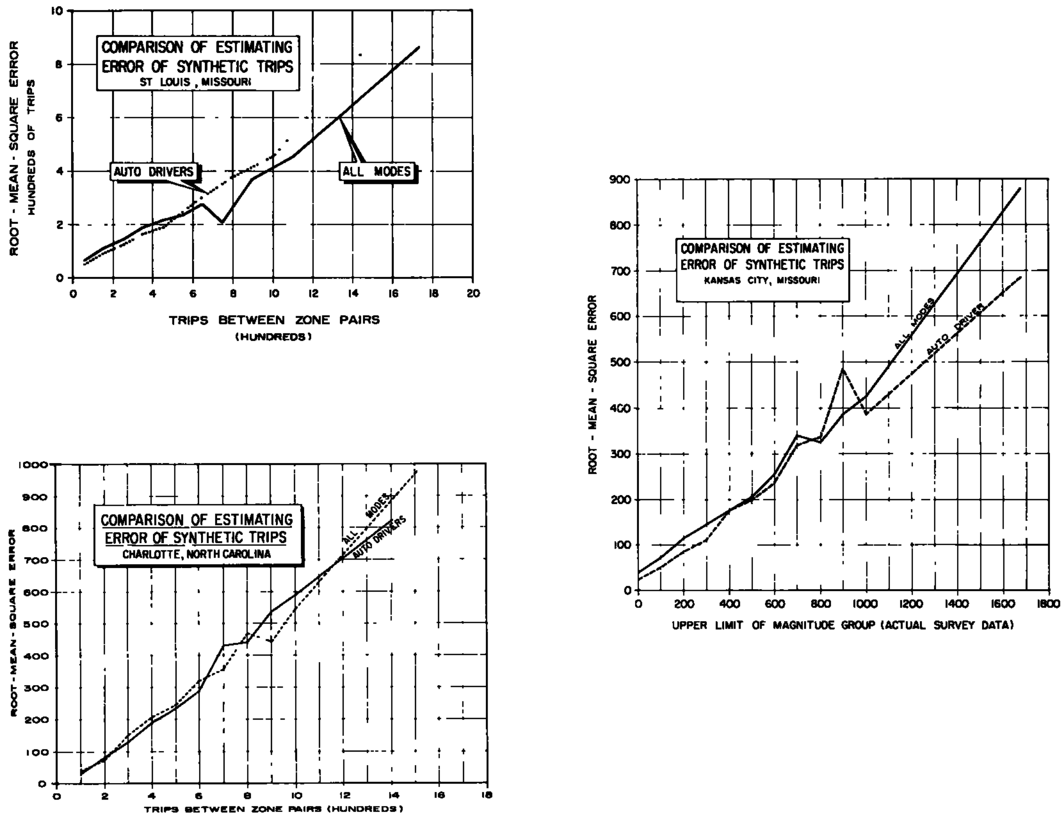



Figure 8. Comparison of estimating error of synthetic trips— St. Louis, Mo.; Kansas City, Mo.; and Charlotte, N.C.

only 8 represented travel to zones beyond the immediately adjacent tier.

There were, of course, even fewer movements of large magnitude for travel by automobile drivers. Only 114 zone pairs generated more than 1, 000 auto driver trips per day in the 1957 study. Of these movements, 107 involved adjacent zones and 7 represented travel to zones beyond the adjacent tier.

Comparison of the root-mean-square errors for different volume classes (Table 5), reveals that the average proportional error developed in the synthetic estimates decreases in the larger trip volume groups, up to volumes of 1, 000 trips. Although relatively few in number, the larger inter-zonal movements, composed primarily of trips between adjacent zones, showed more variability between survey data and theoretical data than observed in smaller movements.

The interactance formulas are sensitive to the measure of distance. Unless the intensity of land-use development in each zone is uniform throughout, each zone is devoted to only one use, and zones are all of approximately the same diameter, there is bound to be some distortion in the use of travel times related to zone centroids. The zoning in most cities will not meet these criteria unless a vast number of small zones is created. In the studies reported, zoning was designed to produce areas with an average diameter of about 3-min driving time. Exceptions were series of small zones in the Central Business Districts and small numbers of larger, low-density zones in peripheral areas. Large parks, cemeteries, railroad yards, etc., were separately identified in zoning so that reasonably uniform land-use intensities were encompassed by most of the important zones.

The interactance formulas produce their best estimates for trips with lengths

TABLE 10

COMPARISON OF ACTUAL AND THEORETICAL TRIP LENGTH DATA
St. Louis Central Business District—Truck Trip Lengths

| Miles | Truck Driver Trips | | | | | Vehicle-Miles | |
|---|---|---|---|---|---|---|---|
| | Survey No. of Trips | Theoretical No. of Trips | Ratio | Survey Cum. % of Total | Theoretical Cum. % of Total | Survey Data | Theoretical Data |
| 0 - 1 | 10,026 | 5,535 | 0.55 | 41.7 | 26.3 | 5,013 | 2,768 |
| 1 - 2 | 4,021 | 5,209 | 1.30 | 58.4 | 51.1 | 6,032 | 7,814 |
| 2 - 3 | 4,041 | 4,399 | 1.09 | 75.2 | 72.0 | 10,103 | 10,998 |
| 3 - 4 | 2,108 | 3,156 | 1.50 | 84.0 | 87.0 | 7,378 | 11,046 |
| 4 - 5 | 1,390 | 1,483 | 1.07 | 89.8 | 94.1 | 6,255 | 6,674 |
| 5 - 6 | 492 | 703 | 1.43 | 91.8 | 97.4 | 2,706 | 3,867 |
| 6 - 7 | 479 | 349 | 0.73 | 93.8 | 99.1 | 3,114 | 2,269 |
| 7 - 8 | 402 | 120 | 0.30 | 95.5 | 99.7 | 3,015 | 900 |
| 8 - 9 | 283 | 36 | 0.13 | 96.7 | 99.9 | 2,406 | 306 |
| 9 - 10 | 273 | 16 | 0.06 | 97.8 | 100.0 | 2,594 | 152 |
| 10 - 11 | 177 | - | - | 98.5 | | 1,859 | - |
| 11 - 12 | 85 | - | - | 98.9 | | 978 | - |
| 12 - 13 | 114 | - | - | 99.4 | | 1,425 | - |
| 13 - 14 | 77 | - | - | 99.7 | | 1,040 | - |
| 14 - 15 | 56 | - | - | 99.9 | | 812 | - |
| 15 - 16 | 28 | - | - | 100.0 | | 434 | - |
| 16 - 17 | 7 | - | - | | | 116 | - |
| Totals | 24,059 | 21,006 | 0.87 | | | 55,280 | 46,794 |
| Average length | | | | | | 2.3 mi | 2.2 mi |

synthetic travel pattern of truck trips developed about 85 percent of the total number of truck vehicle-miles reported in the O-D study.

## Kansas City Trip Length Comparisons

The results of the trip length study for the Kansas City CBD are shown in Figure 9. The projected results of the auto driver trip distributions were in close agreement with the survey data. Although in the category of short trips a slight deficiency was noted in the projected results (8 percent of the projected trips versus 11.5 percent in the survey data were less than 1 mi long), the two sets of data steadily converge until both reveal that slightly more than one-half of all auto driver trips were less than 3 mi in length. A slight excess of trips over 3 mi is observed in the projected results, but at no point do the cumulative distribution curves of the two sets of data vary more than 4 percent. Almost 90 percent of the auto driver trips in both data were less than 7 mi in length. A similar comparison was made of auto passenger trips, where even closer agreement between the two sets of data was observed; at no point does the variation between the two cumulative distribution graphs exceed 1.5 percent of the total passenger trips.

TABLE 11

COMPARISON OF ACTUAL AND THEORETICAL TRIP LENGTH DATA
Kansas City Central Business District—Auto Driver Trip Lengths

| Miles | Auto Driver Trips | | | | | Vehicle-Miles | |
|---|---|---|---|---|---|---|---|
| | Survey No. of Trips | Theoretical No. of Trips | Ratio | Survey Cum. % of Total | Theoretical Cum. % of Total | Survey Data | Theoretical Data |
| 0 - 1 | 9,674 | 7,049 | 0.73 | 11.54 | 8.04 | 4,837 | 3,525 |
| 1 - 2 | 7,839 | 7,360 | 0.94 | 20.89 | 16.44 | 11,759 | 11,040 |
| 2 - 3 | 11,729 | 15,221 | 1.30 | 34.88 | 33.81 | 29,323 | 38,053 |
| 3 - 4 | 12,941 | 15,417 | 1.19 | 50.32 | 51.40 | 45,294 | 53,960 |
| 4 - 5 | 9,642 | 11,326 | 1.17 | 61.82 | 64.32 | 43,389 | 50,967 |
| 5 - 6 | 4,221 | 4,085 | 0.97 | 66.86 | 68.98 | 23,216 | 22,468 |
| 6 - 7 | 9,450 | 10,892 | 1.15 | 78.13 | 81.41 | 61,425 | 70,798 |
| 7 - 8 | 8,712 | 6,984 | 0.80 | 88.52 | 89.38 | 65,340 | 52,380 |
| 8 - 9 | 2,985 | 3,474 | 1.16 | 92.08 | 93 34 | 25,373 | 29,529 |
| 9 - 10 | 3,667 | 3,667 | 1.00 | 96.46 | 97.52 | 34,837 | 34,837 |
| 10 - 11 | 1,897 | 1,559 | 0.82 | 98.72 | 99.30 | 19,919 | 16,370 |
| 11 - 12 | 985 | 529 | 0.54 | 99.90 | 99.90 | 11,328 | 6,084 |
| 12 - 13 | 72 | 78 | 1.08 | 99.99 | 99.99 | 900 | 975 |
| Totals | 83,814 | 87,641 | 1.05 | | | 376,940 | 390,986 |
| Average length | | | | | | 4.5 mi | 4.5 mi |

The comparisons of theoretical and actual trip lengths are summarized for the Kansas City CBD in Table 11. Of even greater significance to the highway engineer than the number of trips is the number of vehicle-miles developed in the study area. It is apparent that the vehicle-miles developed in the projections for each mile increment corresponds closely with the number reported in the survey data, with equally close agreement observed in the cumulative percentage distribution. Although the total number of vehicle-miles developed by to and from the Kansas City CBD by the interactance formulas was about 7 percent more than that derived from the expanded survey data, both distributions revealed an average length of 4.5 mi for auto driver trips. As in St. Louis, comparisons of survey and theoretical trip lengths were made for a limited sample of movements throughout the remainder of the study area. Although the results were not as favorable as those in St. Louis, the study showed that the interactance formulas were producing trip lengths in proportions very similar to those revealed in the survey data.

## Charlotte Trip Length Comparisons

The relatively small number of zone pairs in the Charlotte study area permitted a comparative analysis of survey and synthetic trip lengths throughout the entire area. As given in Table 12, the synthetic distribution of trips resulted in fairly close agreement with the proportions of trips in each 1-mi increment as reported in the Charlotte survey data. The cumulative distribution chart of these data (Fig. 9) shows that short trips are slightly overestimated by application of the interactance formulas. Fewer than 10 percent of the total trips exceeded 5 mi in length in both the synthetic and the survey data. The average auto driver trip length of 2.7 mi in the synthesis corresponds with an average of 3.1 mi derived from the survey data. The total number of vehicle-miles developed in the synthesis is 642,022 or about 90 percent of the total of 719,778 shown in the survey data.

## Evaluation of Trip Length Studies

The trip length studies indicate that trips of various lengths are being projected in approximately correct proportions by use of interactance formulas. Because distance is one of the most significant factors in determining assignable proportions of traffic, this fact is of importance in making an evaluation of the synthetic travel patterns. The proportion of assignable trips increases constantly between zone pairs benefiting from the expressway system as the driving time between the zones increases. Consequently, a large proportion of the longer trips will be diverted to expressways where this service is provided; in the case of expressway systems as comprehensive as those proposed for St. Louis and Kansas City, the great majority of the longer inter-zonal movements will benefit from the system.

TABLE 12

COMPARISON OF ACTUAL AND THEORETICAL TRIP LENGTH DATA

Charlotte Metropolitan Area—Auto Driver Trip Lengths

| Miles | Auto Driver Trips | | | | | Vehicle-Miles | |
|---|---|---|---|---|---|---|---|
| | Survey Trips | Theoretical Trips | Ratio | Survey Cum. % of Total | Theoretical Cum. % of Total | Survey Data | Theoretical Data |
| 0 - 1 | 22,946 | 27,960 | 1.22 | 9.9 | 11.9 | 11,473 | 13,980 |
| 1 - 2 | 49,463 | 60,088 | 1.22 | 31.1 | 37.5 | 74,195 | 90,132 |
| 2 - 3 | 53,908 | 59,134 | 1.10 | 54.3 | 62.7 | 134,770 | 147,835 |
| 3 - 4 | 43,259 | 41,968 | 0.97 | 72.9 | 80.6 | 151,407 | 146,888 |
| 4 - 5 | 27,305 | 22,925 | 0.84 | 84.6 | 90.4 | 122,873 | 103,163 |
| 5 - 6 | 18,440 | 12,888 | 0.70 | 92.6 | 95.9 | 101,420 | 70,884 |
| 6 - 7 | 10,015 | 5,800 | 0.58 | 96.9 | 98.3 | 65,098 | 37,700 |
| 7 - 8 | 4,511 | 2,533 | 0.56 | 98.8 | 99.4 | 33,833 | 18,998 |
| 8 - 9 | 1,813 | 888 | 0.49 | 99.6 | 99.8 | 15,411 | 7,548 |
| 9 - 10 | 674 | 342 | 0.51 | 99.9 | 99.9 | 6,403 | 3,249 |
| 10 - 11 | 199 | 115 | 0.58 | 100.0 | 100.0 | 2,090 | 1,208 |
| 11 - 12 | 70 | 38 | 0.54 | 100.0 | 100.0 | 805 | 437 |
| Totals | 232,601 | 234,679 | | | | 719,778 | 642,022 |
| Average length | | | | | | 3.1 mi | 2.7 mi |

It has been demonstrated in each of the study areas under consideration, that the preponderance of large inter-zonal movements consists of trips between adjacent zones. In most cases, only a very small proportion of trips between adjoining zones is assignable to limited-access facilities. Therefore, the weakness of the interactance formulas in predicting large movements between particular pairs of adjacent zones to a high degree of accuracy loses significance when the projected travel data are used in the estimation of future design-hour expressway volumes. The ability of the interactance formulas to estimate inter-zonal movements of small magnitudes, which are not revealed in the expanded home-interview data because of sample size, has been demonstrated. This, coupled with the fact that underestimation of short trips (when that occurs), is compensated for by trips in the intermediate range of trip lengths rather than by long trips, confirms the applicability of theoretical travel patterns to assignment purposes.

It would have been desirable to make assignments of the synthesized data to the expressway systems proposed in St. Louis and Kansas City. A direct comparison of the expressway volumes thus estimated with the volumes developed in the 1957 assignments made in these cities would have been of great value, but the time and expense involved precluded this analysis.

### Screenline Comparison

Comparisons were made of the numbers of internal truck and person trips crossing the St. Louis screenline. The synthetic trip distribution resulted in about 95 percent of the total person movements crossing the screenline, as recorded in the ground counts given in Table 13. Comparable 24-hr expanded survey movements accounted

TABLE 13

COMPARISON OF SCREENLINE CROSSINGS
Survey and Synthesized Data—St. Louis, Missouri, 1957

| Item | Total Internal Person Trips | Internal Truck Trips |
|---|---|---|
| Ground count (24 hr) | 643,803[1] | 33,468 |
| Expanded survey data | 548,635 | 31,585 |
| Ratio: survey data/ground count | 0.852 | 0.943 |
| Synthetic data | 611,574 | 32,482 |
| Ratio: synthetic data/ground count | 0.950 | 0.971 |

[1] Estimated by applying average occupancy factor of 1.52 persons per vehicle and adding transit ground counts. External trips have been eliminated.

for 85 percent of ground counts. Truck movements crossing the screenline, as revealed in the synthetic and survey data, accounted for 98 and 97 percent of the actual ground counts, respectively. These results indicate that the synthetic trip distribution is capable of producing a screenline check within the degree of accuracy required of survey data by the U.S. Bureau of Public Roads and also of giving close comparison with the expanded survey data.

## SUMMARY AND CONCLUSIONS

In the last several years, interactance formulas have been developed from O-D data collected in urban areas of widely different characteristics. Although it has not been possible to test all of these relationships to synthetically duplicate existing travel patterns, a series of checks has been made on the St. Louis, Kansas City, and Charlotte studies to evaluate the validity of these formulas.

Comparison of individual movements, by computation of the root-mean-square error, showed that the interactance formulas were producing average deviations which were less, in nearly all cases, than those reported in similar research projects by other investigators. Inasmuch as these deviations were large, however, additional comparisons of actual and theoretical trip lengths were made, as a better index of the suitability of using synthetic travel patterns in assigning volumes to urban expressways. The results of these tests were highly satisfactory, revealing the ability of this synthetic approach to approximate the total number of vehicle-miles within 10 percent in most cases. In addition, a screenline check of the synthetic travel pattern developed for St. Louis showed closer agreement with ground counts than was obtained from the expanded survey data.

The insight provided by these successful tests should prove of inestimable value in devising further improvements in the development and application of interactance models. The possibility of analyzing other combinations of trip purposes, special consideration for adjacent zone pairs, and additional land-use variables, are only a few of the possible factors to be studied toward the ultimate goal of developing a universally applicable method for synthesizing travel patterns.

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