Analysis of Stresses in Flexible Pavements and Development of a Structural Design Procedure

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A procedure is developed for the analysis and design of flexible pavements, based on the premise that the lateral stress produced in the pavement structure by surface loads must not exceed the passive lateral resistance provided by the components of the pavement structure. Stress constants of cohesion, c, and angle of internal friction, ϕ, obtained from a strength envelope for the flexible paving material developed from triaxial or direct shear test data, are the predominant factors influencing the resistance of the paving material to deformation. The procedure is applicable to the stability analysis of bituminous mixtures or to the thickness design of the pavement. Examples are given to illustrate the application of this design procedure.

●THERE ARE many factors influencing the design of flexible pavements. Included among the design considerations should be (a) the shearing resistance of the components of the total pavement structure for both dynamic and static loading, (b) the stresses produced both at the surface and within the pavement by surface wheel loads, and (c) the resistance of the pavement structure to the effects of moisture, frost action, and abrasion from vehicle tires. The author feels that the first two of these could and should be based on an analysis of the stresses produced in the components of the pavement structure and of the resistance offered by the pavement to these stresses. It is realized that changes in moisture content will influence the strength properties of subgrade, subbase, and base materials in which a soil binder functions. It is therefore necessary that the strength properties of these materials be determined for the material in its critical service condition. The control of frost action in the pavement must come from a control on the placement of frost susceptible materials and a control of moisture, with consideration to both surface and subsurface water. This paper presents a method for flexible pavement design which gives primary consideration to the theoretical analysis of the strength properties of the pavement materials.

STRESSES IN PAVEMENT

Load Over Circular Area

The vertical intensity of stress along the axis below a circular loaded area has been determined by integration of unit loads from the Boussinesq point load equation over the surface area to be (1)

$$p_z = p\left[1 - \left(\frac{1}{1 + \left(\frac{a}{z}\right)^2}\right)^{3/2}\right] \qquad (1)$$

in which

p = intensity of contact pressure,

a = radius of equivalent circular area,
z = vertical distance, and
p_z = vertical stress at depth z.

Eq. 1 gives the intensity of vertical stress below a circular load due to a surface contact pressure. The total vertical stress at some depth below the pavement surface would include the unit stress due to the weight of the overlying material, and is included in this analysis.

The Boussinesq equation, modified for loading over a circular area, may be employed to determine the maximum vertical stress intensity within a flexible pavement structure, as has been demonstrated by load tests (2).

Pavement Resistance

An element of soil triaxially loaded will provide resistance to failure in accordance with the expression (3)

$$p_z = p_x \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2c \tan\left(45^\circ + \frac{\phi}{2}\right) \tag{2}$$

If it is assumed that p_x is equal to the passive lateral pressure provided by a wedge of soil subjected to a vertical load equal to the overlying weight of pavement (Fig. 1), then (3, p. 237)

$$p_x = \gamma z \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2c \tan\left(45^\circ + \frac{\phi}{2}\right) \tag{3}$$

and, by substitution

$$p_z = \left[\gamma z \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2c \tan\left(45^\circ + \frac{\phi}{2}\right)\right] \tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2c \tan\left(45^\circ + \frac{\phi}{2}\right)$$

Since $\tan^2\left(45^\circ + \frac{\phi}{2}\right) = \frac{1 + \sin\phi}{1 - \sin\phi}$, the above becomes

$$p_z = \gamma z\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^2 + 2c\left\{\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}} + \left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}}\right\}$$

$$= \gamma z\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^2 + 2c\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}}\left\{1 + \left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]\right\}$$

$$= \gamma z\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^2 + 2c\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}}\left\{\frac{1 - \sin\phi + 1 + \sin\phi}{1 - \sin\phi}\right\}$$

$$= \gamma z\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^2 + \frac{4c}{1 - \sin\phi}\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}} \tag{4}$$

In the development of Eq. 4 confinement due to the wheel load itself has been neglected as such confinement would tend to reduce the effective passive lateral pressure of the pavement component. It is considered that the maximum lateral resistance to deformation which can be developed is the passive lateral pressure mobilized from the surcharge effect of the pavement layer only.

It will be noted that the resistance formula as developed here is for strip loading whereas the equation for vertical stress is for circular loading and possible circular failure. This would represent a conservative estimate of the resistance provided, except for moving loads where failure would approach that for strip loading, as has been observed from wheel tracking.

DESIGN PROCEDURE

The Basic Formula

Eq. 1, modified to include unit stress from overlying material, gives the intensity of vertical stress at a point beneath the center of a circular area acted on by a load "p" per unit area. Eq. 4 gives the resistance which must be developed in the pavement at that point if failure of the element is prevented. Failure is assumed to occur along a plane within the element at an angle of $(45^\circ + \frac{\phi}{2})$ with the pavement surface, and the wedge resisting this action slips along a plane making an angle of $(45^\circ - \frac{\phi}{2})$ with the pavement surface, as indicated in Figure 1. Equating Eq. 1, as modified, and Eq. 4 gives the basic structural design formula

$$\gamma z + p\left[1 - \left(\frac{1}{1 + \left(\frac{a}{z}\right)^2}\right)^{3/2}\right] = \gamma z\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^2 + \frac{4c}{1 - \sin\phi}\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}} \quad (5)$$

in which

p = surface contact pressure,
a = radius of equivalent circular contact area,
z = thickness of flexible pavement structure,
γ = bulk density (unit weight) of surcharge material,
ϕ = angle of internal friction of bearing material, and
c = cohesion of bearing material.

Eq. 5 is applicable to any layer of the flexible pavement structure. For instance, if $z = 0$, the equation could be applied to asphaltic surfaces, and would be reduced to

$$p = \frac{4c}{1 - \sin\phi}\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}} \quad (6)$$

Pavement Failure

Surface loads produce a squeezing action of the flexible pavement surface layer. The critical area at the tire contact with the pavement is at the perimeter of the surface contact, and failure is produced by forcing the resisting wedge to slip. Actually, failure is a relative thing, as applied to flexible pavements, and vertical stresses of intensity lower than that for massive slippage may cause some deformation within the structure. This may be observed in the triaxial test where considerable deformation of a specimen may occur prior to the actual development of a failure plane, or before a peak strength is reached. It is assumed in this analysis that the passive lateral resistance provided by the components of the flexible pavement structure will prevent element or wedge slippage. Consequently, the number of repetitions of loads of magnitude lower than that required for slippage may cause pavement deformation or distress.

Study by Paquette

It is interesting to note that Eq. 6 is identical to that developed by McLeod (4) from the geometry of the Mohr diagram for the stability design of bituminous

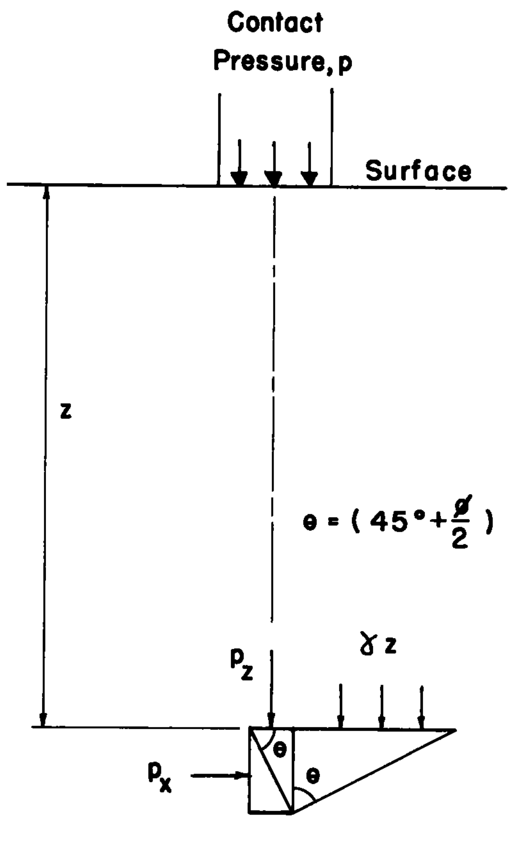

Figure 1. Stresses on an element of flexible pavement below a wheel load.

mixtures. This equation has been found by Paquette (5) to give a rather good correlation with results of the Marshall and Hveem stabilometer methods for contact pressures of 100 psi, which is near the maximum required for highway design. Paquette tested six samples at each of the four asphaltic contents for seven different gradations of aggregate, two for the Marshall Method, and four for the Hveem Method. Values of ϕ were determined from the Hveem stabilometer test data and c from the cohesiometer test data for the computation of "p" from formula 6. A summary of test results for one gradation of aggregate (the ideal gradation for New York State specification 1-A mixture) is extracted from Paquette's thesis and given in Table 1.

TABLE 1

EVALUATION OF IDEAL GRADATION MIXTURE FOR NEW YORK STATE SPECIFICATION 1-A MIX[1]

	Percent Asphalt Cement			
	4	5	6	7
Marshall:				
Voids 2 - 8%	8.85	7.17	4.93	1.27
Flow < 16	13	16	15	23
Stability > 1000	2414	1788	1906	2051
Preference	-	2	1	-
Hveem:				
Voids 3 - 8%	8.85	7.17	4.93	1.27
Cohesion > 250	244	361	368	429
Stability > 25	25.1	32.9	27.2	14.8
Preference	-	1	2	-
Stability Formula:				
Voids 2 - 8%	8.85	7.17	4.93	1.27
Stability 100	131	248	220	142
Preference	-	1	2	-

[1] From Paquette.

Table 2 presents data relating to angle of internal friction, cohesion, and stability which were used by Paquette to develop the summary in Table 1.

There are two aspects of the test procedure and computations which need clarification. First, all samples were compacted using the Marshall method of compaction. This procedure has been found to give lower stability values when using the Hveem stabilometer test procedure than when the kneading compactor is used for the preparation of samples. This fact is believed to be the reason for the relatively low values (as given in Table 1) for Hveem stability. Second, Paquette considered the extreme fiber stress in flexure to be a better measure of cohesion than the intercept of the Mohr diagram and used this in his computations for stability, employing Eq. 6. It will be noted from Table 2 that there is not a great difference between the values for cohesion by the two methods.

Comparison With Smith Triaxial Method

A chart for determining the suitability of bituminous mixtures tested by the Smith triaxial method has been developed (6). The author has superimposed curves on this chart for various contact pressures (Fig. 2) using Eq. 6 for the computations. Also, a curve for ultimate failure as determined by the log spiral (McLeod) analysis (7) for a contact pressure of 200 psi is presented. It should be pointed out that the Smith

TABLE 2

FRICTION, COHESION, AND STABILITY DATA[1]

Percent Asphalt Cement	Friction Angle, ϕ	Cohesion, C			Stability			
		Mohr Diagram	Cohesiometer, gm in.	psi	Marshall lb	Flow	Hveem, %	Equation, psi
4	-	-	-	-	2733	12	-	-
	-	-	-	-	2095	14	-	-
	41.4	6	232	5.5	-	-	29.8	144
	39.4	5	260	6.3	-	-	25.0	146
	34.1	6	232	5.5	-	-	24.4	94
	38.9	6	252	6.2	-	-	21.3	139
5	-	-	-	-	-	-	-	-
	43.0	6	397	9.4				
	42.8	6	350	8.3	-	-	37.4	238
	-	-	-	-	1625	17	-	-
	-	-	-	-	1950	15	-	-
	42.6	7	336	8.3	-	-	29.2	234
6	38.4	5	393	9.6	-	-	31.6	210
	-	-	-	-	2175	17	-	-
	32.3	8	382	8.9	-	-	24.8	134
	45.1	7	354	9.1	-	-	25.7	303
	-	-	-	-	1637	13	-	-
	41.0	9	345	8.7	-	-	26.4	232
7	25.1	9	498	12.3	-	-	13.1	134
	14.7	8	453	11.6	-	-	6.2	81
	-	-	-	-	2233	24	-	-
	-	-	-	-	1870	22	-	-
	29.7	8	464	11.1	-	-	20.1	152
	42.1	7	302	7.4	-	-	20.7	202

[1] From Paquette.

triaxial method specifies testing at 75 F, whereas the work of Paquette and others testing by the Marshall and Hveem procedures employ 140 F as a test temperature.

An interesting observation of the Smith triaxial evaluation chart is that all mixtures having an angle of internal friction below 25 are unsatisfactory for flexible pavement surfaces. Also, inasmuch as most asphaltic concrete mixtures have a value of cohesion between 5 and 15 psi, the chart indicates the need for a high angle of internal friction, approaching 45 degrees.

It is apparent from this study that many asphaltic mixtures which have performed well under existing service conditions on our highways may not have performed so satisfactorily under contact pressures of 200 or 300 psi, which is a requirement imposed on asphaltic surfaces with modern high pressure plane tires. It is also apparent that the answer to greater stability in asphaltic surfaces must come from an increase in the resistance to deformation by greater cohesion in the asphaltic mixture. Many of the asphaltic mixtures which meet present standards of design have an angle of internal friction approaching 45 degrees. A small increase in cohesion to such a mixture will produce a considerable increase in stability (Table 3).

Thickness Design

The basic thickness design formula (Eq. 5) may be expressed

$$pQ = \gamma z\,(R-1) + cS \qquad (7)$$

in which

$$Q = \left[1 - \left(\frac{1}{1 + \left(\frac{a}{z}\right)^2}\right)^{3/2}\right]$$

$$R = \left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^2$$

$$S = \frac{4}{1 - \sin\phi}\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}}$$

TABLE 3

INFLUENCE OF COHESION ON STABILITY (BY COMPUTATION FROM EQ. 6)

Cohesion, c, psi	Stability, p, psi	
	$\phi = 30^0$	$\phi = 45^0$
5	70	165
10	140	330
15	210	495
20	280	-

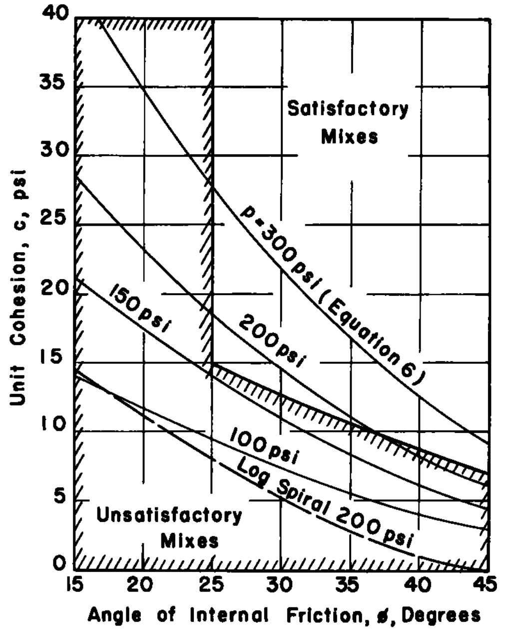

Figure 2. Smith closed-system triaxial compression test evaluation chart for asphaltic concrete, with curves superimposed for other stability equations.

Values of Q have been computed for a number of values of the ratio $\left(\frac{z}{a}\right)$ by Barber (8) and are presented in both graphical and tabular form in Figure 3. Values of R and S have been computed by the writer for a number of values of angle of internal friction (ϕ) and are shown in Figure 4 in both tabular and graphical form.

Eq. 7 may be solved by trial for the determination of thickness for a given design situation. A few examples follow.

Example 1

Given: P = 12,000 lb
p = 90 psi
$\phi = 10^0$
c = 2 psi

Determine thickness of cover required to protect given subgrade.

$$\frac{\pi d^2}{4} = \frac{12,000}{90}$$

$$d^2 = 170$$

$$d = 13$$

$$a = 6.5 \text{ in.}$$

From charts:

R = 2.0
S = 5.78

Assume density of surcharge material to be 130 lb/cu ft = 0.075 lb/cu in.

Trial 1.

$$z = 15 \text{ in.}; \frac{z}{a} = \frac{15}{6.5} = 2.3, \text{ and } Q = 0.23$$

then 90 x 0.23 = 0.075 x 15 x 1.0 + 2 x 5.78

20.7 = 1.12 + 11.56 = 12.68 ----- z is too small.

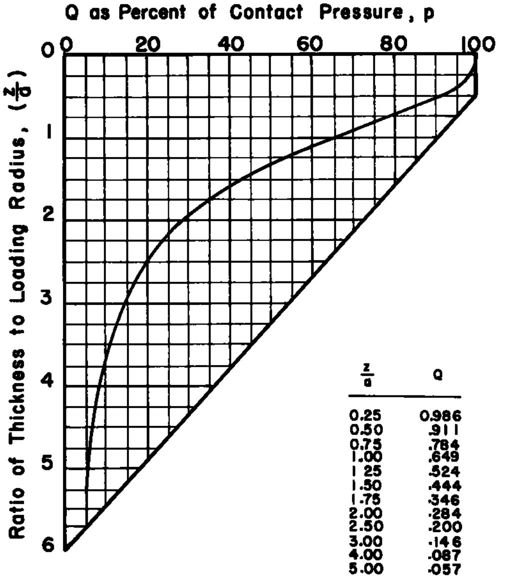

Figure 3. Vertical stress transmitted to a point on the axis in a semi-infinite mass from a surface load uniformly distributed over a circular area, expressed as a percent of surface load intensity.

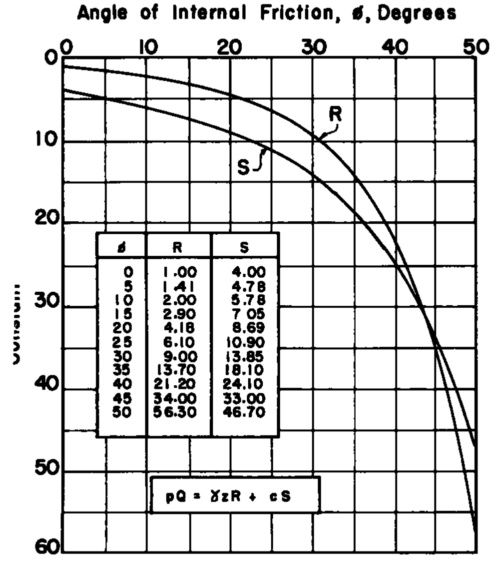

Figure 4. Values of "R" and "S" for use in flexible pavement design formula $pQ = \gamma zR + cS$, as a function of angle of internal friction, ϕ.

Trial 2.

$z = 20$ in.; $\frac{z}{z} = 3.08$, and $Q = 0.14$

then $90 \times 0.14 = 0.075 \times 20 \times 1.0 + 11.56$

$12.6 = 1.5 + 11.56 = 13.06$ ----- z is near correct value.

Trial 3.

$z = 19$ in.; $\frac{z}{a} = 2.92$, and $Q = 0.16$

then $90 \times 0.16 = 0.075 \times 19 \times 1.0 + 11.56$

$14.4 = 1.43 + 11.56 = 12.99$ ----- z is too small.

The design thickness of pavement structure over given subgrade would be 20 in., to the nearest inch.

	Examples				
	2	3	4	5	6
Load, P, lb	15,000	15,000	150,000	15,000	15,000
Pressure, p, psi	100	100	125	100	100
Radius, a, in.	6.9	6.9	19.6	6.9	6.9
Cohesion, c, psi	2.0	0	0	5.0	0
Friction angle, ϕ	0	30	10	40	40
Pavement thickness, z	28	22	42	0	5

The road materials in the foregoing examples might be generally classified as follows:

Example	Material Classification
1	Cohesive silt
2	Saturated soft clay
3	Cohesionless sand
4	Cohesionless silt
5	Asphalt concrete or crushed stone with binder
6	Crushed stone without binder

CONCLUSIONS

The basic design equation appears to give results in fairly close agreement with practice. It is evident, however, that in using this design equation a factor for abrasion from vehicle wheels is not included in the pavement resistance determination. On this basis, some granular materials with binder would satisfy the basic criteria for resistance to shear at the surface, as in example 5 for crushed stone; however, experience dictates that for roads of even moderate traffic the surface would not withstand the abrasion. Many soils may also be tested in a dry condition and show considerable shearing resistance but when subjected to the infiltration of water their shearing resistance would be reduced drastically.

There has been considerable discussion on the topic of stress distribution through various materials in a flexible pavement. Studies do show that in a layered system the degree of reduction of vertical stresses below the surface is a function of the relative strengths of the two materials. This would indicate that the vertical stress determined by the modified Boussinesq equation is not entirely correct. Without sufficient evidence to determine a modifying factor at this time, the author has eliminated this from the basic design equation. Should evidence in the future justify, the basic equation could be modified as follows:

$$pQD = \gamma z\ (R - 1) + cS \tag{8}$$

in which

D = a stress distribution factor.

Another factor deserving consideration in the design of the pavement would be the anticipated volume of heavy wheel loads and the degree of tracking to be expected on the completed structure. A few wheel loads moving along a fixed line may produce much more ultimate deformation in the pavement than many more load applications appropriately spaced over the pavement surface.

In the first case each application of load may add a small deformation to an already deformed section; whereas, in the second case each application of load may only knead the pavement such that it will retain a fairly smooth surface. As study is continued and experience is gained, the basic equation may be modified further as follows:

$$pQDT = \gamma z\ (R - 1) + cS \tag{9}$$

in which

T = a traffic factor.

Of particular importance in the design of bituminous mixtures is the rate of loading, both in testing operations and in field loading. It is possible that the shearing resistance for asphaltic materials as determined from Eq. 6 may be reduced for static loading and increased for dynamic loading, when the shear constants are determined in the laboratory by standard testing procedures. Eq. 6 may include a factor to adjust for the rate or type of loading, as follows:

$$p = \frac{4cL}{1 - \sin\phi}\left[\frac{1 + \sin\phi}{1 - \sin\phi}\right]^{\frac{1}{2}} \tag{10}$$

in which

L = a factor to adjust for rate of loading.

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