Rapid Method for Estimating Maximum Bending Stress in Simple-Span Highway Bridges

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This paper presents a new and rapid method for estimating maximum combined bending stresses in simple-span beam bridges produced by dead load, live load and impact. The maximum live load and impact stresses are those resulting from any heavy vehicle type or loading found on the highway.

The procedure for estimating the total maximum stress in simple-span bridges—of given construction type and design designation—is accomplished in two simple steps: (a) convert any particular vehicle under consideration into its equivalent H truck loading on any span, as may be desired, by use of the conversion coefficients given in Table 1; and (b) once the H-equivalency of a given vehicle has been determined for a given span then its stress-producing effects may be determined from charts similar to those shown in Figures 3 to 14, inclusive, depending on the span and loading conditions.

THE METHOD presented for estimating the maximum combined bending stresses
oduced by dead load, vehicle loads and impact in simple-span beam bridges results
om three rather simple but important observations. Briefly, these observations are:

1. It has been shown that any heavy vehicle may be converted into an equivalent H
uck loading that will produce the same maximum bending moment on a given span as
particular vehicle under consideration (1, 4, 5, 7). Equivalent H truck loadings,
erefore, provide a convenient means for describing or evaluating the stress-produc-
effects of any particular vehicle on a given span. It might be added that heavy ve-
les also may be converted into any other equivalent design loading on the basis of
ments, shears or any other stress function as may be desired (1, 2, 3).

2. It has been found, for any simple-span beam bridge of given length, that the per-
t of total design moment per beam caused by dead load remains about the same,
espective of the lateral spacing of the beams (4, 7). Similarly, for a given span,
percent of total design moment (or stress) per beam caused by live load plus impact
o remains about the same, irrespective of the beam spacing (4, 7). For a given span,
refore, it follows that the percentage of total design moment (or stress) caused by
and dead loads, respectively, is about the same per foot width of bridge or per 10-
ne as it is per beam, irrespective of the beam spacing. These findings permit the
and dead load design moments (or stresses) per beam or per 10-ft lane to be gen-
lized for simple-span beam bridges of given construction type and design designa-
as shown in Figure 2.

3. For a given bridge the maximum dead load stress is a fixed and definite percent of the total design stress; also, the maximum live load plus impact stress caused by a given vehicle will vary directly with the weight or H-equivalency of that particular vehicle. From these and the preceding observations it has been shown that the total maximum stress caused by dead load, vehicle load and impact in a given bridge may be expressed by a simple straight-line equation in which the total maximum stress is a function of the H-equivalency of the vehicle under consideration (4, 7). It is usually more convenient though to convert this total maximum stress into the ratio that it bears to the allowable stress for which the bridge was designed. This ratio is referred to herein as the design stress ratio. The variations of this design stress ratio with equivalent H truck loadings on various spans are as shown in Figures 3 - 14 inclusive.

DEVELOPMENT OF METHOD FOR ESTIMATING MAXIMUM BENDING STRESSES

The bridges used herein for illustrating the method are of H 15 design and consist of a concrete deck of minimum thickness supported by unencased steel beams. It is also assumed that the supporting steel beams are so spaced that the maximum live load bending stress produced in an interior stringer by a single vehicle in one lane only, will amount to C = 75 percent of that produced by indentical vehicles in each lane simultaneously. This means that if the given bridge were loaded with vehicles having identical H-equivalencies, one in each lane, the maximum live load stress produced in a typical interior stringer would be 4/3 or 133 percent of that produced by only one of these vehicles in one lane only.

The reason for selecting this light type of construction is that the ratio of dead load stresses to total design stresses is smaller than would be the case for any of the heavi types of construction, such as reinforced concrete deck girder spans. Consequently, any conclusions arrived at concerning the stress-producing effect of a given vehicle or vehicles on any particular bridge are on the conservative rather than the unsafe side. Although the discussion and illustrative examples given herein are confined to bending moments and bending stresses in simple-span steel beam bridges of H 15 design, the method is equally applicable to bridges of other construction types and design designation.

Once the percent of total design stresses caused by live load plus impact and dead load have been determined for bridges of a given type and design designation similar t those given in Figure 2 it is convenient to consider the method for estimating total maximum bending stress in a given bridge in two parts, as follows: (1) determination of equivalent H truck loadings, and (2) evaluation of total maximum stress caused by a given equivalent H truck loading on a given span corresponding with specified loading conditions.

The nomenclature and definitions used herein are assembled in Appendix A for convenience of reference.

EQUIVALENT H TRUCK LOADINGS

Any heavy vehicle may be converted into an equivalent H truck loading that will pr duce the same maximum bending moment on a given span as the particular vehicle un consideration. Heavy vehicles also may be converted into any other equivalent desig loading on the basis of moments, shears or other stress function on various span leng as may be desired (1, 2, 3).

The H-equivalency of a given vehicle on a given span may be determined on an exa basis by finding the maximum moment caused by this particular vehicle on the given span and then selecting the standard H truck designation in tons that would produce th same maximum moment. The procedure for any other stress function would be simi

It has been found, however, from numerous investigations of actual vehicles, irrespective of the number of spacing of axles, that any normal distribution of load among the axles of a given vehicle will produce slightly less moment on a given span t the same load would produce if it were uniformly distributed over a length, L, equal the wheel base length of the vehicle under consideration (5, 7). This means that the maximum moment caused by any given vehicle on a given span can be estimated quit easily and accurately—but a little on the safe side—by the moment formula resulting from the uniform load of length, L, and weight, W, on span, S (Fig. 1). This loading results in the following formula for maximum moment:

$$M = \frac{W}{4}\left(S - \frac{L}{2}\right) \qquad (1)$$

Eq. 1 provided the basis for calculating the coefficients given in Table 1 for converting heavy vehicles of given weight and wheel base length into equivalent H truck loadings on various span lengths.

The determination and use of the coefficients given in Table 1 can be illustrated by comparing the maximum moment caused by a heavy vehicle weighing 20 tons and having a total wheel base length of 28 ft with that caused by an H 20 truck on a 50-ft simple span. According to Eq. 1 the moment caused by the heavy vehicle would be 360.0 kip-feet. This compares with a moment of 445.6 kip-feet caused by an H 20 truck on a 50-ft span. The 20-ton heavy vehicle with 28-ft wheel base, therefore, causes 80.79 percent as much moment as the H 20 truck on this 50-ft span.

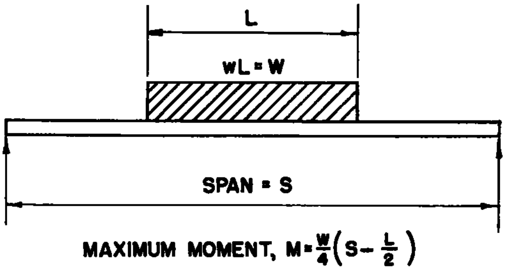


Figure 1. Maximum moment caused by a gross weight of W, uniformly distributed over a length L, on a span length of S.

This means that a vehicle, with a 28-ft wheel base, will cause 0.8079 times as

TABLE 1

COEFFICIENTS FOR CONVERTING ANY HEAVY VEHICLE OF GIVEN WEIGHT AND WHEEL-BASE LENGTH INTO EQUIVALENT H TRUCK LOADINGS ON SIMPLE SPANS

Wheel Base, L (ft)	Coefficient								
	20-Ft Span	30-Ft Span	40-Ft Span	50-Ft Span	60-Ft Span	70-Ft Span	80-Ft Span	90-Ft Span	100-Ft Span
4	1.1250	1.1354	1.0982	1.0772	1.0638	1.0541	1.0473	1.0416	1.0373
8	1.0000	1.0543	1.0404	1.0323	1.0271	1.0231	1.0204	1.0179	1.0161
12	0.8750	0.9732	0.9826	0.9874	0.9905	0.9921	0.9936	0.9942	0.9949
16	0.7500	0.8921	0.9248	0.9425	0.9538	0.9611	0.9667	0.9706	0.9738
20	0.6250	0.8110	0.8670	0.8977	0.9171	0.9301	0.9398	0.9469	0.9526
24	-	0.7299	0.8092	0.8528	0.8804	0.8991	0.9130	0.9232	0.9314
28	-	0.6488	0.7514	0.8079	0.8437	0.8681	0.8861	0.8995	0.9102
32	-	-	0.6936	0.7630	0.8070	0.8371	0.8593	0.8759	0.8891
36	-	-	0.6358	0.7181	0.7704	0.8061	0.8324	0.8522	0.8679
40	-	-	0.5780	0.6732	0.7337	0.7751	0.8056	0.8285	0.8467
44	-	-	-	0.6284	0.6970	0.7440	0.7787	0.8048	0.8256
48	-	-	-	0.5835	0.6603	0.7130	0.7519	0.7812	0.8044
52	-	-	-	-	0.6236	0.6820	0.7250	0.7575	0.7832
56	-	-	-	-	0.5869	0.6510	0.6982	0.7338	0.7621
60	-	-	-	-	0.5503	0.6200	0.6713	0.7102	0.7409

OTE: The H-equvalency of any vehicle of given weight and wheel base length on a iven span, is equal to the weight of the vehicle times the conversion coefficient for at span corresponding to the given vehicle's wheel base length. For example, a 20-n vehicle with a 28-ft wheel base on a 50-ft span would have an H-equivalency of 20.0 0.8079 = 16.16 tons or correspond with an equivalent H 16.16 truck loading.

much moment on a 50-ft span as a standard H truck of equal weight. It also means that a vehicle of given weight, with a 28-ft wheel base, will cause as much moment on a 50-ft span as a standard H truck weighing 80.79 percent as much. This 20-ton vehicle, with a 28-ft wheel base on a 50-ft span, therefore, would have an H-equivalency of 20.0 x 0.8079 = 16.16 tons or correspond with an equivalent H 16.16 truck on that span.

Design Stress Relationships in Simple-Span Steel Stringer Bridges of H 15 Design

As previously stated, it has been found that for any simple-span beam bridge of given length, the percent of total design stress (or moment) per beam caused by dead load remains about the same, irrespective of the lateral spacing of the beams (4, 7). Similarly for a given span, the percent of total design stress per beam caused by live load plus impact also remains about the same, irrespective of the beam spacing. For a given span, therefore, it follows that the percent of total design stress (or moment) caused by live and dead loads, respectively, is about the same per foot width of bridge or per 10-ft lane as it is per beam, irrespective of beam spacing. These findings permit the live and dead load design stresses (or moments) per beam or per 10-ft lane to be generalized for simple-span beam bridges, of given construction type and design designation, similar to those shown in Figure 2.

With design stress information for simple-span bridges, of given construction type and design designation, similar to that given in Figure 2, it has been shown that the total maximum stresses caused by dead load, vehicle load and impact for a given bridge may be expressed by a simple straight-line equation in which the total maximum stress or design stress ratio is a function of the H-equivalency of the vehicle under consideration (4, 7). Design stress ratio for a given member is defined as the ratio of actual total maximum stress caused by dead load, vehicle load and impact in the member, to the maximum allowable stress used for the design of that member.

A further discussion of design stress ratios is given in Appendix B. Also in Appendix B is given the development of the straight-line equations for estimating design stress ratios (or maximum stresses) caused by dead load, vehicle loads and impact for given spans corresponding with various loading conditions. The development of the straight-line equations in Appendix B follows rather closely the development of similar equations previously presented (4, 7).

Estimating Maximum Bending Stresses Caused by Equivalent H Trucks

Inasmuch as Figure 2 gives the ratio of dead load stress to total design stress, R_D, and the ratio of live load plus impact stress to total design stress, R_L, it will be seen that the equivalent H truck loading corresponding with any degree of overstress or understress (design stress ratio, Q) and loading conditions may be determined by Eq. 12. Tables 2 through 5 give the equivalent H truck loadings, in tons, required to produce maximum bending stresses in an interior stringer, cor-

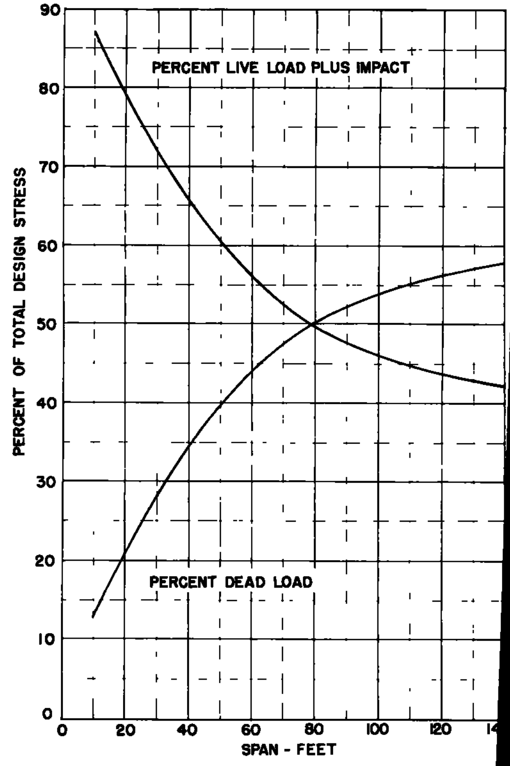


Figure 2. Estimated percent of total d sign stresses represented by live lo plus impact and dead load stresses f simple-span beam bridges of H 15 design

responding to a given design stress ratio, for four different conditions of loading.

Referring to Eq. 13, it will be seen that the design stress ratio, Q, is a linear equation. For any given member of a particular bridge it will also be seen that the change in Q varies directly with the values of H, C, and K' in Eq. 13. This is illustrated in Figures 3 to 14. Figures 3 to 8 give the design stress ratios produced by equivalent H trucks on simple-span steel stringer bridges of H 15 design with one vehicle in each lane and varying allowance for impact. Figures 9 to 14, give the design stress ratios produced by equivalent H trucks on simple-span steel stringer bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

On a 50-ft span, for example, Figure 5 shows that one equivalent H 30 truck in each lane simultaneously (C = 1.00) with full allowance for impact would result in a maximum design stress ratio Q = 1.60. This means that the maximum stress produced in one of the interior steel stringers by such a loading would be 160 percent of the basic allowable design stress, or an overstress of 60 percent. However, if the speed of these equivalent H 30 trucks were reduced to about 5 mph, which would result in little or no impact, it will be seen that the maximum amount of overstress in an interior stringer would be reduced to about 32 percent.

Similarly, on a 50-ft span Figure 11 shows that one equivalent H 30 truck in one lane only (C = 0.75) with full allowance for impact would result in a maximum design stress ratio, Q = 1.31, or an overstress of about 31 percent. However, if the speed of this equivalent H 30 truck were reduced so as to result in little or no impact, the maximum amount of overstress in an interior stringer would only amount to about 10 percent. These illustrations should be sufficient to show the value and utility of the stress data to be obtained from Figures 3 through 14.

Summary of Method

The preceding discussion shows that the maximum combined bending stresses caused by dead load, live load, and impact in simple-span beam bridges may be estimated rather quickly in two simple steps as follows:

TABLE 2

EQUIVALENT H TRUCK LOADING IN EACH LANE WITH FULL ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

= I   K' = 1.00 + I = K   C = 1.00

	Value of Factor									
actor	10-Ft Span	20-Ft Span	30-Ft Span	40-Ft Span	50-Ft Span	60-Ft Span	70-Ft Span	80-Ft Span	90-Ft Span	100-Ft Span
R_L	0.870	0.790	0.715	0.654	0.603	0.558	0.522	0.498	0.478	0.460
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.222
M_D	11.7	41.5	95.9	178.5	283.0	421.0	610.0	820.1	1062.6	1344.8
KM_L	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	89.7	197.5	336.4	515.9	712.8	952.5	1276.1	1633.7	2035.6	2490.4
esign ress tio, Q	Equiv. H Truck Loading									
.50	23.6	24.5	25.5	26.5	27.4	29.1	32.2	35.2	38.2	41.4
.40	21.9	22.6	23.4	24.1	24.9	26.4	29.0	31.7	34.3	37.1
.30	20.2	20.7	21.3	21.9	22.5	23.6	25.9	28.1	30.4	32.8
.20	18.5	18.8	19.2	19.6	20.0	20.9	22.7	24.6	26.5	28.5
.10	16.7	16.9	17.1	17.3	17.5	18.1	19.6	21.1	22.6	24.2
.00	15.0	15.0	15.0	15.0	15.0	15.4	16.4	17.6	18.7	19.8
90	13.3	13.1	12.9	12.7	12.5	12.6	13.3	14.0	14.8	15.5
80	11.6	11.2	10.8	10.4	10.0	9.8	10.1	10.5	10.9	11.2
70	9.8	9.3	8.7	8.1	7.5	7.1	7.0	7.0	7.0	6.9
60	8.1	7.4	6.6	5.8	5.0	4.3	3.8	3.5	3.1	2.6
50	6.4	5.5	4.5	3.5	2.6	1.6	0.7	-	-	-

TABLE 3

EQUIVALENT H TRUCK LOADING IN EACH LANE WITH NO ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

Γ = 0.00 K' = 1.00 + 0.00 = 1.00 C = 1.00

Factor	Value of Factor									
	10-Ft Span	20-Ft Span	30-Ft Span	40-Ft Span	50-Ft Span	60-Ft Span	70-Ft Span	80-Ft Span	90-Ft Span	100-Ft Span
R_L	0.870	0.790	0.715	0.654	0.603	0.558	0.522	0.498	0.478	0.460
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.222
M_D	11.7	41.5	95.9	178.5	283.0	421.0	610.0	820.1	1062.6	1344.8
KM_L	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	89.7	197.5	336.4	515.9	712.8	952.5	1276.1	1633.7	2035.6	2490.4

Design Stress Ratio, Q	Equiv. H Truck Loading									
1.50	30.7	31.8	33.2	34.4	35.3	37.0	40.4	43.8	47.1	50.6
1.40	28.5	29.4	30.4	31.4	32.1	33.5	36.5	39.4	42.3	45.3
1.30	26.3	26.9	27.7	28.4	28.9	30.0	32.5	35.0	37.5	40.1
1.20	24.0	24.4	25.0	25.5	25.7	26.5	28.6	30.6	32.7	34.8
1.10	21.8	22.0	22.2	22.5	22.5	23.0	24.6	26.2	27.8	29.5
1.00	19.5	19.5	19.5	19.5	19.3	19.5	20.6	21.8	23.0	24.2
0.90	17.3	17.0	16.8	16.5	16.1	16.0	16.7	17.5	18.2	19.0
0.80	15.0	14.6	14.1	13.5	12.9	12.5	12.7	13.1	13.4	13.7
0.70	12.8	12.1	11.3	10.6	9.7	9.0	8.8	8.7	8.6	8.4
0.60	10.5	9.6	8.6	7.6	6.5	5.5	4.8	4.3	3.8	3.2
0.50	8.3	7.2	5.9	4.6	3.3	2.0	0.9	-	-	-

TABLE 4

EQUIVALENT H TRUCK LOADING IN ONE LANE WITH FULL ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

Γ = I K' = 1.00 + I = K C = 0.7

Factor	Value of Factor									
	10-Ft Span	20-Ft Span	30-Ft Span	40-Ft Span	50-Ft Span	60-Ft Span	70-Ft Span	80-Ft Span	90-Ft Span	100-F Span
R_L	0.870	0.790	0.715	0.654	0.603	0.558	0.522	0.498	0.478	0.4
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.2
M_D	11.7	41.5	95.9	178.5	283.0	421.0	610.0	820.1	1062.6	1344.8
KM_L	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	89.7	197.5	336.4	515.9	712.8	952.5	1276.1	1633.7	2035.6	2490.4

Design Stress Ratio, Q	Equiv. H Truck Loading									
1.50	31.5	32.6	34.0	35.3	36.6	38.8	42.9	46.9	51.0	55.
1.40	29.2	30.1	31.2	32.2	33.2	35.1	38.7	42.2	45.8	49.
1.30	26.9	27.6	28.4	29.2	29.9	31.5	34.5	37.5	40.6	43.
1.20	24.6	25.1	25.6	26.1	26.6	27.8	30.3	32.8	35.4	38.
1.10	22.3	22.5	22.8	23.1	23.3	24.1	26.1	28.1	30.1	32.
1.00	20.0	20.0	20.0	20.0	20.0	20.5	21.9	23.4	24.9	26.
0.90	17.7	17.5	17.2	16.9	16.7	16.8	17.7	18.7	19.7	20.
0.80	15.4	14.9	14.4	13.9	13.4	13.1	13.5	14.0	14.5	15.
0.70	13.1	12.4	11.6	10.8	10.0	9.5	9.3	9.3	9.3	9.
0.60	10.8	9.9	8.8	7.8	6.7	5.8	5.1	4.6	4.1	3.
0.50	8.5	7.3	6.0	4.7	3.4	2.1	0.9	-	-	-

TABLE 5

EQUIVALENT H TRUCK LOADING IN ONE LANE WITH NO ALLOWANCE FOR IMPACT REQUIRED TO PRODUCE MAXIMUM STEEL STRESS CORRESPONDING TO GIVEN DESIGN STRESS RATIO

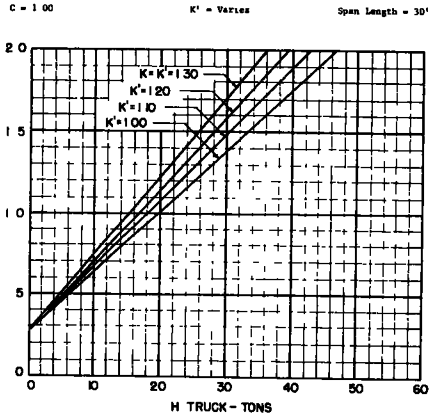
I' = 0.00 K' = 1.00 + 0.00 = 1.00 C = 0.75

	Value of Factor									
Factor	10-Ft Span	20-Ft Span	30-Ft Span	40-Ft Span	50-Ft Span	60-Ft Span	70-Ft Span	80-Ft Span	90-Ft Span	100-Ft Span
R_L	0.870	0.790	0.715	0.654	0.603	0.558	0.522	0.498	0.478	0.460
K	1.30	1.30	1.30	1.30	1.286	1.27	1.256	1.244	1.232	1.222
M_D	11.7	41.5	95.9	178.5	283.0	421.0	610.0	820.1	1062.6	1344.8
KM_L	78.0	156.0	240.5	337.4	429.8	531.5	666.1	813.6	973.0	1145.6
M_T	89.7	197.5	336.4	515.9	712.8	952.5	1276.1	1633.7	2035.6	2490.4

Design Stress Ratio, Q	Equiv. H Truck Loading									
1.50	41.0	42.4	44.2	45.9	47.0	49.3	53.9	58.4	62.8	67.4
1.40	38.0	39.1	40.6	41.9	42.8	44.6	48.6	52.5	56.4	60.4
1.30	35.0	35.9	36.9	37.9	38.5	40.0	43.3	46.7	50.0	53.4
1.20	32.0	32.6	33.3	34.0	34.2	35.3	38.1	40.8	43.6	46.4
1.10	29.0	29.3	29.7	30.0	30.0	30.7	32.8	35.0	37.1	39.3
1.00	26.0	26.0	26.0	26.0	25.7	26.0	27.5	29.1	30.7	32.3
0.90	23.0	22.7	22.4	22.0	21.4	21.3	22.3	23.3	24.3	25.3
0.80	20.0	19.4	18.7	18.1	17.2	16.7	17.0	17.4	17.9	18.3
0.70	17.1	16.1	15.1	14.1	12.9	12.0	11.7	11.6	11.4	11.2
0.60	14.1	12.8	11.5	10.1	8.7	7.4	6.4	5.7	5.0	4.2
0.50	11.1	9.5	7.8	6.1	4.4	2.7	1.2	-	-	-

1. Convert the heavy vehicle under consideration into its equivalent H truck loading n a given span by use of the appropriate coefficient in Table 1.

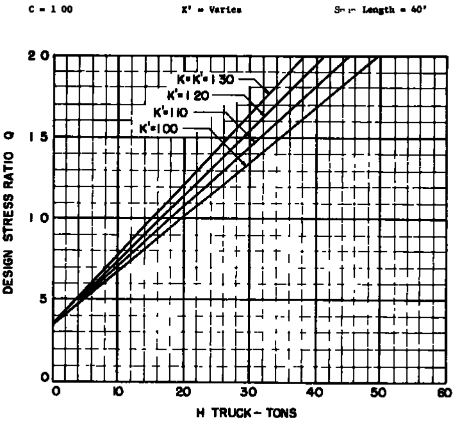
2. With the H-equivalency found in the first step, an estimate of the bending stress-s caused by it on the given span may be read directly from the appropriate chart iven in Figures 3 through 14, depending on the span length and loading conditions.



Equations for Design Stress Ratio

K = K' = 1 300, Q = 0476 H + 285
K' = 1 200, Q = 0439 H + 285
K' = 1 100, Q = 0403 H + 285
K' = 1 000, Q = 0366 H + 285

gure 3. Design stress ratio produced by uivalent H trucks on simple-span bridges H 15 design with one vehicle in each ane and varying allowance for impact.

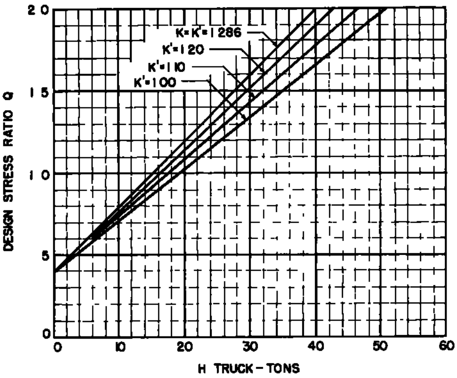


Equations for Design Stress Ratio

K = K' = 1 300, Q = 0436 H + 346
K' = 1 200, Q = 0402 H + 346
K' = 1 100, Q = 0369 H + 346
K' = 1 000, Q = 0335 H + 346

Figure 4. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

C = 1 00 K' = Varies Span Length = 50'

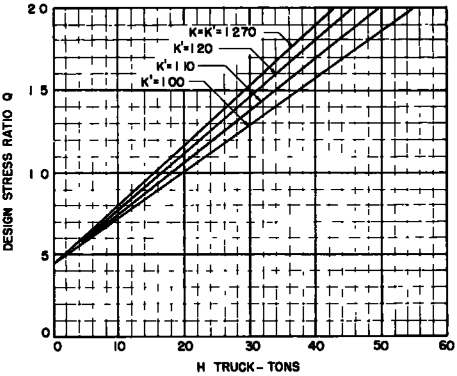


Equations for Design Stress Ratio

K = K' = 1 286, Q = 0402 H + 397
K' = 1 200, Q = 0376 H + 397
K' = 1 100, Q = 0344 H + 397
K' = 1 000, Q = 0313 H + 397

Figure 5. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

C = 1 00 K' = Varies Span Length = 60'

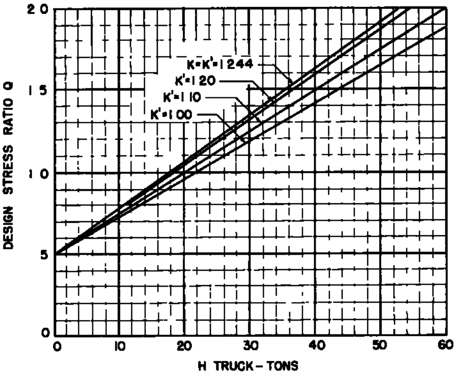


Equations for Design Stress Ratio

K = K' = 1 270, Q = 0364 H + 442
K' = 1 200, Q = 0343 H + 442
K' = 1 100, Q = 0315 H + 442
K' = 1 000, Q = .0286 H + 442

Figure 6. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

C = 1 00 K' = Varies Span Length = 80'

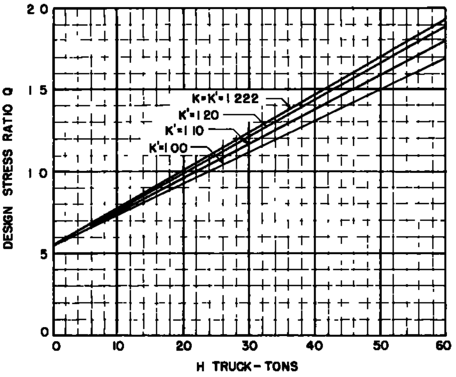


Equations for Design Stress Ratio

K = K' = 1 244, Q = 0284 H + 502
K' = 1 200, Q = 0274 H + 502
K' = 1 100, Q = 0251 H + 502
K' = 1 000, Q = 0278 H + 502

Figure 7. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.

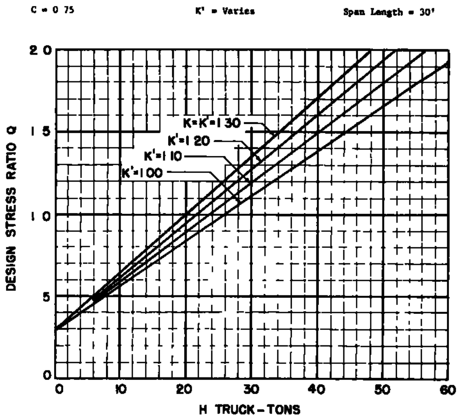
C = 1 00 K' = Varies Span Length = 100



Equations for Design Stress Ratio

K = K' = 1 222, Q = 0232 H + 540
K' = 1 200 Q = 0228 H + 540
K' = 1 100, Q = 0209 H + 540
K' = 1 000, Q = 0190 H + 540

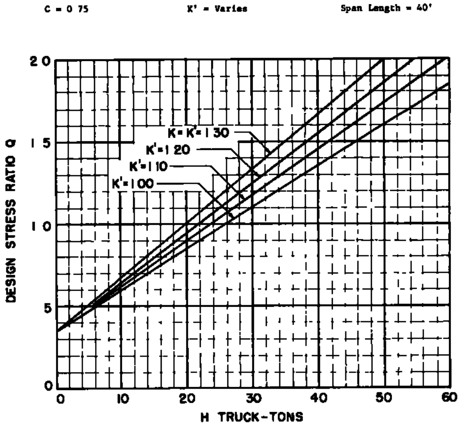
Figure 8. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in each lane and varying allowance for impact.



Equations for Design Stress Ratio

K = K' = 1 300, Q = 0357 H + 285
K' = 1 200, Q = 0330 H + 285
K' = 1 100, Q = 0303 H + 285
K' = 1.000, Q = 0275 H + 285

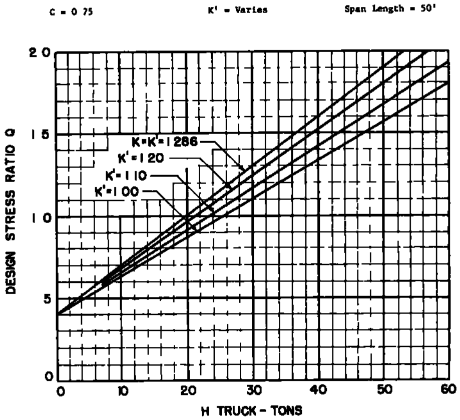
Figure 9. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.



Equations for Design Stress Ratio

K = K' = 1 300, Q = 0327 H + 346
K' = 1 200, Q = 0302 H + 346
K' = 1 100, Q = 0277 H + 346
K' = 1 000, Q = 0252 H + 346

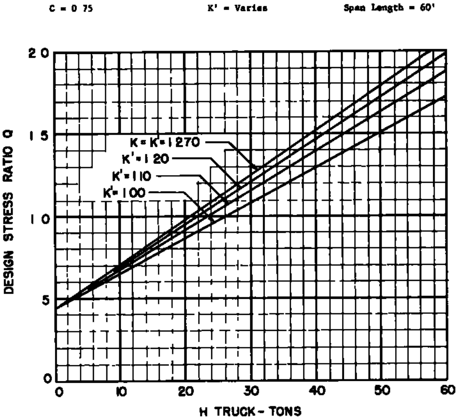
Figure 10. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.



Equations for Design Stress Ratio

K = K' = 1 286, Q = 0302 H + 397
K' = 1 200, Q = 0282 H + 397
K' = 1 100 Q = 0259 H + 397
K' = 1 000, Q = 0235 H + 397

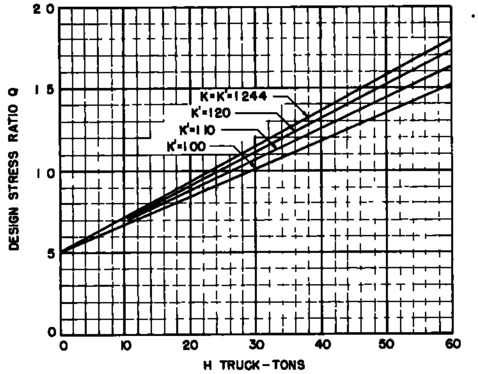
igure 11. Design stress ratio produced by quivalent H trucks on simple-span bridges f H 15 design with one vehicle in one ane only and varying allowance for impact.



Equations for Design Stress Ratio

K = K' = 1 270, Q = 0273 H + 442
K' = 1 200, Q = 0258 H + 442
K' = 1 100, Q = 0237 H + 442
K' = 1 000, Q = 0215 H + 442

Figure 12. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

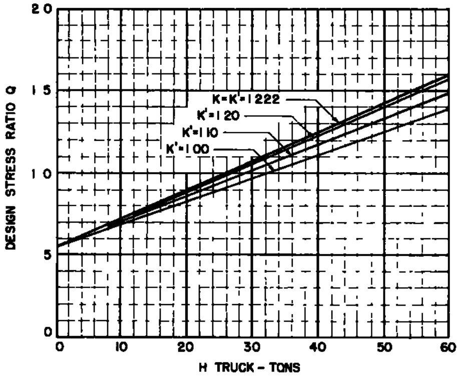


C = 0 75   K' = Varies   Span Length = 80'

Equations for Design Stress Ratio

K = K' = 1 244, Q = 0213 H + 502
K' = 1 200, Q = 0205 H + 502
K' = 1 100, Q = 0188 H + 502
K' = 1 000, Q = 0171 H + 502

Figure 13. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.



C = 0 75   K' = Varies   Span Length = 100'

Equations for Design Stress Ratio

K = K' = 1 222, Q = 0174 H + 540
K' = 1 200, Q = 0170 H + 540
K' = 1 100, Q = 0156 H + 540
K' = 1 000, Q = 0142 H + 540

Figure 14. Design stress ratio produced by equivalent H trucks on simple-span bridges of H 15 design with one vehicle in one lane only and varying allowance for impact.

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Appendix A

NOMENCLATURE AND DEFINITIONS

C = coefficient which represents the fractional part of the total live load stress, in a given member, produced by one or more lanes loaded, C = 1.00 if a stringer bridge is loaded with identical vehicles, one in each lane and so placed to produce maximum stress. For a steel stringer bridge, if one vehicle in one lane only would produce 75 percent as much stress in an interior stringer as identical vehicles in each lane it would mean that C = 0.75.

f = unit stress in pounds per square inch or unit stress as may be defined, f_D = unit stress resulting from dead load; f_L = unit stress resulting from live load; f_T = maximum total design stress; f_H = stress resulting from vehicle or vehicles weighing H tons each.

H = equivalent H truck in tons. For example, if a given vehicle produces the same maximum moment (or other stress function) in a given member as a standard H truck weighing 23.6 tons, it would be rated as an equivalent H 23.6 truck loading in which case H = 23.6 tons.

I = impact fraction (maximum 0.30 or 30 percent) as determined by the AASHO formula $I = 50/(S + 125)$ in which S = length in feet of the portion of the span which is loaded to produce the maximum stress in the member.

I' = impact fraction assumed in connection with the determination of the stress-producing effects of any given vehicle under consideration. For example, if the speed of a given vehicle were limited to about 5 mph, this impact fraction might be considered so small as to be negligible, in which case I' might be assumed equal to zero. Depending on traffic and conditions, therefore, the impact fraction, I' could be assumed at any reasonable value between zero and the full impact allowance, I, as defined by the AASHO design specifications.

K = (1.00 + I) = coefficient by which the design live load moment (shear, or other stress function is multiplied to obtain the live load plus impact moment (shear or other stress function) used for design. Thus, KM_L would be equal to the live load plus impact moment used for design; similarly KV_L would be equal to the live load plus impact shear used for design.

K' = (1.00 + I') = coefficient by which the live load moment (shear, or other stress function) produced by a given vehicle is multiplied to obtain the live load plus impact moment (shear, or other stress function) produced on a given span or in a given member by the vehicle under consideration. Thus, $K'M_H$ would be equal to the live load plus impact moment produced on a given span by any particular vehicle having an H-equivalency of H tons.

M_D = dead load moment as included in total design moment.

M_H = moment in an interior stringer (or other member) resulting from equivalent H trucks weighing H tons each. Likewise M_H represents the moment for 1 lane produced by equivalent H truck weighing H tons.

$M_{H(1)}$ = moment for one lane produced by a standard H truck weighing 1 ton.

M_L = live load moment as included in total design moment.

M_T = moment used for design or total design moment.

Q = design stress ratio. This term refers to the ratio of total maximum stress, caused by dead load, vehicle load and impact, to total design stress in any particular member of part of a given highway bridge.

R_D = (M_D/M_T) = ratio of dead load moment M_D (shear, or other stress function) to total moment M_T used for design. In terms of shear this ratio would be $R_D = (V_D/V_T)$, and for other stress functions it would be similar.

R_L = (KM_L/M_T) = ratio of live load plus impact moment, KM_L, (shear, or other stress function), used for design, (to the total design moment, M_T, or total moment (shear, or other stress function) used for design. In terms of shear, this ratio would be R_L = (KV_L/V_T), and for other stress functions it would be similar.
S = span length (usually in feet).
V = shear force.

Appendix B

DEVELOPMENT OF EQUATIONS FOR DESIGN STRESS RATIOS

Design Stress Ratios

Design stress ratios, Q, refer to the ratios of total actual stresses to total design stresses in any particular member or part of a given highway bridge. For example, consider a 50-ft simple-span steel stringer bridge with concrete deck of H-15 design. If the design calculations for this bridge show that the dead load produces a maximum stress of 7.20 ksi and the design live load plus impact produces a maximum stress of 10.80 ksi in an interior stringer, the total design stress for this stringer is 7.20 + 10.80 = 18.00 ksi.

If further calculations indicate that a particular heavy vehicle load would produce a maximum live load plus impact stress of K f_H = 16.20 ksi (see Appendix A for nomenclature) it will be seen that the maximum total actual stress in this stringer would be 7.20 + 16.20 = 23.40 ksi. So, the ratio of total actual stress to total design stress for this situation results in a design stress ratio of Q = 23.40/18.00 = 1.30. This means that the heavy vehicle under consideration would result in total actual stresses 1.30 times as much as the total basic design stress of 18.00 ksi for which this stringer was designed or an overstress of 30 percent in excess of the basic design permitted by the AASHO design specifications.

Each of the many heavy vehicle types and loadings has one thing in common—the capacity to induce a stress (bending, shear, or direct stress) of definite and calculable magnitude at any particular point in a given bridge. Consequently a bridge of given type and span can be made to serve as a sort of weighing device by which the maximum stress (bending, shear, or direct stress) produced by any given heavy vehicle can be directly compared with that produced by any other vehicle or arbitrarily standardized loading. However, rather than directly comparing the actual stresses produced by a given heavy vehicle with those produced by others, it is more convenient to appraise the stress-producing effects of a given vehicle if they are expressed in terms of some arbitrary or standardized loading on a simple span of given length.

For this purpose a standard H truck, H-S truck, or any other arbitrary loading, could be used. The standard H truck loading is used herein as a basis for measuring the stress-producing characteristics of all other vehicles because the load-carrying capacities of most existing highway bridges are rated in terms of the H loading desig And, as previously mentioned, bending moment is the stress function used to illustrate the method for measuring overstress because it is the bending stresses that ordinarily determine the load-carrying capacity of most highway bridges.

On a 50-ft simple span, for example, if it were determined that a given heavy vehicle produced a maximum live load moment of 445.6 kip-ft, with no allowance for impact, it would be found to be the same as the maximum live load moment produced by H-20 truck on the same span. Based on its capacity to produce bending stresses in a simple span of 50 ft the given vehicle would be converted into or rated as an equivalent H truck load weighing 20 tons, or simply an equivalent H-20 truck loading. In a similar manner, if a given heavy vehicle produced as much direct stress in a particul member of a given through truss bridge as an H-21.6 truck, it would be rated as an equivalent H-21.6 truck loading insofar as its capacity to produce direct stress in tha particular member is concerned. The logic would be similar for any type of stress o stress function at any point that might be of interest in any type of simple-span or continuous bridge. The manner in which these equivalent design loads can be used fo determining the degree of overstress, or design stress ratio, produced by any given vehicle at some particular point in a given bridge is presently explained.

Development of Equations

The stress relationships in the 50-ft simple-span steel stringer bridge of H-15 design referred to previously provides a convenient basis for illustrating the development of equations relating to design stress ratios. A study of the stresses in this bridge, and how they are related to each other, shows how such relationships provide a basic and necessary tool for the further investigation of maximum stresses in highway bridges (also see 4). For this 50-ft bridge the design calculations show that the dead load produces a maximum stress of 7.20 ksi and the design live load plus impact produces a maximum stress of 10.80 ksi, or a maximum total design stress of 7.20 + 10.80 = 18.00 ksi, in a typical interior stringer. In accord with the nomenclature given in Appendix A, it will be seen from these data that the dead load ratio, R_D, which is defined as the ratio that the maximum dead load stress, f_D, bears to the maximum total design stress, f_T, would be

$$R_D = \frac{f_D}{f_T} = \frac{M_D}{M_T} = \frac{7.20}{18.00} = 0.400 \qquad (2)$$

Similarly, it will be seen that the live load ratio, R_L, which is defined as the ratio that the maximum live load plus impact stress, Kf_L (moment, $K\,M_L$; shear, $K\,V_L$; or other stress function), bears to the maximum total design stress, f_T (moment, M_T; shear, V_T; or other stress function) would be

$$R_L = \frac{K\,f_L}{f_T} = \frac{K\,M_L}{M_T} = \frac{10.80}{18.00} = 0.600 \qquad (3)$$

But because the sum of the design dead load, live load and impact stresses for a given member is equal to the total design stress, the sum of the dead load and live load ratios must equal 1.00, and it follows that

$$R_D + R_L = \frac{f_D + K\,f_L}{f_T} = \frac{7.20 + 10.80}{18.00} = 0.400 + 0.600 = 1.00 \qquad (4)$$

Similarly, if these ratios were defined in terms of moments for an interior stringer r in terms of moments for a full lane, which would be proportional to those in the tringer, their values would remain the same and their sum equal 1.00. Thus

$$R_D + R_L = \frac{M_D + K\,M_L}{M_T} = 0.400 + 0.600 = 1.00 \qquad (5)$$

In Eq. 4 the maximum stress, f_L, in one of the interior stringers is produced by the tandard design live loading (without impact), which for this 50-ft span consists of one andard H-15 truck in each lane, simultaneously: K is the coefficient by which the live ad stress, f_L, is increased to include the specified allowance for impact. That is,

$$K = 1.00 + I \qquad (6)$$

which I is the impact fraction as determined by the AASHO formula

$$I = 50/(S + 125) \qquad (7)$$

d S is the length in feet of the span which is loaded to produce maximum stress in e member under consideration (for simple spans, S = full length of span).

For the 50-ft simple span the impact would amount to I = 50/(50 + 125) = 0.286, ich in turn would result in a coefficient K = 1.00 + 0.286 = 1.286. As previously ted, the design live load plus impact produces a maximum stress in an interior inger of $K\,f_L$ = 10.80 ksi. Inasmuch as this value includes an allowance of 28.6

percent for impact, it will be seen that the design live load stress without impact would be $f_L = 10.80/1.286 = 8.40$ ksi.

It was also stated previously that further calculations indicated that a particular heavy vehicle would produce a maximum live load plus impact stress of $K\,f_H = 16.20$ ksi in the most highly stressed interior stringer of that 50-ft simple-span steel stringer bridge. The next question would be: What is the H-equivalence of this particular vehicle on a 50-ft span? In other words, a standard H truck of what weight would be required to produce a live load plus impact stress of 16.20 ksi in the most highly stressed interior stringer? This question can be answered by referring to previous calculations which show that the design live load plus impact produces in this same interio stringer a maximum stress of $K\,f_L = 10.80$ ksi. The design live load in this consists of one H-15 truck in each lane simultaneously. For this bridge, too, it was assumed that a single vehicle in one lane only would produce 75 percent as much stress in an interior stringer as that produced by identical vehicles, one in each lane simultaneousl On this basis, therefore, a single H-15 truck on this bridge in one lane only would produc in the same interior stringer a live load plus impact stress of $C\,K\,f_L = 0.75 \times 1.286 \times 1.286 \times 8.40 = 8.10$ ksi.

Now if a single H-15 truck, in one lane only, produces a maximum live load plus impact stress of this magnitude, by direct proportion one can find the equivalent H truck required to produce a corresponding stress of $K\,f_H = 16.20$ ksi, or $K\,f_H/C\,K\,f_L = 16.20/8.10 = 2.00$ times as much live load plus impact stress as a single H-15 truck. Therefore, this given heavy vehicle would be rated as an equivalent H-30 truck on a 50-ft span. Symbolically, the equivalent H truck rating (KHT) for this particular vehicle would be $EHT = 15(K\,f_H/C\,K\,f_L) = 15(16.20/8.10)$ = Equivalent H-30 truck.

Based on the foregoing discussion of dead load, design live load, impact and actual live load plus impact stresses, and how they may be related for determining the design stress ratios which result from actual vehicle loadings, it is now possible to write a general expression for determining the design stress ratio (4, 7) produced by a vehicle of given H-equivalency on a span of given length. In terms of stress produced by vehicles of given H-equivalency, the design stress ratio would be

$$Q = R_D + R_L \frac{K'\,f_H\,C}{K\,f_L} \tag{8}$$

Similarly, if the stress function were in terms of maximum bending moments produce by vehicles of given H-equivalency, the design stress ratio would be

$$Q = R_D + R_L \frac{K'\,M_H\,C}{K\,M_L} \qquad (9$$

in which

$$K' = 1.00 + I' \qquad (10$$

is the coefficient by which the actual live load stress (moment or other stress functio is multiplied to obtain the live load plus impact stress (moment or other stress function) produced on a given span by a given vehicle under consideration; and I' is the impact fraction assumed in connection with the stress-producing effects of any given vehicle under consideration. Depending on the speed of the vehicle under consideration and other traffic conditions, the impact fraction I', could be assumed at any reasonab value between zero and the full impact allowance, I, as defined by the AASHO design specifications.

In Eq. 8, if f_H represents the maximum live load stress in an interior stringer resulting from identical vehicles of given H-equivalency, one in each lane simultaneou the coefficient C = 1.00 (or 100 percent) of the potential stress that would result fro identical vehicles of given H-equivalency, one in each lane simultaneously. But if o one of these vehicles were placed in one lane only, C would be less than 1.00, and i the foregoing examples it has been assumed that C = 0.75 for the case of one vehicle one lane only. Here, it will be remembered that C is a function of the stringer spacing and for all lanes loaded, C = 1.00. Similarly, in Eq. 9, if M_H represents the maximum live load moment in an interior stringer resulting from identical vehicles of given H-equivalency, one in each lane simultaneously, C = 1.00. But if only one of these vehicles were placed in one lane only, this coefficient would be less than one, say C = 0.75, as has been assumed previously.

Likewise, if M_H represents the moment for one lane produced in a given span by a vehicle of given H-equivalency and M_L represents the live load moment for one lane produced by the design live load, the ratio M_H/M_L would be the same as would obtain if M_H were defined as the moment in an interior stringer resulting from vehicles of given H-equivalency, one in each lane simultaneously and M_L the moment in the same stringer resulting from the design live load in each lane simultaneously. Therefore, Eq. 9 provides a general expression for determining design stress ratios resulting from heavy vehicle loadings.

Use of Eq. 8 or Eq. 9

To illustrate the use of Eq. 8 or Eq. 9 for determining design stress ratios, suppose it is desired to determine the design stress ratio resulting from the live load plus impact stress of 16.20 ksi produced in an interior stringer by the equivalent H-30 truck in one lane only on the 50-ft span referred to earlier. Now is K' = K = 1.286, and this stress of 16.20 ksi is 75 percent of what it would be if each lane were loaded, then if vehicles with identical H-equivalencies were placed one in each lane simultaneously, the maximum actual live load stress in an interior stringer would be $K'f_H = 16.20/0.75 = 21.60$ ksi.

With this information it is now possible by use of Eq. 8 to determine the design stress ratio in an interior stringer of this 50-ft span. Therefore, by Eq. 8 it will be found that the design stress ratio for this situation is

$$Q = 0.400 + 0.600 \left(\frac{21.60 \times 0.75}{10.80}\right) = 0.400 + 0.900 = 1.300$$

This shows that the given vehicle, which turned out to be an equivalent H-30 truck, ould result in a design stress ratio of 1.30 or an overstress of 30 percent in an inerior stringer if this vehicle were the only one on the span at one time.

What would the design stress ratio be if vehicles of idential H-equivalencies (equialent H-30 trucks) were placed one in each lane simultaneously? An equivalent H-30 ruck in each lane simultaneously on this 50-ft span would, by Eq. 8, result in a deign stress ratio of

$$Q = 0.400 + 0.600\left(\frac{21.60 \times 1.00}{10.80}\right) = 1.60$$

n other words, on this 50-ft span of H-15 design, an equivalent H-30 truck in each ne simultaneously would produce a maximum stress in an interior stringer 60 perent in excess of the basic design stress, or a maximum actual stress of 1.60 x 18.00 28.80 ksi.

valuating H-Equivalencies

For any given span, if M_H is the moment for one lane produced by a single equivant H truck weighing H tons and $M_{H(1)}$ the moment for one lane produced by a standard truck weighing 1.0 tons, then the H rating or H-equivalency in tons for any particular hicle on a given span would be

$$H = M_H/M_{H(1)} \tag{11a}$$

$$M_H = H\,M_{H(1)} \tag{11b}$$

bstitution of Eq. 11b in Eq. 9 gives

$$H = \frac{K\,M_L\,(Q - R_D)}{C\,K'\,M_{H(1)}R_L} \tag{12}$$

an equation for determining the equivalent H truck loading that would be required on a given span to produce a design stress ratio, Q, of specified value.

Ratios Resulting from Equivalent H Truck Loadings

By rearranging Eq. 12 or by substituting the value of M_H, as given by Eq. 11b in Eq. 9, it will be seen that the design stress ratios resulting from various weights of equivalent H trucks and other loading conditions would be

$$Q = R_D + R_L \frac{H\, C\, K'\, M_{H(1)}}{K\, M_L} \qquad (13)$$

This shows that the design stress ratio, Q, is a linear equation. Therefore, for any given member of a bridge of given span, Q varies directly with the values of H, C and K' in Eq. 13. Thus, Eq. 13 provides a simple and effective means for estimating the stress-producing effects of heavy vehicle loads on highway bridges of various spans, types of construction and design designation.

The usefulness and variety of information to be obtained from Eq. 13 are discussed and illustrated in the main body of the paper.