

Non-Dimensional Techniques Applied to Rigid Plate Bearing Tests on Flexible Pavements

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Non-dimensional techniques based on the methods of dimensional analysis provide a rational basis for analyzing rigid plate bearing tests on flexible pavements. Test data reported by Benkelman and Williams (1, Table 4) have been successfully analyzed by such techniques. The surface deflection is explicitly expressed as a function of the applied load, bearing plate diameter, pavement thickness, and the strength characteristics of the subgrade. Several illustrative examples are presented using the derived deflection equation to indicate possible applications. For the Hybla Valley data analyzed, the analysis shows that the load-carrying capacity of the flexible pavement as expressed by the surface deflection is dependent on the total pavement thickness and not on its proportion of asphaltic concrete or base course.

● ONE of the most comprehensive field investigations of rigid plates bearing on flexible pavements is a cooperative study conducted by the U.S. Bureau of Public Roads, the Asphalt Institute and the Highway Research Board on a specially constructed track at Hybla Valley near Alexandria, Virginia. The factual test data of the study are presented in tabular form by Benkelman and Williams (1).

Included are static rigid-plate bearing tests on full-scale pavement sections constructed on a minimum embankment of 5 feet of uniform A-7-6 soil (AASHO Classification-1949). The test sections of pavement were built with great care and every precaution was exercised to insure uniformity of thickness, compaction and composition of the various component layers. The soil used in the embankment was secured from a previously prospected area and a high degree of uniformity of the material, both in composition and condition, was obtained. The first stages were completed in 1946, whereas some sections were not placed until 1949.

There are innumerable possible procedures for conducting static load tests. For any given pavement section the various controllable factors that may affect the result of tests of this type include the magnitude of the load and the manner in which it is applied, the number of applications and releases of a given load, the duration of each load application and release, and the size of the bearing plate. The data presented were obtained by the use of four different load-test procedures; namely, the incremental, the incremental-repetitional, the accelerated and the repetitional. The vast majority of the tests were made with the accelerated and repetitional procedures.

The incremental tests were conducted on 3-, 6-, and 9-in. asphaltic concrete surface courses of a 24-in. base section using circular bearing plates of 1.954- and 3.568-in. diameters. The relatively small plate diameters, compared to the thickness of the surface course, confined the effects of the applied load to the surface course. Therefore such tests do not give a true indication of the load-carrying capacity of the pavement section. The most desirable procedure would have been to use the incremental test with 12-, 18-, 24- and 30-in. diameter bearing plates for surface deformations up to approximately 1 or 1.5 in.

The accelerated test procedure was conceived in a search for a method that would produce the data sought and at the same time permit the conduct of a number of test

per day. It consists of two parts, an incremental portion (part a) which is a much abbreviated version of the actual incremental test procedure mentioned, and an accelerated portion (part b). The incremental portion provides for the application and release, once each, of three individual loads of increasing magnitude. The period of application or release is maintained until the rate of movement slows to 0.001 in. per 15 sec.. The load magnitudes are such as to produce gross deflections of approximately 0.20, 0.30 and 0.40 in. for each of the three loads, respectively. Following the release of the third load and after the movement-time criterion of the incremental portion has been satisfied, the accelerated portion of the test is begun. It consists of the continuous application of a load of varying magnitude which is controlled so as to produce a rate of vertical movement of the surface under test of 0.5 in. per min. The application of the load is continued until (a) the material is unable to support a further increase, or (b) the gross deflection exceeds 2.0 in. or (c) the total reaction load is used.

TABLE 1
FLEXIBLE PAVEMENT SECTIONS AND PLATE SIZES ANALYZED

Thickness of Asphaltic Concrete a (in.)	Thickness of Base Course b (in.)	Bearing Plate Diameter d (in.)			
		12	18	24	30
3	6	x	x	x	x
3	12	x	x	x	x
3	18	x	x	x	x
3	24	x	x	x	x
6	6	x	x	x	-
6	12	x	x	x	-
6	18	-	x	x	x
6	24	-	x	x	x
9	6	x	x	x	-
9	12	x	x	x	-
9	18	-	x	x	x
9	24	-	x	x	x

Because the accelerated procedure is actually two types of test—an incremental, and creep test (quasi static) followed by a constant rate of deformation test—it is not desirable to use the results for deformation greater than 0.4 in. which is the limiting deformation for the incremental portion. It would be possible to analyze a constant rate of deformation-type test if the incremental had not been conducted first.

The incremental-repetitional tests were conducted on subgrades and the repetitional tests covered only small deformations. In addition, the tests conducted on the base course of the asphaltic concrete removed were influenced by the confining effect of the surface course.

After carefully examining the various test procedures, it was decided that the incremental part of the accelerated tests given in Table 4 of HRB Special Report 46 (1) for the complete pavement section is the most meaningful data and as such is the only data analyzed in this paper.

One method of analysis using the results of the constant rate of deflection portion of the accelerated procedure was presented by Ingimarrson (3), in which the linear equation of Housel's perimeter-shear theory is shown to be applicable. Ingimarrson's modification of the constant rate of deflection data to eliminate the effects of the preceding incremental portion is questionable. The results of Ingimarrson's paper are presented in terms of Housel's "perimeter-shear constant" and "developed-pressure constant" which are plotted as functions of the surface deflection. However, these re-

TABLE 2

PHYSICAL QUANTITIES CONSIDERED FOR THE RIGID BEARING PLATE TESTS
ON FLEXIBLE PAVEMENTS

Physical Quantities	Symbol	Fundamental Units
Surface deformation	x	L
Total applied force	F	F
Thickness of asphaltic concrete	a	L
Thickness of asphaltic concrete plus subbase	h	L
Cross-sectional area of the bearing plate	A	L ²
Perimeter of the bearing plate	c	L
Time	t	T
Maximum unconfined compressive strength of the soil	τ	FL ⁻²
Viscosity of the soil	η	FL ⁻² T
Characteristic strength parameter of the asphaltic concrete	k ₁	FL ⁻²
Characteristic viscosity of the asphaltic concrete	c ₁	FL ⁻² T
Characteristic strength parameter of the subbase	k ₂	FL ⁻²
Characteristic viscosity of the subbase	c ₂	FL ⁻² T

sults are not expressed explicitly in terms of the parameters pertinent to the study.

It is the purpose of this paper to study this same data by non-dimensional techniques based on the variables involved in the investigation, and to develop an explicit functional relationship among these variables.

THEORETICAL DEVELOPMENT

Although some of the concepts of dimensional analysis go back to the time of Galileo and have been used in various ways by such investigators as Mariotte, Newton, Fourier, Stokes, Froude, Reynolds, Rayleigh, and others (3), the basic theorem was not formally presented and proved until 1914 by Buckingham (4) in his famous Pi Theorem. A more general proof has more recently been given by Martinot-Lagarde (5). The general theory of dimensional analysis has been illustrated by numerous authors, particularly in the field of fluid mechanics, and several books have been written on the subject; for example, Bridgman (6), Murphy (7) and Langhaar (8). At present the senior author has been applying such techniques to a variety of problems in the field of soil mechanics (9, 10, 11, 12, 13, 14, 15). Because of the complex properties of the various pavement materials and the complicated interaction of these various layers with the loads being supported, it is felt that the use of non-dimensional techniques in both model and prototype research investigations of pavement problems would offer definite advantages with regard to the cost, scope, and time for completion of such studies.

Thus the study reported in this paper not only provides another analysis of a portion of the Hybla Valley test data, but of more importance, it illustrates and calls attention to the possible advantageous use of such a well-known general research tool as dimensional analysis in the field of pavement design. The authors are certainly not proposing any new theoretical methods, but are only calling attention to an existing research technique and illustrating one way in which such techniques can be extended into the practical aspects of pavement design.

Examples of the practical use of non-dimensional techniques, based on the method of dimensional analysis, in the area of soil mechanics have been given by Kondner (10, 12, 14, 15), Kondner and Edwards (11) and Kondner and Krizek (13).

The methods of dimensional analysis as used to determine relationships between physical quantities may be briefly summarized as follows: there are m physical quantities, containing n fundamental units, which can be related by an equation, then there are $(m-n)$ and only $(m-n)$ independent, non-dimensional parameters, called π terms, which are arguments of an indeterminate, homogeneous function F .

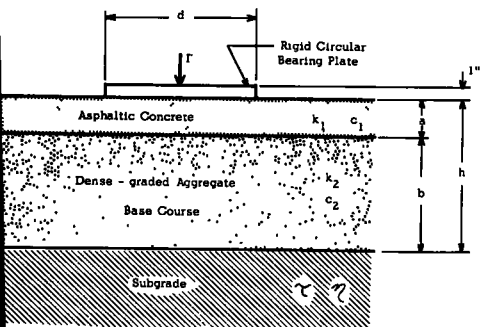


Figure 1. Cross-section of flexible pavement.

$$F(\pi_1, \pi_2, \pi_3 \dots \pi_{m-n}) = 0 \quad (1)$$

The physical quantities given in Table 2 have been selected for use in the dimensional analysis of the problem of the rigid plate bearing test on the surface of a flexible pavement. A force, length, time system of fundamental units has been used. Figure 1 is a typical cross-section of a flexible pavement showing the bearing plate, asphaltic concrete layer, base course and subgrade.

It is assumed that the material constants needed to describe the deformation characteristics of the cohesive soil subgrade are implicit in a characteristic soil strength parameter and the viscosity. The characteristic soil strength parameter used is the maximum unconfined compression strength of the soil. It may very well be that for the range of surface deflections being considered in this paper that the problem is primarily one of deformation and not of failure. As such the soil moduli in compression and shear should be used instead of the shearing strength as given by the unconfined compressive strength, but with regard to practical application these quantities are not as easily obtainable as the unconfined compressive strength. In addition previous work by the senior author (10) on stress relaxation and creep characteristics of a cohesive soil indicate that compression and shear moduli tend to be proportional to the maximum unconfined compressive strength. The viscosity controls the rate at which the deformation takes place and may include non-Newtonian effects. It is also assumed that the deformation characteristics of the asphaltic concrete and the subbase are each controlled by characteristic strength parameters and viscosities. The duration of loading is important in creep and viscous response. The effect of the geometry of the bearing plate is expressed by the cross-sectional area and the circumference.

Because there are thirteen physical quantities and three fundamental units, there must be ten independent, non-dimensional π terms. By a methodical process previously described by Kondner (9, 10, 11, 12) the following π terms can be obtained:

$$\pi_1 = \frac{F}{A\tau}, \quad \pi_2 = \frac{c^2}{A}, \quad \pi_3 = \frac{x}{c}, \quad \pi_4 = \frac{c}{h}, \quad \pi_5 = \frac{a}{c}, \quad \pi_6 = \frac{\tau t}{\eta}, \quad \pi_7 = \frac{k_1}{\tau}, \quad \pi_8 = \frac{k_2}{\tau},$$

$$\pi_9 = \frac{k_1 t}{c_1}, \quad \pi_{10} = \frac{k_2 t}{c_2} \quad (2)$$

The above π terms can be substituted into Eq. 1 to obtain the function F . A general interpretation of these non-dimensional parameters has previously been given by Kondner (10, 12). The terms π_1 , π_7 , and π_8 express the strength ratios of the subgrade, asphaltic concrete, and base course, respectively. The ratios of the time of loading to the relaxation time for the subgrade, asphaltic concrete, and base course are expressed by π_6 , π_9 , and π_{10} , respectively. The term $\frac{c^2}{A}$ is a shape factor, and $\frac{c}{h}$ and $\frac{a}{c}$ are characteristic length ratios. For circular- and square-shaped plates the value of $\frac{c}{h}$ is 4π and 16 , respectively, regardless of the size. The settlement parameter is expressed as $\frac{x}{c}$ and is the dependent parameter for the study.

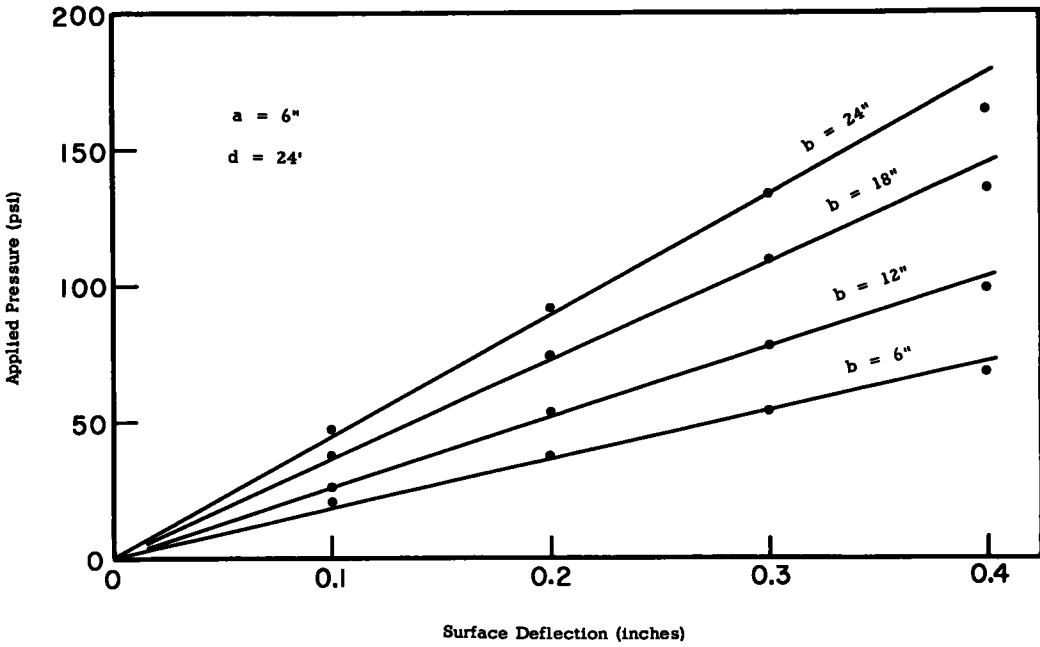


Figure 2. Applied pressure versus surface deflection.

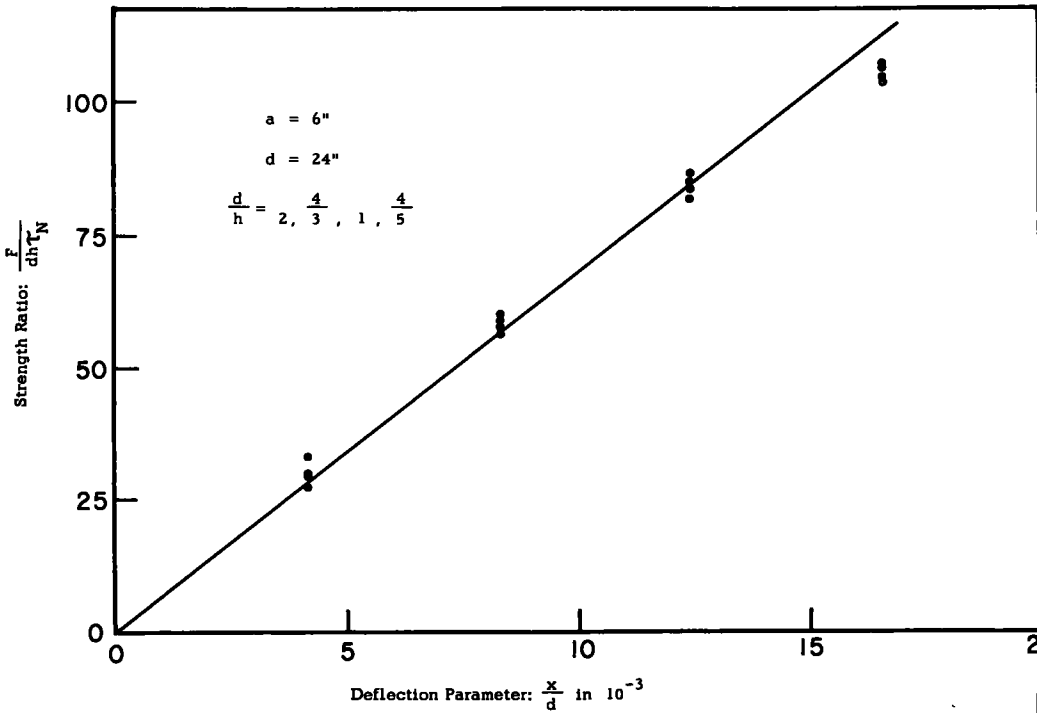


Figure 3. Non-dimensional plot: strength ratio versus deflection parameter.

The functional relationship given by Eq. 1 can be written as:

$$\frac{x}{c} = \theta \left[\frac{F}{A\tau}, \frac{c^2}{A}, \frac{c}{h}, \frac{a}{c}, \frac{\tau t}{\eta}, \frac{k_1}{\tau}, \frac{k_2}{\tau}, \frac{k_1 t}{c_1}, \frac{k_2 t}{c_2} \right] \quad (3)$$

EXPERIMENTAL RESULTS

For all of the Hybla Valley tests reported (1), τ , η , k_1 , c_1 , k_2 and c_2 were maintained constant and hence π_7 , π_8 , π_9 , and π_{10} were also constant for the investigation and can be eliminated from Eq. 3. This does not mean that the load-deflection relation is independent of the type and quality of pavement materials, but only that the pavement materials were constant for the data analyzed. It is to be expected that the curves given would in general be different for different pavement materials and perhaps even for different types of loading. Thus the analysis that follows is for the particular values of π_7 , π_8 , π_9 , and π_{10} used at Hybla Valley. The tests were conducted in such a manner as to minimize time effects and hence π_6 is relatively constant and can be dropped. Because only circular bearing plates were used in the study, the diameter, d , expresses the geometry of the bearing plate and replaces the perimeter and cross-sectional area. This leaves one dependent and three independent variables which can be algebraically transformed into the form given by Eq. 4. It is important to note that the new π terms are the variables under consideration and not the individual physical quantities composing the π terms.

$$\frac{x}{d} = \theta \left[\frac{F}{dh\tau}, \frac{d}{h}, \frac{a}{d} \right] \quad (4)$$

Figure 2 is a typical conventional plot of applied pressure versus surface deflection for various thicknesses of base course with a bearing plate of constant diameter and a constant thickness of asphaltic concrete. The four different straight lines for various values of b indicate the apparent effect of the base course thickness; however inasmuch as a is a constant, the variation in b is reflected as a variation in the total pavement thickness h and, because d is a constant, the variation of b is expressed as a variation of the ratio $\frac{d}{h}$. The same test results are plotted in Figure 3 in the non-dimensional form $\frac{F}{dh\tau N}$ versus $\frac{x}{d}$. Comparison of Figures 2 and 3 clearly illustrates the advantages of dimensional analysis as an experimental guide and the advantages of expressing experimental data in non-dimensional form. Because τ was constant for all tests, the parameter $\frac{F}{dh\tau}$ is proportional to $\frac{F}{dh}$ and is given for τN , a normalized value of τ equal to unity. Figure 3 is not affected by the variation of $\frac{d}{h}$ and hence $\frac{d}{h}$ can be eliminated from Eq. 4. For these data the ratio $\frac{a}{d}$ was a constant value of 0.25.

Another conventional method of presenting the data is shown in Figure 4 where the applied pressure is plotted against the surface deflection for various values of the thickness of the asphaltic concrete with constant values for the plate diameter and the base thickness. Note the apparent influence of the thickness of asphaltic concrete. Because the diameter of the plate is constant, this variation can be expressed in terms of $\frac{d}{h}$.

Figure 5 is the same data plotted as $\frac{F}{dh\tau N}$ versus $\frac{x}{d}$. The three curves of Figure 4 are reduced to one curve in Figure 5. The same linear relationship of Figure 5 is obtained for base courses of 12, 18 and 24 in. with a constant plate diameter of 18 in. Repeating this analysis for plate diameters of 12, 24 and 30 in., a single resultant curve can be obtained for each plate diameter (Fig. 6). Thus, the non-dimensional parameter $\frac{a}{d}$ exerts very little influence on the phenomena and can be dropped from Eq. 4.

The results of Figure 6 can also be obtained by plotting $\frac{F}{dh\tau N}$ versus $\frac{x}{d}$, as shown in Figure 3 for $d = 24$ in., for all the plate diameters.

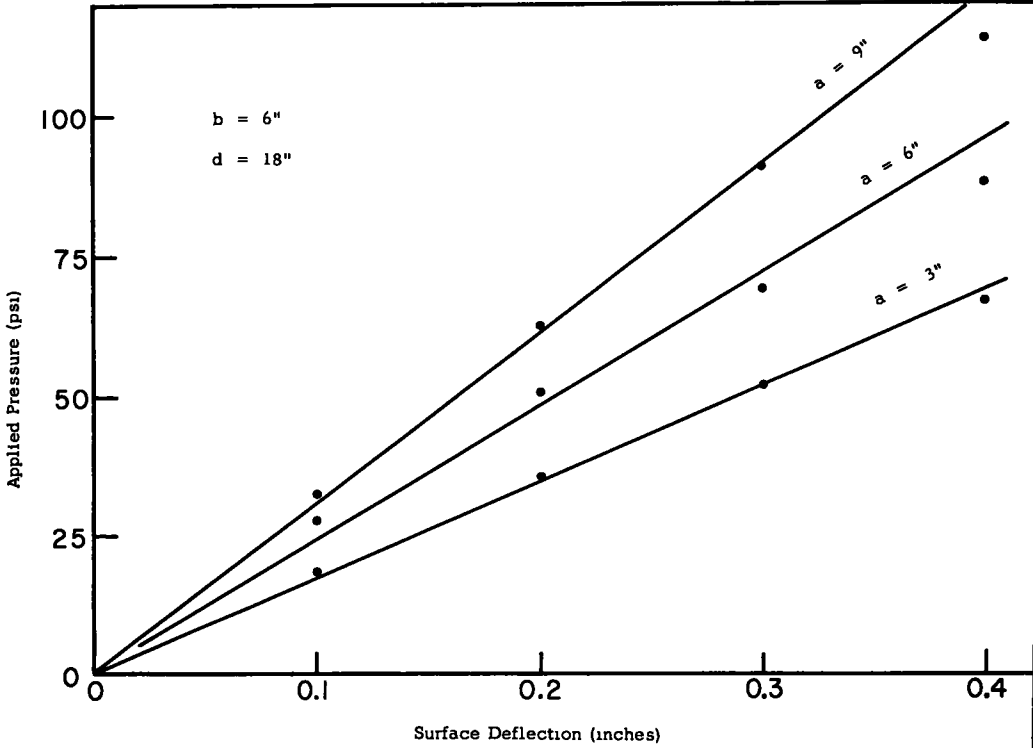


Figure 4. Applied pressure versus surface deflection.

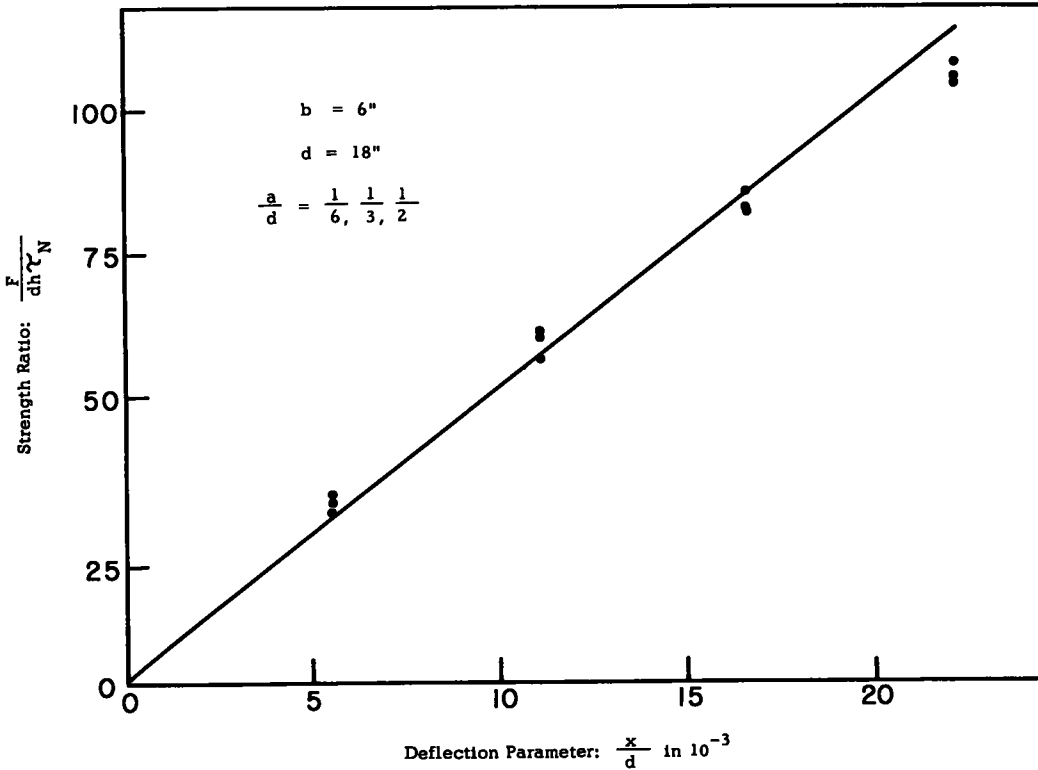


Figure 5. Non-dimensional plot: strength ratio versus deflection parameter.

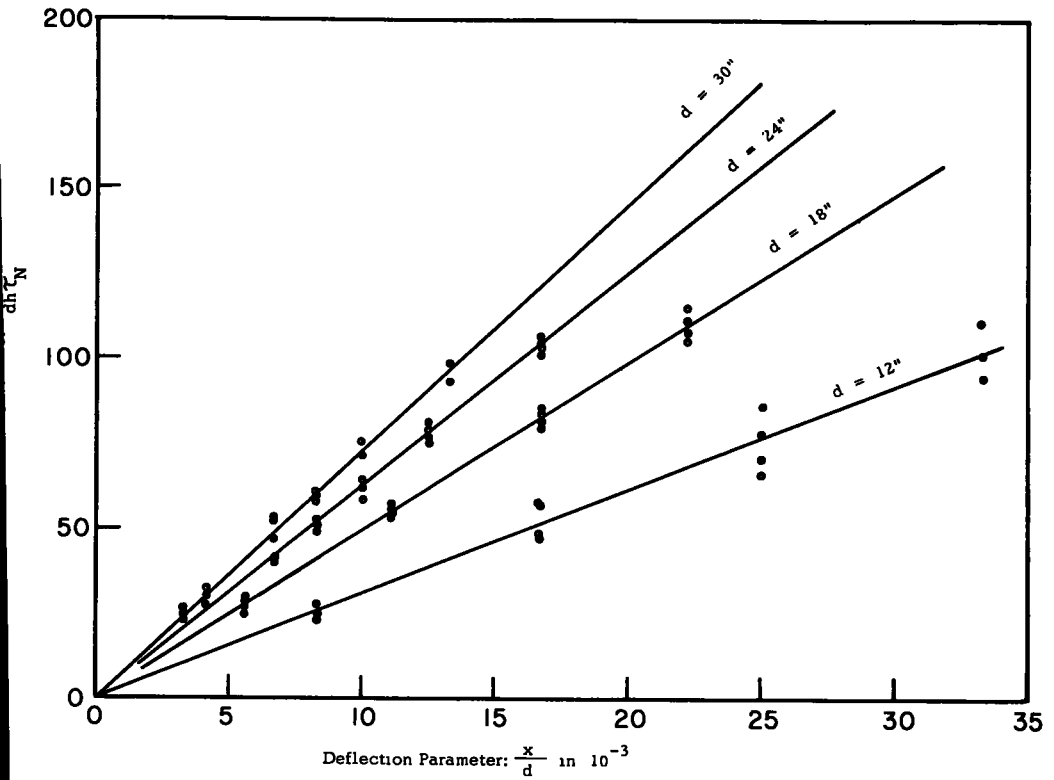


Figure 6. Strength ratio versus deflection parameter: variable plate parameter.

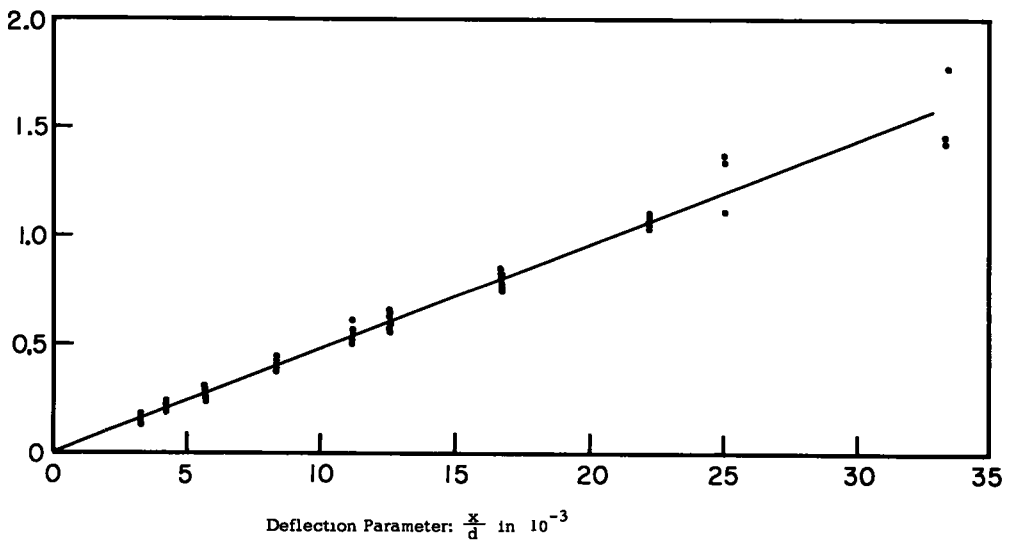


Figure 7. Strength ratio versus deflection parameter: normalized diameter.

The curves of Figure 6 can be reduced to the straight line of Figure 7 by normalizing with respect to the bearing plate diameter. Thus, the data of Table 4 (1) can be reduced to the relation given in Figure 7. Because the curve of Figure 7 is a straight line from the origin, the surface deflection as a function of the applied load, bearing plate size, subgrade strength characteristics, and pavement characteristics can be written as

$$x = M \frac{F}{dh\tau} = \frac{F}{4dh\tau} \quad (5)$$

where M includes the effect of normalizing with respect to the diameter as well as the slope of the straight line. For the Hybla Valley study considered in this paper the factor M in Eq. 5 was found to be $1/4$. Because of the normalization process the factor $1/4$ in Eq. 5 has the units of inches. Eq. 5 is also based on an estimated value of the maximum unconfined compressive strength of the subgrade obtained from another report on the Hybla Valley Study (16). Because $\frac{F}{dh\tau}$ is non-dimensional, any system of compatible units may be used in Eq. 5 and the value of the deflection will be given in inches.

Because of possible variations in the properties of the subgrade, asphaltic concrete and base course, the results expressed by Eq. 5 or by Figure 7 for the Hybla Valley Study may not apply to pavement sections in all localities of the country. It is felt that the basic method of analysis given in Eq. 3 and applied to the Hybla Valley data could also be applied in other localities to determine the necessary relationship to replace Eq. 5.

If the pavement response is linear as indicated in Figures 2 and 4, and if the present techniques are applicable in other localities under various conditions, the procedure required to determine the factor M would be as follows. Determine the maximum unconfined compressive strength of the subgrade and then conduct several rigid plate bearing tests using several plates of different diameter, each with several applied loads. For each diameter plate used, plot $\frac{F}{dh\tau}$ versus $\frac{x}{d}$. To reduce these plots into a single relationship, select a convenient diameter as a normalizing factor and apply it to each plot. If the single resultant plot is a straight line, its slope can be determined and divided into the plate diameter which was used as the normalizing factor in order to obtain the factor M to replace $\frac{1}{4}$ in Eq. 5. If the resultant plot is not a straight line and its equation cannot be determined, the resultant plot itself should be used.

For the case of large surface deflections involving non-linearities Eq. 3 could also be used, but the procedure involved in determining the explicit form of Eq. 3 might be considerably different.

ILLUSTRATIVE EXAMPLES

The following examples are given to illustrate the possible use of Eq. 5.

Example 1

Predict the surface deflection of a flexible pavement consisting of a 6-in. asphalt concrete surface course and a 12-in. dense-graded aggregate base course supported by a subgrade with a maximum unconfined compressive strength of 64 psi when tested in rigid plate bearing under an applied pressure, p , of 78 psi and a bearing plate diameter of 24 in.

Solution:

The total applied load is 35,400 lb and Eq. 5 gives

$$x = \frac{F}{4dh\tau} = \frac{35,400}{4(24)(18)(64)} = 0.32 \text{ in.}$$

This problem was randomly selected from Table 4 of Special Report 46 (1) and had a field deflection of 0.3 in. The predicted deflection value given by Eq. 5 is 6.7 per cent higher than the recorded value.

Example 2

Determine the applied pressure, p , necessary to cause a surface deflection of 0.2 in. on a flexible pavement section of a 3-in. asphaltic concrete and a 24-in. base course layer on a subgrade with a maximum unconfined compressive strength of 64 psi when tested with an 18 in. diameter, rigid bearing plate.

Solution:

$$p = \frac{4F}{\pi d^2} = \frac{4(4dh\tau x)}{\pi d^2} = 5.09 \frac{h\tau x}{d} = 5.09 \frac{27(64)(0.2)}{18} = 98 \text{ psi}$$

The predicted value of 98 psi is 6.7 percent lower than the measured value of 105 psi given in Special Report 46 (1).

Example 3

The following hypothetical problem can be solved. It is necessary to design a flexible pavement on a cohesive subgrade with an unconfined compressive strength of 64 psi. A certain design criteria states that the desired pavement section must be able to support a rigid bearing plate of 24-in. diameter under an applied pressure, p , of 136 psi such that the surface deflection does not exceed 0.4 in. Determine the minimum pavement section.

Solution:

$$h = \frac{F}{4d\tau x} = \frac{p \pi d^2}{4d\tau x(4)} = \frac{pd}{5.09\tau x} = \frac{136(24)}{5.09(64)(0.4)} = 25 \text{ in.}$$

The field tests indicate a pavement thickness of 24 in. Thus, the predicted value is higher by approximately 4.2 percent.

The preceding examples illustrate some possible applications of the results developed in this paper. It may be possible to use these results, or other results developed by the methods presented, as a basis for a design criteria for flexible pavements. It is important to point out that the present results indicate that the load-deflection characteristics of flexible pavements are dependent on the total thickness of the section and not on the ratio of asphaltic concrete surface course to aggregate subbase. From the viewpoint of riding characteristics and durability, under both normal wear and the adverse conditions of water and frost action, the thickness of the surface course will be quite important.

CONCLUSIONS

Non-dimensional techniques based on the methods of dimensional analysis seem to provide a rational basis for analyzing rigid plate bearing tests on flexible pavements. The test data reported by Benkelman and Williams (1, Table 4) has been successfully analyzed by such techniques. The surface deflection, x , in inches can be expressed in equational form as a function of the applied load, F , bearing plate diameter, d , pavement thickness, h , and the unconfined compressive strength, τ , of the subgrade in the following form:

$$x = \frac{F}{4dh\tau}$$

Several illustrative examples have been presented using this equation to indicate its possible application. Because of the test procedure used in the Hybla Valley Study this application is restricted to surface deflections of approximately $\frac{1}{2}$ in. for flexible pavements on cohesive subgrades. A significant result of the analysis is that the load-carrying capacity of the flexible pavement as expressed by the surface deflection is dependent on the total pavement thickness and not on its proportion of asphaltic concrete subbase. With regard to the durability of the pavement the thickness of the asphaltic concrete would be important.

The results also indicate that it may be possible to use the non-dimensional method

in conjunction with durability studies to develop design criteria for flexible pavements. The authors recommend that additional field studies be conducted, using load creep procedures with greater surface deflections, on flexible pavement sections supported by subgrades of different unconfined compressive strengths subjected to various environmental conditions.

This study and other studies conducted by the senior author (11, 12, 13, 14, 15) indicate that both model and prototype research investigations designed and conducted on the basis of non-dimensional techniques can help prevent unnecessary duplication of costly, time-consuming experimental work. Many times, tests which seem to be different because of different values of the physical quantities involved, are in reality duplicate tests giving the same results when examined in non-dimensional form. The reason for this is that in the search for an explicit relation expressing a physical phenomenon, it is the values of the non-dimensional parameters, which are the new variables that are important and not simply the magnitudes of the individual physical quantities. Thus it is felt that if such a program is designed and conducted on the basis of non-dimensional techniques, there is a better chance of developing rational design criteria with a minimum of expended effort.

Although the method of analysis is a general research tool and some recommendations are made for future work, the quantitative results of this paper were obtained solely from the results of the cooperative study of flexible pavements conducted at Hybla Valley.

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