Lateral Distribution of Load in Multibeam Bridges

C. L. HULSBOS, Research Professor of Civil Engineering, Fritz Engineering Laboratory, Lehigh University

•THIS PAPER presents a summary of the research conducted since 1954 at Fritz Engineering Laboratory, Lehigh University, on the lateral distribution of load in multibeam bridges. This type of bridge is constructed from precast rectangular beams made of either reinforced or prestressed concrete. These beams are placed side by side on the abutments and the interaction between the beams is developed by continuous longitudinal shear keys and lateral bolts that may or may not be prestressed.

The investigation has included a field test (1), a theoretical study (2), and a series of tests on a large-scale model bridge (3). The results of this work are summarized and an example application of the proposed design procedure is included.

THEORETICAL INVESTIGATION

The multibeam bridge can be analyzed as an orthotropic plate that considers the bending stiffness in the longitudinal direction, $(EI)_x$, as different from the bending stiffness in the lateral direction, $(EI)_y$. The stiffness in the lateral direction is dependent on the efficiency of the shear keys and lateral bolts. If slip occurs between adjacent beams, the problem of determining the lateral bending stiffness becomes more complex. Due to this discontinuity (slip), deflection and stress distribution do not follow the rules of the plate theory. It was therefore necessary to evaluate the change of the internal forces by empirical approximations.

The basic assumptions in the theory of orthotropic plates are the following:

1. The thickness of the plate is small compared with its other dimensions.
2. The deflections are small compared with the thickness of the plate.
3. The transverse stresses σ_z are small and their influence on the deformation can be neglected.

For a right-hand coordinate system (x, y, z) where x and y are the plane of the plate and parallel to the two distinct directions of the orthotropic plate, the differential equation for the deflection w parallel to the z direction can be written as

$$\frac{\partial^4 w}{\partial x^4} + 2\beta \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha \frac{\partial^4 w}{\partial y^4} = \frac{P(x, y)}{(EI)_x}$$

in which $\alpha = \frac{(EI)_y}{(EI)_x}$; and

β = coefficient of torsional rigidity.

It is possible to determine the deflections w and the internal forces under a given loading P(x, y) by the use of three parameters (2):

$$\frac{B}{L} = \frac{\text{half the bridge width}}{\text{total bridge length}}$$

$$\frac{a}{h} = \frac{\text{width of one beam}}{\text{depth of one beam}}$$

$$\alpha = \frac{(EI)_y}{(EI)_x} = \frac{\text{lateral bending stiffness}}{\text{longitudinal bending stiffness}}$$

The first and second parameters are dependent on the geometry of the bridge and individual beams respectively. The third parameter is difficult to evaluate theoretically because the lateral stiffness is not a constant. In fact, it varies from point to point in the bridge and it is also dependent on the magnitude and location of the applied load as well as the amount of lateral prestressing. The procedure used to evaluate α is described later.

The coefficient of torsional rigidity, β, can be expressed as a function of α and K (2)

$$\beta = 3K(1 - \alpha^{3/2}) + \alpha$$

in which K is a numerical factor for torsion of rectangular beams depending on a/h (4).

The differential equation for an orthotropic plate for two free edges and supported by knife edges at two opposite sides was solved (2). The solutions were obtained with a UNIVAC Digital computor for various combinations of B/L, a/h and α.

The design of multibeam bridges is governed principally by the longitudinal bending moments. Hence, the most important characteristic of such a structure is the lateral distribution of these moments over the cross-section of the bridge. However, in a plate the load is transmitted not only by the longitudinal bending but also by the lateral bending moment, the twisting moments, and the shear forces. Figure 1 shows the qualitative distribution of various quantities over the cross-section.

Because internal forces cannot be measured directly, it is more convenient to compare the measured deflection from the tests with the theoretical calculations. However, the moments are not proportional to the deflections; therefore, the distribution curves for the moment coefficients and the deflection coefficients are not identical. (It is convenient to present distribution coefficients, rather than the actual moments, shear forces, or deflections. The coefficient for a particular point is defined as the ratio of the moment — shear force or deflection — at this point to the average moment — shear force or deflection — of the entire cross-section. These coefficients are dimensionless and, in the range of elastic deformation, independent of the amount of load). This can be visualized by considering the following— with a concentrated load at the center point of the bridge, the deflected shape of an edge beam is somewhat similar to the shape of a uniformly loaded beam, whereas, the middle beam acts more or less like a simple beam under a single concentrated load. The moment-deflection ratio for these two cases differs by 25 percent. In the practical case of a multibeam bridge these effects lead to a deviation between the coefficients of the longitudinal bending moments and the deflection coefficients which may be as high as 50 percent.

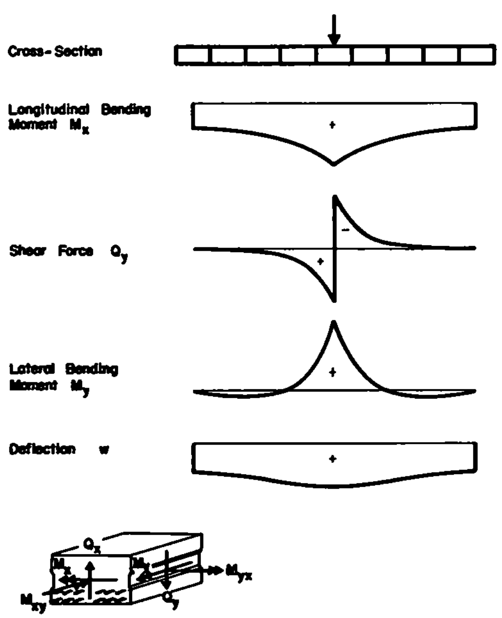


Figure 1. Distribution of moments, shear forces, and deflections at the midspan cross-section (center loading).

COMPARISON BETWEEN THEORY AND TESTS

A series of tests (3) was conducted on the laboratory model bridge shown in Figure 2. A direct comparison was made between the theoretical and measured deflection distribution. It was assumed that if the theoretical and experimental deflections were in good agreement, then the moments in the beams could be calculated by the theory of orthotropic plates.

Because it was not possible to obtain a

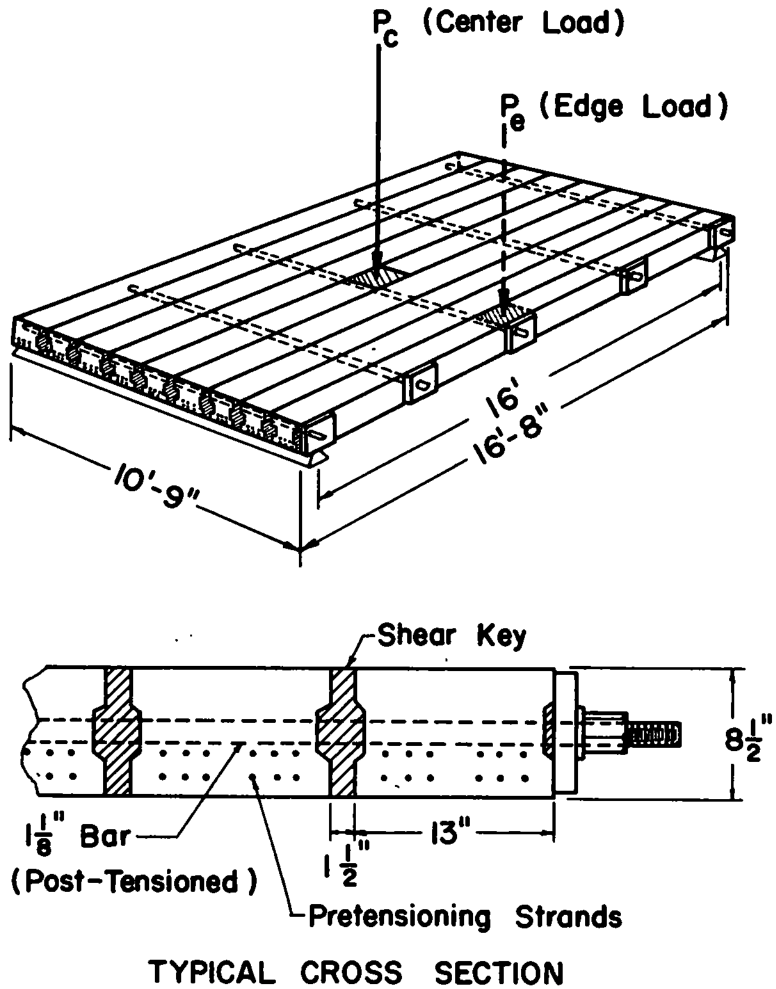


Figure 2. Test bridge.

theoretical value of α, and the α values have to be derived from the same tests that serve to check the adaptability of the theory. This means that only the deflected shape, and not the magnitude of the deflections, can be compared for this particular purpose. For this comparison the distribution of the deflection coefficients was calculated as a function of α; the other two parameters, B/L and a/h, are constant for a given bridge. By varying α, the theoretical curve with the same maximum deflection coefficient as the experimental one could be found and the correlation between the distribution of the measured and theoretical coefficients could be checked. A comparison of the theoretical and experimental deflection coefficients for four tests (3, 5) is shown in Figure 3. The theoretical moment coefficient distribution is also shown in Figure 3. The deflection coefficient S_W is defined as

$$S_{wi} = \frac{\Delta^i}{\Sigma\Delta^i/m}$$

S_{wi} = deflection coefficient for i^{th} individual beam in bridge.

in which Δ_i = deflection of an individual beam at a given cross-section of bridge;

$\Sigma\Delta_i$ = sum of all deflections of individual beams at same cross-section of bridge; and

m = number of individual beams in bridge.

The moment coefficient S_m is defined as

$$S_m = \frac{M_x}{(M_x)_{av.}}$$

in which S_m = moment coefficient for a point in a cross-section of bridge;

M_x = longitudinal bending moment per unit of width for a point in same cross-section of bridge;

$(M_x)_{av.}$ = total bending moment of same lateral cross-section of bridge divided by width of bridge.

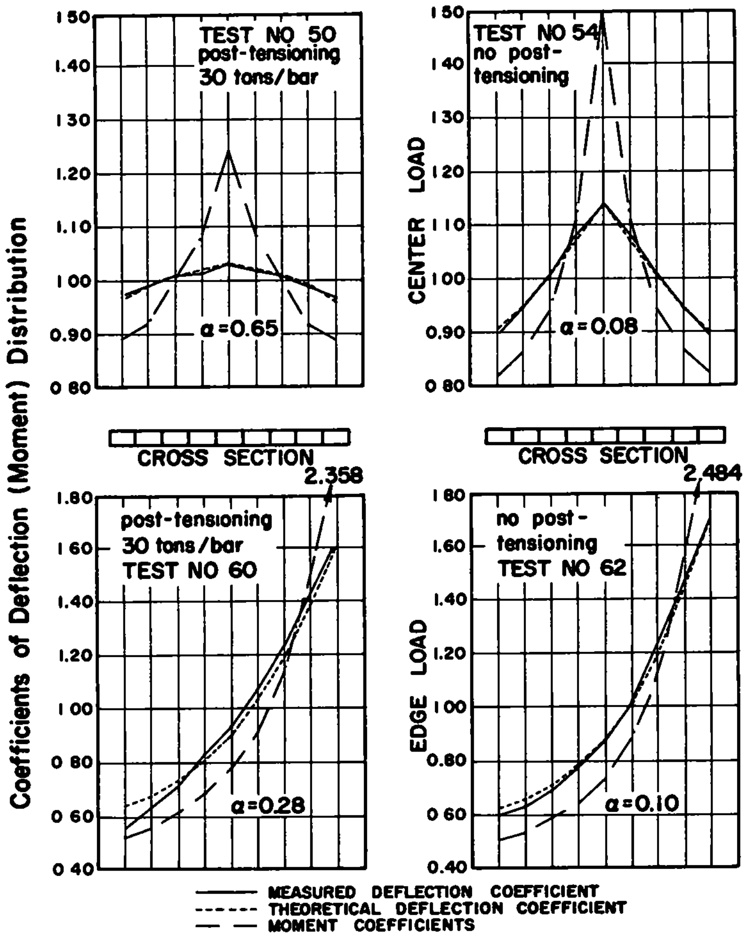


Figure 3. Theoretical and measured deflection distribution.

Figure 3 shows that if the proper value of α is used in the calculation, the theory yields a solution for the deflection distribution which agrees very closely with the experimental results.

To analyze a given multibeam bridge, the value of α as well as the geometric properties must be known. For an isotropic and homogeneous plate, $(EI)_x = (EI)_y$, $\alpha = 1$. For an articulated plate that is, one formed of strips with adjacent sides connected by continuous hinges $(EI)_y = 0$, $\alpha = 0$. For other intermediate conditions α will vary between the limits of 0 and 1. It was found that α could be expressed as a function of F and P:

For a center load: $\alpha = 0.23 \sqrt{F/P_c}$

For an edge load: $\alpha = 0.1 \sqrt[3]{F/P_e}$

in which F = lateral post-tensioning force; and

P = load applied at the center or edge of the bridge.

The equations are intentionally conservative and represent the lower boundaries of the values of α determined experimentally (see Figs. 4 and 5). If a bridge is subjected to a load resulting from several trucks, P (center or edge) is not the total load at the cross-section. For highway loading, P_c is equal to two wheel loads and P_e is equal to one wheel load.

In comparing the theoretical solutions for distribution of moment it was found that the distribution can be considered as dependent on $(S_m)_{max}$ only, no matter from what combination of B/L, a/h, and α this $(S_m)_{max}$ may result. Hence, the distribution curve is known if $(S_m)_{max}$ is known. The values of $(S_m)_{max}$, dependent on B/L, a/h, and α, are given in Figures 6 to 9. Linear interpolation or extrapolation will be necessary if a/h is not equal to either 1.0 or 1.7.

The values of $(S_m)_{max}$ were computed on the assumption that no slip occurred. Data from the model (3) and field (1) tests indicated that slip may occur and will increase the maximum moment coefficient. The amount of increase can be approximated with the curve shown in Figure 10. The data for the curve drawn for no shear keys were obtained from the model tests and the data for the curve drawn for shear keys were obtained from both the model and field tests.

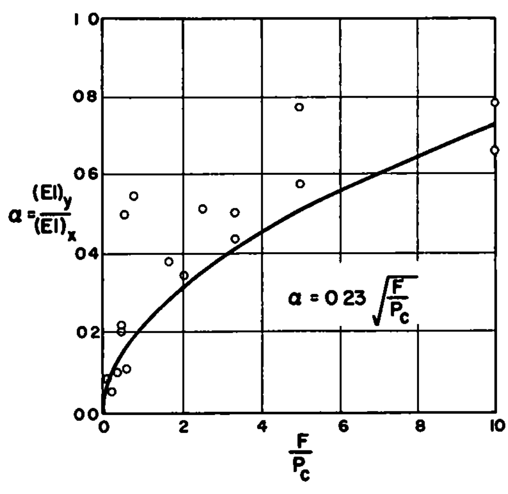


Figure 4. Relation between α and $\frac{F}{P_c}$ (center load).

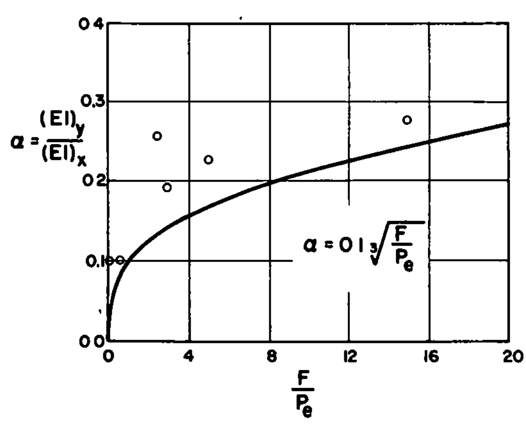


Figure 5. Relation between α and $\frac{F}{P_e}$ (edge load).

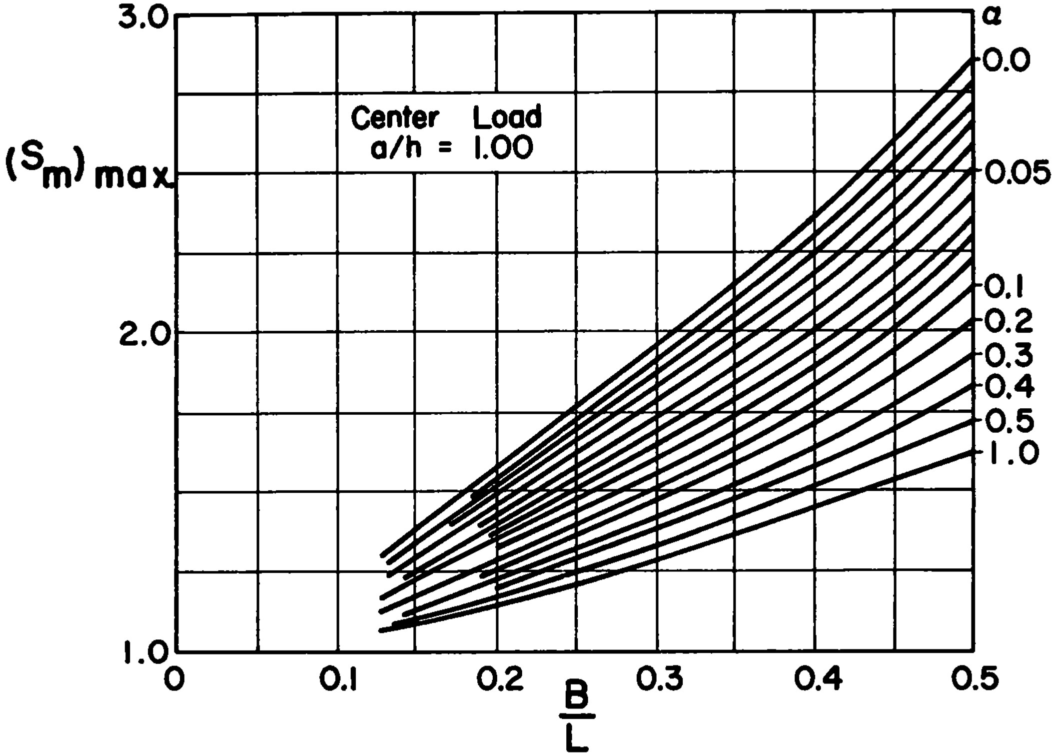


Figure 6. Maximum values of coefficient of lateral distribution of moment.

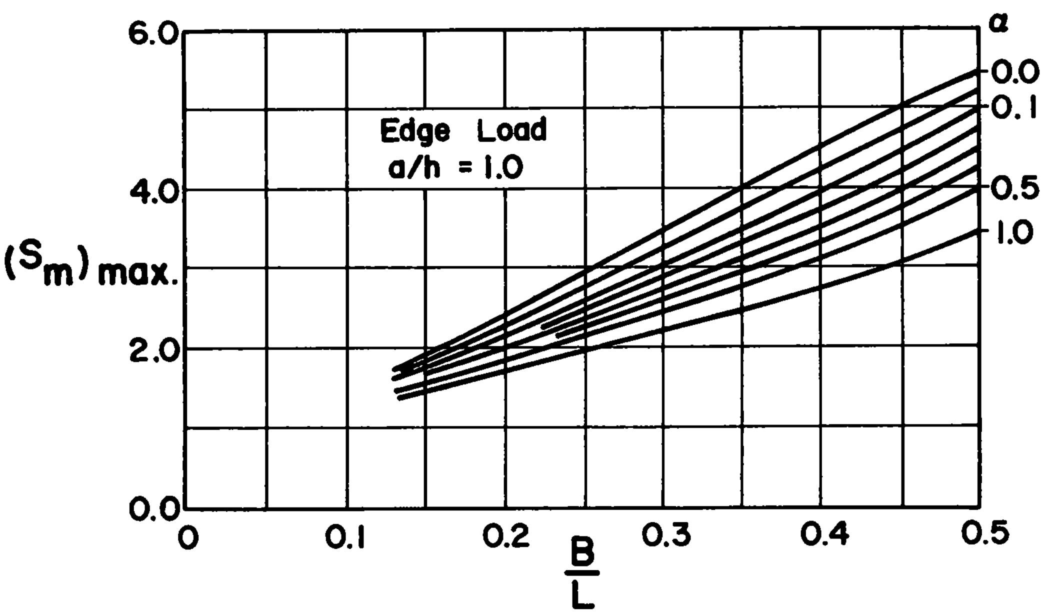


Figure 7. Maximum values of coefficient of lateral distribution of moment.

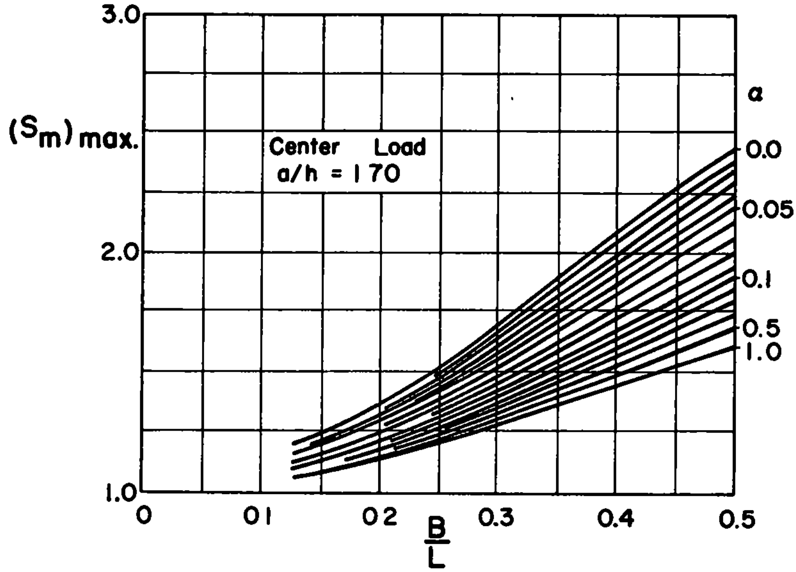


Figure 8. Maximum values of coefficient of lateral distribution of moment.

Once the maximum value for S_m is calculated, the distribution curve can be taken from Figures 11 and 12 for center and edge load, respectively. These distribution curves can also be considered as influence lines for the longitudinal bending moments.

The equivalent load used for design of either the center or edge individual beam can be obtained as

$$W_{eq} = W \frac{S_m}{m}$$

or for a series of loads at one cross-section of the bridge

$$W_{eq} = \sum_{o}^{n} W \frac{S_m}{m}$$

when the individual loads at one cross-section are equal, as the case for standard highway loading,

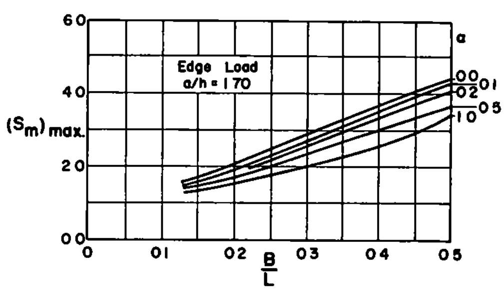


Figure 9. Maximum values of coefficient of lateral distribution of moment.

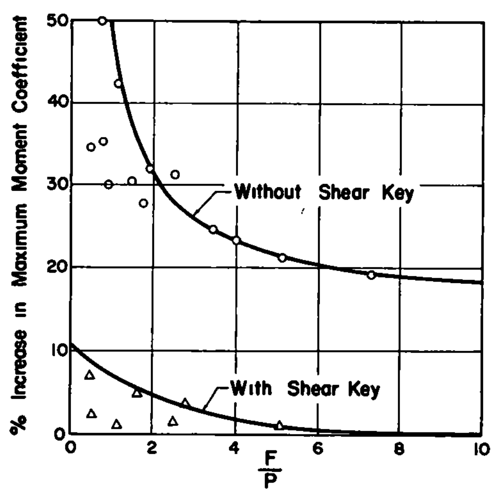


Figure 10. Increase in moment coefficient due to slip and incomplete interaction of shear key (center or edge load).

$$W_{eq} = W \sum_{o}^{n} \frac{S_m}{m}$$

in which W_{eq} = equivalent load applied to one individual beam;

W = one wheel load (all other wheel loads across a lateral section are equal);

S_m = moment coefficient (defined previously);

m = number of beams; and

n = number of wheel loads at one cross-section of the bridge.

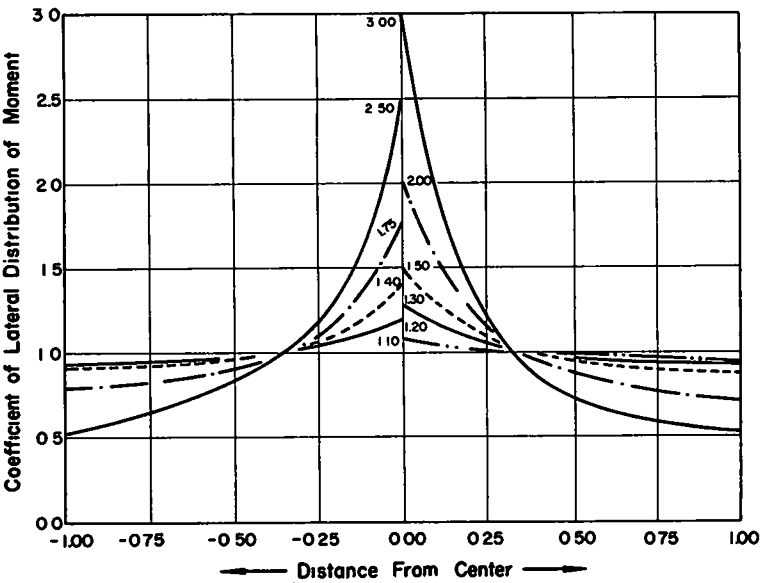


Figure 11. Distribution of moment coefficients for center load as a function of $(S_m)_{max}$.

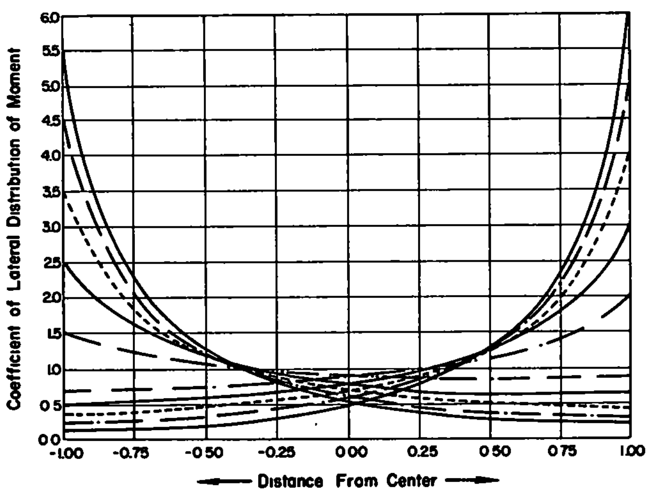


Figure 12. Distribution of moment coefficients for edge load as a function of $(S_m)_{max}$.

The quantity $\sum_{o}^{n} \frac{S_m}{m}$ is, in other words, the fraction of a wheel load to be applied to an individual beam making up the complete bridge.

The lateral distribution of moment is least uniform at midspan; therefore, W_{eq} becomes greatest at the midspan section. If the factor $\sum_{o}^{n} \frac{S_m}{m}$ is used for all loads along one beam, the resulting design will be conservative. Although this factor was derived for bending, it may also be used to calculate the shear for an individual beam.

EXAMPLE

Given:

Length of bridge = 60 ft
Width of roadway = 48 ft
AASHO specifications
Loading = H20-S16

Chosen:

Assumed cross-section = 36 by 36 in.
Lateral post-tensioning = 3 ⅝-in. diameter bars; working force per bar = 26,000 lb
Shear keys are used
Use 17 box beams at 3.0 ft plus 16 joints at ½ in. = 51 ft 8 in.

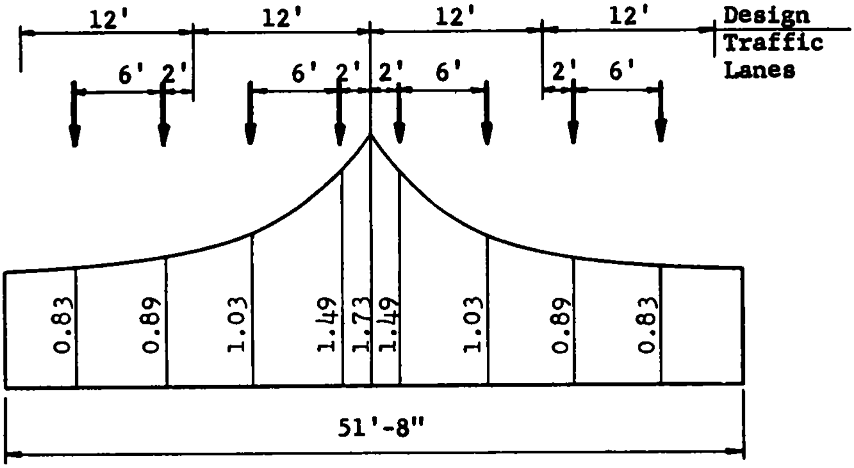
Calculations:

1. Determination of α
   (a) Center loading
   $$\frac{F}{P_c} = \frac{26,000\ (3)}{(2)\ (16,000)} = 2.44$$
   from Figure 4, $\alpha = 0.36$
   (b) Edge loading
   $$\frac{F}{P_e} = \frac{26,000\ (3)}{16,000} = 4.88$$
   from Figure 5, $\alpha = 0.17$
2. Determination of the maximum coefficient of lateral distribution of moment
   $$\frac{a}{h} = \frac{\text{width of beam}}{\text{depth of beam}} = \frac{36}{36} = 1$$
   $$\frac{B}{L} = \frac{\text{half of the bridge width}}{\text{length of the bridge}} = \frac{51.67/2}{60} = 0.43$$
   (a) Center loading
   from Figure 6, $S_m = 1.67$
   Effect of slip and incomplete interaction of the shear keys from Figure 10, 3.5 percent increase
   $S_m = 1.67\ (1.035) = \underline{\underline{1.73}}$
   (b) Edge loading
   from Figure 7, $S_m = 4.10$
   Effect of slip and incomplete interaction of the shear keys from Figure 10, 1.3 percent increase
   $S_m = 4.10\ (1.013) = \underline{\underline{4.15}}$
3. Final lateral distribution of moment curves.

The desired distribution moment curves are obtained from Figures 11 and 12. These curves are also the influence lines for the longitudinal bending moments for the centerline and edge beam.

In using an influence line to determine the maximum effect on a member, a concentrated load should be placed to coincide with the maximum ordinate. However, the AASHO specifications require that the lane loadings or standard trucks be assumed to occupy any position within their individual design traffic lane which will produce the maximum effect. A solution will be made two ways to show the effect of these requirements. Also, the AASHO specifications permit a 25 percent reduction in load intensity when four or more lanes are loaded and a 10 percent reduction when three lanes are loaded to produce the maximum stress.

(a) Center loading



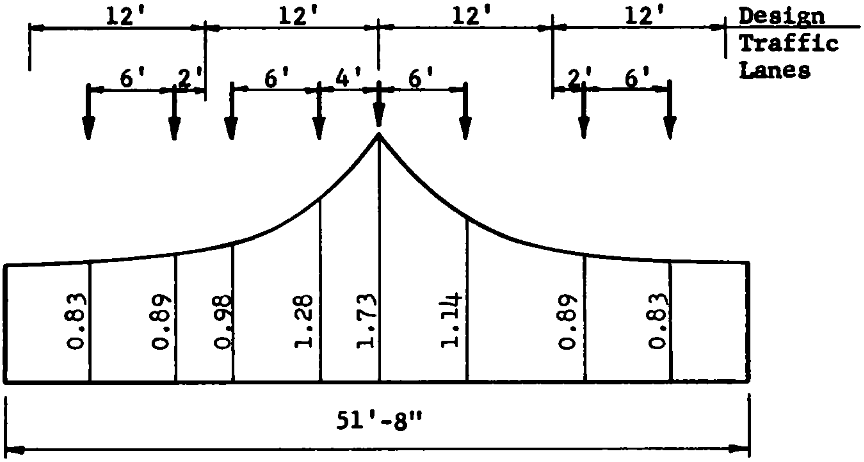
The percentage of a wheel load to be carried by the center beam is

$$\frac{\Sigma s_m}{m} = \frac{8.48}{17}(0.75) = 0.374 \text{ for four loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{6.76}{17}(0.90) = 0.358 \text{ for three loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{5.04}{17} = 0.296 \text{ for two loaded lanes}$$

or

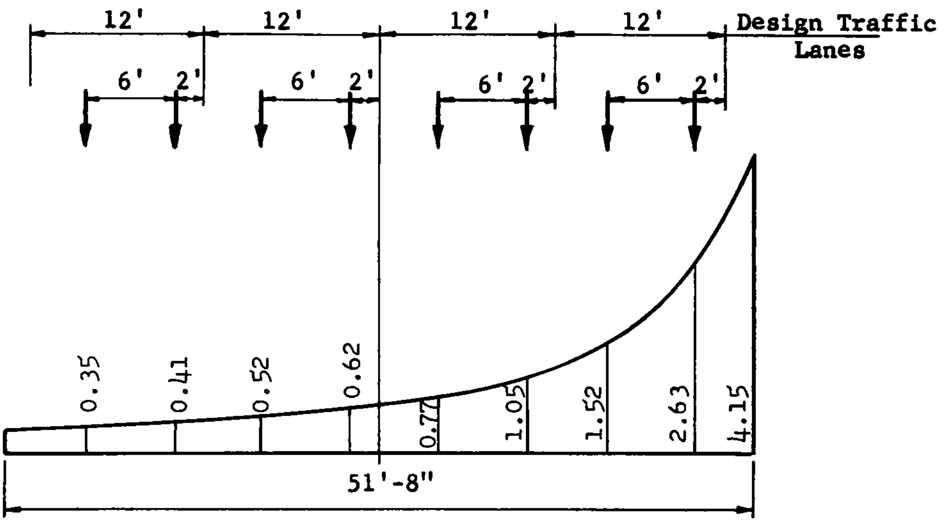


$$\frac{\Sigma S_m}{m} = \frac{8.57}{17}\ (0.75) = 0.379 \text{ for four loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{6.85}{17}\ (0.90) = 0.363 \text{ for three loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{5.13}{17} = 0.302 \text{ for two loaded lanes}$$

(b) Edge loading

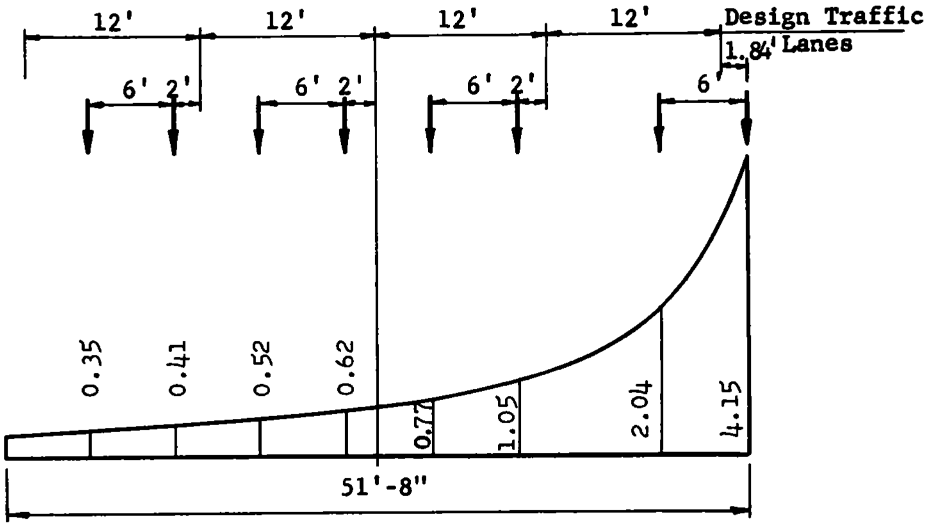


$$\frac{\Sigma S_m}{m} = \frac{7.87}{17}\ (0.75) = 0.347 \text{ for four loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{7.11}{17}\ (0.90) = 0.376 \text{ for three loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{5.97}{17} = 0.351 \text{ for two loaded lanes}$$

or



$$\frac{\Sigma S_m}{m} = \frac{9.91}{17}(0.75) = 0.436 \text{ for four loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{9.15}{17}(0.90) = 0.485 \text{ for three loaded lanes}$$

$$\frac{\Sigma S_m}{m} = \frac{8.01}{17} = 0.471 \text{ for two loaded lanes}$$

4. If the loads are confined to the design traffic lanes, the live load distribution factor to be multiplied times the vehicle wheel loads is 0.376. However, if one vehicle is placed out of its design lane at the maximum ordinate of the influence line, the distribution factor is 0.485. The latter value is recommended for this particular example.

NOTATION

a = width of one beam
B = one-half width of bridge
$(EI)_x$ = longitudinal bending stiffness
$(EI)_y$ = lateral bending stiffness
F = total lateral post-tensioning force
h = depth of one beam
L = length of bridge
m = number of individual beams
M_x = longitudinal bending moment
M_y = lateral bending moment
M_{xy} = twisting moment
n = number of wheel loads at one cross-section
P = applied concentrated load (P_c, center; P_e, edge)
Q_x = longitudinal shear force
Q_y = lateral shear force
S_m = moment coefficient
S_w = deflection coefficient
w = deflection
α = $(EI)_y/(EI)_x$
β = coefficient of torsional rigidity

ACKNOWLEDGMENTS

This paper is based on the theoretical and experimental investigations presented in Fritz Engineering Laboratory Reports 223.9, 223.10, and 223.14 by C. E. Ekberg, Jr., W. J. Eney, A. Roesli, A. Smislova, and R. E. Walther. The research was sponsored by the Pennsylvania Department of Highways; U. S. Bureau of Public Roads; Concrete Products Division, American-Marietta Company; American Steel and Wire Division, United States Steel Corporation; John A. Roebling's Sons Corporation; Reinforced Concrete Research Council; and Lehigh University.

REFERENCES

1. Roesli, A., Smislova, A., Ekberg, C. E., Jr., and Eney, W. J., "Field Tests on a Prestressed Concrete Multibeam Bridge." Fritz Engineering Laboratory Report 223.9, Lehigh Univ. (Jan. 1956).
2. Roesli, A., "Lateral Load Distribution on Multibeam Bridges." Fritz Engineering Laboratory Report 223.10, Lehigh Univ. (July 1955).
3. Walther, R. E., "Investigation of Multibeam Bridges." Fritz Engineering Laboratory Report 223.14, Lehigh Univ. (Aug. 1956).
4. Timoshenko, S., "Theory of Elasticity." 2nd ed., McGraw-Hill (1951).
5. Walther, R. E., "Investigation of Multibeam Bridges." ACI Jour. (Dec. 1957).