Use of Stress Loci for Determination of Effective Stress Parameters

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Recent investigations and studies have shown that the laboratory-measured values for internal friction and cohesion C for sensitive clays are much larger than the calculated field values. The need for a better correspondence in measured and calculated field values in such soils is evident. The study described herein was designed to investigate the use of stress loci for determination of the shear parameters not only for sensitive soils but also for normally consolidated clays, because it is realized that the state of stress in a soil specimen at any one condition of strain is an indication of the nature of stress resistance derived therein.

In consolidated undrained triaxial tests on both laboratory consolidated and field-undisturbed clays, the curve joining the points on the Mohr diagram representing shear and effective normal stress on the rupture plane at varying stages of strain has been used in this study to indicate the stress locus. By defining the friction parameter $\overline{\varphi}$ as that parameter in the shear stress vs normal stress region wherein the shear stress increases with the effective normal stress, and the cohesion parameter $\overline{C}$ as the parameter in the same region where the shear stress increases despite a decrease in the effective normal stress, it has been possible to arrive at values for these parameters which correspond to the difference measured by recent investigators. Using conventional methods involving effective stresses, the measured angle of friction for an illitic clay was 20°. However, using the stress locus, this angle was found to be 10°. Although $\overline{C}$ was found to be a function of the consolidation history for normally consolidated clays, its magnitude can also be determined from the stress locus and evaluated for the corresponding consolidation or overburden pressure.

•IN CONSOLIDATED undrained triaxial tests on fine-grained soils, pore pressure measurements taken during the period of tests reflect the effective stress condition within the soil specimen—assuming that effective stress is the difference between the total stress and the measured pore pressure. Under the application of a deviator stress, corresponding measured pore pressures can be used to indicate the effective stress condition on a prescribed plane at any one condition of axial strain. Casagrande and Hirschfeld (2), and Holtz and Ellis (4) among others have used this technique of data presentation to compare effective stress and pore pressure build-up under deviator stress application for various soil types and at varying confining pressures. Crawford (3) has used a slightly different technique to arrive at a stress locus, and in doing so, offers a method for interpretation of the effective stress parameters.

Inasmuch as there is reasonable doubt as to the interpretation and significance of the strength parameters (internal friction and cohesion) when applied to both recompacted and undisturbed soils, it would seem most appropriate to use the stress locus as a means of observing the stress condition on a prescribed plane within a soil specimen, and to deduce therefrom the parameters giving rise to the components of soil strength. Recognizing that the definition of both "cohesion" and "friction" may not be semantically correct as used in soil testing, the physical significance of these terms and parameters may well be taken as descriptive of observed soil behavior. If the normal stresses and shear stresses on the surface of ultimate failure as obtained from triaxial data show that the component of shear resistance increases as the effective normal stress increases, it seems most reasonable to deduce that this constitutes a friction shear component. With this in mind, it is felt that the stress locus that describes the stress condition on a prescribed plane within a soil mass provides a very useful tool for use in interpretation of fundamental shear strength components or parameters.

The purpose of this paper is to present a method for interpretation of effective shear strength parameters giving rise to their respective strength components, based on a knowledge of the stress conditions on the failure plane during stress application. It is recognized, however, that the "friction" and "cohesion" parameters are primarily descriptive and that although the method described defines a technique for obtaining quantitative values for these parameters, it is understood that both may be operative over the entire range of shear resistance. However, in accordance with the definition of these parameters, the predominant shear component is used to define the corresponding operative parameter.

THE STRESS LOCUS

In general, the "stress-vector" technique used by Casagrande and Hirschfeld (2) has been used with certain deviations in selection of the failure plane and subsequent approximations following the initial stress locus determination. The stress locus defines, in terms of measured and calculated quantities, the shear and effective normal

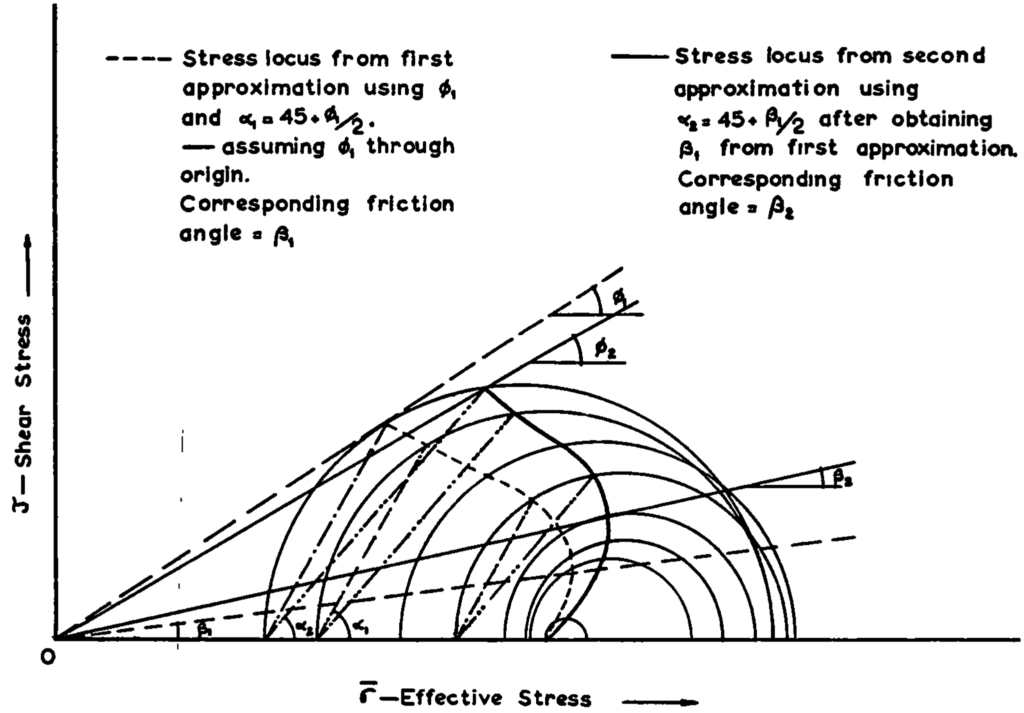

Figure 1. Stress locus from Mohr circles at varying strain conditions.

stresses on a prescribed plane. The dependency of stress on both pore pressure and strain can also be clearly seen in this system of presentation.

In the ordinary interpretation of the shear strength parameters of a cohesive soil, φ is taken as the slope angle of the rupture line derived from the envelope of failure, (Mohr's circles) and C is the intercept of the extended rupture line on the vertical shear stress axis. Such a rupture line depends on (a) the load under which the soil has been consolidated and (b) whether or not total stresses or effective stresses are used in plotting the Mohr's circles. If it is recognized that the shear strength of any normally consolidated clay soil increases with consolidation pressure from zero at zero consolidation pressure and that only effective stresses have meaning in this build-up of strength, then it is apparent that the true envelope of Mohr's circles must pass through the origin.

In the following discussion φ_1 (Fig. 1) is the angle that the rupture line makes with the horizontal. The point of tangency of the rupture line with the failure Mohr's circle defines the slope angle $\alpha_1 = 45^0 + \varphi_1/2$ of the plane in the specimen on which failure would occur if the shear strength could be attributed entirely to interparticle friction. If, however, this strength is partly derived from cohesion (developed from the consolidation pressures), φ_1 will not be the true friction angle and α_1 will not necessarily define the true failure plane. If now the shear stress vs effective normal stress curve on the α_1 plane is plotted over the range of stresses from $\overline{\sigma}_c$ (effective lateral stress) to the maximum σ_1 (effective axial stress), the stress locus (broken curve) is obtained. In examining the manner in which the shear stress on this plane changes with normal effective stress, it can be seen that the shear stress increases with normal stress to the point where the tangent to the curve is vertical. Beyond this point the shear stress increases as the normal effective stress decreases. A line through this point and the origin (slope β_1) defines a new friction angle which in turn defines a new failure plane (slope $\alpha_2 = 45^0 + \beta_1/2$). The stress locus for this new plane defines still another friction angle β_2. This process may be repeated to give other α angles but it can be shown that all additional α angles will lie between α_1 and α_2 and the additional β angles will not differ greatly from the β_2 angle. A new φ angle (φ_2) corresponding to α_2 is also shown. This is the slope of a line through the origin and the terminal point of the stress locus. If the total shear strength is actually composed partly of friction and partly of cohesion, neither φ_1 nor φ_2 is actually a true angle of internal friction. The angle that most nearly approximates a true friction angle is β and because successive approximations such as already described will yield a β not differing much from β_2 the latter may be taken as representing the proper friction angle or friction parameter $\overline{\varphi}$. The $\overline{C}$ is that part of the shear strength which is independent of the normal stress on the particular plane. In most cases, the $\overline{C}$ value will vary with the normal consolidation pressure σ_c. The particular values of $\overline{C}$ given in Figures 4, 5, 6, and 7, however, are for $\overline{\sigma}_c = 0$. At a particular $\overline{\sigma}_c$ the actual $\overline{C}$ would be the vertical distance from the $\overline{\varphi}$-plane to the φ-plane.

SAMPLE PREPARATION AND TEST PROCEDURE

Both laboratory consolidated and field undisturbed samples were used as test samples for the consolidated undrained triaxial tests. All laboratory samples (artificial soil) were consolidated from a slurry to a water content of about 45 percent and at a corresponding void ratio of about 1.3 in consolidation tubes provided with both end and side drainage. The solids used were illitic with a liquid limit of 39.0 and a plasticity index of 18.0 (94 percent of the particles were less than 105 μ in size), and a kaolinitic clay with a liquid limit of 70.2 and plasticity index of 29.4. The tube-consolidated soils were further consolidated in the triaxial cell at the predetermined cell pressure under incremental pressure loadings. Final water contents varied from 29 to 23 percent for the illite clay, and 44 to 49 percent for the kaolinite clay, depending on final cell pressure.

To reduce possible particle interaction arising from interparticle forces, artificial soil samples were also consolidated using ethylene glycol ($C'_2H_4(OH)_2$) as the pore fluid. In a clay-liquid system there exist attractive and repulsive forces between the particles. In general, the attractive forces are insensitive to system characteristics but the repulsive forces can be altered considerably depending on the system polarity

In replacing water with ethylene glycol in a soil-liquid system, the repulsive forces are reduced. Because ethylene glycol is less polar than water, this also permits the attractive forces to play a much larger role in the development of shear strength, allowing more contact between particles and thereby increasing the "frictional" contribution to strength. The purpose of the tests with ethylene glycol was therefore (a) to compare the shape of the stress locus curve with that obtained using water, (b) to see if the "frictional" part and "cohesive" part of the curve as previously described still existed, and (c) to compare the friction angles obtained in the two cases. The preparation procedure was similar to that used for samples with distilled water as the pore fluid. In this instance, ethylene glycol was also used in both the saturation and pore pressure line for the test series.

As still another test of the stress locus characteristics, any "undisturbed" samples of Chicago clay were tested in the same manner. These were obtained from freshly taken Shelby tube samples. The properties of this clay corresponded to those described by Peck and Reed (10). Because of the possibility of a slight precompression, the test samples were not fully consolidated in the triaxial cell prior to load application. Thus, it may be argued that $\bar{\varphi}$ obtained from this series may be somewhat low.

Because complications can arise as a result of partial and incomplete saturation of the test samples, all samples were allowed to take in pore fluid in the triaxial chamber under a low confining pressure —3 psi. Pore pressure response was determined as a function of cell pressure increase to ascertain the degree of saturation of the test sample. Both side filter strips and double membranes were used as standard procedure. A rate of loading of 0.001 in. per min was selected as a reasonable rate of strain loading corresponding to the results obtained by Osterberg and Perloff (9) and with due cognizance of the effect of other possible factors involved as suggested and discussed by Seed (12).

RESULTS AND INTERPRETATION

The results of the tests reported herein are not meant to be used specifically as quantitative data but to point out and highlight the role and application of stress loci in evaluation of the significant effective stress parameters. Because the technique involved defines the stress conditions on a predetermined plane, it is necessary to explain the significance of the stress locus in terms of components of shear strength. Following the method previously described for determination of the stress locus, Figure 2 shows the stress loci for two extreme conditions. These are the conditions defined primarily by the response of the pore pressure under application of the deviator stress.

In Figure 2, stress locus B represents the condition where there is no pore pressure response under deviator stress application. The specimen is consolidated in the cell to a consolidating pressure of $\bar{\sigma}_c$ before load application. When the load is applied, because pore pressure response is zero, $\bar{\sigma}_c$ is equal to $\bar{\sigma}_3$ (lateral pressure for shear test) at all times and hence remains constant

$$\bar{\sigma}_1 = \bar{\sigma}_3 + \Delta\sigma$$

in which $\Delta\sigma$ = deviator stress (because pore pressure μ is zero). The stress locus defined is coincident with the line determined from the tangent of the rupture envelope to the maximum deviator stress circle (not shown in the figure). The angle resulting therefrom is $45^0 + \varphi/2$. In tests conducted on saturated Ottawa sand at close to the critical void ratio, little or no pore pressure was recorded except very close to failure where condition of incipient failure creates irregular strains in the specimen. The stress loci drawn from these tests were similar to that defined by case B in the figure, except close to the rupture point. The total strength component operative over the entire stress region is then the friction stress component.

Stress locus A in Figure 2 defines the condition for full pore pressure response—assuming that such a condition can be achieved in an ideal case. Following consolidation under the consolidation pressure of $\bar{\sigma}_c$ any application of the deviator load results in a full response of the pore pressure. In consequence, $\bar{\sigma}_c = \bar{\sigma}_1$ = constant, and $\bar{\sigma}_3 = \bar{\sigma}_c - u$. Therefore, the condition defined indicated that $\varphi=0$, the rupture envelope for

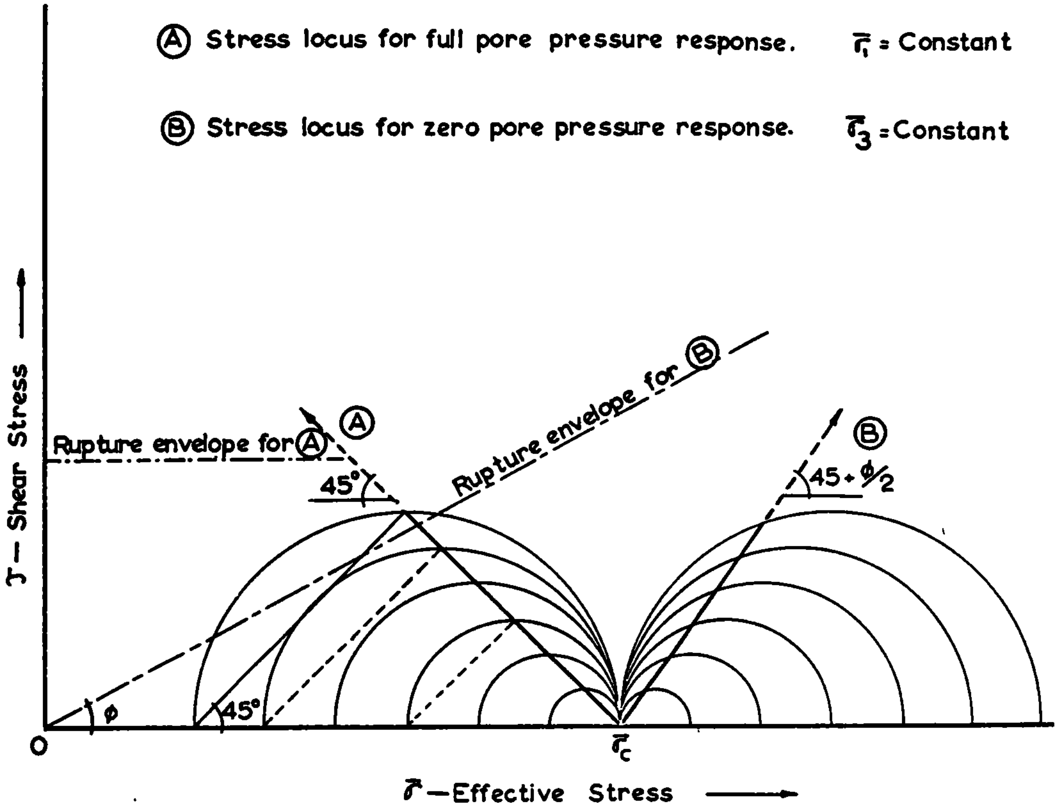

Figure 2. Stress loci for full pore pressure and zero pore pressure response.

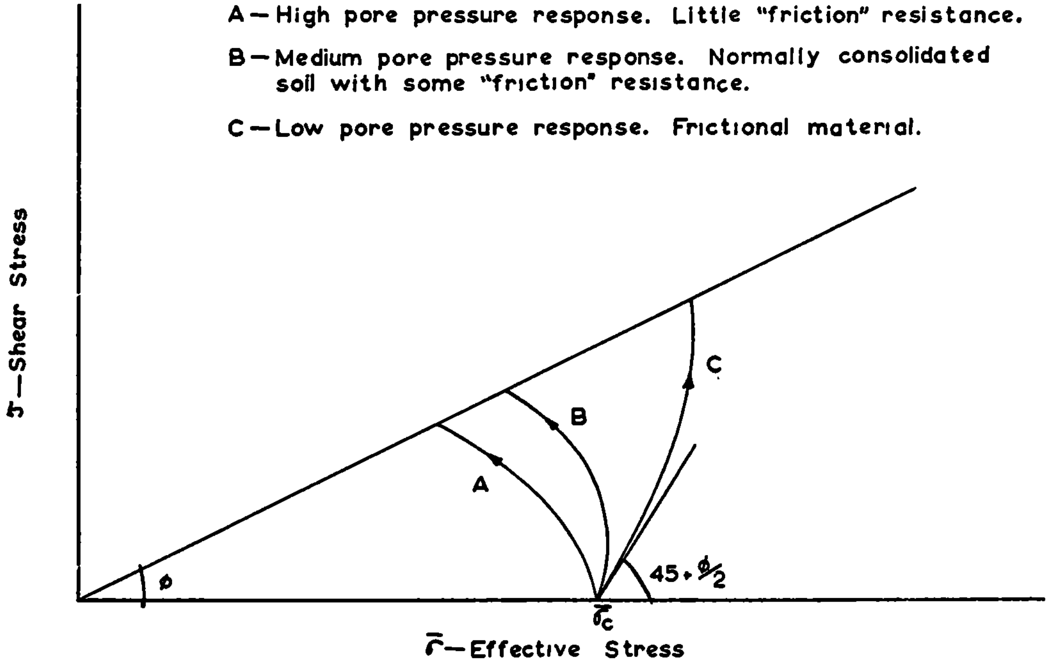

Figure 3. Stress loci for soils with varying pore pressure response.

case A, would be a horizontal line. In this instance, the inclination of the plane of failure is 45° and the stress locus defined is shown at 45° in the figure. Assuming that the ideal case for pore pressure response can be found, this would then indicate that the total shear strength component is the "cohesion" component.

Except for soils where pore pressure response is neither of the two extremes, the shear strength components will be operative throughout the entire test. Figure 3 shows the stress loci for soils with varying pore pressure response going from stress locus A, which is for a soil with a small frictional strength component and with high pore pressure response under load application, to stress locus B, which is for a soil of a higher frictional quality with a lesser pore pressure response, and final stress locus C illustrating the case for a still smaller pore pressure response and a much higher friction resistance component. Holtz and Ellis (4) have shown stress loci for artificial clay soils mixed with sand and gravel, and their results indicate the trend shown in cases A, B, and C in the figure. The higher the proportion of sand and gravel, the lesser is the pore pressure response and the more the stress locus trends from A to C, indicating the increasing influence of the friction strength component with proportional increases of sand and gravel.

The results of the consolidated undrained triaxial tests on both laboratory consolidated and natural soils are shown in Figures 4 through 7. Both the first and second approximations for the stress loci are shown.

In Figures 4 and 5, the soil used was illite clay and the variable component was the pore fluid. As expected, the shear strength for the illite clay with distilled water as its pore fluid was much lower than that of the same clay with ethylene glycol as its pore fluid—for corresponding densities and void ratios. Without going into the mechanism of particle interaction possibly arising as a result of interparticle forces, the argument can be resolved around the development of a higher friction strength component in the case of glycol in the absence of clay-water interaction. In comparing the friction angle φ between the two cases, there is a difference of about 2.5° between the φ established by the terminal points of the stress loci (and the tangents of the effective stress circles). $\varphi = 22^0$ for the illite clay with glycol and $\varphi = 19.5^0$ for the same clay with distilled water. However, in making the comparison on the basis of the effective friction angle, defined by the maximum effective normal stress on the stress locus, the difference between the two $\bar{\varphi}$'s is 5°; i.e., $\bar{\varphi} = 14^0$ and $\bar{\varphi} = 9^0$ for the illite clay with glycol and distilled water, respectively. The void ratios for the samples ranged from 0.83 at a cell pressure of 20 psi to 0.67 at a cell pressure of 56 psi. Variation in void ratio between companion samples was ± 0.02 at the same cell pressure following final consolidation in the cell and prior to load application.

The case for kaolinite is shown in Figure 6 where $\bar{\varphi}$ is reported as 11.5°. In the test on Chicago clay, results show $\bar{\varphi}$ to be equal to 11° (Fig. 7). Peck and Reed (10) report the value of 11° for consolidated quick tests on a Chicago clay with slightly higher consistency limits and water contents. The result for the slow tests given in the same report was 20°. The comparison is made here to show the use of the stress locus for evaluation of $\bar{\varphi}$. However, because there was a difference in consistency limits and natural water contents, presumably there would also be a small difference in the evaluated 'friction' angle.

The rupture envelopes for all soils were established by the terminal points of the stress loci. These in general defined a straight line for the same soil tested. The angle φ resulting from the establishment of the rupture envelope may be questioned in view of the technique used to obtain the second approximation for the stress locus as shown in Figure 1. However, the difference in φ values by the two procedures is relatively small. In Figure 8, only two cases are shown for the rupture envelopes drawn from the common tangent to the stress circles for illite clay with both types of pore fluid (glycol and distilled water). In comparing the effect of pore fluid on the friction angle φ in this figure with those in Figures 4 and 5, it can be seen that the percent difference is less than 1 percent. Although this correspondence may seem fortuitous, it can be shown that, by successive approximations for determination of the stress locus as shown in Figure 1, the final answer derived for φ will be identical with that shown in Figure 8. In this instance, the second approximation for stress locus determination has been found both adequate and sufficiently accurate.

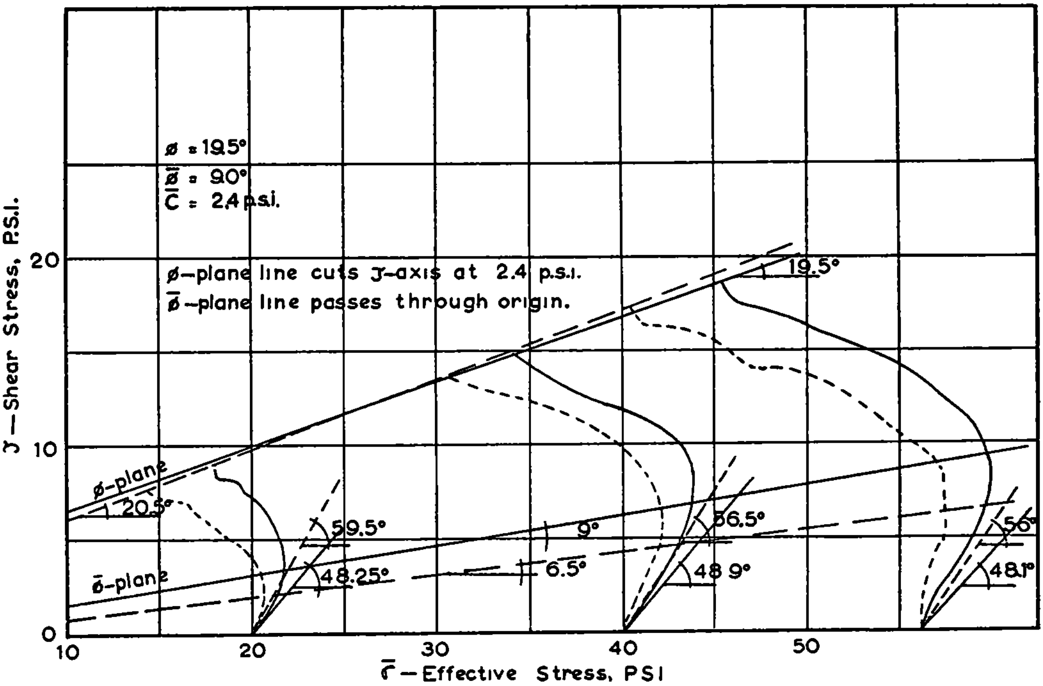

Figure 4. Stress loci for illite clay with distilled water.

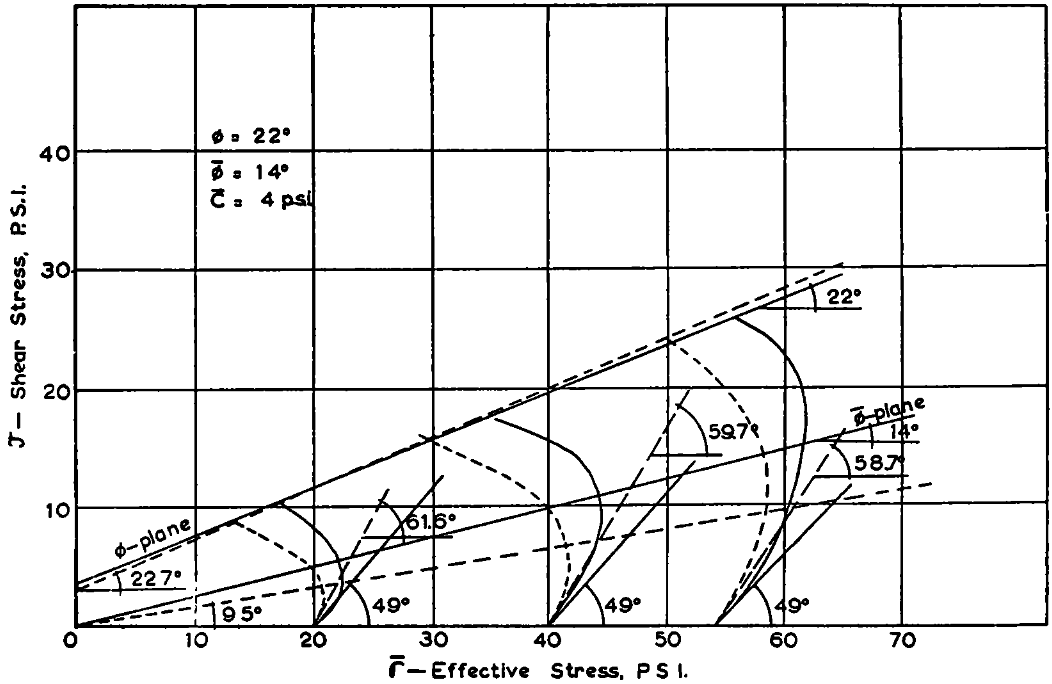

Figure 5. Stress loci for illite clay with $C_2H_4(OH)_2$.

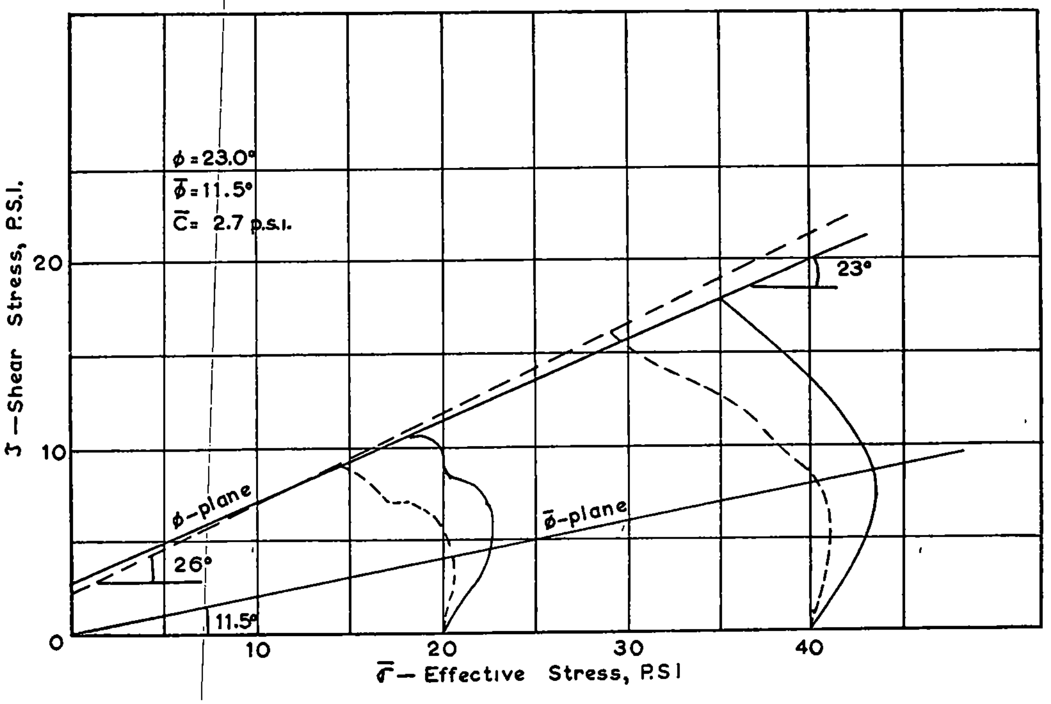

Figure 6. Stress loci for kaolinite clay with distilled water.

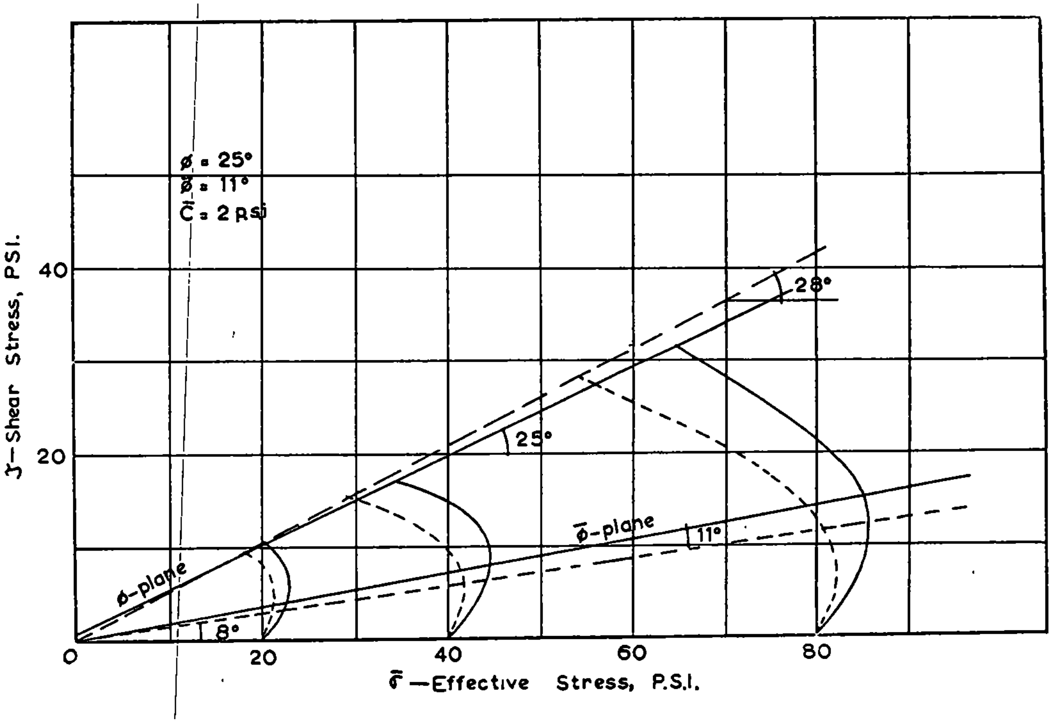

Figure 7. Stress loci for Chicago clay.

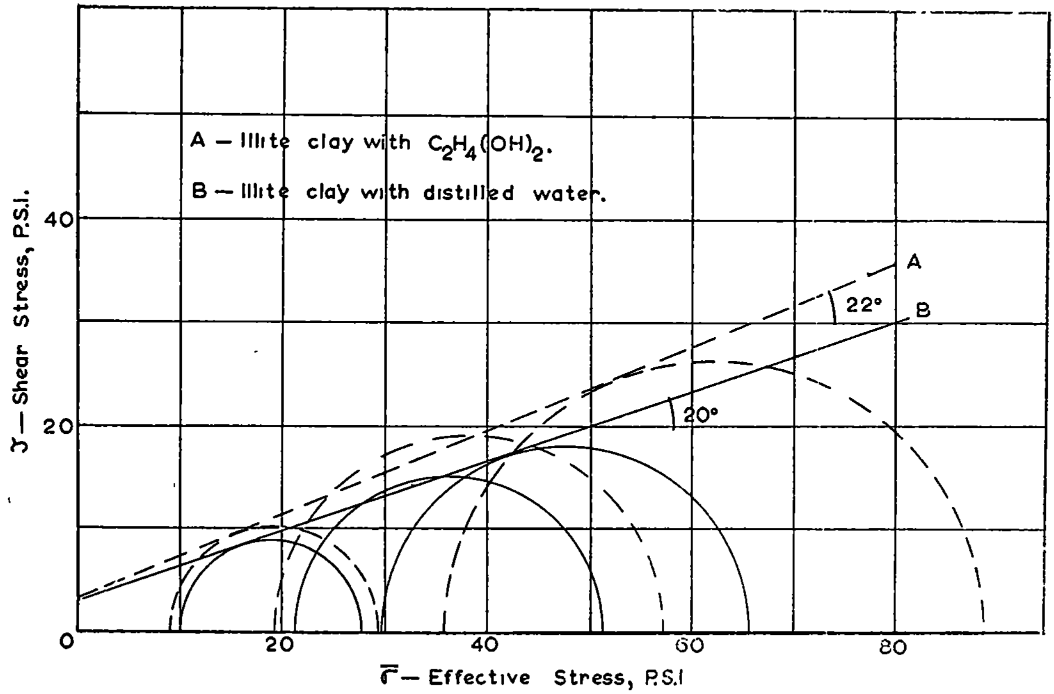

Figure 8. Effective stress circles for illite clay.

The maximum effective normal stress points on the stress loci for identical soils tested at varying confining pressure can be joined approximately by a straight line passing through the origin. The deviation from the straight line is slight.

SIGNIFICANCE OF STRENGTH PARAMETERS DERIVED FROM STRESS LOCI

It must be emphasized here that although both $\bar{\varphi}$ and $\bar{C}$ may be obtained from the stress loci, there is no possible way to define the actual boundaries of operation for these two parameters as limiting factors. Crawford (3) has shown that the friction angle varies with strain and increases as strain increases when measured in the conventional manner. In line with this, the cohesion parameter will also vary with strain. It is now well accepted that both these parameters are not constant during the entire test for measurement of shear strength, and it is further accepted that they are dependent on both pore pressure and strain—although it can be argued that strain may in turn be dependent to some extent on pore pressure.

The stress locus does not pretend to define the limits of operation of the parameters because it is well understood that both shear strength components are operative throughout the state of stress. Rather, it defines the relation between the shear stress and the effective normal stress for any one specimen subject to load application under the prescribed conditions. It is believed that the friction strength component increases rapidly in the initial stages of load application and reaches a peak while the cohesion strength component is still increasing. Beyond the peak friction component strength, total shear strength increases due to the continued increase of the cohesion strength component. This is shown in Figure 9. The stress locus in previous figures shows the shear stress increasing as effective normal stress increases (not necessarily as a linear function), to a point where shear stress increases while effective normal stress decreases. The friction strength component in the figure is drawn on this basis to interpret the variation of this component with strength. This does not necessarily mean a decreasing friction angle with increasing strength because the friction strength component is dependent on

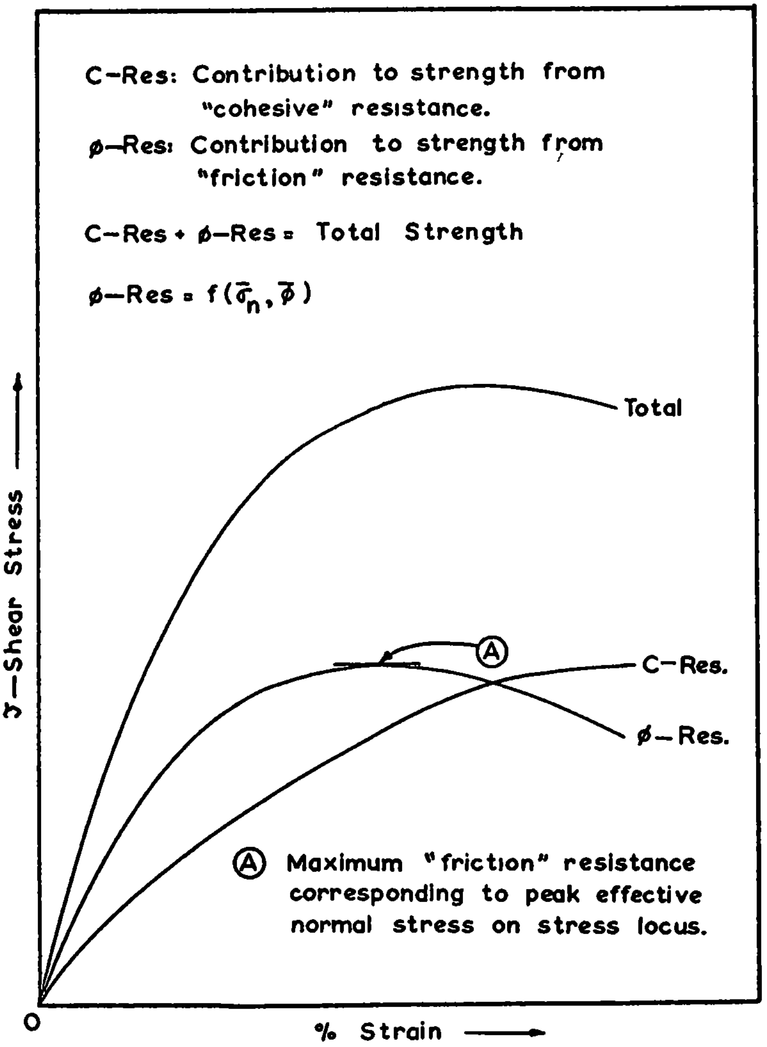

Figure 9. Strength parameter contribution to total strength.

both φ and effective normal stress. Schmertmann and Hall (11) also give data to show that φ increases with increasing strain as determined from their cohesion-friction-strain technique in which the pore pressure is controlled.

The total shear strength remains the same regardless of the method or technique used for interpretation of the strength component. Interpreting Crawford's (3) data on the basis of stress locus, $\bar{\varphi}$ = 9° instead of 17° as reported. The soil considered in this reported study was a sensitive marine clay. As pointed out by Bjerrum (1) in conventional consolidated undrained triaxial tests on sensitive marine clay, the effective friction angle measured does not correspond to the effective friction angle calculated from the ratio of undrained shear strength to effective overburden pressure. Although laboratory tests gave values of effective friction angle ranging from 32° to 25° for increasing plasticity index, the calculated values for actual field conditions gave values of from 6° to 8°. On this basis, the stress locus technique as applied to Crawford's data seems to bear out the value suggested by Bjerrum. However, whether this is the actual effective friction parameter seems to be more a question of definition.

For overconsolidated or partially saturated clay soils, the stress locus derived from the consolidated undrained test will give a characteristic "frictional" performance not unlike case C in Figure 3. The significance and interpretation of these loci will be discussed at a later date inasmuch as the present study is restricted to normally consolidated soils.

CONCLUSIONS

The stress loci technique used in this study shows the stress condition on a plane in the soil specimen at any one condition of strain. Although it is understood that the shear strength parameters are operative over the total stress range, a method is proposed here that defines the effective parameters, $\bar{C}$ and $\bar{\varphi}$, on the basis of their dominance in component strength contribution to total strength. By defining the effective friction angle $\bar{\varphi}$ in terms of a dependency on effective normal stress increase, a means for arriving at the parameter for normally consolidated soils is established.

In terms of the strength components, it is believed that the dominant component should define the operative parameter at its maximum point. The method suggested by this study has not been tested in terms of field application where it is believed verification or contradiction of the value of the parameters derived therefrom can be obtained.

ACKNOWLEDGMENTS

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Discussion

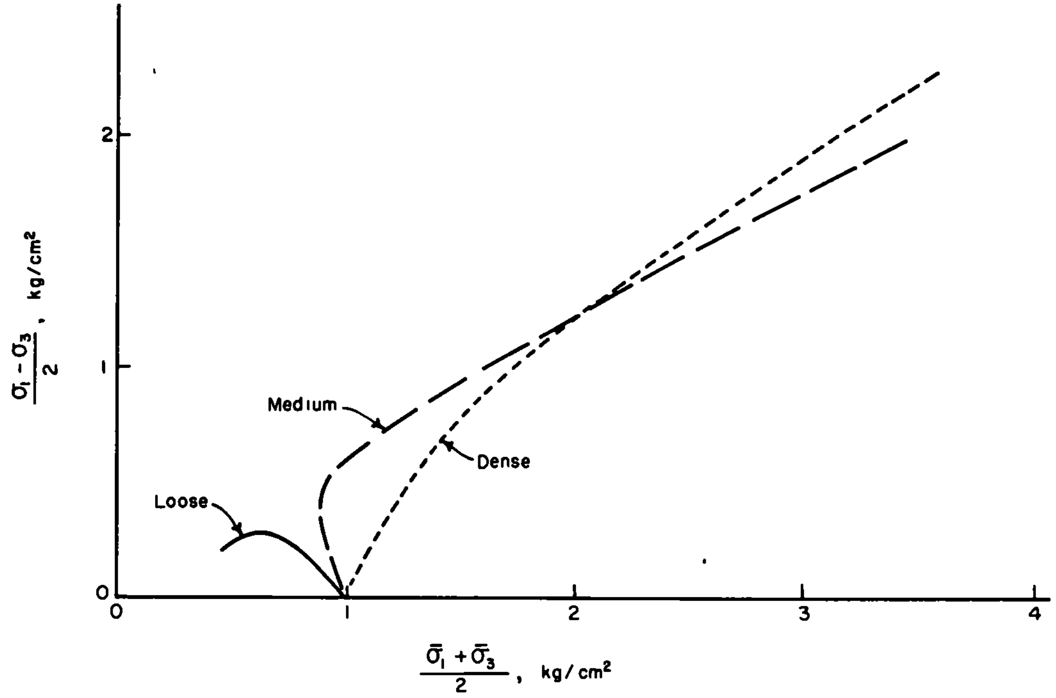
CHARLES C. LADD, Assistant Professor of Soil Engineering, Department of Civil Engineering, Massachusetts Institute of Technology.—There is no doubt that better methods need to be developed for relating the shear strength parameters in terms of effective stresses as measured by triaxial tests to those that describe actual strength behavior in the field. The parameters developed by the authors could be a step in the right direction. However, the interpretation of the physical significance that the authors have attached to these parameters is considered by the writer to be incorrect.

Referring to Figures 2 and 3, which show effective stress vector curves from consolidated undrained triaxial tests with pore pressure measurements, the authors state that if there is "zero" or "low pore pressure response" (in other words, Skempton's A factor is close to zero), the resistance to shear is primarily frictional in character because if "the component of shear resistance increases, it seems most reasonable to deduce that this constitutes a friction shear component." On the other hand, if there is a "full" or "high pore pressure response" (i.e., the A factor is close to unity), the material has little frictional resistance and hence the resistance must be predominately cohesional in character because the shear stress is increasing while the effective normal stress is decreasing.

Test data on a saturated Ottawa sand close to the critical void ratio are quoted to support the preceding argument. Because the vector curve showed an increasing effective normal stress as the shear stress increased (the A factor was very low), the "friction stress component" was, by definition, operative over the entire stress region. One cannot question that the strength of a sand is predominately frictional in character. However, if the authors had run a consolidated undrained test with pore pressure measurements ($\overline{\text{CU}}$ test) on a sample of loose sand, the proposed criteria for defining frictional behavior would be inconsistent. This is shown in Figure 10 which presents vector curves from $\overline{\text{CU}}$ tests on dense, medium, and loose specimens of a fine sand. The curves have been plotted from stress-strain data reported by Bjerrum, et al. (2). (The curves are not exact because the figure from which the data were scaled was very small. However, the general shapes of the curves are correct.) Although the strength of all three specimens is certainly frictional in character, the proposed criteria would define the strength as cohesional, cohesional and then frictional, and frictional for the loose, medium, and dense specimens, respectively.

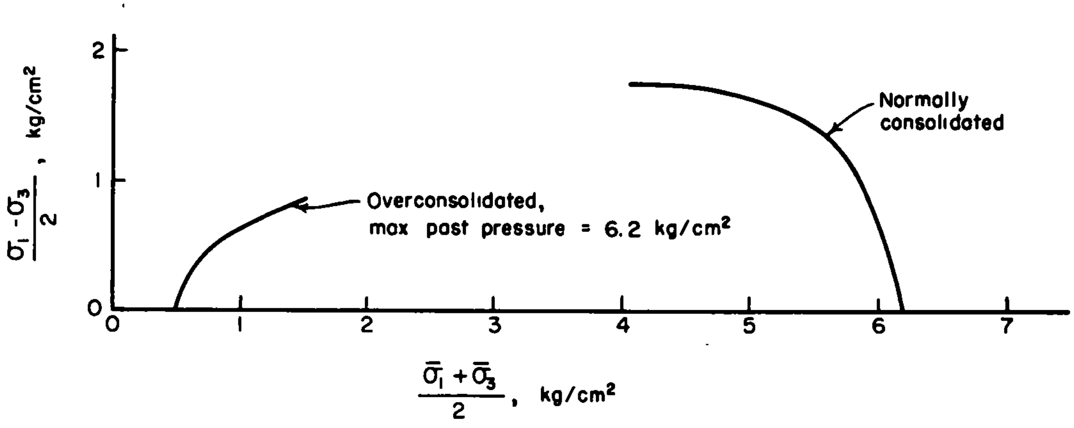
Similar inconsistencies arise if the criteria are applied to clays. Vector curves from $\overline{\text{CU}}$ tests on normally consolidated and overconsolidated specimens of a plastic clay (P.I. = 40 percent) are shown in Figure 11. By the authors' criteria, the strength of the normally consolidated specimen, where A approaches unity, is predominately cohesional and that of the overconsolidated specimen, where A becomes negative, is predominately frictional. The Hvorslev parameters and hypotheses concerning the mechanism of shear strength in clays (for example, Lambe (4) and Ladd (3)) suggest, if anything, that the opposite is more plausible.

Although the writer disagrees with the interpretation that the authors have attached to their parameters, he feels that their analysis of strength via the shape of vector curves may develop into a most promising avenue of research. For example, $\overline{\text{CU}}$ tests by Bailey (1) have shown that leached samples of Boston blue clay that had been consolidated in sea water show very low friction angles at maximum stress difference and



(Data replotted from Bjerrum, et al, 1961)

Figure 10. Vector curves from $\overline{\text{CU}}$ tests on a fine sand.



(Liquid Limit = 63%, Plasticity Index = 39%)

Figure 11. Vector curves from $\overline{\text{CU}}$ tests on Vicksburg buckshot clay.

marked decreases in strength at strains in excess of the strain at maximum stress difference. Such a soil might well exhibit a low friction angle in the field under conditions of undrained shear. (The samples were leached in the triaxial cells so that no disturbance occurred prior to shear.)

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R. YONG and E. VEY, Closure—The test results reported by Dr. Ladd for saturated sand and overconsolidated clays corroborate very well the current authors' findings of their initial study of the restrictions and limitations of the stress locus technique. The importance of this restriction was demonstrated in similar tests by the writers and evaluation procedures for overconsolidated and partially saturated clays. Included also, as a means for further definition of the nature of the effective stress parameters were tests on saturated sands and silts. Extreme care was taken in the paper to restrict the proposed method of evaluation to normally consolidated clays subjected to consolidated undrained triaxial tests.

The initial test results obtained from $\overline{CU}$ tests on various soil types showed that the stress locus can indeed vary over the entire range of forms shown in Figure 10 by Dr. Ladd for dense, medium, and loose fine sand. However, this same variation can be obtained with any soil type, depending on pore pressure response under load. In the case of fine sand, this response varies with the density at which the specimen was prepared. In the case of clay, it varies principally with prestress history, method of specimen preparation, and testing procedure. The effect of specimen preparation and testing procedure on the character of the stress locus has been well illustrated in other recent studies.

Because the technique proposed is essentially one that includes the variation of pore pressure coefficient A, it can be said that the stress locus is actually a measure of A for any particular specimen. Perhaps the significance of the stress locus is better viewed in this manner, because in the final analysis it is the performance of the specimen that is the object of strength evaluation studies. Based on the intrinsic dependency of the stress locus on pore pressure coefficient A, and on other recent studies it is fairly apparent that the stress loci shown in Figures 4 through 7 of the paper are acceptably those derived from normally consolidated clays. In particular, it is interesting to note from Henkel's studies that plotting his stress paths on the same basis as the stress loci demonstrated in the paper, the stress loci for normally consolidated and overconsolidated clays agree well with the current authors' suggestions. As stated in the paper, "For overconsolidated or partially saturated clay soils, the stress locus derived from the consolidated undrained test will give a characteristic 'frictional' performance not unlike case C in Figure 3."

To apply the working hypothesis to cases other than normally consolidated clays would be extending it beyond the defined limits of the test and the restrictions imposed by the writers. It is perhaps unfortunate that all the different varieties of stress loci were not included in the original presentation. However, it was purposely intended to restrict the paper to a consideration of normally consolidated clays and to defer consideration of both overconsolidated and partially saturated clays until more data are available.