Vertical Stresses in Subgrades Beneath Statically Loaded Flexible Pavements

GEORGE F. SOWERS and ALEKSANDAR B. VESIC, Respectively, Professor and Associate Professor of Civil Engineering, Georgia Institute of Technology

Full-scale static load tests have been made of single, dual, and dual tandem truck tire loads on flexible pavements to determine the stress distribution in the subgrade and the relative load spreading ability of different base course materials now in use by the Georgia Highway Department.

The measured stresses below topsoil and soil bound macadam base pavements are comparable to those computed by the Boussinesq theory. The measured stresses below the sand asphalt are comparable to or slightly more than those computed by the Boussinesq. The stresses found below the soil cement base are much lower than those given by Boussinesq and are comparable to the stresses found by the two layer elastic theories. Asphaltic concrete overlays reduce the stresses to the same degree as an equal thickness of the topsoil or soil bound macadam.

•THE RATIONAL DESIGN of any structural system, including pavements, requires a knowledge of the stresses induced by the imposed loads. However, little information is available regarding the stresses developed in the underlying soils by wheeled vehicles supported by pavements. The purpose of this research was to investigate the stresses produced in soil subgrades by wheel loads such as truck tires on flexible pavements.

A major function of any pavement is to spread the concentrated load delivered to it by the wheel sufficiently so that the stress ultimately transmitted to the underlying soil will not cause shear nor excessive deformation. The rigid pavement does this by beam or slab action. The flexible pavement system action is more complex—the load is spread through a mass of discrete particles that react like an elastic continuum when they are confined. The flexible pavement is composed of layers, all of which contribute to the load spreading, but some with more specialized functions. The surface resists the vertical and tractive loads as well as the wear of the wheels and provides a smooth roadbed. The base course is the main load spreading member. It is usually the thickest layer and the one that involves the greatest variety of materials and methods of construction. The subgrade, either natural soil or compacted fill, furnishes the ultimate support for the load. Some pavements include a subbase course that serves as a transition between base and subgrade. Resurfacing or overlaying with asphaltic concrete restores a damaged surface. It also adds another load-spreading layer to the pavement system.

Of course, the stresses at any point in the subgrade consist of combinations of shear and normal stress. Whether all the components of stress or only certain ones need be evaluated depends on the criteria established for pavement performance and design. The earliest methods of design were related to shear and shear failure in the subgrade; therefore, in these methods shear stresses have been considered the most important. Recent studies indicate that deflection may be the better index to design; and therefore, the vertical normal stress is of greatest significance. In this project, only the vertical normal stress has been investigated.

Although flexible pavements enjoy widespread use, little information has been available regarding the way in which vertical stresses are transmitted through them into the subgrade. Various simple assumptions have been proposed. One such method assumes

that the wheel is concentrated at a point on the pavement surface. The vertical component of the load spreads uniformly over an area defined by a cone whose vertex is at the pavement surface and whose sides slope at 45° with the vertical. A second method assumes the contact area of the tire to be a circle and the vertical component of the load is spread uniformly over an area defined by the frustum of a cone whose upper base is the circle of tire contact and whose sides slope at an angle of 30° with the vertical.

Various elastic theories have also been proposed for evaluating subgrade stresses beneath pavements. In all of these the pavement is represented by a simplified model whose physical properties can be described mathematically. The stresses in the model are analyzed by the laws of mechanics and assumed to be the same as those in the real pavement it represents. Certainly their validity depends on how accurately the model represents the real pavement.

Few data are available to confirm or reject any of the proposed theories for determining vertical stresses in the subgrade. One purpose of this investigation was to compare the measured stresses beneath pavements with those computed theoretically and if possible verify the theoretical methods.

The loads encountered in highway work are predominantly moving and vertical (except at points of braking and acceleration). The rate varies from a standstill to 70 mph or more. However, present knowledge of the behavior of soils and similar fragmental materials indicates that maximum deflections and stresses are more likely to occur with sustained load rather than rapidly changing load. Therefore, only static loading was investigated experimentally.

Theoretical Stress Distribution

The analysis of stress distribution in loaded soil masses or pavements is, generally, a problem that is being solved, for ideal materials, by the theory of elasticity.

The basic solution of this problem is the well-known Boussinesq solution (1) for a single, vertical point load acting on the horizontal surface of a semi-infinite, homogeneous, isotropic, elastic solid. This solution was extended to the case of load uniformly distributed over any finite area by Love (2). Particular solutions have been worked out, evaluated and presented in tables or graphs by many authors. The work

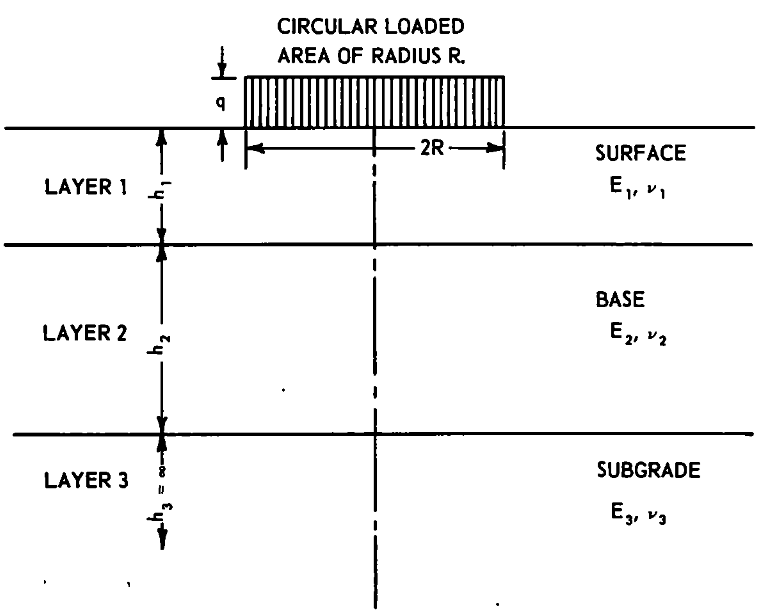

Figure 1. Flexible pavement system.

by Steinbrenner (3), Newmark (4, 5), Fadum (6), Fergus and Miner (7), Foster and Ahlvin (8), and Deresiewicz (9) should be cited as perhaps the most important for stress evaluation in pavements.

A semi-empirical modification of the Boussinesq solution by introduction of the concentration index has been proposed by Griffith (10) and Fröhlich (11) for semi-infinite soil masses that are nonhomogeneous in a vertical direction or nonisotropic. Their work was further extended by Ohde (12).

The stress distribution in semi-infinite, homogeneous, orthotropic solid, having different deformation moduli in horizontal and vertical directions was investigated by Buisman (13), Wolf (14), Jelinek (15, 16, 17) and Koning (18). Buisman also investigated the case of an orthotropic and nonhomogeneous mass having a modulus of deformation linearly increasing with depth. The problem of a semi-infinite solid reinforced by horizontal perfectly flexible membranes was solved by Westergaard (19) and integrated by Fadum (6).

Of particular interest for stress distribution in pavement systems is the problem of a multi-layered elastic solid (Fig. 1). The basic solution of this problem is that found by Burmister for a circular, uniformly distributed load at the surface of a two-layer system (20). This solution was extended by the same author to the case of a three-layer system (21) and generalized by Schiffman for any case of surface loading (22). Numerical evaluation of stresses in a two-layer system was performed by Fox (23) and Hank and Scrivner (24). Evaluation of stresses at the interfaces of a three layer system was made by Acum and Fox (25). All mentioned evaluations of stresses were performed for points beneath the center of the loaded circular area only.

Previous Experiments

The mentioned theories of stress distribution are based on several simplifying assumptions which, to a greater or lesser degree, always deviate from the real behavior of materials. Assumptions are made, for instance, that the materials are perfectly elastic and have linear relationship between stresses and strains defined, generally, by constant Young's moduli E and Poisson's ratios ν. However, soils and other pavement materials are only partly elastic and do not have linear stress-strain relationship. Neither E nor ν are constant but vary with the applied load. Both E and ν may have quite different values in tension than in compression (many soils and base materials have practically no tensile strength at all). Also, most solutions are based on assumption of perfect homogeneity and isotropy of different layers. Whereas pavement layers are usually reasonably homogeneous, they normally possess a structural aniostropy. Subgrades often display a decrease of compressibility with depth; they also may be stratified or laminated.

Consequently, discrepancies have to be expected between theoretical and actual stress in loaded soil masses or pavement systems. Several investigators have undertaken so far the task of checking to what extent the actual stresses follow the stress pattern indicated by the theories.

The early investigations of this kind, made on homogeneous sand fills in large boxes (26 to 31) as well as on layered pavement models (32), gave somewhat misleading results concerning the concentration of stresses under the applied loads, as compared with the Boussinesq theory for homogeneous solids. Namely, due to the limited dimensions of test boxes, the bottom acted as a rigid base and caused additional stress concentration.

Thorough experimental studies of stress distribution in homogeneous silt and sand masses were made at the Waterways Experiment Station at Vicksburg, Miss. (33, 34, 35). It was found that the pattern of measured stresses for both kinds of homogeneous soil followed closely the general shape indicated by the Boussinesq theory. There was somewhat higher concentration of stresses under the loads, particularly in sand; however, the use of the Fröhlich concentration index did not improve the agreement between the theoretical and measured stresses.

Experimental studies were made by McMahon and Yoder (36) on stress distribution in homogeneous clay as well as in a two-layered mass consisting of crushed stone base of variable thickness and a clay subgrade. This investigation showed within the homo-

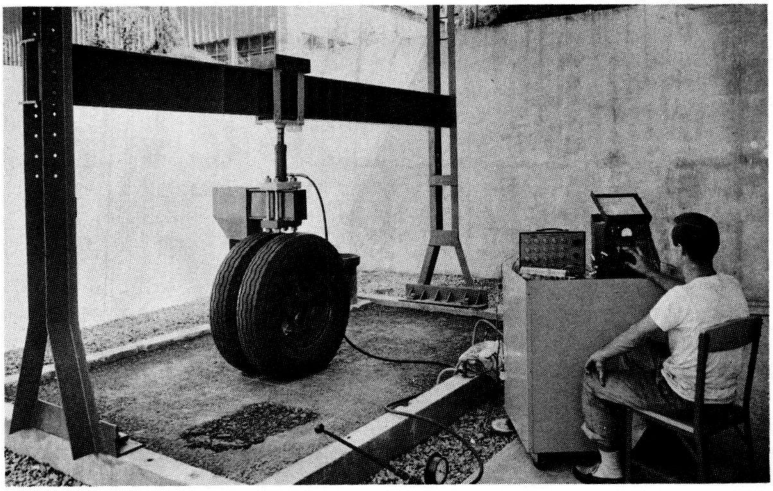

Figure 2a. Pavement load test in progress.

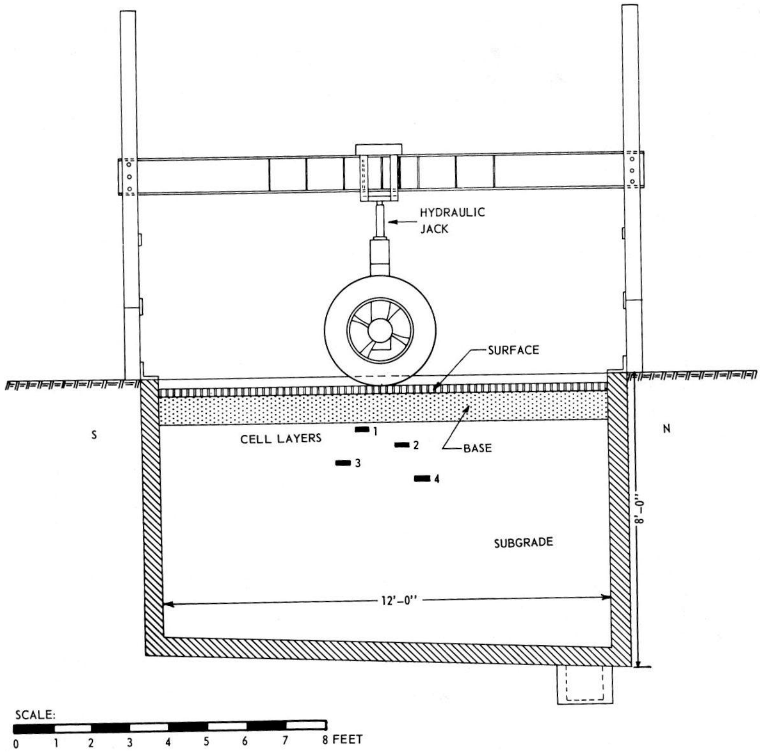

Figure 2b. Cross-section of the test pit showing position of layers of pressure cells and the loading equipment.

geneous clay mass a stress pattern close that predicted by the Boussinesq theory. At the same time the stresses in the two-layered system were somewhat reduced directly under the interface; however, only with a distance not greater than the radius of the loaded plate. Nevertheless, stresses were considerably higher than those predicted by the Burmister two-layer theory, and in general closer to values obtained by the Boussinesq theory for homogeneous soil.

TEST APPARATUS

Full-scale models of flexible pavement systems, including the subgrade, were constructed in a test pit. Static loads were applied to the pavement by truck tires and the vertical stresses in the subgrade were measured by pressure cells.

Test Pit and Load Frame

To control moisture content changes in the subgrade the entire model was built in a test pit (Fig. 2a and 2b). The inside dimensions are 8 ft wide, 7 ft deep and 12 ft long, so that the volume of the model was 25 cu yd. At a 9,000-lb wheel load the approximate diameter of the tire contact area was 11 in. The depth and width of the pit were 7.6 and 8.7 diameters respectively which means that the rigid boundary effects should not be significant A steel frame, fastened to the ends of the pit with a heavy beam spanning the centerline, furnished the reaction for loading the tires.

A hydraulic jack mounted beneath a carriage riding on the beam supplied the load. The load was measured by the hydraulic pressure with a calibration error of less than 2 percent.

Wheel Assemblies

Single, dual, and dual tandem wheel assemblies were employed, with 9- by 20-in. heavy duty truck tires inflated to pressures between 70 and 90 psi. These tires are designed for a maximum load of 9,000 lb each although they are occasionally overloaded in practice. The tire spacings were 10.5 in. center to center in the dual configuration

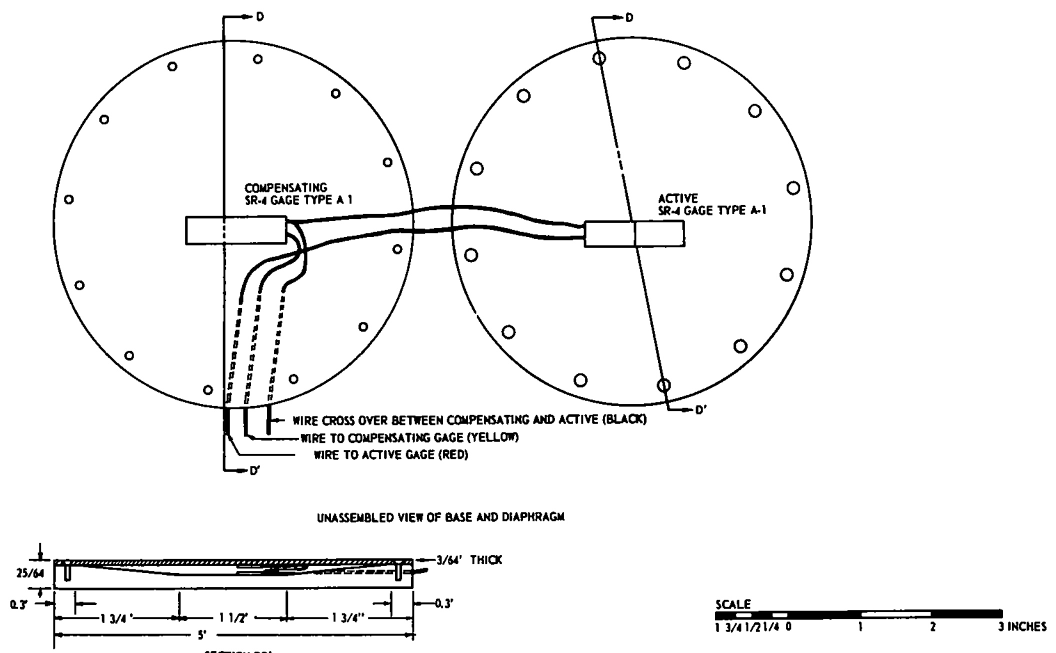

Figure 3. Typical pressure cell.

and 54 in. axle to axle in the dual tandem. These are standard spacings in U. S. Highway trucks.

Pressure Cells

A simple diaphragm type pressure cell (Fig. 3) previously developed in the Georgia Tech soil laboratory was employed to measure the vertical normal stresses in the subgrade. It consists of an elastic membrane (a thin disk of aluminum) fixed at its perimeter to a thicker circular base plate. The membrane responds to normal stresses by bending and stretching slightly. A SR-4 electrical strain gage, bonded to the membrane, measures the strain induced by the pressure. A second compensating gage is mounted in the base plate at a point where it undergoes little strain under load. The individual gages are internally waterproofed with petrosene wax. The entire assembled cell, with its vinyl-insulated lead wires, is waterproofed by dipping in a solvent-type vinyl cement, From three to four coats of the cement provide a waterproof, flexible jacket that can resist immersion in water for a month or more. Three different cell diameters were used (4, 5, and 6 in.) with diaphragms 0.19 and 0.12 in. thick to provide a range in sensitivities. The cells were 0.4 in. thick or less. The cell diameter to thickness ratios were between 10 and 15 which minimized the variable effects of soil arching over the deflecting diaphragms.

Each cell was individually calibrated by the application of a uniform pressure by means of an air-loaded rubber membrane in a soil-filled calibration chamber. Initially the cells were calibrated completely surrounded by subgrade soil compacted to the same density as in the test section. Comparative tests, however, showed that virtually the same results were obtained for pressures within the middle range for each cell if the cell were placed on a surface of compacted soil and loaded directly with the rubber membrane. Subsequent calibration was performed in this manner.

From 25 to 30 cells were employed, arranged in 4 (or 3) straight lines perpendicular to the axis of the pit, with each line at a different depth below the surface and at a different position along the pit axis. In this way no cell was directly below another so as to minimize any arching or load concentration caused by their rigidity. The exact depths varied from test to test. In the first series, on the topsoil base, the upper layer was in the base at a depth of 9 in. below the surface, and the remainder in the subgrade at depths of 15.7, 27.6, and 45.6 in. below the surface or respectively 0.8, 1.4, 2.5, and 4.1 diameters (based on an equivalent circular 9,000-lb wheel load). In the remainder of the tests the upper cell layer was in the subgrade, just below the base course at a depth of 11 to 13 in. (1 to 1.2 diameters) with a 3-in. asphaltic surface or 15 in. (1.4 diameters) with a 6-in. asphaltic surface. The other layers were at depths of about 17, 23, and 29 in. (1.5, 2.1, and 2.7 diameters) with a 3-in. asphaltic surface and correspondingly greater depths with the 6-in. thick surface. The exact depths for each test series are shown on the plots of the test results.

PAVEMENTS SYSTEMS

The pavement systems tested were typical of those currently employed by the Georgia State Highway Department. The subgrade soil for all tests was a micaceous sandy silt, a residual material derived from granite gneiss This type of soil is widespread in the northern half of the State and is characterized by a low resistance to deformation, particularly under load. It is far from an ideal subgrade and is typical of the poorer materials in the State on which roads must be built. Four types of bases were used: topsoil, (silty, well-graded sands from two locations in Central Georgia); soil-bound macadam consisting of 40 percent by weight of the topsoil and 60 percent by weight of size 467 crushed granite; soil cement, consisting of 4 percent by weight Type 1 portland cement and 96 percent of the soil-bound macadam; and sand asphalt, a uniform subangular quartz sand with 5 percent RC-3 cut-back asphalt.

Standard laboratory tests were run on all these materials to determine their physical properties. Because of the coarse aggregate size it was impossible to run the ordinary compaction test on the soil-bound macadam and soil-cement macadam materials. These materials were compacted in an 8-in. diameter by 16-in. high mold in 2-in. layers by

a 5.5-lb hammer filling 12 in. using sufficient blows on each layer to provide the same 12,400 ft-lb per cu ft as in the Standard AASHO test. The physical properties of these materials are summarized in Table 1. The surface in all cases was a plant mix asphaltic concrete.

Pavement Construction

Different combinations of base, base thickness and pavement thickness were tested as summarized in Table 2. The initial work employed 8-in. thick base courses and

TABLE 1

PROPERTIES OF SUBGRADE AND BASE MATERIALS

Material	Description	Liquid Limit	Plastic Index	Percent Passing No. 10	Percent Passing No 200	Maximum Density[a] (pcf)	Class.
Silt subgrade	Micaceous fine sandy silt	45	8	98-100	36-40	95-97	A-5
Topsoil	Silty, well-graded sand	14 5	0	98	12-15	128	A-1
Soil-bound macadam	Subangular to angular granite gneiss (sizes 467)[b] 60 percent; 40 percent topsoil			40	6-8	131	
Soil cement	4 percent Type 1 cement, 96 percent soil-bound macadam above					135	
Sand asphalt	Uniform subangular quartz sand 95 percent, 5 percent RC-3 cutback asphalt	0	0		0	105	A-3[c]
Asphaltic concrete	Plant Mix, 5 percent Bitumen					140	

[a]Std ASTM D 698-58T Method C for sand, topsoil, mica silt. For others a special compaction test was employed (see text)

[b]ASTM D 448

[c]Sand alone.

TABLE 2

TEST CONDITIONS

Test Series	Subgrade Soil	Surface-Plant Mix Asphalt Conc Thickness (in)	Base Course Composition	Base Course Thickness (in)	Loading Wheel	Loading Total Load (kips)
I	Mica silt	3	Topsoil I	8	Single	5, 9, 13.5
					Dual	5, 9, 13 5, 18
					Dual tandem	18, 27, 36
II	Mica silt	3	Soil-bound macadam	8	Single	5, 9, 13 5
					Dual	9, 13 5, 18
III	Mica silt	3	Soil-cement macadam	8	Single	5, 9, 13 5
					Dual	9, 13 5, 18
IV-1	Mica silt	3	Sand asphalt	8	Single	5, 9, 13 5
					Dual	9, 13 5, 18
IV-2	Mica silt	6[a]	Sand asphalt	8	Single	5, 9, 13 5
					Dual	9, 13 5, 18
V-1	Mica silt	3	Topsoil II	8	Dual	9, 13.5, 18
V-2	Mica silt	6 5[a]	Topsoil II	8	Single	8 5, 12 5, 17
					Dual	9, 13 5, 18
VI-1	Mica silt	3	Soil-cement macadam	6	Single	5, 9, 13 5
					Dual	9, 13.5, 18
VI-R	Mica silt	3	Soil-cement macadam	6	Single	13 5[b]
VI-2	Mica silt	6 5[a]	Soil-cement macadam	6	Single	5, 9, 13 5
					Dual	9, 13 5, 18
VI-F	Mica silt	6 5[a]	Soil-cement macadam[c]	6	Dual	9, 13 5, 18

[a] 3-in overlay

[b] 1,000 cycles

[c] Subgrade and base inundated

3-in. asphaltic concrete surface courses (laid in two layers). Later tests included 3- to 3.5-in. thick asphaltic concrete overlays on the 3-in. surfaces, as well as 6-in. base thickness.

The subgrades were constructed in accordance with the current Georgia Highway Department practice for embankments. The lower 3 ft was compacted to 90 percent of the maximum density as specified by ASTM D-698 - 58T - C Compaction test and to the upper 3 ft to a density of between 95 and 100 percent of the same maximum. Both were tamped in 2-in. thick layers with a gasoline-driven dynamic device, the "Jay Tamp" at a moisture content equal to or slightly below the optimum. The topsoil and sand asphalt base courses were compacted to 100 percent of the ASTM D698-58T-C max.; the soil-bound macadam and the soil-cement macadam were compacted to 100 percent of the maximum density found on the large compaction molds. The asphaltic surface was compacted with the Jay Tamp in 1.5-in. thick layers to as great a density as possible. Tests of cores from one series showed a mean density of 131 pcf which is slightly less than that obtained by rolling the same mix on a highway job.

Engineering Properties of Pavement Components

Tests were run on each of the pavement components to determine their modulus of elasticity and strength properties. Triaxial shear tests were made of all the materials in which both stress-strain characteristics and stresses at failure were determined. The tests of the subgrade, topsoil, and sand asphalt were run on both laboratory specimens compacted in a 4-in. diameter 8-in. high mold and on undisturbed samples cut

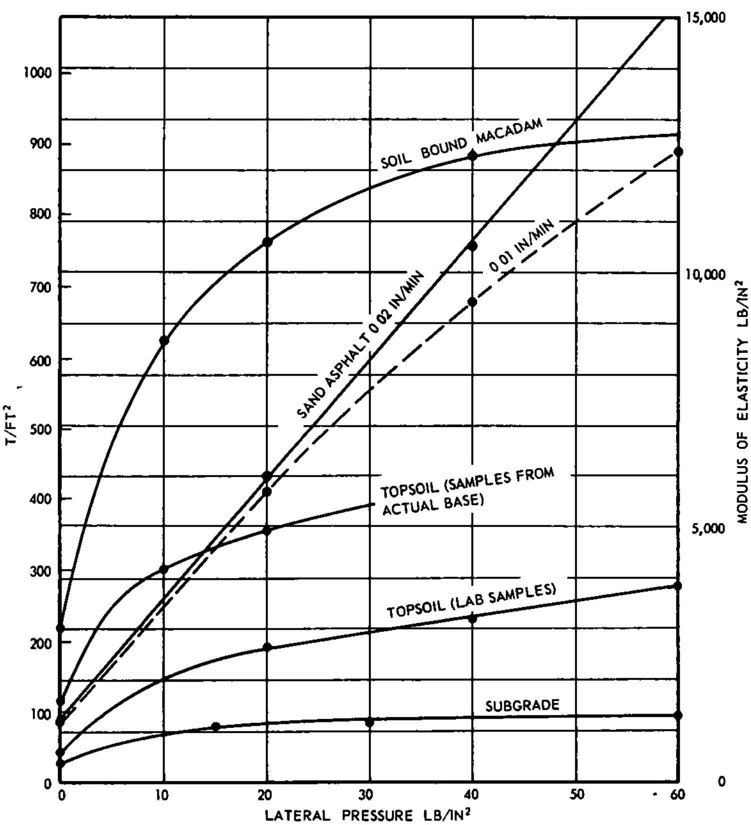

Figure 4a. Modulus of elasticity, E, of base and subgrade materials obtained by triaxial tests at different confining pressures.

from the test section. The tests on the soil-bound macadam were made on specimens compacted in the 8-in. diameter 16-in. high mold in the laboratory. All were tested in a large triaxial cell with interchangeable 4-, 6-, and 8-in. diameter sample bases. In all cases the undrained (quick) procedure was used, with a constant axial deformation rate of 0.02 in. per min. The sand asphalt was also tested at a rate of 0.01 in. per min to determine whether shear rate had any effect on its deformation properties. It was tested at temperatures comparable to those measured in the base course at the time of stress measurement. The elasticity test results are shown in Figures 4a, 4b, 5a, and 5b. They show the initial or tangent modulus of elasticity of each material as a function of the minor principal stress and the ratio of the modulus of elasticity of the base to that of the silt subgrade at equal minor principal stresses. The strength and elasticity data are summarized in Table 3.

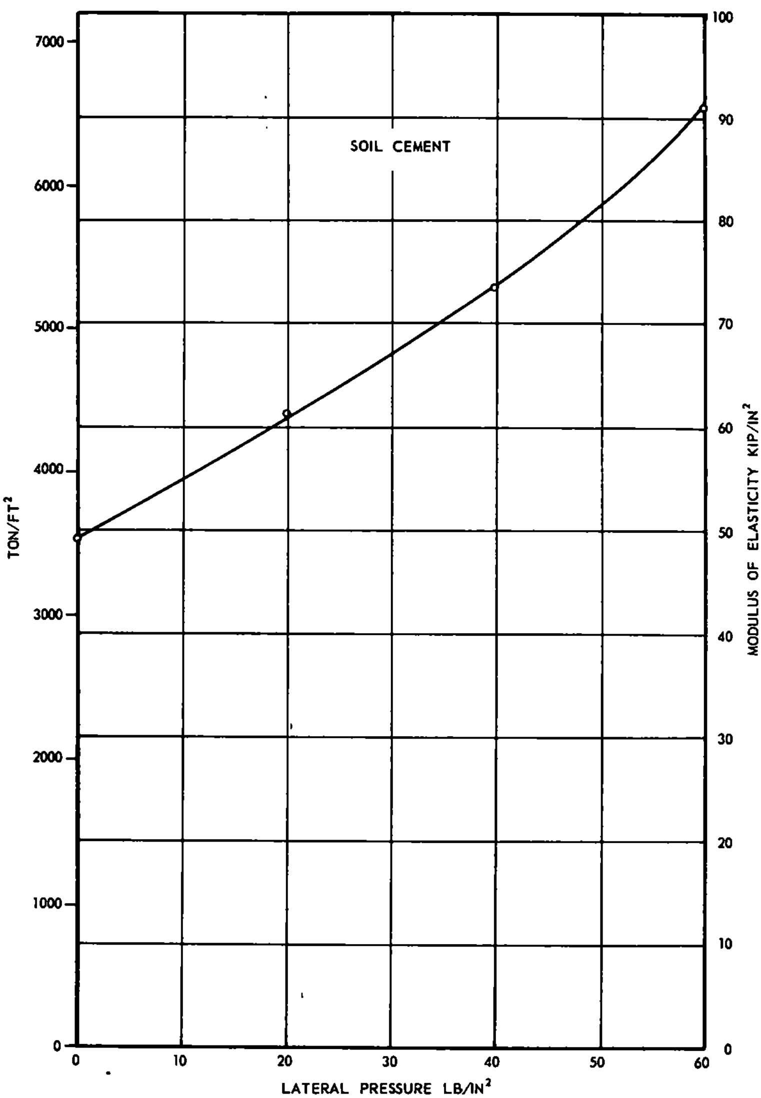

Figure 4b. Modulus of elasticity, E, of soil cement base material obtained by triaxial tests at different confining pressures.

TABLE 3

STRENGTH AND DEFORMATION CHARACTERISTICS OF MATERIALS

Material	Weight (pcf)	Water Content (%)	Strength Characteristics c (psi)	Strength Characteristics φ (°)	Deformation Characteristics from Triaxial Tests: Elasticity Modulus (psi) At 0 Psi	At 20 Psi	At 40 Psi	At 60 Psi	Strain at Failure (%) At 0 Psi	At 20 Psi	At 40 Psi	At 60 Psi	Elastic Modulus from Plate Load Tests (psi)	Tangent Modulus from CBR Tests (psi) No Surcharge	0.40-Psi Surcharge	CBR Value No Surcharge	0.40-Psi Surcharge
Subgrade	79 1	26 8	9 0	23	328	1,104[a]	1,160[b]	1,346	2 5	13.7	14 3	17.6	1,300	691	1,237[c]	3 4	4.2
Topsoil samples																	
From actual base	121.2	10 2	20 2	33	1,640	4,910	4,800	-	4 9	6 3	8 2	-	10,400[d]	4,400	6,650	29.1	44 1
Prepared in laboratory	123 0	10 6	5 1	33	545	2,700	3,140	3,970	3 8	10 4	13 2	23.9	-	-	-	-	-
Soil-bound macadam	131 1	3 9	2 5	37	2,940	10,520	12,360	-	0 6	2 9	8 2	-	11,200[d]	12,660	-	34 5	-
Soil cement	134 5	3.5	51 3	50	49,400	61,500	74,000	91,000	0.8	0.7	0 9	0.9	130,000	-	-	-	-
Sand asphalt																	
0.02 in./min loading rate	103 4	8 5	2 2	35	1,245	5,970	10,520	15,380	1.4	4.8	4 4	5.9	5,590	2,370	3,820	9.2	19 3
0 01 in /min loading rate	105.2	8 8	1 0	33	1,080	5,670	9,450	12,350	2 9	6.2	8.2						

[a]At 15 psi. [b]At 30 psi. [c]Surcharge 0 80 psi [d]With 3 in of pavement above

The modulus of elasticity curves all show an increase in E with an increase in confining pressure. The curve for the sand asphalt shows a continuing, nearly linear increase, whereas for the others E increases rapidly at low pressures but approaches a constant of higher pressures. The ratios of the E of the base to the E of the subgrade, however, are nearly constant regardless of confining pressure for all except the sand asphalt base. For the latter, the ratio increases with increased confinement. These variations in E are significant because most theoretical analyses, including the Boussinesq and the two-layer and three-layer theories assume that E is a constant that is independent of confinement.

Two types of in-place tests were conducted on the pavement components—California Bearing Ratio (CBR) and plate load tests. The CBR tests were made on the upper surfaces of the subgrade and base courses using the standard methods of the Corps of Engineers with a nominal surcharge load equivalent to 3 in. of pavement.

Plate load tests were made on the subgrade, all base courses, and the surface course for all but the sand-asphalt base pavement. An 18-in. diameter plate was employed that was rigidly reinforced so as to have negligible deflection.

An effective modulus of elasticity was computed from each CBR and plate load test using the theory of the deflection of a rigid circular load on an elastic medium. The plate load modulus of elasticity data for the base courses were computed from the two layer elastic theory using the elastic modulus of the subgrade as computed from the subgrade load tests. The results of these computations, which must be considered as rough indications at best, are also given in Table 3.

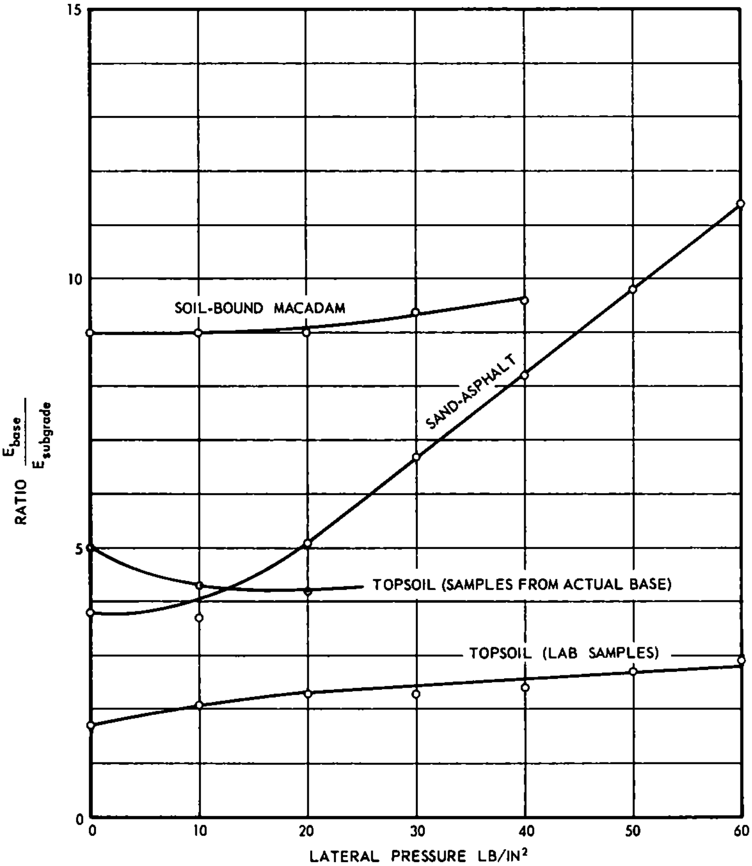

Figure 5a. Ratio of modulus of elasticity of base material to subgrade soil at different confining pressures.

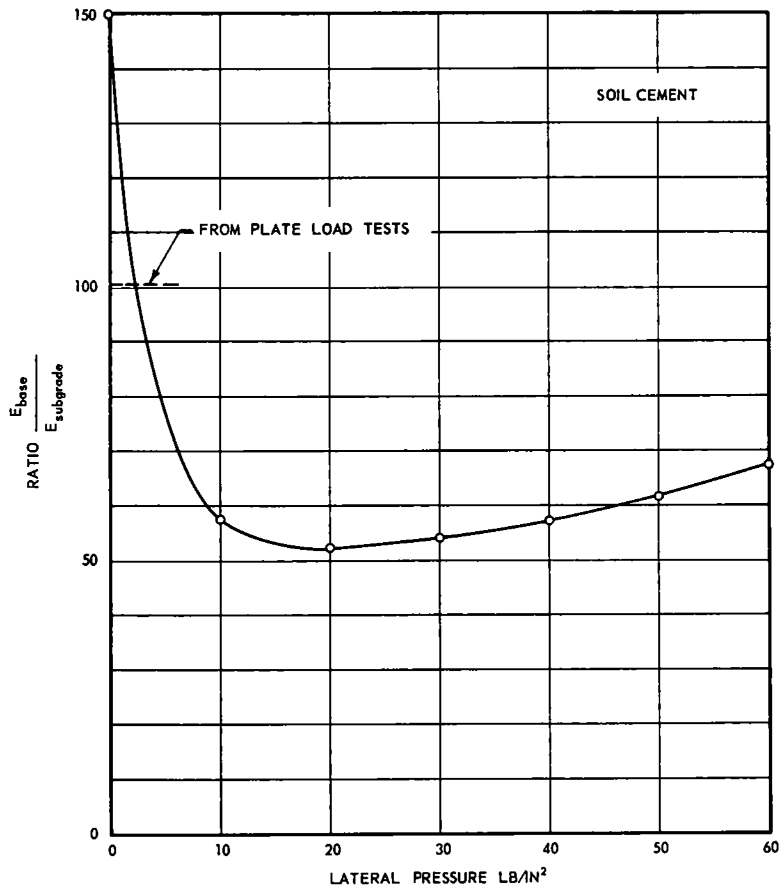

Figure 5b. Ratio of modulus of elasticity of soil cement base to subgrade at different confining pressures.

WHEEL LOAD-SUBGRADE STRESS TESTS

The full-scale model tests of truck tires on the pavement included more than sixty combinations of load, wheel configuration, and pavement design. In general the single tire was subjected 5, 9, and 13.5 kips equivalent to 10-, 18- and 27-kip axle loads and the dual tires were subjected to 9, 13.5, and 18 kips total or 18, 27, and 36 kips per axle. The range selected includes the legal loads permitted in many States and anticipates possible larger maximum loads of the future.

Tire Contact Area

The tire selected is designed for a maximum load of 9,000 lb at an inflation pressure between 80 and 90 psi. The size and shape of the tire contact was determined for loads of 4,500, 6,750, 9,000, and 13,500 lb in order to compute the stresses in the soil theoretically. The tire prints and the load-contact pressure curve are shown in Figures 6 and 7. The prints show a nearly circular contact at one-half the design tire load and a rectangle whose length is 2.5 times the width for 1.5 times the design tire load.

Test Procedure

Each load was placed on the pavement of ten different positions on the longitudinal

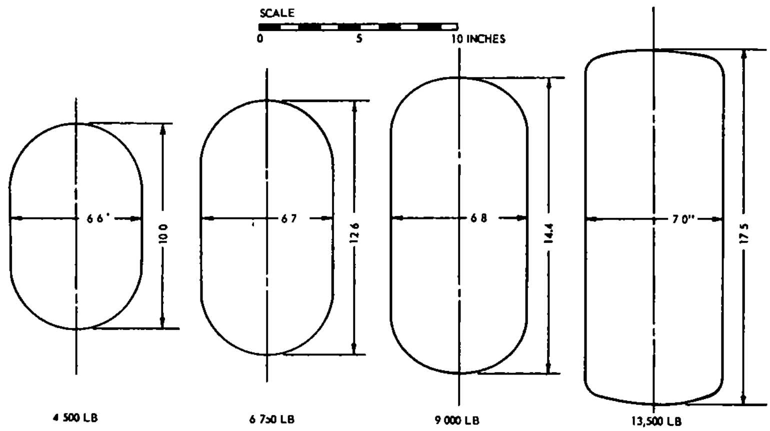

Figure 6. Typical tire prints for new 9 X 20 truck tire inflated to recommended pressure of 86 psi.

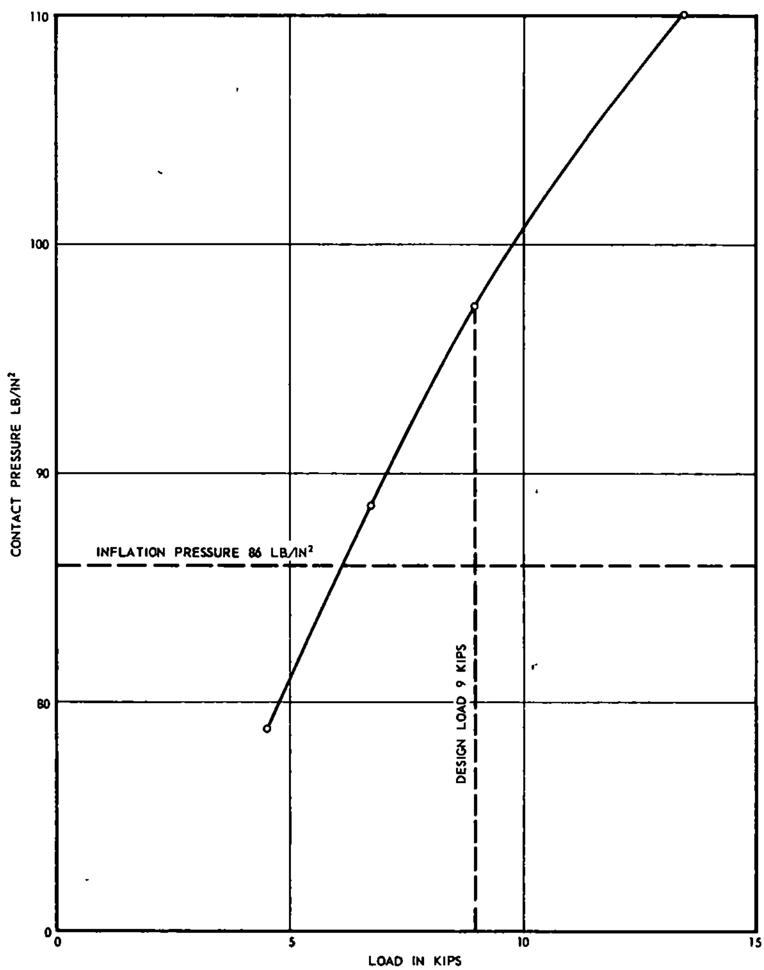

Figure 7. Variation of average contact pressure of a 9 x 20 truck tire as a function of tire load, at recommended inflation pressure of 86 psi.

axis of the pit—center, 3, 6, 12, 24 in. each side of center, and 36 in. north of the center. The vertical pressures were measured by each cell in each position. The results are shown graphically on the attached charts. They are plotted with vertical stress as a function of distance from the center of the load, regardless. The variable positioning of the tire made it possible for each cell to contribute more than one point to the curve. Only the center pressure with a distance from the load of 0 was measured by one cell alone at each depth. At least two and sometimes five sets of identical independent loadings and stress measurements were made for each tire load. All the individual stress measurements are plotted rather than averages to show the range of variability in the readings. In this paper only the data for the 9, 000-lb single tire

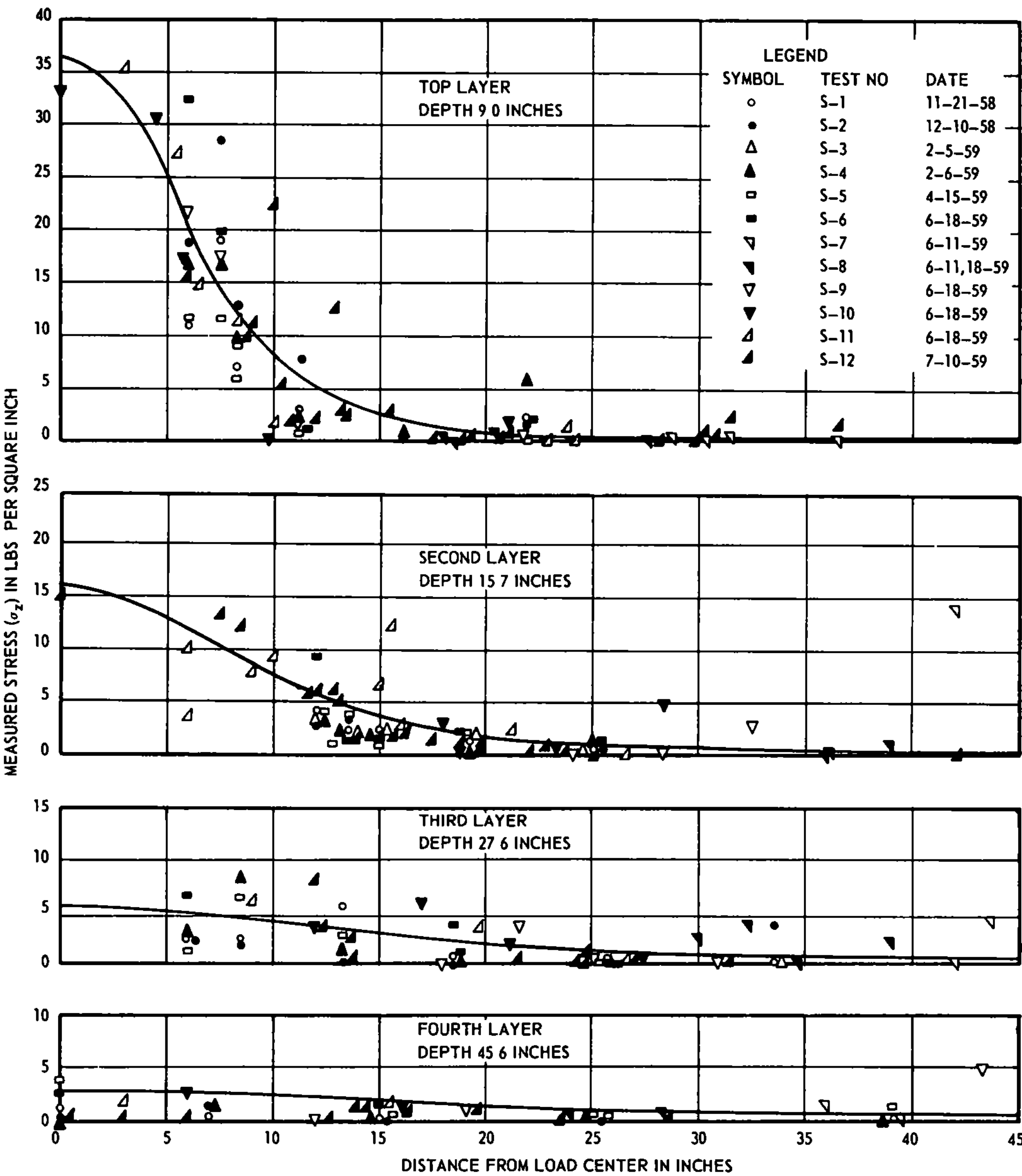

Figure 8. Measured stresses; single load 9,000 lb topsoil I base 8 in. thick; 3-in. surface. Solid line is Boussinesq stress distribution.

and the 13,500-lb dual are included. The data from the other loads are in direct proportion.

Theoretical Stresses

The theoretical stresses for each loading were computed from the load-tire contact area data using an equivalent rectangular or circular loaded area. The Boussinesq analysis of a semi-infinite homogeneous isotropic elastic solid was used for all. In addition the two-layer theory was employed for the soil-cement pavements by assuming that the modulus of elasticity of the asphaltic concrete surface was the same as that of the soil cement. Three different ratios of the elasticity of the upper layer to that of the lower layer were assumed: 1 to 1 (which is the same as the Boussinesq), 10 to 1,

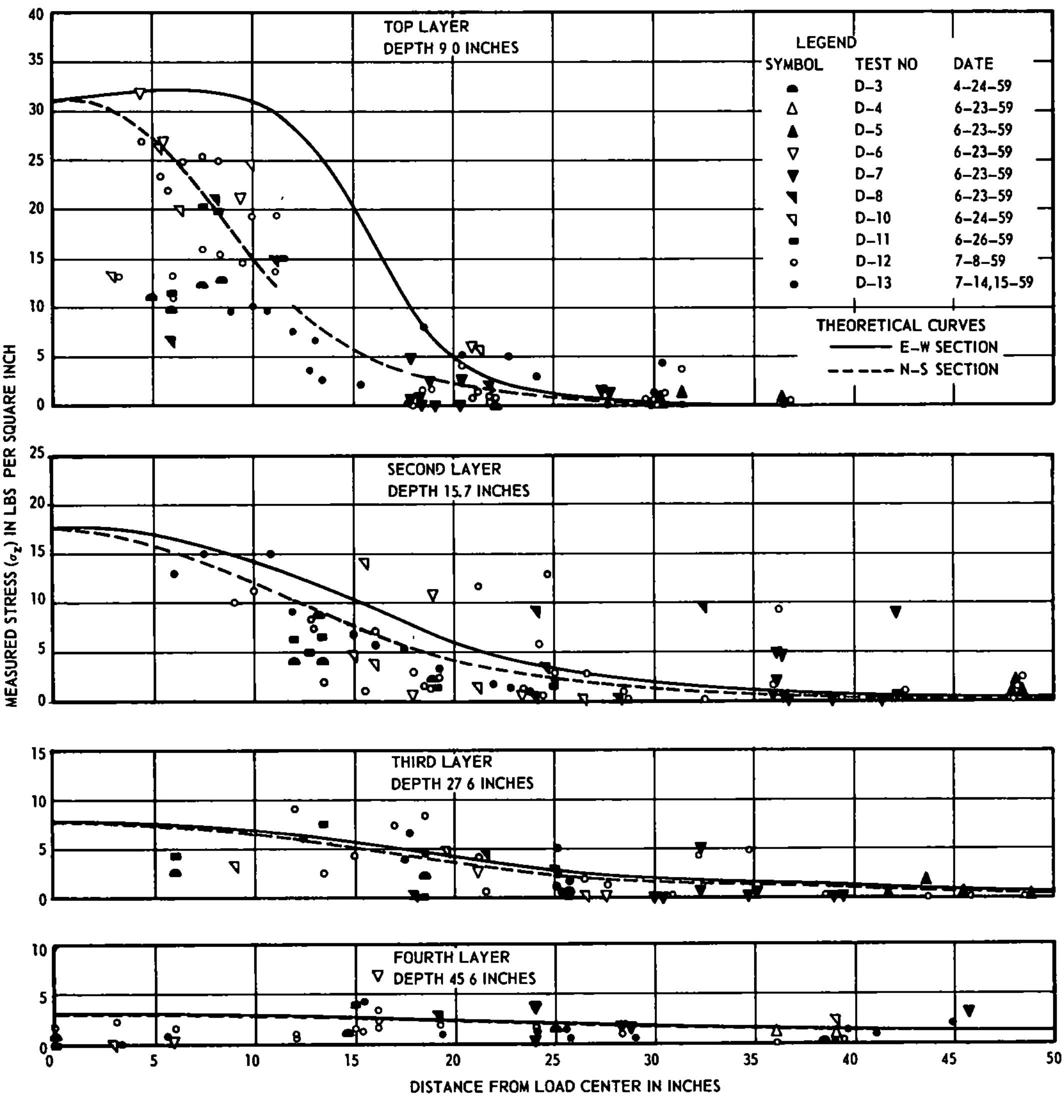

Figure 9. Measured stresses; dual load 13,500 lb topsoil I base 8 in. thick; 3-in. surface. Solid line is Boussinesq stress distribution parallel to axle; dotted line is Boussinesq stress perpendicular to axle.

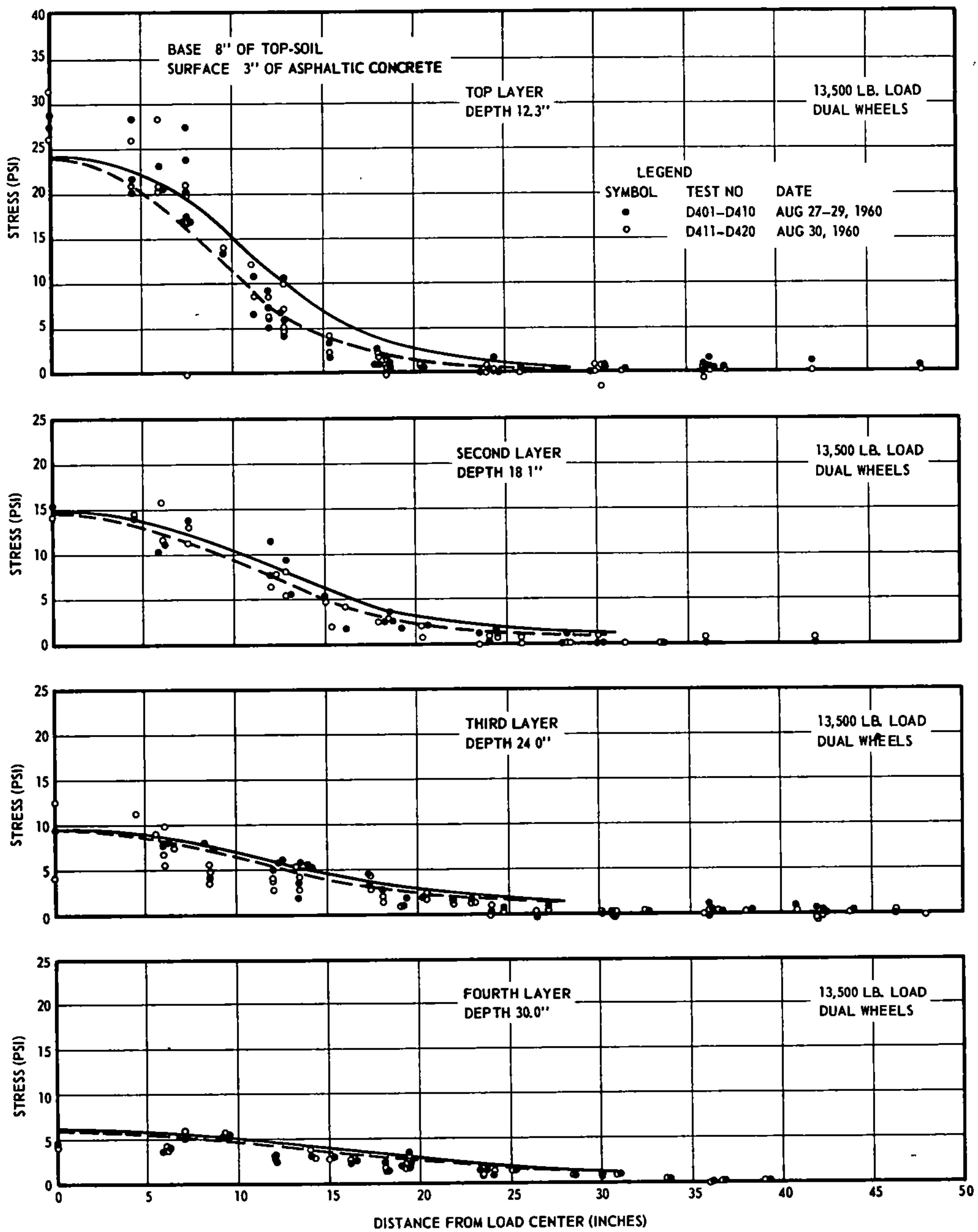

Figure 10. Measured stresses; dual load 13,500 lb topsoil II base 8 in. thick; 3-in. surface.

and 100 to 1. The stress computations for the dual tires were made in two directions: (a) along the axis of the pit—in the direction of wheel travel; and (b) on the line of the axle at right angles to the direction of wheel travel. (The actual stress measurements include both these directions and many others in between, and are not differentiated in the graphical presentation.) The theoretical stress distribution is shown in Figures 8 through 26 by the continuous curves. In the dual-tire results, the solid curve represents the stress distribution in line with the axle and the dashed curve at right angles to the axle.

Topsoil Base Pavements

The measured vertical stresses for the topsoil base pavement systems are shown in Figures 8 through 12 as individual points on the graphs. They show that the vertical

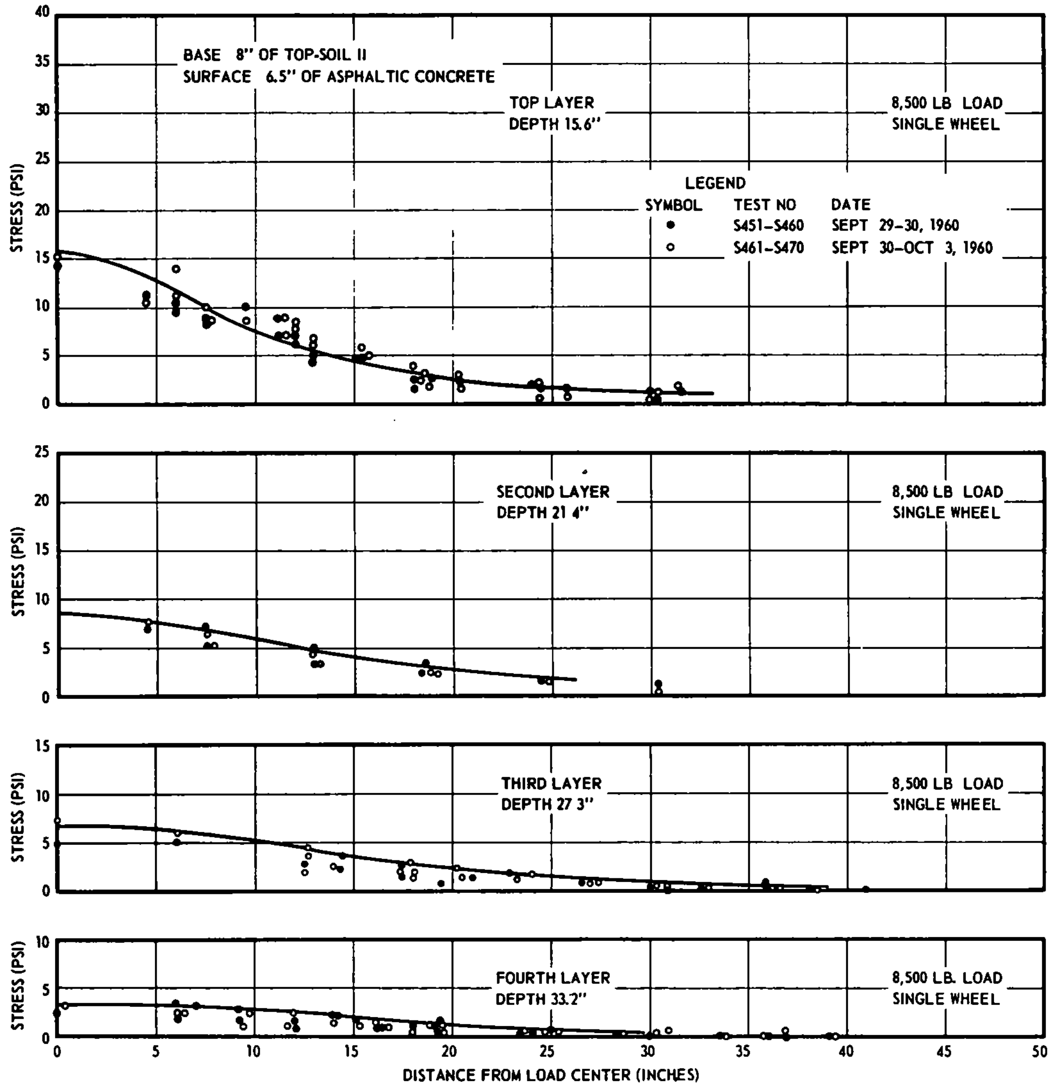

Figure 11. Measured stresses; single load 8,500 lb topsoil II base 8 in. thick; 3-in. asphaltic concrete surface and 3.5-in. asphaltic concrete overlay.

pressures in the subgrade decrease rapidly with increasing depth The pressure distribution is seen to be close to that given by the Boussinesq theory. The maximum pressure in the subgrade, under the center of the load is equal to the Boussinesq for Topsoil I and equal to or slightly greater (up to 15 percent more) for Topsoil II. For the topsoil subgrades, therefore the Boussinesq theory, based on a semi-infinite homogeneous isotropic elastic mass, is found to be a reasonably accurate representation. This is somewhat surprising because the ratio of the modulus of elasticity of the base to the subgrade is 3 to 4 which, by the two layer theory, should yield slightly lower stresses directly beneath the center of the load. Two factors are undoubtedly responsible. First, in order for the two layered theory to be theoretically valid, there must be tensile stresses in the interface between the two layers. The Mohr envelopes show a cohesion of only 15 psi for the topsoil (based on a curved envelope) or 20 psi (based on an approximate straight line) for the samples taken from the model pavement and 5

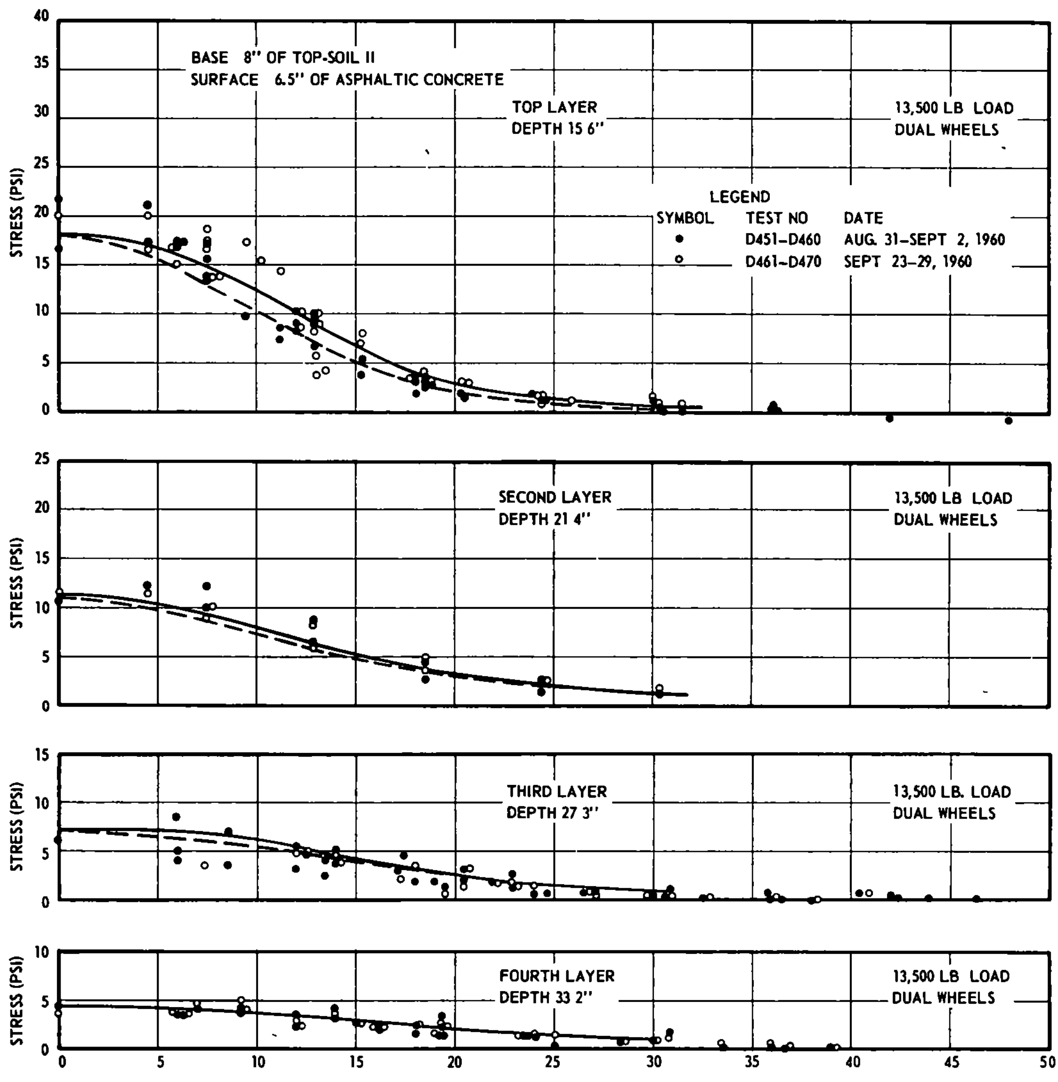

Figure 12. Measured stresses; dual load 13,500 lb topsoil II base 8 in. thick; 3-in. thick asphaltic concrete surface with 3.5-in. thick asphaltic concrete overlay.

psi for the laboratory compacted samples. The cohesion in the subgrade is 9 psi. Neither material, therefore, is capable of resisting much tension, which invalidates, to some degree, the two-layer theory. Second, neither modulus of elasticity is a constant although both the two-layer theory and the Boussinesq theory assume unchanging E values. It is possible the increase in E with increasing confinement causes a stress concentration, as noted by Griffith and Fröhlich, and partially offsets the gain in load spreading produced by the more rigid base. However, it also may be argued that the ratio of the E of the base to that of the subgrade is nearly constant so that the variation in E with confinement is not so significant. A theoretical analysis of the effects of varying E would be instructive, but is not available.

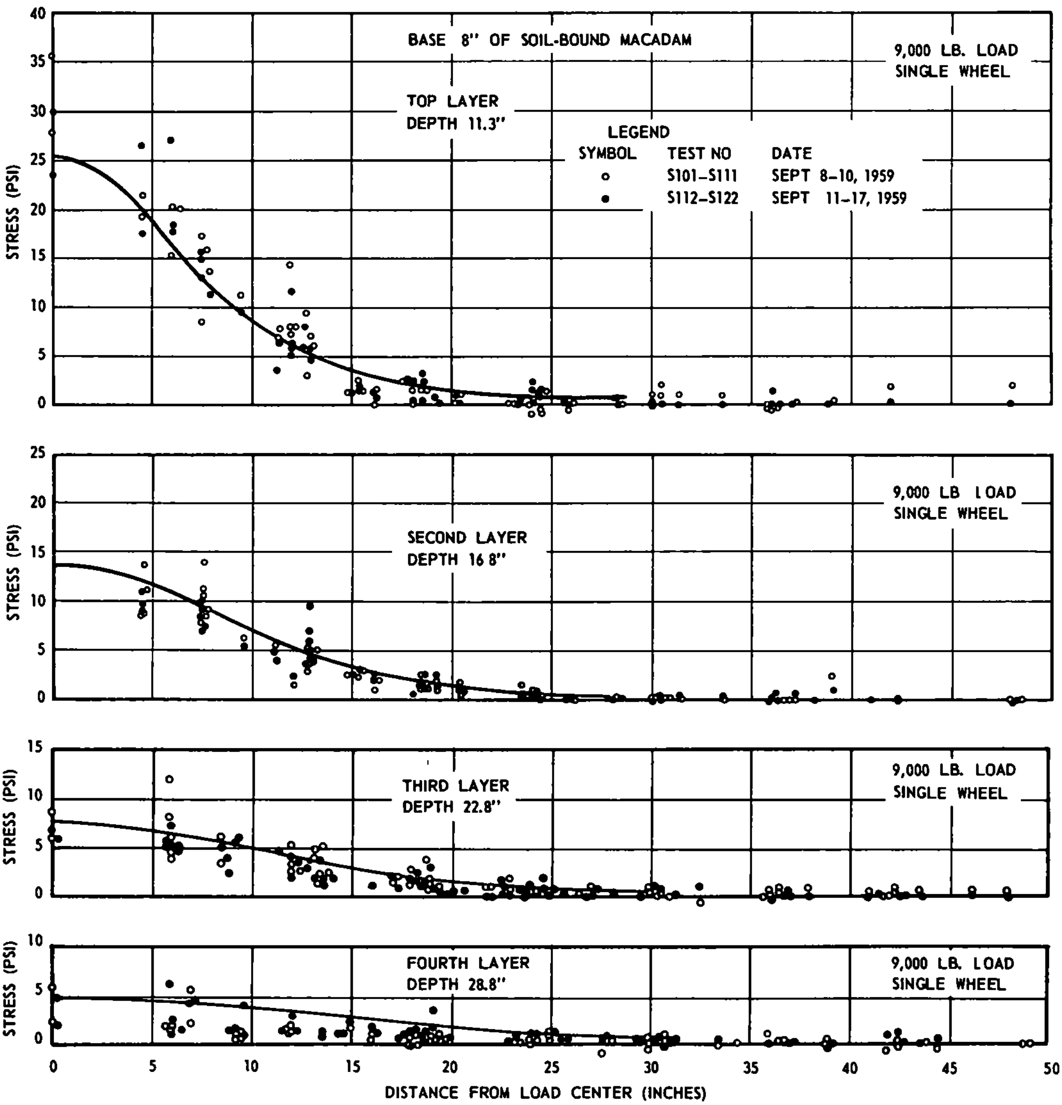

Figure 13. Measured stresses; single load 9,000 lb soil-bound macadam base 8 in. thick; 3-in. asphaltic concrete surface.

Soil-Bound Macadam Base Pavements

The test results for the wheel loads on the macadam base pavement are shown in Figures 13 and 14. The results are almost identical to those of the topsoil base—a marked decrease in vertical stress with increasing depth and a pressure distribution that is very close to that given by the Boussinesq theory. The modulus of elasticity data (Fig. 5a) shows that the soil-bound macadam base has a modulus of elasticity 9 times that of the subgrade. According to the two-layer elastic theory the stresses in the upper surface of the subgrade should be slightly less than one-half the measured vertical stresses (or slightly less than one-half the Boussinesq). As in the case of the topsoil base, two factors appear responsible. First, the cohesion of the soil-bound macadam is only 2 psi which with its high angle of friction means negligible tensile resistance. Tensile cracks develop at the bottom of the base course along the interface

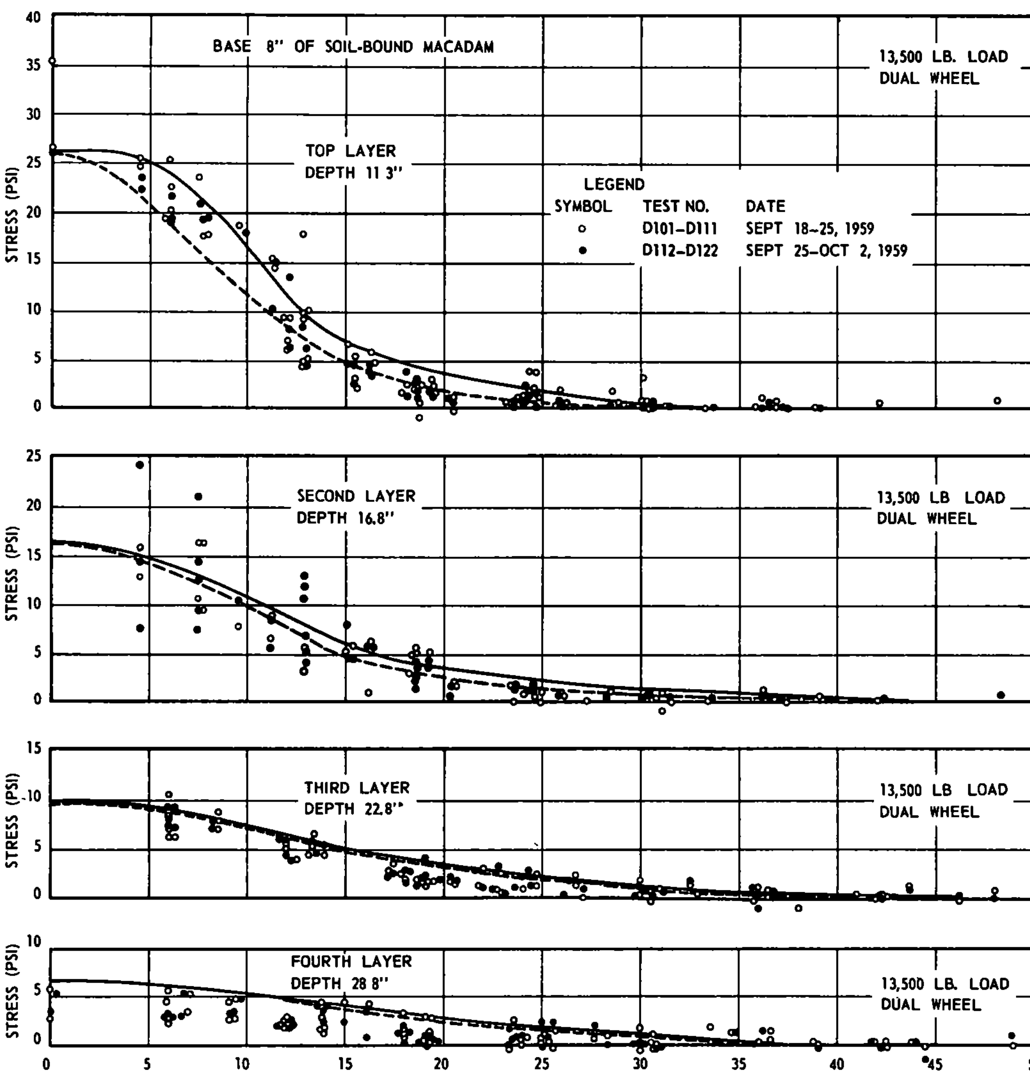

Figure 14. Measured stresses; dual load 13,500 lb soil-bound macadam base 8 in. thick; 3-in. asphaltic concrete.

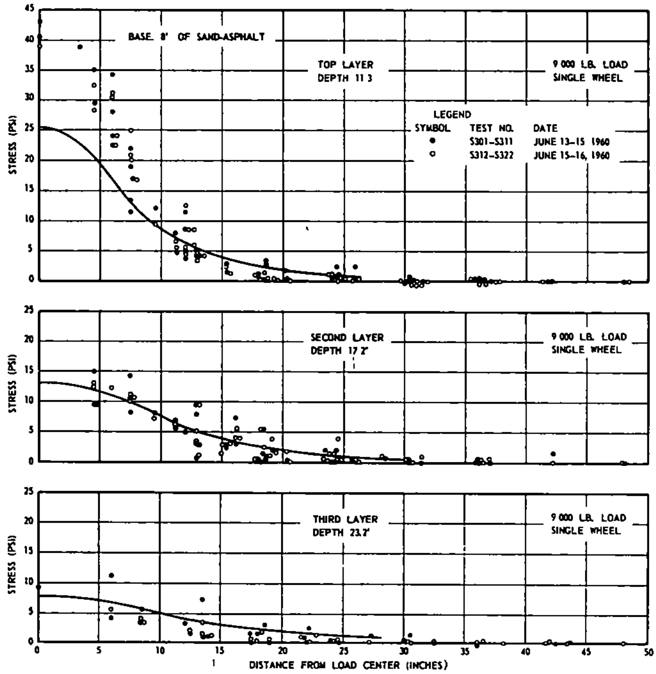

Figure 15. Measured stresses; single load 9,000 lb sand-asphalt base 8 in. thick; 3-in. thick asphaltic concrete surface.

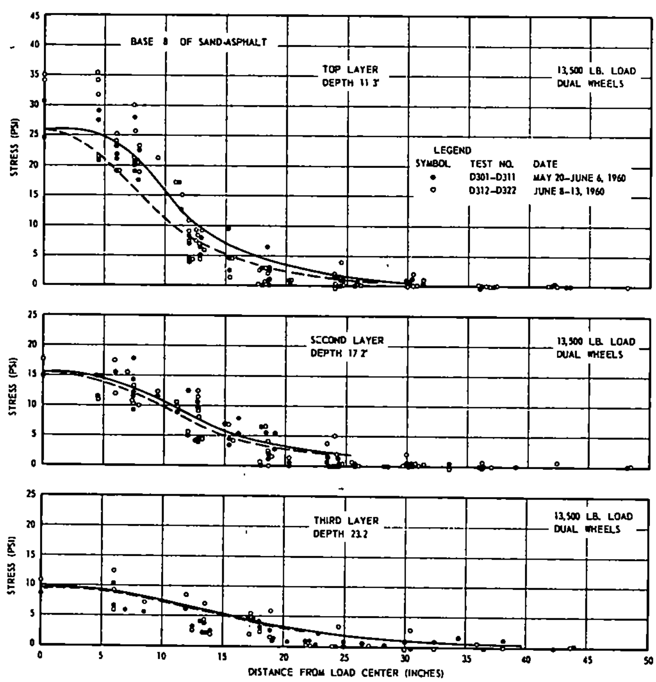

Figure 16. Measured stresses; dual load 13,500 lb sand-asphalt base 8 in. thick; 3-in. thick asphaltic concrete surface.

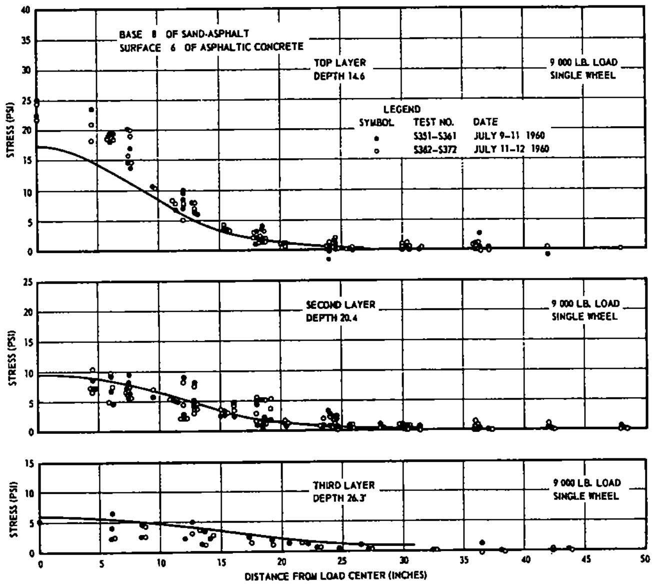

Figure 17. Measured stresses; single load 9,000 lb sand-asphalt base 8 in. thick; 3-in. asphaltic concrete surface with 3-in. asphaltic concrete overlay.

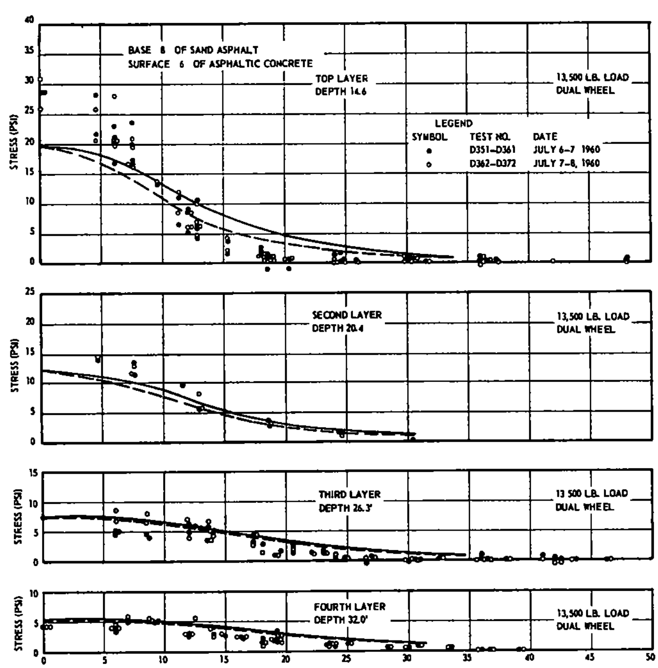

Figure 18. Measured stresses; dual load 13,500 lb sand-asphalt base 8 in. thick; 3-in. asphaltic concrete surface and 3-in. asphaltic concrete overlay.

TABLE 4
RATIOS OF MAXIMUM MEASURED STRESSES

	Single Tire		Dual Tire	
Load (lb)	3-In. Surf.	6-In. Surf.	3-In. Surf.	6-In. Surf.
5,000	1.72[a]	1.52[a]	---	---
9,000	1.61	1.42	1.31[a]	1.33[a]
13,500	1.38[a]	1.23[a]	1.23	1.36
18,000	---	---	1.25[a]	1.32[a]
Avg.	1.57	1.39	1.26	1.34

[a]Not shown in the figures included with this report.

which destroy the continuity of the elastic layer and invalidate the theory. Second, the modulus of elasticity increases with increasing confining pressures, although the ratio of the E of the base to that of the subgrade is nearly constant. The result may be a stress concentration that offsets the gain in load spreading of the more rigid base, as also may be in the case of the topsoil.

Sand Asphalt Base

The stresses beneath the pavements with sand asphalt bases on the micaceous silt subgrade are shown in Figures 15 through 18. Although all the tests show a marked reduction in vertical stresses with increasing depth, the maximum vertical stress directly under the load center and 0.3 in. below the base-subgrade interface was found to be considerably greater than that at an equivalent location in the topsoil and soil-bound macadam base pavements. The ratios of these maximum measured stresses just below the interface to the theoretical stresses as computed by the Boussinesq theory at the same point are given in Table 4. As can be seen, the ratios are greater for the single tire than for the dual and for the smaller loads than for the larger ones. Deeper in the subgrade (6 in. below the interface) the measured stresses are approximately the same as those computed by the Boussinesq theory.

The modulus of elasticity data (Fig. 5a) show that the E of the sand asphalt base is 4 or more times that of the subgrade and on that basis it would be presumed that the subgrade stresses, in accordance with the two-layer theories would be appreciably less than those computed by the Boussinesq theory. Instead, they are considerably greater. The Griffith-Fröhlich theories demonstrate that in a material whose modulus of elasticity increases in direct proportion to the confining pressure there is a concentration of stress immediately below the load. A concentration factor of 4 in the G-F equations (which has been verified by limited tests of cohesionless sands) gives a maximum stress 1.33 times the Boussinesq; a concentration factor of 5 gives a maximum stress 1.63 times the Boussinesq. Of all the base materials tested, the sand asphalt has a modulus of elasticity that most nearly resembles that of a cohesionless sand—a nearly linear increase of E with increasing confining pressure. Further, it is the only base material in which the ratio of the base E to the subgrade E increases substantially with increasing pressure. It is the authors' opinion that the stress concentration found just below the interface of the sand asphalt base is similar to the stress concentration described by the Griffith-Fröhlich equations. Deeper in the elastic subgrade, the stresses return to the Boussinesq as might be expected.

Soil-Cement Bases

The test results for the 8-in. soil-bound macadam cement are given in Figures 19 and 20, and the 6-in. soil-bound macadam cement in Figures 21 through 26. The stresses in the subgrade are considerably less than those found below the topsoil, soil-bound macadam, and sand-asphalt bases. The reduction is greater beneath the 8-in. thick base than the 6-in. thick base as might be expected. Each graph shows the stresses

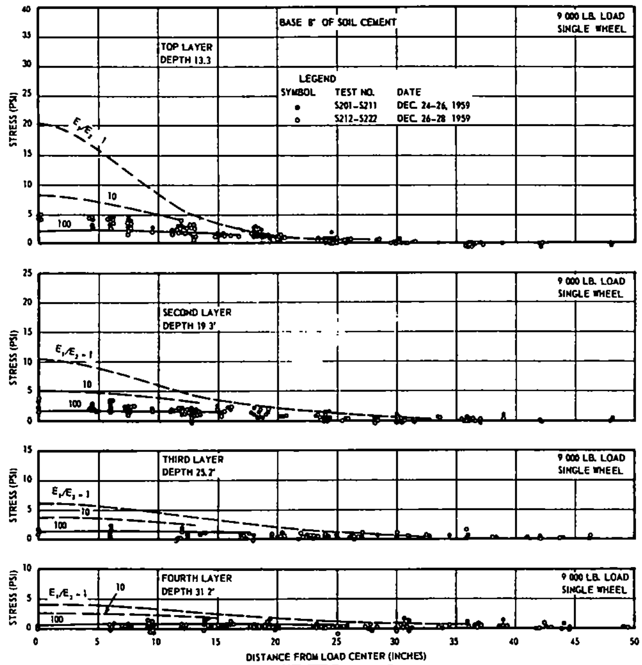

Figure 19. Measured stresses; single load 9,000 lb soil-cement base 8 in. thick; 3-in. asphaltic concrete surface. Curves are for theoretical stresses computed by the two layer theory, for different elasticity ratios; upper curve is equivalent to Boussinesq distribution.

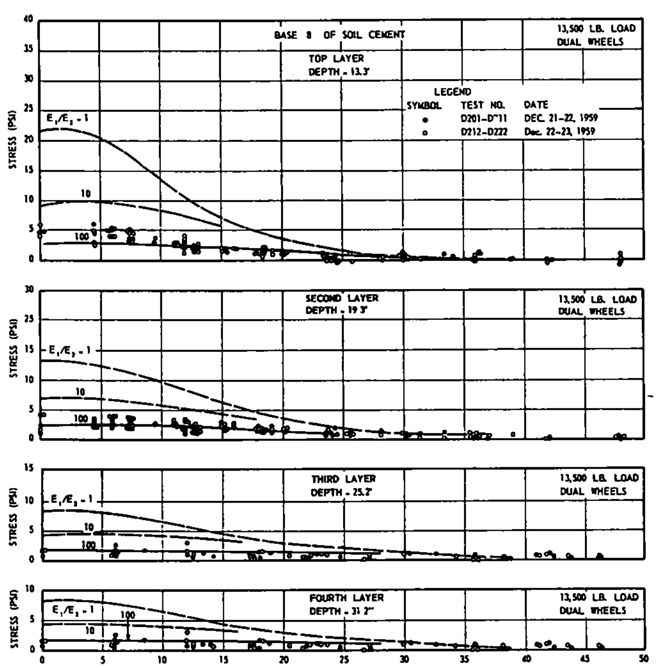

Figure 20. Measured stresses; dual load 13,500 lb soil-cement base 8 in. thick; 3-in. asphaltic concrete surface.

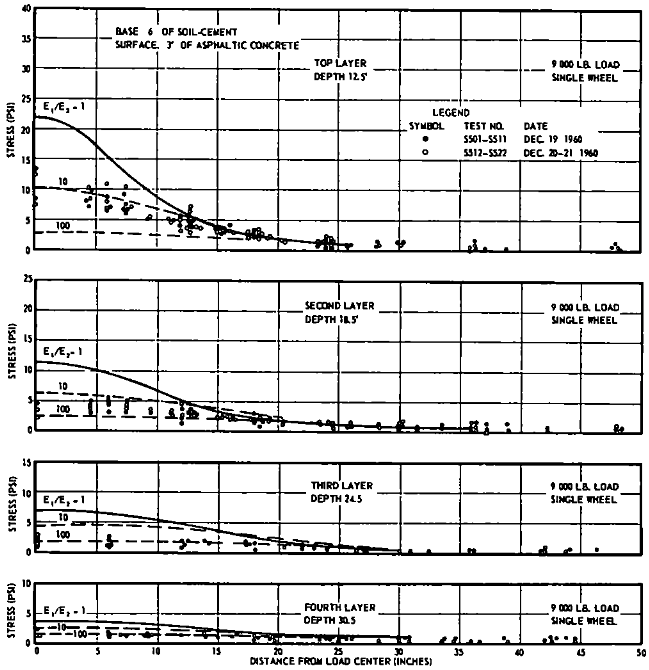

Figure 21. Measured stresses; single load 9,000 lb 6-in. soil-cement base; 3-in. asphaltic concrete surface.

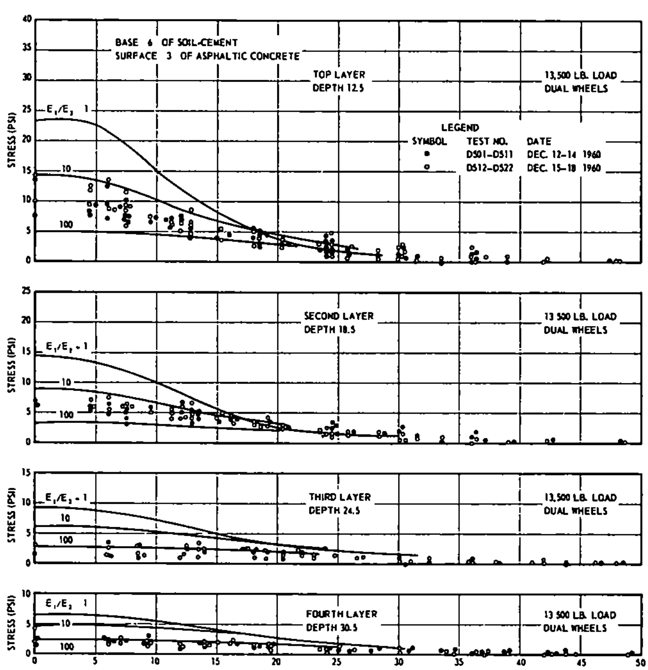

Figure 22. Measured stresses; dual load 13,500 lb 6-in. soil-cement base; 3-in. asphaltic concrete surface.

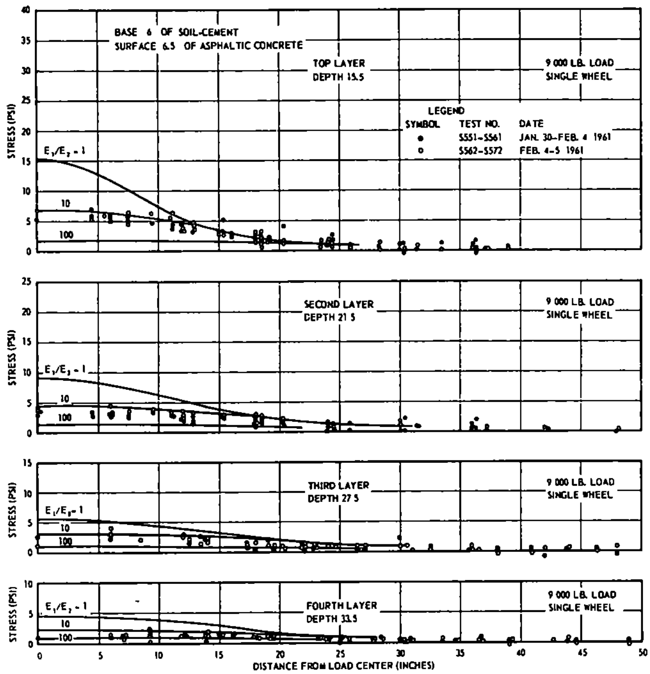

Figure 23. Measured stresses; single load 9,000 lb 6-in. soil-cement base; 3-in. asphaltic concrete surface and 3.5-in. asphaltic concrete overlay.

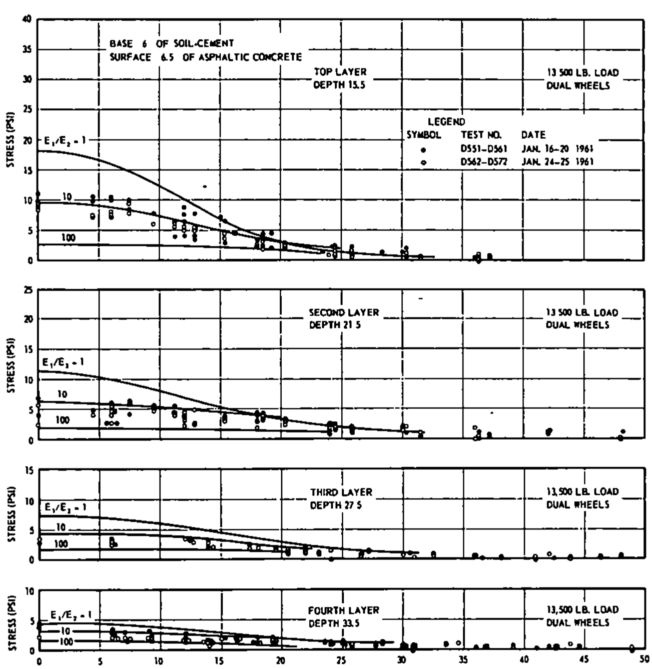

Figure 24. Measured stresses; dual load 13,500 lb 6-in. soil-cement base; 3-in. asphaltic concrete surface and 3.5-in. asphaltic concrete overlay.

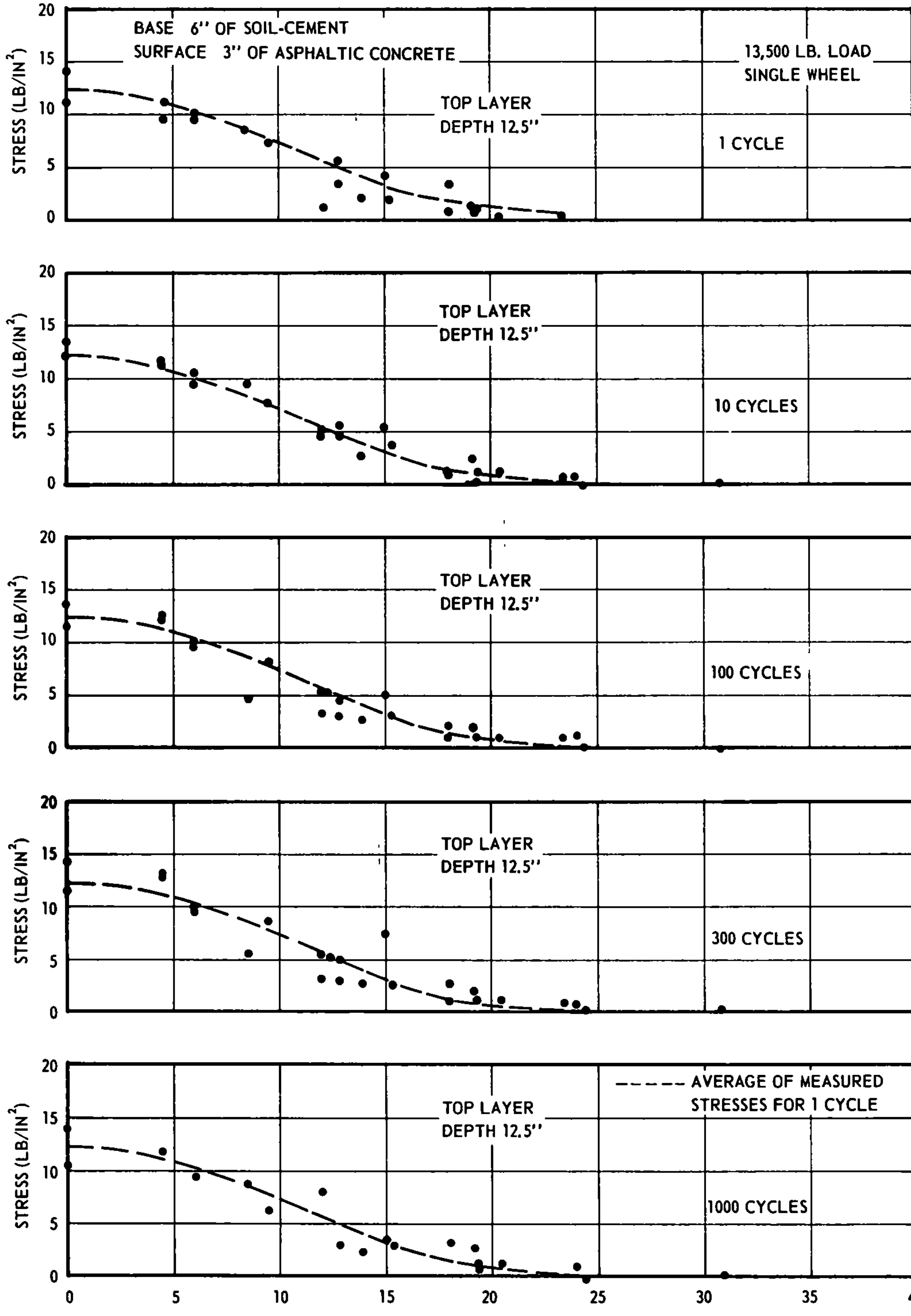

Figure 25. Measured stresses; repeated single wheel load 13,500 lb on 6-in. soil-cement base; 3-in. thick asphaltic concrete surface. Curve is average of measured stresses for one load cycle.

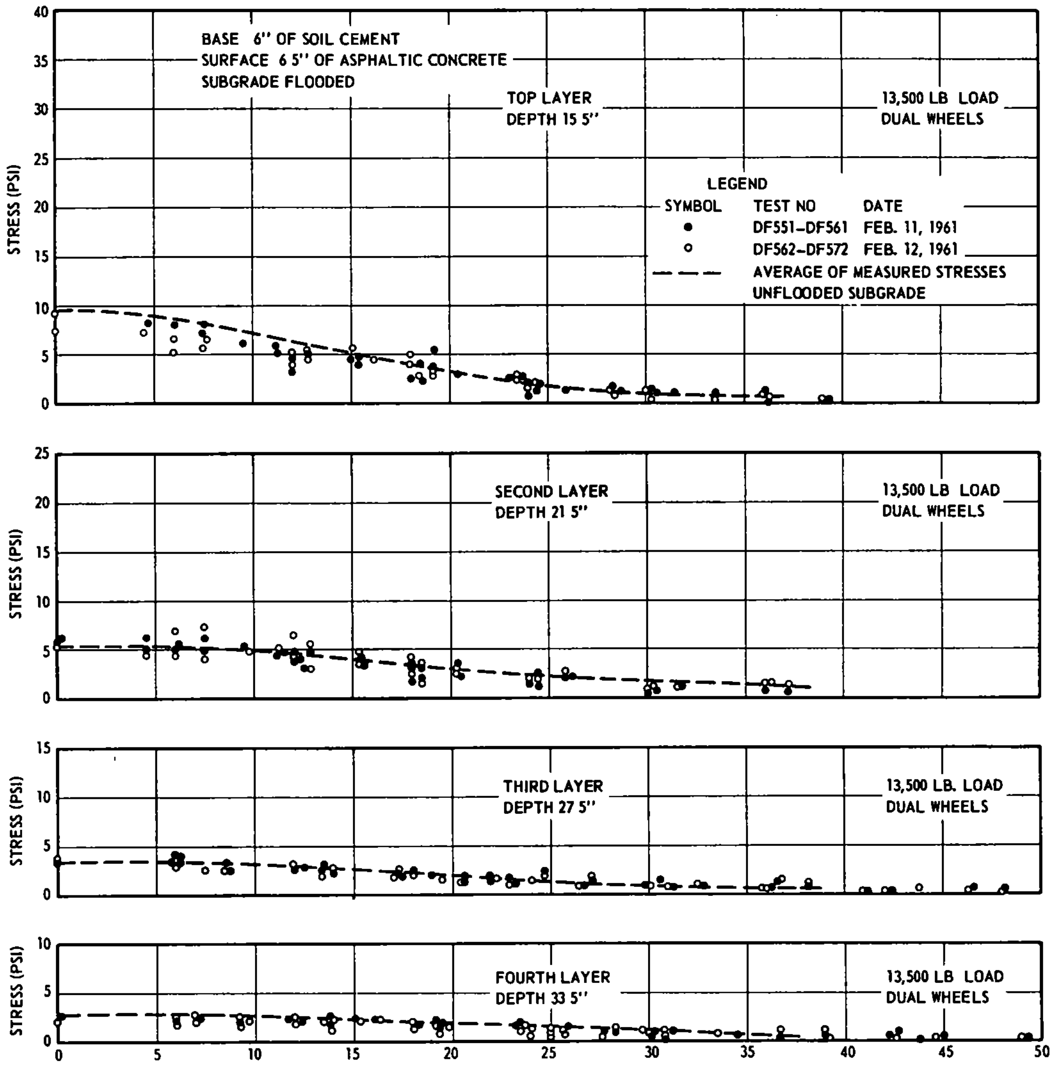

Figure 26. Measured stresses; dual load 13,500 lb 6-in. soil-cement base; 3-in. thick asphaltic concrete surface and 3.5-in. asphaltic concrete overlay, subgrade flooded. Curves are average measured stresses in unflooded subgrade.

TABLE 5

Base	E (ksi) At 0 Psi	E (ksi) At 20 Psi	ΔE (ksi)	Increase (%)
Topsoil (actual)	1.6	4.9	3.3	206
Soil-bound macadam	3.0	10.6	7.6	254
Sand asphalt	1.2	5.8	4.6	384
Soil cement	49	62	13	26

computed by the two-layer elastic theory by assuming that the asphaltic surface has the same modulus of elasticity as the base. This is not strictly valid because the asphaltic concrete is less rigid than the soil cement, but it will serve as an index for comparison. The stresses beneath the 8-in. thick soil cement base (where the base thickness is 73 percent of the total pavement) are between those computed for a ratio of base E to subgrade E of 10 and those for a ratio of 100. The laboratory tests (Figs. 5, 6) show an E ratio of 100 or more for very small confining pressures dropping to about 55 for confining pressures between 10 and 40 psi. Plate load tests on the base found a ratio of 100 (computed by the elastic layer theory). Because the surface is not as rigid as the base, the effective ratio would appear to be 50 or slightly less which agrees with the test results.

The results for the 6-in. base and 3-in. surface are equivalent to an elasticity ratio between 10 and 100, but closer to 10. This is to be expected because the less rigid surface is one-third the total pavement thickness. The measured stresses with the 6-in. base and 6-in. thick surface correspond to a slightly lower elasticity ratio, approximately 10. These differences are to be expected because the effective rigidity of the pavement (base plus surface) becomes less as the proportion of surface to base increases.

The agreement between the measured stresses and the stresses computed by the two-layered elastic theory for the soil cement tends to verify the authors' explanation of the lack of validity of the layered theory for the other bases. The shear tests show that the soil cement is capable of withstanding appreciable tensile stresses, whereas the others are not. In addition, the modulus of elasticity of the soil cement, while increasing with increasing confining pressure, does not change as much relatively as the others. This can be seen by the percentage increase of E with respect to E measured at 0 confining pressure produced by increasing the confining pressure from 0 to 20 psi (Table 5).

Repeated Load-Soil Cement Base

Some concern was felt about the continued load-spreading efficiency of the soil cement after repeated loading. It has been observed that soil-cement pavements tend to develop hair cracks that divide the surface into large polygonal blocks. To investigate this possibility the 6-in. thick soil cement base 3-in. surface pavement was subjected to 1,000 cycles of load-unload with the 13,500-lb single tire. In employing this gross overload on a single tire, equivalent to a 27,000-lb axle, it was hoped to magnify any effects of base cracking. The results (Fig. 25) show no change of stresses in the subgrade 3 in. below the subgrade-base interface.

Effect of Inundation

All the tests had been made with the moisture content of the soil subgrade and the base near the respective optimum moistures Holes were drilled through the 6-in. base soil cement, 6.5-in. surface pavement, and through the entire depth of the subgrade. These were filled with water and kept full so as to inundate the subgrade and base. Moisture tests made at regular intervals in observation holes between the inflow holes showed an increase in saturation from the original value of about 75 percent to an average of 96.5 percent in one week, after which it remained constant. Tests were conducted with dual wheels and loads of 9,000, 13,500, and 18,000 lb. The results (data for the 13,500-lb load are given in Figure 26) showed possibly a slight reduction in the stresses just below the subgrade-base interface. Because inundation caused a small reduction in the modulus of elasticity of the subgrade but little change in the soil-cement base, the elasticity ratio was increased. Theoretically, the stress should have decreased slightly, and the tests verify it.

Effect of Overlay

A number of the pavement systems were tested with both 3- and 6-in. thick asphaltic surfaces to determine the load spreading effects of a 3-in. thick overlay. The results can be seen by comparing the graphs of the 3- and 6-in. surfaces for the topsoil II, sand asphalt, and 6-in. soil-cement base pavements. They show that the overlay causes a

stress reduction slightly less than that produced by an equal thickness of a homogeneous, isotropic elastic solid. In other words, the stress reduction was comparable to that produced by an equal thickness of topsoil or soil-bound macadam base and somewhat more effective than that of an equal thickness of sand asphalt.

ACKNOWLEDGMENTS

This work has been sponsored by the Georgia State Highway Department and the U. S. Bureau of Public Roads, through the Engineering Experiment Station of the Georgia Institute of Technology. The authors are indebted to C. E. Hedges, W. H. Johnson, F. Chabrol, C. E. Snepp, W. L. Boyd, A. Schwartz, and D. Wheeless, Graduate Research Assistants, for their part in these investigations.

REFERENCES

1. Boussinesq, J., "Application des Potentiels a l'Etude de l'Equilibre et du Mouvement des Solides Elastiques." Gauthier-Villars, Paris (1885).
2. Love, A. E. H., "The Stress Produced in a Semi-Infinite Body by Pressure on Part of the Boundary." Phil. Trans. Roy. Soc., Series A, 228:377-420 (1928).
3. Steinbrenner, W., "Tafeln zur Setzungsberechung." Die Strasse, Vol. 1 (1934).
4. Newmark, N. M., "Influence Charts for Computation of Stresses in Elastic Foundations." Univ. of Illinois Eng. Exp. Sta. Bull. 338 (1942).
5. Newmark, N. M., "Influence Charts for Computation of Vertical Displacements in Elastic Foundations." Univ. of Illinois Eng. Exp. Sta. Bull. 367 (1947).
6. Fadum, R.E., "Influence Values for Vertical Stresses in a Semi-Infinite Solid Due to Surface Loads." Harvard Univ. (1941).
7. Fergus and Miner, "Distributed Loads on Elastic Foundation." HRB Proc. 34: 582-597 (1955).
8. Foster and Ahlvin, "Stresses and Deflections Induced by a Uniform Circular Load." HRB Proc., 33: 467-470 (1954).
9. Deresiewicz, H., "The Half-Space Under Pressure Distributed over an Elliptical Portion of Its Plane Boundary." Trans. ASME, Jour. Appl. Mech. (1960).
10. Griffith, J. H., "The Pressures Under Substructures; Engineering and Contracting." 1:113-119 (1929)..
11. Fröhlich, O.K., "Druckverteilung im Baugrunde." J. Springer, Berlin (1934).
12. Ohde, J., "Zur Theorie der Druckverteilung im Baugrunde." Der Bauingenieur, 20:451-459 (1939).
13. Buisman, A. S., "Druckverdeeling in Bouwgrond in Verband met Ongelijke Samendrukbaarheid in Horizontale en Verticale Richting." De Ingenieur, 47: B175-180 (1932).
14. Wolf, K., "Ausbreitung der Kraft in der Halbebene und im Halbraum bei Anisotropem Material." Zeitschift Angew. Math. und Mech., 15:249-254 (1935).
15. Jelinek, R., "Der Boden als Querisotropes Medium." Abhandlungen über Bodenmechanik und Grundbau, E. Schmidt, Berlin, pp. 19-24 (1948).
16. Jelinek, R., "Die Kraftausbreitung im Verallgemeinerten Ebenen Spannungszustand für Querisotrope Böden." Abhandlungen über Bodenmechanik und Grundbau, E. Schmidt, Berlin, pp. 24-27 (1948).
17. Jelinek, R., "Die Kraftausbreitung im Halbraum für Querisotrope Böden." Abhandlungen über Bodenmechanik und Grundbau, E. Schmidt, Berlin, pp. 28-33 (1948).
18. Koning, H., "Stress Distribution in a Homogeneous, Anisotropic, Elastic Semi-Infinite Solid." Proc., 4th Int. Conf. Soil Mech. and Found. Engrg. London, 1:335-338 (1957).
19. Westergaard, H. M., "A Problem of Elasticity Suggested by a Problem in Soil Mechanics, Soft Material Reinforced by Numerous Strong Horizontal Sheets, Contribution to Mechanics of Solids." Stephen Timoshenko 60th Anniversary Volume, MacMillan (1938).

20. Burmister, D. M., "The Theory of Stresses and Displacements in Layered Systems and Application to the Design of Airport Runways." HRB Proc., 23: 126-148 (1943).
21. Burmister, D. M., "The General Theory of Stresses and Displacements in Layered Systems." Jour. Appl. Physics, 16: 296-302 (1945).
22. Schiffman, R. L., "The Use of Integral Transforms in the Solutions of Three-Layer Soil Problems." (Unpublished).
23. Fox, L., "Computation of Traffic Stresses in a Simple Road Structure." Road Research Laboratory (England), Road Research Paper 9, London (1948). Also, Proc. Sec. Inf. Conf. Soil Mech. and Found. Engrg., 2: 236-246.
24. Hank and Scrivner, "Some Numerical Solutions of Stresses in Two and Three Layered Systems." HRB Proc., 28: 457-468 (1948).
25. Acum, W. E. A., and Fox, L., "Computation of Load Stresses in a Three-Layer Elastic System." Geotechnique, 2: No. 4, pp. 293-300 (1951).
26. Steiner, "Handbuch der Ingenieurwissenschaften." Der Bruckenbau, Leipzig 2: 195, (1882).
27. Strohschneider, O., "Elastische Druckverteilung und Drüchüberschreitung in Schüttungen." Sitz. Berichte der K. K. Akad. Wiss. Wien, 121: 301 (1912).
28. Moyer, J. A., "Distribution of Vertical Soil Pressure." Eng. Record, 69: 608 (1914) and 71:330 (1915).
29. Enger, M. L., "High Unit Pressures Found in Experiments on Distribution of Vertical Loading Through Sand." Eng. Record, 73: 106 (1913).
30. Goldbeck, A. T., "Distribution of Pressures Through Earth Fills." Proc. ASTM, 17: 640 (1917).
31. Kögler, F., and Scheidig, A., "Druckverteilung im Baugrunde." Bautechnik, 5: 418 (1927); 6: 205 (1928); 7: 268 (1929).
32. Spangler, M. G., and Ustrud, H. O., "Wheel Load Stress Distribution Through Flexible Type Pavements." HRB Proc., 20: 235-257 (1940).
33. Waterways Experiment Station, "Investigations of Pressures and Deflections for Flexible Pavements." Report 1, Homogeneous Clayey-Silt Test Section, Vicksburg, Miss. (1951).
34. Waterways Experiment Station, "Investigations of Pressures and Deflections for Flexible Pavements." Report 4, Homogeneous Sand Test Section, Vicksburg, Miss., (1954).
35. Turnbull, W., Maxwell, A. A., and Ahlvin, R. G., "Stress Distribution in Homogeneous Soil Masses." Proc., 5th Int. Conf. Soil Mech. and Found. Engrg., Paris, 2: 337-345 (1961).
36. McMahon, T. F., and Yoder, E. J., "Design of a Pressure-Sensitive Cell and Model Studies of Pressures in a Flexible Pavement Subgrade." HRB Proc., 39: 650-682 (1960).

Discussion

ROBERT L. SCHIFFMAN, Associate Professor of Soil Mechanics, Rensselaer Polytechnic Institute—It is gratifying to see experiments directed towards the validity of layered system analyses. All too often, theories are accepted or rejected without adequate experimental evidence. On the other hand, the interpretation of the experiment must always consider the alteration of the theory due to the measuring instruments.

The experimental results presented showed a general trend of normal stresses being in excess of the layered theory. The system tested was geometrically a three-layer system. For most cases the recorded results were in the range of, or higher than, the homogeneous analysis of Boussinesq. In the case of the soil-cement bases, the stresses were compared to a two-layer theory. The actual geometry, a three-layer theory, would produce stresses of smaller magnitude than actually recorded in all cases.

A general conclusion can be drawn that the measured normal stresses were, on the whole, higher than the predications of the appropriate theory being tested.

An interpretation of this conclusion can fall into two patterns. In the first instance, one is tempted to conclude that the theories use gross idealizations of field conditions

and thus are insufficient to predict pavement behavior. It is possible to judge this interpretation by considering the effects of an improved theory. In a linear elastic theory, two major considerations can be the effect of adhesive restraints between the tire and the pavement (1) and the effect of anisotropy (2). These two influences would tend to reduce the theoretical stresses, and thus increase the deviation between theory and experiment. The influences of nonlinear stress-strain properties and "large" strains are more difficult to access. The Vicksburg tests (3), however, have indicated that these influences do not seriously affect the magnitude and distribution of normal stresses.

The result of an "improved" theoretical analysis would thus give greater "errors" than are presented. Inasmuch as every physical phenomenon must have a theoretical description (known or unknown), the reasons for the difference between theory and experiment must be sought, elsewhere than in the analysis of the assumptions of the theory.

A second interpretation is that the experiment performed does not geometrically conform to the theory tested. It is the writer's opinion that the experimental set-up was sufficiently geometrically different from the theories tested to cast doubt on the comparisons used. The gages used, with relationship to the surrounding soil are, in fact, rigid inclusions in the soil mass. The inclusion effect will produce higher stresses at the gage than the corresponding "non-inclusion" theory. The fact that there were several gages in a line would accentuate the effect. The inclusion effect for a line of gages can crudely be compared to the stresses at a rigid boundary. Biot's (4) analysis for a single layer with a rigid boundary underlying the elastic layer shows that the normal stresses (particularly under the load) are substantially higher than the corresponding Boussinesq stresses.

It is the writer's interpretation that the deviations between the experimental results and the theory used are largely due to the influence of the gage as a rigid inclusion in a layered system.

The inclusion effect is one that will affect all stress measurement work. It is a most difficult problem, because the effect depends on the type of gage, the geometry of the gage, the gage characteristics the gage spacing, etc. It is a problem that must be solved, however, if one wishes to evaluate the validity of stress distribution theories for soil.

This discussion is not intended to criticize the excellent experimental data obtained by the authors. It is hoped that the discussion will provide a possible explanation for the lack of agreement between the theories used and the experiments reported. Furthermore, it is hoped that this discussion will point up certain deficiencies in the theoretical knowledge which make an assessment of layered system theories difficult. If and when layered system theories with inclusions are developed, the authors' data will provide an excellent means of evaluating the theory.

REFERENCES

1. Schiffman, R. L., "The Influence of Adhesion on the Stresses and Displacements in an Elastic Semi-Infinite Solid." Rensselaer Polytechnic Institute Report N 11/9 (March 1962).
2. Koning, H., "Stress Distribution in a Homogeneous, Anisotropic, Elastic Semi-Infinite Solid." Proc., 4th Int. Conf. on Soil Mech. and Found. Eng., 1: 335-338 (1957).
3. Turnbull, W. J., Maxwell, A. A., and Ahlvin, R. G., "Stresses and Deflections in Homogeneous Soil Masses." Proc., 5th Int. Conf. on Soil Mech. and Found. Eng., 2: 337-345 (1961).
4. Biot, M. A., "Effect of Certain Discontinuities on the Pressure Distribution in a Loaded Soil." Physics, 6: 367-375 (1935).

GEORGE F. SOWERS and ALEKSANDAR B. VESIC, Closure—In his discussion Professor Schiffman expressed the opinion that the reasons for the differences observed between the theoretical and experimental values of stresses in the pavements tested must be sought not in the analysis of the assumptions of the theory but in the effect of rigidity of cells included in a layered system. He based his conclusions on the premise that

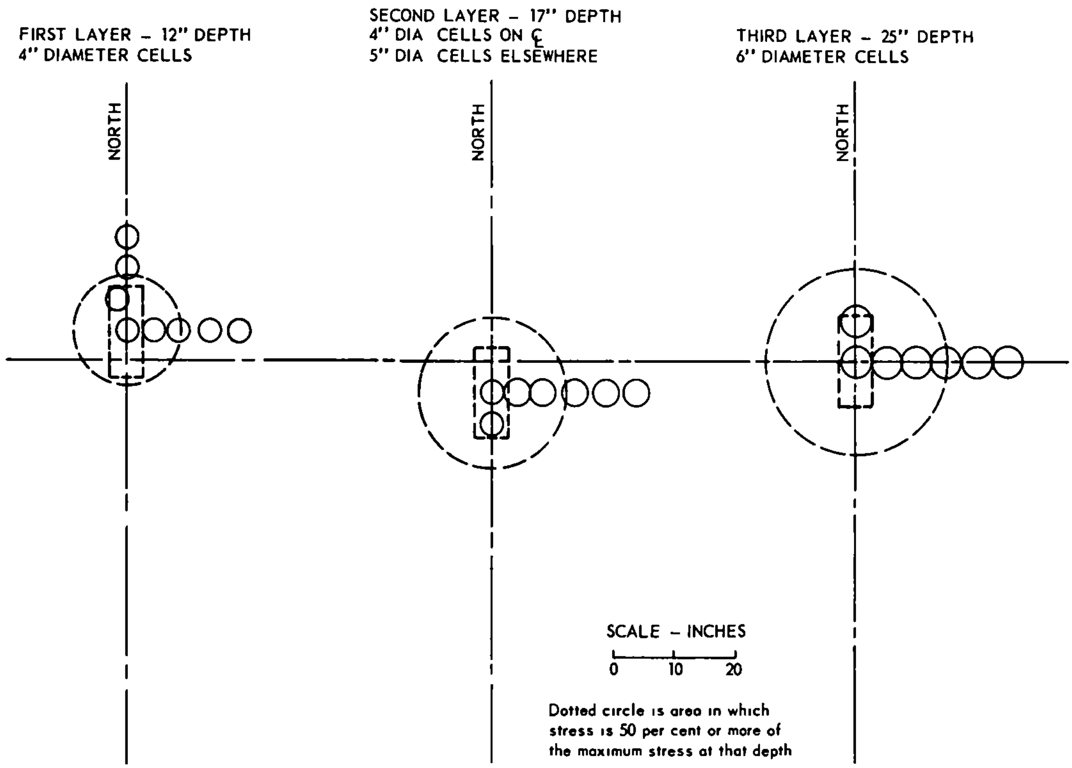

Figure 27. Pressure cell layout test series -600 showing imprint of single wheel under 13,500 lb.

two major factors not considered in the theory should be the adhesion between the tire and pavement and the anisotropy of the pavement materials. However, he does not consider one of the basic assumptions of the layered soil theory—constant modulus, E, equal in both tension and compression and independent of strain.

An analysis of lateral stresses in an ideal two-layer system shows that the upper layer (Fig. 27) will have tension at the interface as soon as $E_1/E_2 > 1.5$ and that, in the cases tested, tension should be of the order of 30 psi in the flexible bases and of the order of 60 psi in the soil-cement base. The tensile strength of the flexible bases was most likely not higher than 5 psi.

How much effect on stress distribution this lack of tensile strength will have has not yet been rigorously investigated. In support of the authors' conclusions, it should be mentioned that Ruckli (37) reports, for a two-layer system with $E_1/E_2 = 10$ and $h_1 = 12$ in. (data very close to authors' test conditions), a stress concentration factor n (after Fröhlich) of 2.82, which is quite close to the Boussinesq case (n=3) and would mean a reduction under the load of only 6 percent compared to Boussinesq stresses.

Professor Schiffman believes that the tests did not conform to the theories in that the pressure cells were rigid inclusions that produced stress concentrations at the point of stress measurement. It is his opinion that the rigid cells were responsible for the differences between the test data and the theoretical stresses.

Stress concentration due to cell rigidity is a fault of all pressure cells. It may be minimized by making the cell as small as possible compared to the volume of soil undergoing stress and by minimizing the ratio of cell thickness to cell diameter. The cells employed in these tests were far smaller in diameter than those of the U. S. Waterways Experiment Station to whose test results Professor Schiffman refers.

Furthermore, the ratio of cell thickness to diameter was less than 1/10, as small as those of the USWES. The cells were individually calibrated in soil so that the cali-

bration would reflect the effect of cell rigidity. Also, if there were any significant stress concentrations due to cell rigidity, it would be apparent in the tests on soil-cement base as it was in the other flexible bases. Therefore, the authors doubt that stress concentration at the cells was a significant source of the stress difference.

A second effect of cell rigidity brought out by Professor Schiffman is the rigid boundary effect as described by Biot. Biot investigated the stresses on an infinitely rigid boundary of infinite lateral extent beneath an elastic layer of finite thickness. The vertical stress directly beneath the load was 1.56 times the Boussinesq stress, if a perfectly rough interface is assumed. The authors do not believe that the pressure cells, arranged close together in a line, resemble the Biot condition. First, the area covered by the group of cells at any one depth is a small fraction of the area in which the vertical stress is one-half or more of the maximum stress, as shown in Figure 27. Second, the cells are individually free to tilt or deflect downward in the subgrade soil rather than being rigidly supported.

The authors contend that the test results for the soil-cement base, which are in agreement with the layered theory, demonstrate that the stress measurement system is not the cause of the lack of agreement between the layered theory and certain of the test results. Instead, it is the inappropriate use of the layered theory that is at fault.

A few additional comments should be of interest concerning some of Professor Schiffman's remarks. Stating that the asphaltic surface, the soil-cement base, and the subgrade form a three-layer system, he suggests that the theoretical stresses are even less when computed by the three-layer theory than by the two-layer theory, and that the authors' test results for soil-cement bases, therefore, exceed the theoretical stresses. The authors would agree with this suggestion if the asphaltic surface were significantly more rigid than the soil-cement base. However, the modulus of deformation of the asphaltic concrete tested at corresponding temperature was only about 20 percent of that of the base. It can be shown that under such conditions the theoretical stresses computed by the three-layer theory should be approximately 8 percent greater than that computed by the authors using the two-layer theory and assuming the surface course and the base to have the same modulus. The authors, therefore, must reaffirm their conclusion that the soil cement base test results are in substantial agreement with the layered theory.

The writer also contends that the effect of anisotropy would be to reduce stresses, as compared with those in an isotropic medium and that the differences between the measured stresses and those computed by the layered soil theory would therefore be even greater. It should be mentioned that anisotropy of materials does not necessarily reduce the σ_z stresses. As it has been shown by Buisman (13) the effect of anisotropy depends on the ratio of the moduli of deformation of the soil in horizontal and vertical directions, E_h/E_v. If $E_h/E_v < 1$, an increase of σ_z stresses has to be expected. However, an E_h/E_v ratio as low as 0.4 brings an increase of only 30 percent compared to the case of isotropic materials, whereas the measured stresses are several hundred percent higher than those obtained for isotropic layered solids.

The authors wish to thank Professor Schiffman for his discussion and hope he will continue his extension of the layered theory into real situations.

REFERENCE

Ruckli, R., "Discussion on Pavement Bearing Capacity Computed by Theory of Layered Systems." Trans., ASCE, 116: 767 (1951).