Evaluation of Gravity Model Trip Distribution Procedures

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•DURING THE past decade there has been increasing realization that to plan transportation systems for dynamic urban areas properly it is necessary to be able to forecast changes in travel demands resulting from anticipated or proposed changes in the land use patterns and transportation systems of these areas. This realization has resulted in a concerted effort to develop an urban transportation planning process capable of providing quantitative information on future traffic movements of sufficient precision to enable cities to make an informed choice between the many alternate land use and transportation programs open to them. The heart of such a process is a procedure capable of synthesizing zone-to-zone movements for alternate configurations of land use and transportation facilities.

Several such procedures, generally referred to as "traffic models" have been developed by various organizations through the country. A trip-opportunity model developed by the Chicago Area Transportation Study has been utilized in both Chicago and Pittsburgh (1). Another procedure is currently being developed by the Penn-Jersey Study for use in that area (2). Yet another procedure, and the one which has been most widely applied, is the so-called gravity model (3, 4). Studies based on this procedure have been conducted in Hartford, Conn.; Baltimore, Md.; seven cities in Iowa; and is presently being used in a transportation study of the Los Angeles region.

For the most part, comprehensive origin-destination studies were not conducted in conjunction with these applications of the gravity model. Rather, selected travel data (such as screenline crossings, volume counts, employee data, and a limited number of home interviews) were utilized to calibrate the model to local conditions and check the resulting estimated travel demands. A comparison and evaluation of gravity model procedures using a complete home interview survey was carried out by the Iowa State Highway Department in Cedar Rapids. The findings are reported in an unpublished paper (5).

The Office of Research, Bureau of Public Roads, initiated a research project in the fall of 1960 to evaluate more fully this traffic estimating procedure. The project used the Washington, D.C., metropolitan area as a study area. This selection was made for several reasons, the most important being the availability of comprehensive origin-destination data for both 1948 and 1955, thereby making it eventually possible to test directly the applicability of this as well as other traffic models over a period of time. The first phase of this project, the one described by this paper, was an evaluation of the trip distribution characteristics of a gravity model based on analysis of and comparison with 1955 origin-destination travel data. Additional investigations into the trip production and attraction aspects of the gravity model traffic estimating procedures, as well as application and evaluation with 1948 travel data, are being conducted by the Bureau of Public Roads as a part of its continuing urban research program.

GRAVITY MODEL

The general formulation of the gravity model is

$$T_{i-j} = P_i \frac{A_j/d_{i-j}^{\,b}}{A_i/d_{i-i}^{\,b} + A_j/d_{i-j}^{\,b} + \ldots A_n/d_{i-n}^{\,b}} \qquad (1)$$

in which

T_{i-j} = number of trips produced at zone i attracted by zone j;

P_i = total trips produced by zone i;

A_j = total attraction of zone j;

d_{i-j} = measurement of the separation between zones i and j normally expressed in terms of driving time; and

b = empirically determined exponent.

Eq. 1 was changed to the following form for computational purposes and obtaining more flexibility in the research program:

$$T_{i-j} = \frac{P_i A_j F(d_{i-j}) K_{i-j}}{A_1 F(d_{i-1}) K_{i-1} + A_j F(d_{i-j}) K_{i-j} + \dots A_n F(d_{i-n}) K_{i-n}} \qquad (2)$$

in which

T_{i-j}, P_i, A_j = same as in Eq. 1;

$F_{(d_{i-j})}$ = an empirically determined set of friction factors replacing the inverse of distance d_{i-j} raised to a power b; and

K_{i-j} = a specific zone-to-zone adjustment factor to allow for the incorporation of the effect on travel patterns of factors not otherwise accounted for in Eq. 1.

The use of a set of friction factors rather than the inverse function of distance greatly simplified the computational requirements of the model. In addition, it did not commit the research project to a constant exponential function of distance which previous studies have shown to vary for a given trip purpose.

The K_{i-j} factor was incorporated into the model to permit adjustment of the model for social and economic linkages not accounted for by the other parameters of trip production and attraction and interzonal separations. Previous research has indicated the importance of such factors on urban travel patterns, and one of the objectives of this project was to determine the effect and magnitude of such factors on urban travel patterns in the Washington area (6).

This study stratified the total travel demands of the area into the following six trip purpose categories:

1. Home-based work.
2. Home-based shopping.
3. Home-based social-recreational.
4. Home-based school.
5. Home-based miscellaneous.
6. Non-home-based.

The specific definition of these purpose categories is the same as those used by the 1955 origin-destination survey. The home-based miscellaneous category is composed of the personal business and medical-dental-eat trips. The non-home-based category includes all trips with neither their origin nor destination at home.

It should be pointed out that before this study was made, the 1955 survey data were processed so as to link trips that were originally coded as change mode and serve passenger trips. The procedures used were similar to those developed by the Pittsburgh Area Transportation Study. This was done to obtain a more precise picture of the ultimate destinations and purpose of the travel demands in the area. As shown by the studies in Pittsburgh and Chicago, although this procedure reduces the total number of trips it does not change the vehicle-miles of travel in an area significantly.

The distribution by trip purpose of the total trips and travel in the Washington area is shown in Table 1. Over 90 percent of the total trips and travel in the Washington area in 1955 had either their origin or destination at home. The trips and travel are expressed in terms of person-movements. This study, unlike previous research in this field, deals with total person-movement rather than with the movement of vehicles.

As was previously stated, this phase of the research was concerned with only the trip distribution aspects of the gravity model. Therefore, the zonal trip productions and attractions (P_i and A_j) by purpose for each of the 436 zones comprising the Washington metropolitan area were taken directly from a summary of 1955 origin-destination survey.

The measurement of zonal separation used by this study was off-peak minimum path driving time plus terminal time. Interzonal driving time was obtained through the use of the standard tree-building computer program. Intrazonal driving times were estimated on the basis of the interzonal driving times to the adjacent zones.

In the development of a model for forecasting person-movements, the determination of interzonal separation involves considerable compromise. This is due to the sizable range between the levels of service (speed of travel) offered by the various modes of travel. Although it would have been possible in this study to construct separate models for transit and auto travel based on their respective levels of service, such a procedure would require that the mode split be determined in developing the zonal production and attraction figures when estimating future travel. From an over-all view of urban traffic forecasting, it was felt that the determination of mode split should be made after the zone-to-zone distribution of person-movements.

An estimate of terminal time was made for each zone. This estimate was based on the type and intensity of land development within each zone. Although this estimate was quite subjective, it was incorporated in this study for two reasons. First, it was felt that people consider the total travel time (driving time plus terminal time) rather than only the driving time associated with a contemplated trip. Second, it was felt that one of the reasons that previous research in this field has indicated that the exponent of distance for a given purpose varies with trip length was a result of not incorporating terminal time into this measurement of zonal separation. The estimated terminal times varied from 6 min within the central portion of the region to 3 min in the outlying suburban residential areas.

TABLE 1

DISTRIBUTION OF TRAVEL BY TRIP PURPOSE, WASHINGTON, D.C., 1955[a]

Trip Purpose	Trips		Person-Hours[b]	
	Number (X 10^3)	Percent	Number (X 10^3)	Percent
Home-based:				
Work	1,075	43.4	255	53.7
Shopping	335	13 5	40	8.5
Social-rec.	326	13.1	63	13.2
School	217	8.7	29	6.1
Miscellaneous	247	9.9	44	9.3
Non-home-based	282	11.4	44	9.2
Total (all purposes)	2,482	100.0	475	100.0

[a]Source: Home Interview Origin-Destination Survey, Washington, D.C., 1955.
[b]Based on minimum path zone-to-zone driving time.

CALIBRATION OF MODEL

After the information on zonal trip production and attraction and zonal separation was prepared, the next phase of the research was the calibration of the model to reflect the over-all travel characteristics of the Washington metropolitan region. This was accomplished through a step-by-step process:

1. Determining friction factors.
2. Balancing zonal trip attractions.
3. Adjustment for systematic geographical variations.

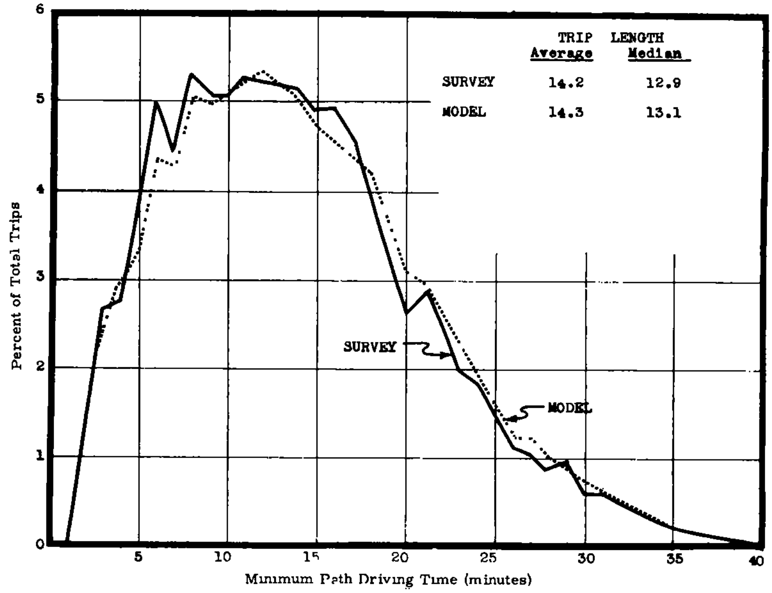

Figure 1. Comparison of work trip length frequencies, Washington, D C., 1955.

TABLE 2

COMPARISON OF TRIP LENGTH FREQUENCIES, WASHINGTON, D.C., 1955

Trip Purpose	Trip Length (min of driving time)[a]			
	Average		Median	
	Survey	Model	Survey	Model
Home-based:				
Work	14.2	14.3	12.9	13.1
Shopping				
Social-rec.	11.6	11.6	10.0	10.0
School	7.9	7.8	6.8	6.6
Miscellaneous	10.7	11.2	10.0	10.1
Non-home-based	9.3	9.3	7.0	7.0

[a]Based on minimum path zone-to-zone driving time

Determining Friction Factors

The best set of friction factors associated with each purpose category was determined through a process of trial and adjustment. This was done by assuming a set of friction factors for each trip purpose and calculating all zone-to-zone movements. The resulting estimated over-all trip length frequency distribution was then compared to that obtained from the O-D survey and the assumed friction factors were adjusted accordingly. The process was then repeated. Four such iterations were required in this study. This was because for the initial run it was assumed that all friction factors were equal to 1; in other words, it was assumed that travel time had no effect on travel patterns. This was done to determine how fast the procedure would close on the desired trip length frequency. Operationally, this step of the calibration process should take no more than two runs using reasonable first approximations.

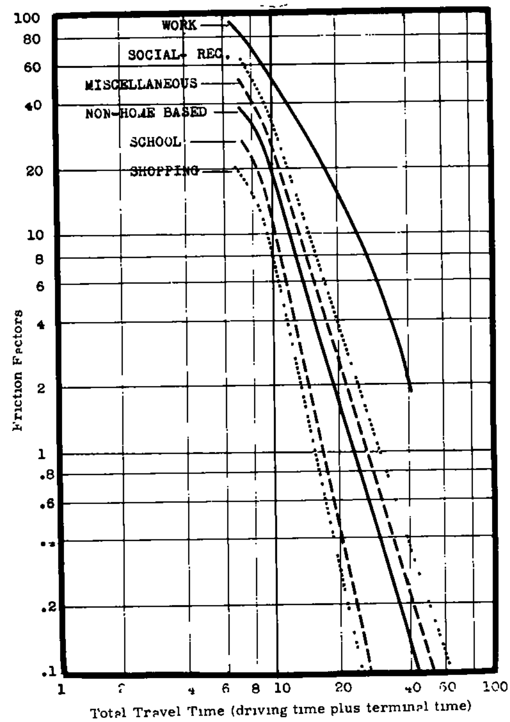

Figure 2. Friction factors, Washington, D. C., 1955.

The results of this step are shown in Figure 1 and Table 2. Figure 1 shows the close correspondence of the estimated trip length frequency for home-based work trip with that obtained by the O-D survey. Measures of this comparison for all of the purpose categories are given in Table 2. As can be seen, the close correspondence shown in Figure 1 for work trips exists for all trip purpose categories.

The sets of friction factors resulting from this step are shown in Figure 2. These are shown as a function of total travel time, and it can be seen that the friction factors for each purpose, with perhaps the exception of work, could be approximated very closely with a constant exponent. (Care should be taken in comparing the friction factors developed by this study with the findings of previous research which used driving time alone as a measurement of zonal separation.) The tendency to curve down at the low travel time is probably a result of too low an estimate of intrazonal times. These sets of friction factors were used throughout the remainder of the study.

Balancing Zonal Trip Attractions

A review of the gravity model formulation will show that there is no guarantee that the number of trips assigned to each zone by the gravity model will necessarily equal the attractions that were originally used. Therefore, the next step in the calibration process was the adjustment of the model to bring the number of trips assigned to a zone into balance with the trip attraction of that zone. This was accomplished by adjusting the attraction of each zone in accordance with the amount of under attraction or over attraction resulting from the previous estimate. The model was then rerun with these adjusted zonal attractions.

The extent of this adjustment varied considerably by purpose of trip. Work trips and non-home-based trips required the smallest amount of adjustment. Most of the areas were within ± 15 percent. There was, however, a discernible pattern to these adjustments. The central area of the city had received too many trips, whereas the zones in the outlying sections of the area were generally too low.

One factor creating this overestimate was the use of off-peak driving times. By using off-peak driving times the accessibility and therefore the attractiveness of the central area was overstated due to the high peak-hour congestion and relatively high transit usage associated with this area.

The pattern for shopping trips was just the reverse of that observed for work trips. For this purpose, the central area of the region received too few trips and the suburban areas too many. The extent of the underestimate to the central area was quite large (40 percent). It is felt that this situation would be considerably improved if shopping trips were further stratified into convenience and shopping goods trips. That is, a separate model should be used to distribute the larger, less frequent travel related to the purchase of specialized major items found only in the central area and major competing suburban shopping centers.

Social-recreation trips and miscellaneous trips exhibited a pattern similar to shopping trips, but to a lesser extent. School trips tended to vary considerably, primarily as a result of the small volumes involved.

The results obtained in this phase of the calibration process would vary considerably from city to city. In smaller cities, where decentralization of employment and shopping facilities is normally not as pronounced as it is in the larger metropolitan areas, the extent of the adjustments required would be considerably less than required in this analysis. For example, in the previously cited study in Cedar Rapids, Iowa, no adjustment of this type was required.

Final Review and Adjustments

The estimated zone-to-zone movements resulting from the balancing of trip attractions were then assigned to a zone centroid network on a minimum distance basis. The O-D information was assigned in an identical manner and the two were compared to determine whether any systematic geographical differences existed. Two patterns were discernible.

The first was the fact that the gravity model was overestimating the trips across the Potomac River. This is by no means an isolated case. Similar results have been obtained by application of the gravity model in Hartford, Boston, and New Orleans. An analysis was made to determine specifically the types of trips producing this overestimate. The results are given in Table 3 which shows that the model was consistently high for all trip purposes and for travel by both the residents of Maryland and D. C., and Virginia. This indicates quite clearly that the factor creating this overestimate is directly associated with the river, as this is the only factor common to these various types of trips.

Also, work trips were overestimated to a greater extent than were the other trip purposes. Based on these observations, it was concluded that this overestimation was

TABLE 3

COMPARISON OF TRIPS CROSSING THE POTOMAC RIVER

Trip Purpose	Person-Trips by Residents of					
	Maryland and D. C			Virginia		
	Survey	Model	% Diff.	Survey	Model	% Diff.
Home-based:						
Work	71,496	83,169	+16	99,730	120,034	+20
Shopping	1,945	3,549	-	6,588	6,855	+ 3
Social-rec.	12,416	13,227	+ 7	18,869	21,471	+14
Other	8,801	9,028	+ 3	12,238	15,394	+26
Non-home-based	13,561	14,043	+ 3	12,120	13,676	+12
Total (all purposes)	108,219	123,016	+14	149,545	177,430	+19

due to high peak-hour congestion associated with the present river crossings, and that off-peak travel times did not reflect a true relative measure of the transportation service offered in this area.

A study of peak and off-peak travel times made by the AAA showed that travel time for routes from the central area crossing the Potomac River to Virginia increased approximately 5 min more during the peak periods relative to the other routes from the central area to Maryland. Therefore, 5 min additional travel time was placed on all Potomac River crossings, and the model was rerun. This 5-min constraint brought the model crossings into agreement with those obtained by the O-D survey.

The second factor noted in this examination was a systematic difference with respect to work trips attracted to the Central Business District. This pattern is shown in Figure 3. This figure indicates the amount of increase or decrease required to bring the gravity model calibrated as previously described into agreement with the surveyed data.

To those familiar with Washington, this pattern is no surprise, as it is the logical result of the economic and social patterns that exist in the Washington area. It results from the fact that the bulk of the employment opportunities within the central area are middle or upper income, white collar jobs. However, the labor force, concentrated adjacent to and extending east of the central area, are low income, blue collar workers. As a result of this heterogeneous residential pattern, the model overestimated the trips from the close-in low income areas and underestimated those from the higher income outlying areas.

To incorporate the influence of this stratified occupational-economic pattern, specific zone-to-zone adjustment factors were determined and used to adjust the model.

RESULTS

The results obtained by the gravity model calibrated as described to the over-all characteristics of the Washington area are shown in Figure 4 in the form of comparisons of screenline crossings. Examination of this figure shows that there are no significant systematic differences in the model. This is further indicated by Figure 5. In only eight of the screenline comparisons were there differences greater than 10 percent, as indicated by the dashed lines.

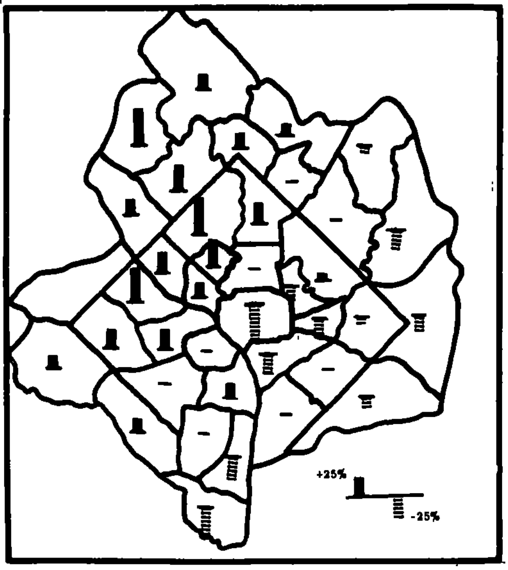

Figure 3. Comparison of work trips attracted to CBD, Washington, D. C., 1955.

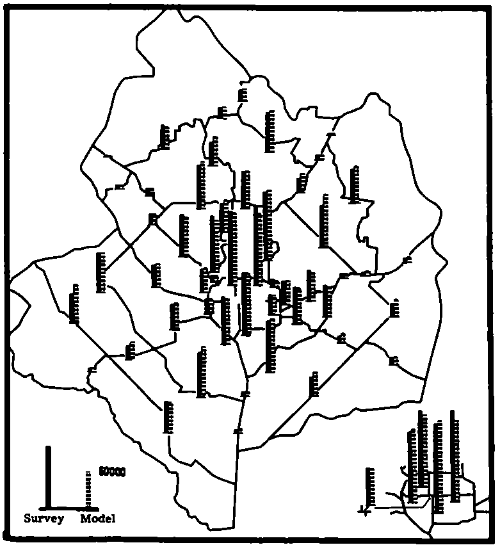

Figure 4. Screen line crossings, all purposes, Washington, D. C., 1955.

Similar comparisons are shown in Figures 6 and 7 for non-work and work trips respectively. The estimated work trips show a much closer correspondence to the O-D figures than do the estimated non-work trips. A review of the information showed that much of the differences shown in Figure 6 were a result of the shopping trip estimates. Again, it is felt that additional stratification of this category of trips would have substantially improved these results.

The information shown in Figure 7 is for work trips after the adjustments were applied to correct for the effect of the social-economic factors previously discussed. The information on Figure 8 is a similar comparison without the adjustments. A

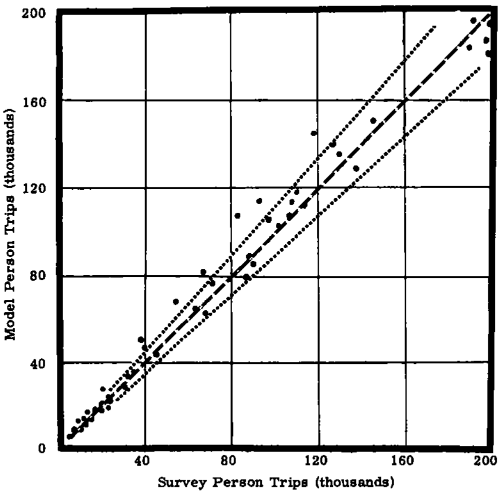

Figure 5. Comparison of screen line crossings, all purposes, Washington, D. C., 1955.

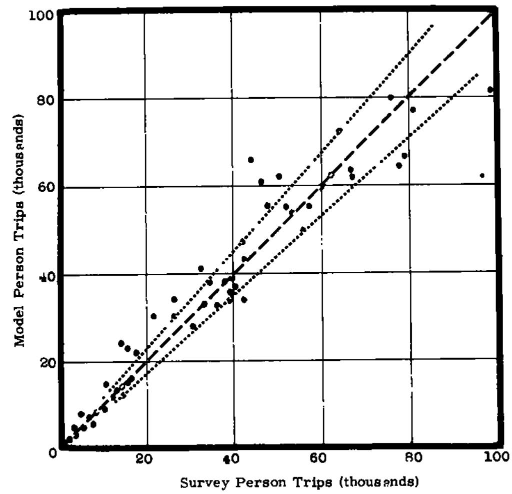

Figure 6. Comparison of screen line crossings, non-work trips, Washington, D. C., 1955.

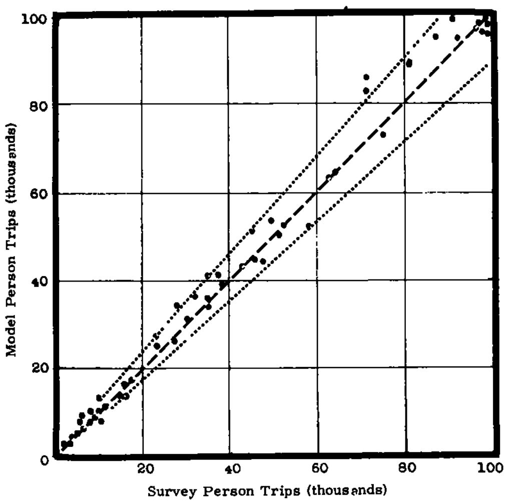

Figure 7. Comparison of screen line crossings, work trips, adjusted, Washington, D. C , 1955.

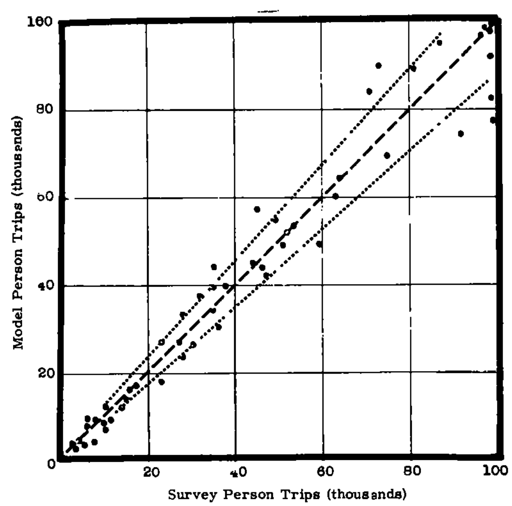

Figure 8. Comparison of screen line crossings, work trips, unadjusted, Washington, D. C., 1955.

comparison of these two charts indicates the importance of these factors on the travel patterns of the Washington area and the need to consider them in estimating urban traffic patterns.

The gravity model estimates and O-D survey of work and non-work travel were assigned in an identical manner to a zone centroid network and a statistical evaluation of the resulting comparisons was made. The results of this comparison, in terms of absolute differences, are shown in Figure 9. As shown, only 20 percent of the resulting loadings exhibited differences greater than 2,000 trips. Again, the work trips show closer correspondence to the survey data than do the non-work trips.

Table 4 gives an evaluation of these loadings by volume group. The percent of differences in the estimated volumes decreases as volume goes up, and for volumes greater than 10,000 persons the differences are less than 15 percent.

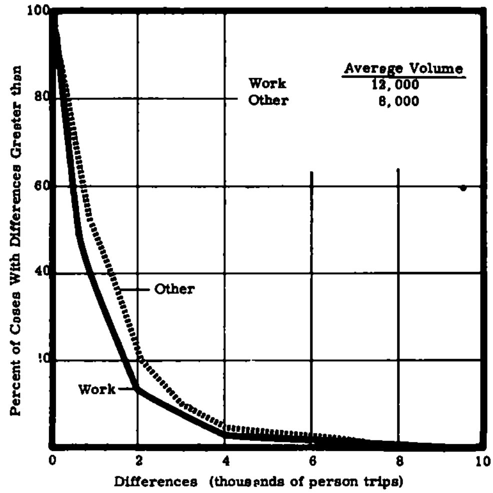

Figure 9. Differences between model and survey trips loaded on zone centroid network, Washington, D. C., 1955.

One of the major problems faced by anyone evaluating traffic estimates is the determination of what constitutes sufficient reliability; in other words, how good is good enough. A review of the screenline crossings indicates that with the possible exception of two cases the differences would not be likely to influence a transportation decision. In addition, the results of the statistical evaluation are as good, if not better, as those associated with the other phases of the transportation planning process.

TABLE 4

ANALYSIS OF DIFFERENCES GRAVITY MODEL VS O-D SURVEY LOADED ON SAME NETWORK, WASHINGTON, D. C., 1955

Volume Group	Work			Non-Work		
	Obs.	Standard Deviation	% Standard Deviation[a]	Obs.	Standard Deviation	% Standard Deviation
0- 499	34	144	54	22	219	72
500- 999	40	271	28	30	437	62
1,000- 1,999	101	408	27	76	506	32
2,000- 2,999	94	701	28	72	849	34
3,000- 3,999	65	734	21	84	968	27
4,000- 4,999	63	838	18	80	1,202	26
5,000- 5,999	47	816	15	77	1,324	24
6,000- 7,999	61	1,145	16	109	1,531	22
8,000- 9,999	55	1,045	12	78	1,815	21
10,000-14,999	116	1,541	12	148	1,873	15
15,000-19,999	80	1,880	11	62	2,776	16
20,000-24,999	39	2,072	9	33	2,222	10
25,000-49,999	96	2,711	8	42	3,645	11
50,000-74,999	14	5,126	8	-	-	-
Over 75,000	10	4,505	5	-	-	-

[a] Expressed as a percent of the mean O-D volume for each volume group.

Based on the results of this examination, it is concluded that the basic gravity model formulation can serve as a framework for forecasting urban traffic. Further, properly calibrated to the over-all travel characteristics of an urban area, the gravity model provides estimates of interzonal trip distribution sufficiently precise for transportation planning purposes.

One of the major findings of this study was the measurement of the influence that factors other than those of trip generation and travel time have on travel patterns in the Washington area, and the need to analyze, understand, and incorporate the impact of these factors when estimating urban travel demands.

Unfortunately, at the present time, these factors can only be incorporated into the model and future traffic forecasts as over-all adjustment factors based on a comparison with surveyed travel data. This does not stem from a lack of understanding of the factors creating these travel patterns, or an inability to express these relationships in a quantitative form. Continuing research has developed several relationships between the K-values and social-economic data for the Washington area. The real limitation lies in the inability, with present techniques, to forecast with any degree of confidence the socio-economic factors creating these travel patterns. Therefore, major improvements in this area of traffic estimation must come from the development of more knowledgeable land-use forecasting techniques.

Although it is not possible to generalize on the applicability of the specific adjustments made to the basic model in this study, it is felt that the general calibration process as outlined provides an analytical framework for developing a gravity model for any city.

ACKNOWLEDGMENTS

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The interpretation of this research and the conclusions drawn are solely those of the author. Neither approval or disapproval of these findings on the part of the Bureau of Public Roads is intended or implied.

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