A LaGrangian Approach to Traffic Simulation on Digital Computers

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A technique for simulating traffic movement on digital computers is described. In this technique computations are performed on an as required basis rather than on an incremental time basis. This treatment of the time parameter may reduce the computational effort required to simulate traffic movement.

• SINCE PUBLICATION of the traffic simulation studies of Gerlough (1) and of Goode, Pollmar, and Wright (2) in 1956, there has been increasing interest in the simulation method of analyzing traffic flow. Subsequent investigations (3, 4) in this area have been concerned with the simulation of increasingly complex traffic situations, based on the simulation techniques presented by Gerlough and by Goode, et al.

Briefly, techniques employed heretofore may be classified according to the way in which vehicles are represented in a computer, and from the manner in which motion and the time parameter are handled.

In the technique of physical representation a roadway or system of roadways is represented by a group of storage locations within a computer. Individual vehicles are represented by binary ones and the spacing between vehicles by binary zeros. The binary ones, representing vehicles, are moved stepwise along the simulated roadway by simple mathematical operations. Before each movement of a simulated vehicle, the existing "traffic" and "roadway" conditions are examined to determine the permissible movement. The result of repeated application of the process is a flow of binary ones along a system of computer locations in a manner analogous to a flow of vehicles along a roadway.

In the memorandum method each vehicle is represented by information stored in a computer word or words. This information includes the vehicle's velocity, its location, its desired velocity, and any other characteristics attributed to an individual vehicle and driver. The parameter of time is treated as an incremental function so that movement is simulated by adjusting, at specified increments of time, the position of each vehicle and such other characteristics attributed to the vehicle that may be influenced by time and position.

Both the memorandum method and the method of physical representation, as used, have been based on an Eulerian viewpoint of motion in that traffic is represented in each as it would appear to outside observers in positions fixed with respect to the roadway. Such observers would see each vehicle moving along the roadway and could describe the phenomena by recording, at successive instants of time, the position, velocity, and attitude of individual vehicles.

LAGRANGIAN APPROACH

In a LaGrangian approach to the simulation of traffic flow, the traffic is described as it would appear to an observer in each vehicle. By moving the observer from outside to within the moving traffic system, the necessity of considering the behavior of each vehicle at each of regular intervals of time is eliminated. Computer programs written from the La Grangian viewpoint need be concerned only with those values of the time parameter when the behavior of a vehicle would change and only with those vehicles actually affected by the change in behavior.

As employed by the authors, (5), the LaGrangian treatment of the time parameter has been incorporated in the memorandum method of simulation. The following information is stored for an individual vehicle:

1. The characteristics assigned to an individual vehicle and its driver.
2. An index number corresponding to the vehicle's relative position in its lane of travel.
3. The equation of motion of the vehicle.
4. The value of the time parameter at which the equation of motion may change.

Boundary conditions at the beginning of a simulation period may be established with the roadway system either empty or bearing vehicles. If vehicles are in the system, their equations of motion and their individual characteristics will be stored in appropriate storage locations. Also stored will be a value of time, T_e, when the first additional vehicle is to enter the roadway. These additional vehicles are introduced into the system by means of a function generator sub-routine of either random or controlled nature. In the remainder of the discussion, it is assumed that vehicles are on a roadway when simulation begins.

The beginning of a period of simulation is taken as time zero, T_0. At T_0, each vehicle's equation of motion is solved for the time, T_i, when the first condition will exist that will require a change in the vehicle's behavior. The computations required involve the solution of each vehicle's equation of motion to determine the time a certain position will be reached; and/or the time at which the vehicle's velocity or acceleration reaches some critical value; and/or the simultaneous solution of several equations of motion for the time when the distance between the vehicle and other vehicles attains some preset critical value. When more than one of these computations is performed for a single vehicle, the resulting set of times is examined and the least value taken as T_i.

The value selected as T_i is stored with the information pertaining to a particular vehicle, and represents the time at which, under the existing traffic and roadway conditions, something may cause the behavior of the vehicle to change.

The least value, T, of the stored times, considering T_e and all the T_i, then determines the first event that will cause a change to be made in the stored information relating to the individual vehicles. To determine the changes required, the situation corresponding to time T must be identified, either by assigning, at the time of the original computation of the T_i, code numbers identifying the events predicted, or by examining the equation of motion and desired behavior pattern of the vehicle whose T_i became T.

If, at T, the event is a vehicle entering the system, the individual characteristics of the vehicle are generated and its equation of motion formed. This new equation of motion then is solved for the T_i of the vehicle. Also determined and stored is a new value T_e which, with the new T_i, is compared with all the previously determined T_i to find the time T of the next event.

When the event at T is a vehicle leaving the system, any desired information relating to that vehicle is stored for compilation or print-out and remaining information is removed from further consideration.

If, at time T, any event other than a vehicle entering or leaving the system is to take place, the change called for is introduced into the vehicle's equation of motion and a new time, T_i determined and stored for that vehicle.

Any vehicle entering the roadway, leaving the roadway, or changing its behavior, may cause changes in the times when other vehicles are to alter their behavior. These changes will effect any vehicle for which the computation of T_i involved the previously affected vehicle. Frequently, only the immediately following vehicle and the immediately oncoming vehicle are involved. In any case, the values of T_i of the vehicles affected are recomputed and stored.

The behavior of all vehicles, as defined by the new set of equations of motion, remains constant in the interval from the original value of T until the least value of the new set of stored times. The event associated with the new T is identified and, again, any required changes are made in the equations of motion of the affected vehicles. The process of determining the time of occurence, T of an event, of adjusting equations of motion dependent upon that event, and determining a new value, T, is continued until the value of T exceeds some predetermined time of simulation and the process is stopped.

SUMMARY

The LaGrangian approach to the simulation of traffic may be recognized as an inspection of events rather than a continuous survey of traffic. Here, computations are performed only for those times when interactions occur, and only for those vehicles affected by the interactions. If the traffic situation being simulated is extremely complex (i.e., a large number of interactions is involved), the computational effort required by this technique approaches that of the Eulerian techniques. If, however, the number of interactions is reduced, as is the case with either moderate or very high traffic density, this technique is indicated.

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