Calculation of Depth of Freezing and Thawing Under Pavements

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Test installations of thermocouples measuring ground temperatures beneath runway pavements at three sites in Alaska are described. Data obtained from these installations over a period of several years are utilized to determine depths and rates of penetration of frost and thaw. A theoretical method of calculating frost and thaw penetration utilizing soils data, air temperature data, and predetermined thermal conductivity values and surface correction factors is explained. Results of calculations by this method are compared with the frost and thaw depths indicated by the thermocouple readings. In certain instances frost conditions obtained by borings and test pits are available for comparisons with the theoretical results. It is concluded that the theoretical method is a useful tool for making estimates of frost conditions in runway sections. The depth of frost and thaw penetration may be computed with a resulting error of 1 to 3 ft. in soils where actual penetration is from 6 to 15 ft., provided adequate soil and air temperature data are available.

● THE performance of roads and runways in any northern climate is affected greatly by frost action. In arctic and subarctic regions the effects of frost action may be particularly severe. The presence of permafrost in such regions may bring about special problems which must be considered.

In order to meet problems introduced into road or runway design and construction by frost action, a knowledge of the rate and depth of frost penetration and the rate and depth of thawing is highly desirable. As a part of a comprehensive investigation of the design and construction of airfields in arctic and subarctic regions being conducted by the St. Paul District, Corps of Engineers, Department of the Army, field measurements of temperatures in runway sections are being made. These observations have been accompanied by theoretical studies of frost penetration and thawing. Comparisons between the depth of frost or depth of thaw as determined by the temperature observations and those calculated by theoretical methods can be made. Although this work is still continuing, it is felt that the results obtained thus far are of sufficient value and interest to be reported. Results to date indicate that if one has adequate information on the soils materials, principally the water contents of the various strata, and also the climatological data, it is possible to make calculations for the location of the frost line with a reasonable degree of accuracy needed for stability considerations of the pavement section.

The investigations reported were conducted by the Permafrost Division, St. Paul District, Corps of Engineers, U. S. Army. Colonel A. H. Bagnulo is the district engineer. Acknowledgement is made of the assistance of D. E. Johnson of the Permafrost Division for assistance in calculations and preparation of data.

THEORETICAL CALCULATION METHODS

In attempting to select a method of calculation for frost depths, a review of both foreign and American literature was made (1). Study of the various methods indicated that an adaptation of the so-called Stefan equation was the most prom-

ising. This equation is essentially predicated on the hypothesis that the latent heat of fusion is the only heat that need be conducted to or from a point in the process of thawing or freezing. Heat quantities involved in change of temperature above or below the freezing point are considered of minor importance, and are ignored. The flow of heat during the freezing or thawing process is between the ground surface and the freeze or thaw line. Modifications of the "Stefan" equation used in these calculations include: (1) use of a correction factor to convert degree days of air temperature to degree days of surface temperature and (2) calculation of freezing or thawing by increments based on soil strata of varying physical characteristics.

Restricting our discussion for the moment to a freezing action, or penetration of frost, the equations utilized in this method are as follows:

For a uniform soil (nonstratified),

$$h = \sqrt{\frac{48kF}{L}}$$

h = depth of frost penetration in feet

k = thermal conductivity of frozen soil in Btu. per square foot per degrees Fahrenheit per foot per hour

F = surface freezing index in degree days Fahrenheit

L = latent heat of fusion in Btu. per cubic foot

For a layered system of soil, such as a pavement section which might consist of a bituminous concrete pavement, a gravel base course, and a subgrade (which might be layered in itself), the following method is utilized:

The partial-freezing index required to freeze the top layer may be calculated by a rearrangement of the above equation

$$F = \frac{L_1 h_1^2}{48\,k} = \frac{L_1 h_1}{24} \cdot \frac{h_1}{2k} = \frac{L_1 h_1}{24} \cdot \frac{R_1}{2}$$

in which R_1 is termed the thermal resistance of the soil layer and is equal to its thickness divided by the thermal conductivity.

In freezing the second layer of soil, of thickness h_2, the heat must travel through all of Layer 1, represented by the thermal resistance R_1, and through some part of Layer 2. For the freezing of all of Layer 2, an average resistance in that layer of $R_2/2$ is utilized. The partial-freezing index required for freezing of this layer is

$$F_2 = \frac{L_2 h_2}{24} \left(R_1 + \frac{R_2}{2}\right)$$

Likewise, for the nth layer, the partial freezing index would be

$$F_n = \frac{L_n h_n}{24} \left(R_1 + R_2 + \ldots R_{n-1} + \frac{R_n}{2}\right)$$

$$= \frac{L_n h_n}{24} \left(\Sigma R + \frac{R_n}{2}\right)$$

in which ΣR is the summation of the thermal resistances of all layers above Layer n. Knowing the total degree-days of freeze for any season, or particular time period, the depth of freezing may be determined by selecting those layers whose sum of partial freezing indexes equals the total of the period.

To calculate thawing in soil, similar equations are used except I, the surface-thawing index, is utilized instead of F, and the k value should be the thermal conductivity of thawed rather than frozen soil, since in this instance the heat required to thaw the ice is travelling from the surface to the point of melting through thawed soil.

Each of the factors in these equations is discussed more fully in the following paragraphs.

F, the surface-freezing index, is determined by a summation of degree-days below 32 F. at the pavement surface. Since air temperatures are usually available and surface temperatures are not, a study has been made from data obtained at Corps of Engineers installations in Alaska to determine the relation between a freezing index calculated from air temperatures and from different types of pavement surfaces. A surface correction factor of 0.6 has been found for bituminous, portland-cement-concrete, and gravel surfaces, i.e.,

Surface F = 0.6 x F based on air temperatures.

For the thawing index, I, which is a summation of degree-days above 32 F., the correction factor is 1.4 for the same three types of surfaces, i.e.,

Surface I = 1.4 x I based on air temperatures.

These correction factors are based on more complete temperature data than those previously reported (1). It is emphasized that the correction factors may be different for other climates or localities and for other types of surfaces.

L, the latent heat of fusion in Btu. per cubic foot is calculated by the formula

$$L = 143.4 \frac{wd}{100} = 1.434\ wd$$

in which w = water content of soil in percent of the dry weight

d = dry density in pound per cubic foot.

Values for k, the coefficient of thermal conductivity have been obtained from charts derived from tests of thermal properties made at the University of Minnesota under a contract with the St. Paul District, Corps of Engineers (2, 3).

An example of a calculation of depth of thaw for an actual pavement section is given later.

FIELD TEMPERATURE MEASUREMENTS

General

In this report temperature-measurement installations at three different Alaskan sites will be considered. Two of these were in actual airfield runways and the third in a series of specially constructed runway test sections with a variety of base-course thicknesses.

Northway Airfield

Investigational work at Northway Airfield, about 230 mi. southeast of Fairbanks, was initiated in 1945 to collect basic physical data on soil characteristics, ground water, foundation designs, ground temperatures, and other factors affecting facilities at the site, with particular reference to permafrost. During 1945, ground-temperature-measuring equipment consisting of mercury-thermometer strings was installed in a series of holes along the edge of the runway. After about a year of observations, the thermometer strings were replaced with thermocouple installations in eight holes. Temperatures were read at approximately weekly intervals until early in 1949.

The soils beneath the runway are principally fine, medium, and coarse-grained sands with some layers of silt or sandy silt near the surface. Unfortunately the soil logs kept during the boring are not as complete, particularly concerning water contents, as would be desired for frost calculations. It has been necessary to make certain assumptions on soil conditions for purposes of this study.

Eielson Air Force Base

Five temperature observation holes were drilled in the runway at Eielson Air Force Base, 26 mi. southeast of Fairbanks, in April 1948. Ground temperatures were measured by thermocouples at weekly intervals from October 1948 through January 1951.

The soil beneath this runway is a gravel with an occasional layer of fine sand. No permafrost was encountered to a depth of 30 ft. The runway has a bituminous-concrete pavement and a crushed-rock-and-gravel base.

Fairbanks Research Area

The Fairbanks Research Area is located 2½ mi. northeast of Fairbanks and was constructed for observing various types of structures erected on permafrost under conditions known and recorded from the time of construction. Included in the extensive research installations at this site are a series of 26 runway test sections, each 50 ft. square consisting of various types and thicknesses of base and pavement. In some instances different types of insulation are included in the base course. The subgrade soil is silt. Thermocouple strings were installed in the center of each of these test sections and readings were taken at approximately weekly intervals from October 1947 to April 1951. In this report data from nine of the test sections are utilized.

Interpretation of Temperature Data

In utilizing plots of ground-temperature data to locate the position of the so-called frost line, or 0-deg.-centigrade isotherm, the results are dependent on the accuracy of the temperature measuring apparatus. The need for accurate measurements is great, because in many instances the thawed ground exists at a temperature

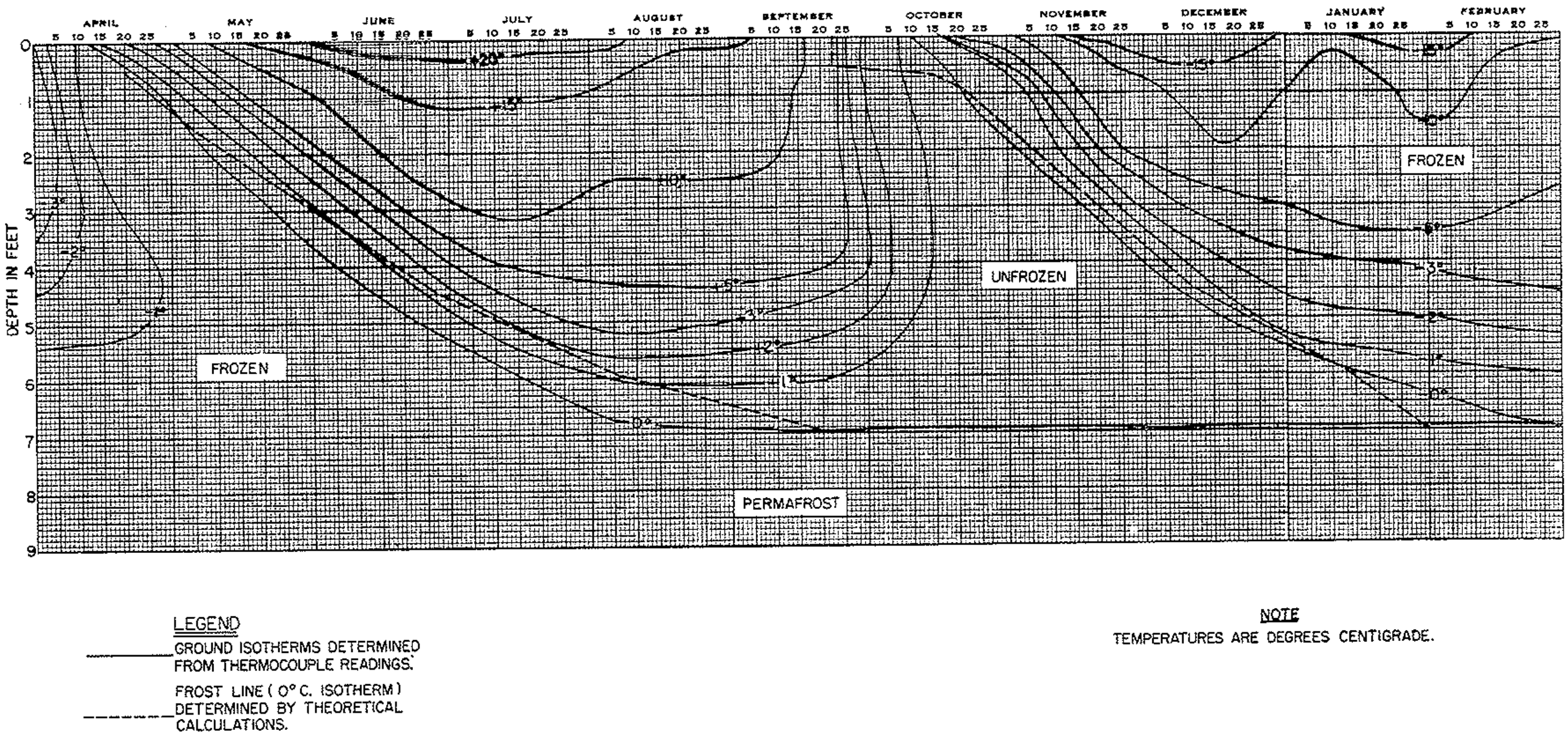


Figure 1. Ground isotherms and theoretical frost line - Northway Airfield, Alaska. Hole No. 2

of only a fraction of a degree above freezing and the frozen ground is likewise only a fraction of a degree below the freezing point. The thermocouple installations in the three test locations presented above consisted of $1\frac{1}{4}$-in. steel pipe filled with SAE 10 motor oil. The thermocouple circuits led from the pipe to a selector switch and measurements were made with a potentiometer. Because of the conductivity of the steel pipe and oil, the thermocouples did not necessarily measure the exact temperature that would exist in the soil at the same depth. In many instances it is known that this resulted in a change from just below to just above the freezing point of the soil, or vice versa. However, by judicious interpretation of the temperature values and comparisons with known conditions, it is considered that location of the frost line can be obtained from the readings by noting the temperature gradients which exist rather than the exact result obtained from a single thermocouple. Determination of the frost line by this method is explained and illustrated in succeeding paragraphs. Wherever possible, the steel pipe is being removed from ground-temperature wells and the thermocouples are being placed in direct contact with the soil or in a pipe which is a poor heat conductor.

CORRELATION OF FIELD OBSERVATIONS AND THEORETICAL CALCULATIONS

To check on the degree of accuracy with which theoretical calculations can be used to predict frost or thaw depths in pavement sections, calculated curves for the three test locations are compared with the results obtained from the thermocouple readings. These comparisons are affected by the accuracy with which the temperature readings in the steel pipes portray the ground temperatures, as has already been discussed. They are also affected by the completeness of the soil information available for use in the theoretical calculations. In this respect, the logs of some of the holes at Northway Airfield are incomplete; those at Eielson Air Force Base are fairly good, although the locations of soil strata changes are not complete; and those at the Fairbanks Research Area are entirely adequate. The basic soil information desired is texture, water content, and dry density; good water content values are of first importance. The dry density can usually be estimated within reasonable limits for the calculations if it has not been measured.

Northway Airfield

To illustrate the interpretation of the temperature data at Northway Airfield, Figure 1 portrays the period of April 1946 to February 1947 for Hole 2. By means of observations of thermocouples of 0 to 6 in. and 1, 2, 4, 7, 11, 15, 20, 25, and 30 ft. at approximately weekly intervals, the 1-, 2-, 3-, 5-, 10-, 15-, and 20-degree isotherms are sketched in as shown. Numerous observations have indicated that the temperature gradient above the frost line during freezing or above the thaw line during thawing approximates a straight line, i.e., there is a uniform increase or decrease in temperature with an increase in depth to the frost or thaw line. Thus the -3, -2, -1 and zero or +3, +2, +1 and zero isotherms are at equal spacings. Utilizing this fact the zero isotherm, or freeze or thaw line, is sketched in below the +1 or -1 C. line. There are certain periods, namely at the start of the freeze or thaw, when the zero isotherm can be quite easily drawn between positive and negative readings. During the latter part of both the freezing and thawing seasons, however, the use of the above scheme is often needed.

The zero-isotherm lines determined by these methods for the period from 1945 to 1949 for all nine test holes are shown in the full lines in Figure 2. In certain instances the full line curves are not complete. This is usually where the temperature information is incomplete, or of such a nature that the zero isotherm cannot be reasonably estimated. Also plotted on Figure 2 are the theoretical curves of the freeze and thaw lines. The calculation of the latter are explained by the example in the following paragraphs.

The log of Hole 1, as drilled in March 1945, is given in Table 1. Nearly all of the soils were in a highly saturated condition. For purposes of calculation where densities were not given, densities which would be saturated by the given water contents in a frozen condition were assumed.

TABLE 1

LOG OF HOLE 1, NORTHWAY AIRFIELD, ALASKA

Depth ft.	Material	Water content %
0 - 0.5	Bituminous pavement	
0.5 - 3.0	Well-graded sand, black (SW)	21[a]
3.0 - 4.0	Gray silt (ML)	28
4.0 - 7.0	Gray silt (ML)	47
7.0 - 8.0	Gray silt (ML)	64 & 77
8.0 - 13.5	Gray silt (ML)	56
13.5 - 17.0	Gray silt (ML)	58

[a] Assumed value, based on data from other holes.

TABLE 2

VALUES FOR USE IN FROST CALCULATIONS HOLE 1, NORTHWAY AIRFIELD

Depth ft.	Material	Water Content %	Dry Density lb. per cu. ft.	Frozen k, per ft.	Thawed k, per ft.	L Btu. per cu. ft.
0.0-0.5	Bit.	0.0		0.83	0.83	0
0.5-3.0	SW	21.0	104	2.00	1.21	3130
3.0-4.0	ML	28.0	92	1.17	0.77	3690
4.0-7.0	ML	47.0	74	1.25	0.60	4990
7.0-8.0	ML	70.0	58	1.25	0.41	5810

Values of the thermal conductivity of the soil, both in a frozen and thawed condition were selected from the diagrams previously mentioned. The latent heat of fusion per cubic foot could be calculated utilizing the density and water contents. The pertinent values for the upper layers of the above section are given in Table 2.

The solution of the degree-days required to thaw the various layers by the equation can be best

$$I = \frac{L_n b_n}{24} \left(\Sigma R + \frac{R_n}{2}\right)$$

carried out in Table 3. The last column in this table is the conversion from the summation of the surface thawing indexes to the corresponding air index, utilizing the correction factor of 1.4. To find the particular dates of a given year on which the thaw penetrates to any given depth, a cumulative tabulation of the degree-days of thaw is made, starting with the advent of the thawing season. From such a tabulation the day on which the thawing index first reaches the totals in the last column of Table 3 can be determined. In the spring of 1946, for example, the thawing season started on April 20, reached 383 on May 25, 747 on June 8, and reached a maximum of 3,280 on September 22. Thus the thaw would not reach the 7-ft. depth, but some value between 4 and 7 ft. To calculate the maximum depth of thaw, the following procedure is used:

Let h = depth of thaw below the 4.0-ft. depth

Degree days available to thaw below 4.0-ft. depth, based on air temperatures = 3280-747 - 2533

Changing to surface thawing index = 2533 x 1.4 - 3546

$$\text{Then } I_n = \frac{Lh}{24}\left(\Sigma R + \frac{R}{2}\right) = \frac{Lh}{24}\left(\Sigma R + \frac{h}{2k}\right)$$

$$3546 = \frac{4990h}{24}\left(3.97 + \frac{h}{2 \times 0.60}\right)$$

Solving for h,

h = 2.8 ft.

Or the total depth of thaw on September 22 is 4.0 + 2.8 = 6.8 feet.

The dates and depths of thaw thus determined are used to construct a "theoretical" curve as shown by the dashed line of Figure 1. This curve may be compared with the zero isotherm to check the theoretical procedure against actual observetions.

Theoretical calculation of the freeze of this thawed depth of 6.8 ft. is given in Table 4. Other intermediate points can be computed to define more closely the theoretical curve. The freeze curve is also shown by a dashed line in Figure 1.

TABLE 3

THAW CALCULATIONS, HOLE 1, HORTHWAY AIRFIELD

Depth ft.	Thickness h, ft.	k, thawed per ft.	L Btu. per cu. ft.	Therm. Resist R = h/k	ΣR	ΣR+ R/2	I, deg.-days	ΣI	$\Sigma I_{air} = \Sigma I/1.4$
							0	0	0
0 - 0.5	0.5	0.83	0	0.60	0	0.30	536	536	383
0.5 - 3.0	2.5	1.21	3130	2.07	0.60	1.64	511	1047	747
3.0 - 4.0	1.0	0.77	3690	1.30	2.67	3.32	4030	5077	3630
4.0 - 7.0	3.0	0.60	4990	5.00	3.97	6.47	2465	7542	5390
7.0 - 8.0	1.0	0.41	5810	2.44	8.97	10.19			

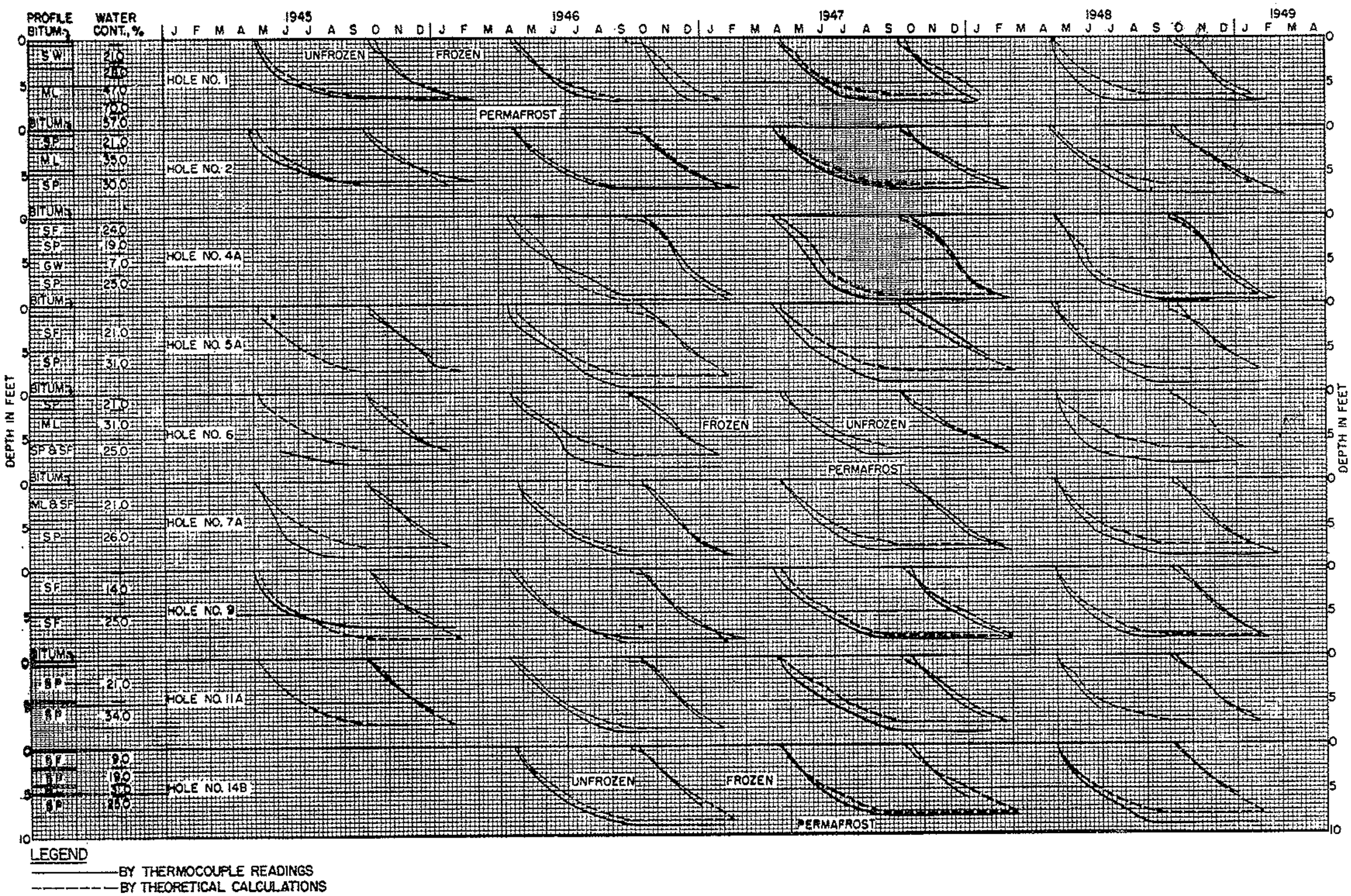


Figure 2. Freeze and thaw lines as determined by thermocouple readings and theoretical calculations, Northway Airfield, Alaska.

TABLE 4

FREEZE CALCULATIONS, HOLE 1, NORTHWAY AIRFIELD

Depth ft.	Thickness h, ft.	k, Frozen per ft.	L Btu. per cu. ft.	Therm. Resist. $R = h/k$	ΣR	$\Sigma R + \frac{R}{2}$	F Degree days	ΣF	$\Sigma F_{air} = \Sigma F/0.6$	Date
0 - 0.5	0.5	0.83	0	0.6	0	0.3	0	0	0	Sept. 23
0.5 - 3.0	2.5	2.00	3130	1.25	0.6	1.23	401	401	668	Nov. 16
3.0 - 4.0	1.0	1.17	3690	0.86	1.85	2.28	351	752	1253	Nov. 28
4.0 - 6.8	2.8	1.25	4990	2.24	2.71	3.83	2230	2982	4970	Feb. 1

Inspection of the calculated theoretical curves together with the curves determined from the thermocouple readings for all nine holes for the entire period of observations in Figure 2 indicates the degree of agreement of the two. The basic shape of the curves, or the rate of freeze and thaw, is substantially the same. The depth of the freeze or thaw at a particular time as calculated theoretically is, on the average, not more than 1 ft. different than the observed depth, and rarely differs by as much as 2 ft. from the observed value. In nearly all instances, the depth of the frost line determined by the thermocouple readings is greater than the calculated depths. This might be accounted for in some part by the error which is introduced into the thermocouple readings by the conductivity of the oil-filled pipe. For example, during the period of thaw, the top part of the pipe and the oil are at temperatures which may be several degrees above freezing. This column may tend to warm the pipe at greater depths in the vicinity of the frost line. Thus at the depth below which the soil is frozen, the temperature in the pipe may be a fraction of a degree above freezing and the change to below freezing in the pipe would be at a depth slightly greater than the frost line in the soil itself. The same possible error occurs during penetration of frost. The upper portion of the pipe located in the frozen depth of soil may be at temperatures several degrees below freezing. This cold column tends to draw some heat from the pipe and oil below and the changes from above-freezing to below-freezing temperatures in the pipe will be at a somewhat greater depth than the actual frost line in the soil.

The soil at depth in the test holes shown in Figure 2 is permafrost. Toward the end of the freezing season, i.e., in March or early April, the entire depth covered by the thermocouples is frozen. During the summer thaw occurs to depths of 6 to 9 ft. This depth is usually refrozen by some time in January or February of the following freezing season. The depth of 6 to 9 ft. which thaws and freezes annually is called the annual frost zone. In this instance it also represents the suprapermafrost.

Eielson Air Force Base

The test installation at Eielson Air Force Base consisted of five holes spaced on a transverse section across the runway. The holes were 30 ft. deep and had thermocouples at 0 and 6 in., every 2 ft. from a 4- to 12-ft. depth, and every 3 ft. from 15 to 30 ft. The section at all holes consisted of a bituminous pavement about 6 in. thick, a crushed rock base to a depth of 1.8 ft., and a compacted gravel fill to a depth of from 5 to 8 ft. The subgrade was essentially a well-graded sand and gravel. The water table was at about 11 ft. at the time of the borings.

Thermocouple readings were taken at approximately weekly intervals between October 1948 and January 1951 in three of the test holes. In the other two, readings were discontinued in April 1950, due to damage to the installations. The zero isotherm can be distinguished as the dividing line between positive and negative centigrade temperatures during most of the test periods. The exact depth of frost penetration is not always distinctly defined within 3 ft., plus or minus, because of the 3-ft. thermocouple spacing used below 15 ft.

In the logs of the holes as bored water contents were reported at 5-ft. intervals. Closer tests would have been desirable. However, utilizing the information at hand and selecting what were deemed as reasonable density and base course water

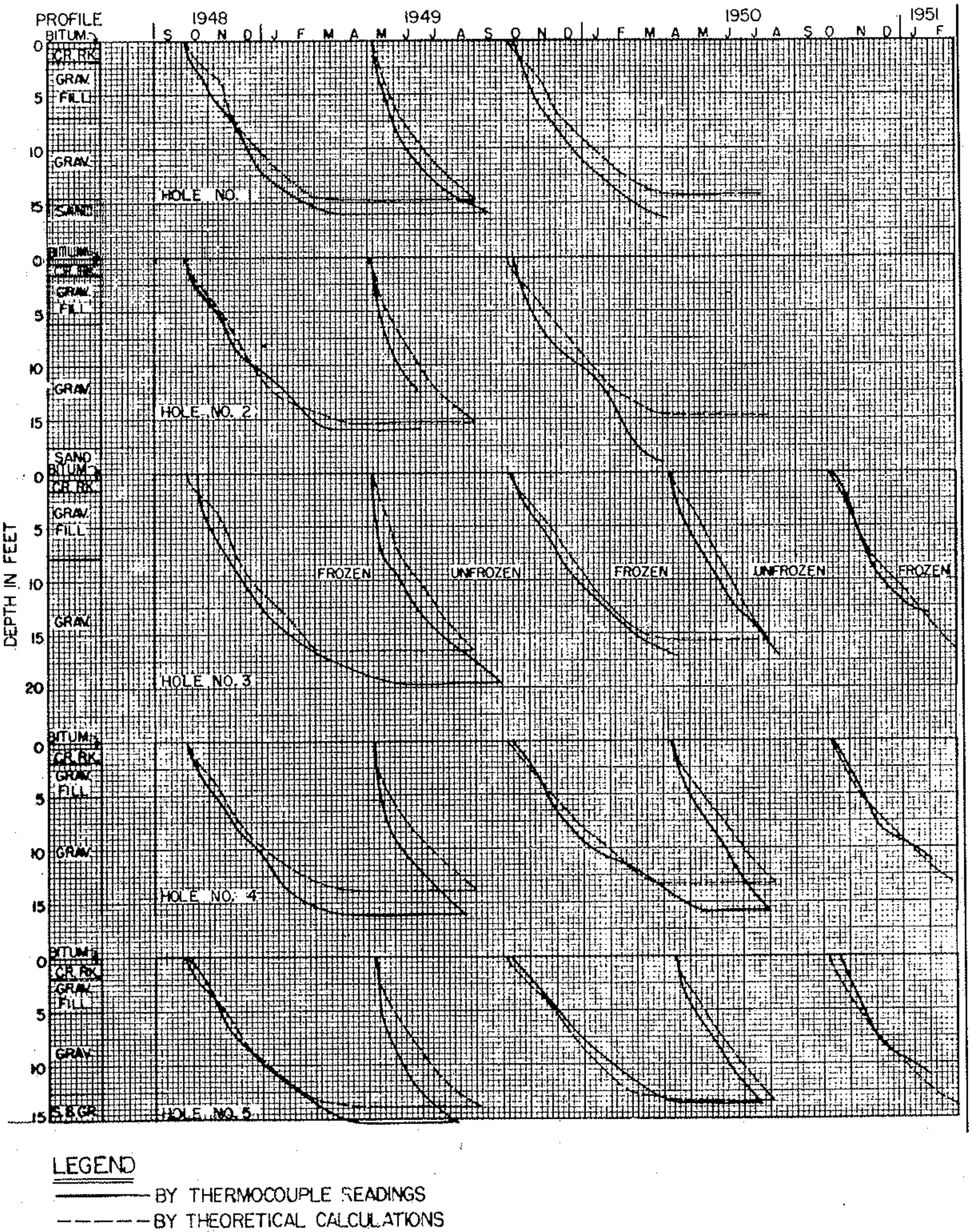


Figure 3. Freeze and thaw lines as determined by thermocouple readings and theoretical calculations, Eielson Air Force Base, Alaska.

content values, theoretical calculations of freeze and thaw were made. Air temperatures at Fairbanks were utilized and the surface correction factors previously mentioned (0.6 for freezing, 1.4 for thawing) were applied to the air indexes. Plots of the frost line as determined by thermocouple readings and by calculation are shown together in Figure 3.

Depths of freeze and subsequent thaw, as shown in Figure 3, were to a depth of 15 ft., plus or minus. This is much greater than the freeze in any of the holes at Northway Airfield. The reason for the difference is essentially in the differences in water contents. The sandy-gravel subgrade at Eielson had a water content of only about 8 to 10 percent in most strata. Frost penetration through such materials is rapid because of the relatively low volumetric heat of fusion.

The comparison of the theoretical and thermocouple-determined curves in Figure 3 is considered to be good. The differences for the most part do not exceed 2 or 3 ft. Again it may be noted that the frost depths determined by the thermocouple readings are in general deeper than the theoretical calculated values. This may be due in part to the conductivity of the oil-filled pipe, as explained previously for the Northway tests.

It will be noted that, although the general appearance of the frost-line loops, or continuous zero isotherms, in Figure 3 is similar to those of Figure 2, the areas inclosed by the zero isotherm are reversed. In Figure 3 the area within the loop represents a frozen condition, in Figure 2 a thawed soil. This is due to the fact that whereas permafrost existed at Northway Airfield, there is none in the test holes at Eielson. The depth of seasonal frost increases during the entire freezing season at Eielson, reaching a maximum in late April or early May. The thaw then starts and continues from the surface down until the entire layer is thawed, which usually occurs during August.

Present plans call for making some borings in the runway during the winter of 1952-53 to obtain a visual check on the frost penetrations indicated by the theoretical calculations and by the thermocouple readings.

Fairbanks Research Area

Two types of analysis of the pavement test sections of the Fairbanks Research Area are presented. In the first, comparisons are obtained between the frost line determined by thermocouple readings and by theoretical calculations, as was done with the Northway and Eielson sections. This has been done for seven different pavement sections, all with a bituminous-concrete pavement and a sand-and-gravel base course. The thicknesses of the base course vary from 2.8 to 12.0 ft. The second study presented is for four sections, two with bituminous-concrete pavements and two with portland-cement concrete, in which borings were made five times between October 1951 and July 1952 to locate the seasonal freeze and thaw lines and surface of the permafrost. These actual observations are compared with theoretical calculations.

The subgrade soils beneath the pavement test sections to a depth of 35 ft. are principally silts. The boring notes indicate frequent inclusions of peat. The water content in the upper 12 ft. of the soil is about 30 to 40 percent, except for some samples which contain considerable ice lenses or peat, in which instance higher water contents are obtained. For purposes of the theoretical calculations, a uniform moisture content of 35 percent, based on an averaging of numerous tests, has been used. A density representing a saturated condition for this water content has been assumed. Tests on the gravel base course material give an average water content of 3.0 percent and an average dry density of 140 lb. per cu. ft. These values have been used for all sections.

Utilizing the soil and base course information above and the degree-day data from Fairbanks, theoretical calculations of the freeze and thaw for the several years covered by the observations were made. These curves are shown as the dashed lines of Figure 4. The frost lines determined from the thermocouple readings are plotted as the full lines. The curves are similar to those of Eielson Air Force Base, Figure 3, in that permafrost is sufficiently deep so that it does not restrict the depth of annual freeze. The permafrost levels are not shown in Fig-

ure 4, but in most instances there is a residual thaw layer from about 2 to 4 ft. thick between the maximum depth of annual freeze and the surface of the permafrost. Thus frost penetration continues to increase during the entire freezing season, and this depth thaws during the first part of the thawing season. Inspection of Figure 4 indicates that the annual frost zone is usually thawed by sometime in July or August. During the remainder of the thawing season there would be a degradation of the permafrost. Calculations for such degradation can be made in the same manner as the computation of thaw of a layer of annual frost. The depth to the permafrost surface must be known; the water content of the permafrost is an important item in the calculations.

Inspection of Figure 4 indicates a good agreement between the theoretical annual depths of freeze and that determined by thermocouple readings. In 23 of the 28 comparisons shown (four-year record for each of seven sections), the difference is 1 ft. or less. The depths of annual frost vary from about 8 to 14 ft. The greatest divergence of the theoretical curves from those plotted from the thermocouple readings is in the rate of thaw of the subgrade soil. The theoretical curves in nearly all instances indicate that the thaw of the subgrade would not be completed until several weeks after the time indicated by the thermocouples. The depth of soil which thaws during this additional time in the theoretical calculation is about 1 ft., plus or minus. It may be that the thermocouples, because of their being in the oil-filled pipe, are incapable of showing the presence of a 1-ft. layer of frost. It is also possible that the simplified theory, based on flow of heat from the thawing layer to the atmosphere, is inadequate for the thaw of this last increment.

An additional check is available on the theoretical frost calculations. As a part of an accelerated traffic test of the runway test sections in 1951, test pits were dug in four of the sections in April or June. The depth of annual frost was measured in these pits. The points are shown in Figure 4, and Table 5 lists the theoretical annual frost depths, those determined by the thermocouple readings, and the depths measured in the pits.

TABLE 5

COMPARISON OF FROST DEPTHS
FAIRBANKS RESEARCH AREA, 1950-51

Section	Depth of Annual Frost		
	Theoretical ft.	Thermocouples ft.	Test Pit ft.
RN-12	7.9	9.0	8.5
RN-4	8.4	9.0	8.3
RN-25	8.4	9.8	9.5
RN-15	9.9	10.0	9.3

The maximum difference between the theoretical and test pit values is 1.1 ft. and the average difference is 0.6 ft. The depths interpreted from the thermocouple readings are all greater than those observed in the test pits; the average difference is 0.6 ft.

Since there was some question as to the accuracy to which the frost line could be determined by means of the thermocouple readings installed in the pipes, a program of periodic borings to determine the frost line was initiated in four of the runway test sections in the fall of 1951. In making borings with a hand auger, exact determinations of the level of seasonal frost or thaw and the position of the top of the permafrost could be made. Also, moisture contents of the base course and subgrade were obtained for use in theoretical calculations.

The theoretical calculated curves for the four sections included in this program for the 1951-52 period are shown in Figure 5, together with frost line positions determined by the borings. Different symbols are used for the depth of seasonal thaw, the depth of seasonal freeze, and the top of the permafrost. The latter is of no particular concern in the present discussion. The check between the theoretical calculation and the boring data may be studied by inspection of Figure 5. The plotted boring points in all four sections fall quite close to the theoretical curves, both for penetration of frost and for thaw. The greatest deviation is about 1 ft. This is considered to be a good check. There are some seemingly inconsistent changes in frost levels as indicated by the hand borings. It would be expected, for example, that the surface of the permafrost would be stable during the time period shown. In two of the sections variations of 1 ft. or more in this level are indicated in different borings. Such differ-

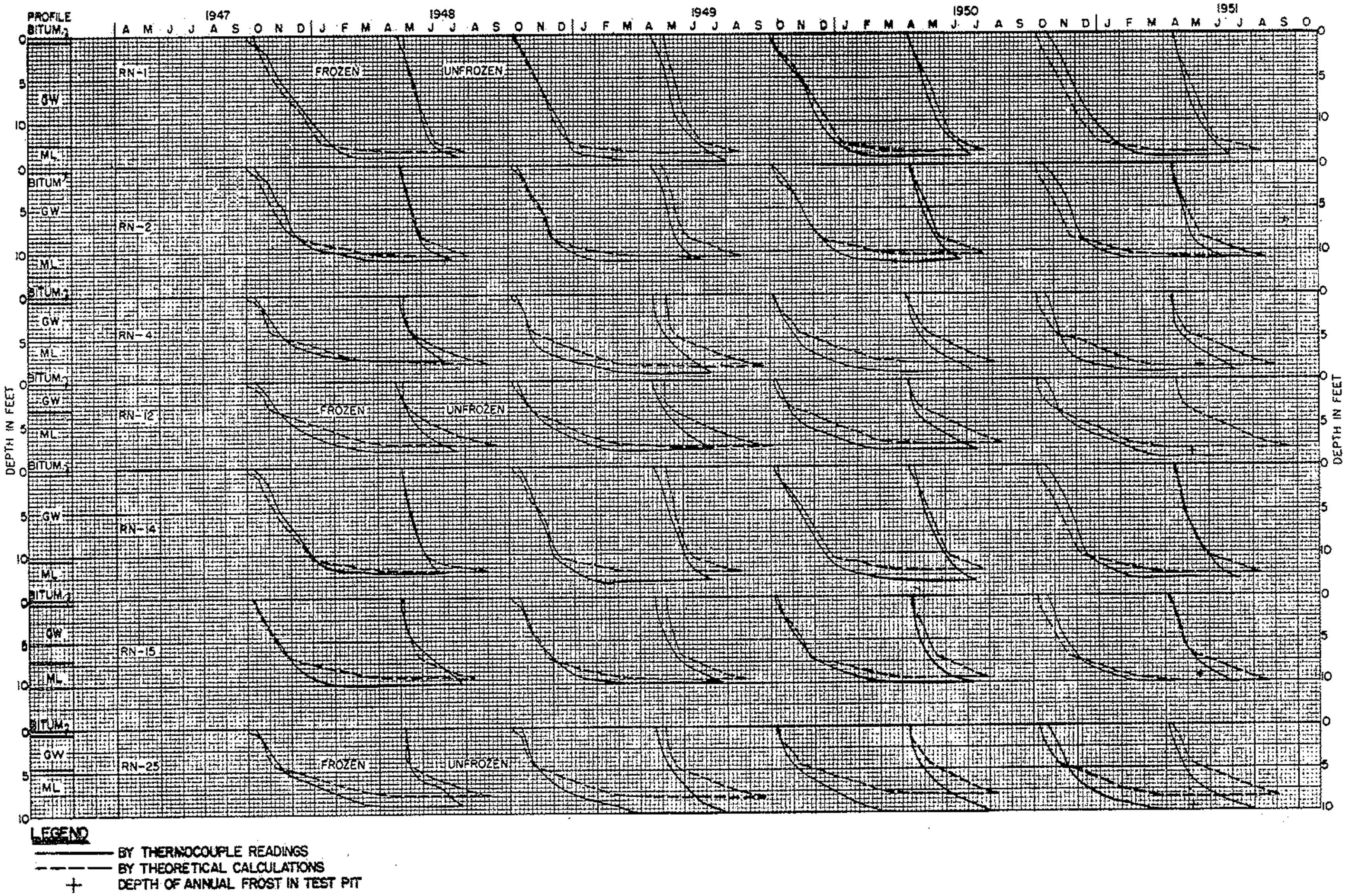


Figure 4. Freeze and thaw lines as determined by thermocouple readings and theoretical calculations, Fairbanks Research Area, Alaska.

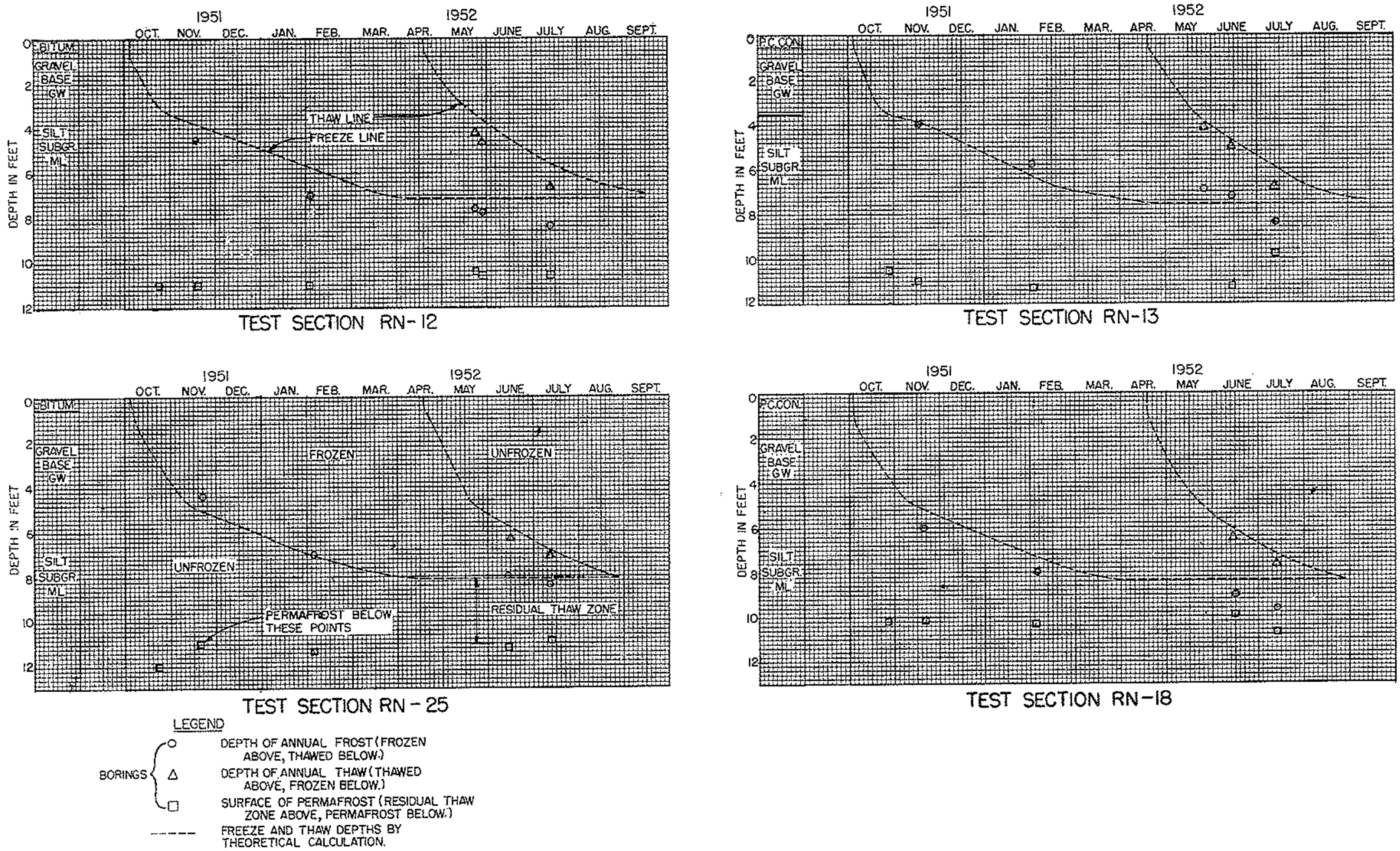


Figure 5. Frost conditions as determined by borings and theoritical calculations - Fairbanks Research Area, Alaska.

ences are due to the fact that the borings were made at various locations in each test section, and local variations in base course thickness, water contents, and other items caused differences.

EFFECT OF BASE THICKNESS ON FROST PENETRATION

The data presented in Figure 4 may be utilized to study the effect of base course thickness on frost penetration. All seven sections shown on this figure had a bituminous concrete pavement about 5 to 6 in. thick. The thickness of the gravel base courses varied from 2. 8 ft. for Section RN-12 to 12 ft. for Section RN-1. Table 6 lists the sections with the average values of frost penetration as obtained from the thermocouple observations for the 4-year period. Although there are some discrepancies in order of the sections, it is readily apparent that an increase in the thickness of the base course results in an increase in the total depth of frost penetration, measured from the surface, but a decrease of frost penetration into the subgrade. Even with a base course 12 ft. thick there is about 1. 6 ft. of the subgrade which is frozen annually.

TABLE 6

EFFECT OF THICKNESS OF PAVEMENT AND BASE COURSE ON FROST PENETRATION, FAIRBANKS RESEARCH AREA

Section	Pavement Plus Base Course Thickness	Average Frost Penetration From Surface	Average Frost Penetration Into Subgrade
	ft.	ft.	ft.
RN-12	3.3	8.3	5.0
RN-4	4.5	8.6	4.1
RN-25	4.5	9.3	4.8
RN-15	7.0	10.0	3.0
RN-2	8.5	11.0	2.5
RN-14	10.5	12.3	1.8
RN-1	12.5	14.1	1.6

SUMMARY AND CONCLUSIONS

Measurement of temperatures over a period of several years in runway-pavement sections indicate the depth and rate of frost penetration and thawing. There are some shortcomings in the thermocouple assemblies as installed, but judicious use of the readings yields a reasonably accurate picture of the thermal regime of the runway sections. Theoretical calculations by the modified Stefan method utilizing soils information from the specific runway locations, air temperature data from the site, and selected thermal conductivity values and surface correction factors have been made and are compared with the results of the thermocouple observations. Borings and test pits to determine depths of frost were available in some sections for comparison with the theoretical calculations. These comparisons lead to the following statements and conclusions:

1. At Northway Airfield, with essentially sandy soils but some silt layers, annual frost depths of 6 to 9 ft. were obtained with the thermocouples in nine test holes and for four winters' observations. The theoretical calculations checked these depths within 1 ft. in most instances. In the majority of cases, the theoretical depth was less than that determined by the thermocouples. The shape of the time-depth curves by observation and by theoretical calculation were very similar.

2. At Eielson Air Force Base, with essentially clean, gravelly soils, annual frost depths of about 13 to 16 ft. were obtained by both thermocouple measurements and theoretical calculations. Differences between the two methods averaged about 2 ft. Rates of freeze and thaw were similar as judged by the shape of the time-depth curves.

3. At the Fairbanks Research Area, thermocouple measurements and theoretical calculations of frost penetration and thawing both yield similar shapes of time-depth curves, and give depths of annual freeze which usually check within 1 ft. The depths of annual freeze in these sections vary from about 8 or 9 ft. for those with 4 ft., plus or minus, of base course and pavement over a silt subgrade, to about 14 ft. for a section with 12. 5 ft. of pavement and base course over the silt subgrade.

4. Both field temperature measurements and theoretical calculations show that increasing the thickness of a high-density, gravel base course over a silt subgrade increases the total depth of frost penetration, as measured from the pavement surface, but lessens the depth of frost penetration into the subgrade.

5. Borings in four pavement test sections in the Fairbanks Research Area with

frost penetrations of as much as 9 ft. checked theoretical calculations for frost and thaw depths within 1 ft. in nearly all instances.

6. On the basis of the reasonable agreement between the frost lines as determined by thermocouple readings and by borings with depths determined by theoretical calculation with the modified Stefan method, it is concluded that this method of calculation will give results reasonably close to actuality. Where frost penetrations are in the range of from about 6 to 15 ft., the error to be expected is about 1 to 3 ft.

7. The agreement between observed and theoretical frost depths is also considered as a verification of the surface correction factors used to change air freezing-and-thawing indexes to surface indexes. It is also a verification of the charts used for thermal conductivity ·lues.

REFERENCES

1. Harry Carlson, "Calculation of Depth of Thaw in Frozen Ground," Frost Action in Soils, A Symposium. Highway Research Board Special Report 2 (1952).

2. Miles S. Kersten, "Laboratory Research for the Determination of the Thermal Properties of Soils - Final Report," Department of the Army, Corps of Engineers, St. Paul District, (September 1948).

3. Miles S. Kersten, "Thermal Properties of Soils," Frost Action in Soils, A Symposium. Highway Research Board Special Report 2 (1952).

Discussion

HARL P. ALDRICH, JR., and HENRY M. PAYNTER, Massachusetts Institute of Technology—The authors are to be commended for obtaining and reporting such excellent field data on the depth of freezing and thawing under pavements in Alaska. There can be no doubt that an important phase of soil mechanics and foundation engineering in the future will be the correlation of theoretical concepts with full-scale field measurements.

The authors have pointed out a significant practical consideration in their Conclusion 4. That is, increasing the thickness of a gravel base course can increase appreciably the total depth of frost penetration. In other words, if the depth of frost in a given locality is three feet, having 3 ft. of gravel below a pavement is no assurance that frost will not penetrate into the subgrade. The gravel base will ordinarily have a smaller water content than the subgrade. Hence, frost will penetrate faster and deeper into the base since the latent heat evolved as the soil freezes is smaller.

The agreement which the authors found between the calculated depth of freezing and thawing using the so-called Stefan formula and the actual depth as determined from thermocouple measurements and test pits, is remarkably good. The writers wish to point out, however, for the benefit of highway and soil engineers working in the United States, that the use of the Stefan equation in the more temperate regions of the United States will generally yield frost penetrations which are too deep. Although this is not mentioned in the paper, other publications by the authors indicate that they are well aware of this probability.

In general, the reason for the discrepancy is that the Stefan equation, while considering the latent heat of fusion of the soil moisture, neglects the effect of the volumetric heat of the frozen and unfrozen soils. This effect is relatively small when the mean annual temperature is near the freezing point as in Alaska. In the United States, however, the mean annual temperature varies from perhaps 40 to 60 F in localities where frost occurs in the winter months. In this instance the volumetric heat of the soil is an important factor. This can be demonstrated mathematically as follows.

Figure A shows the conditions which must hold for the general solution of a rational formula for the determination of the depth of frost penetration. Nearly all formulas are based on the assumption that the soil is a semi-infinite mass of uniform properties and having initially a

uniform temperature. Let it be further assumed that the surface temperature is suddenly changed from its initial value v_o above freezing to a temperature v_s below freezing. This temperature value is then maintained constant and uniform over the entire surface to yield a one-dimensional problem.

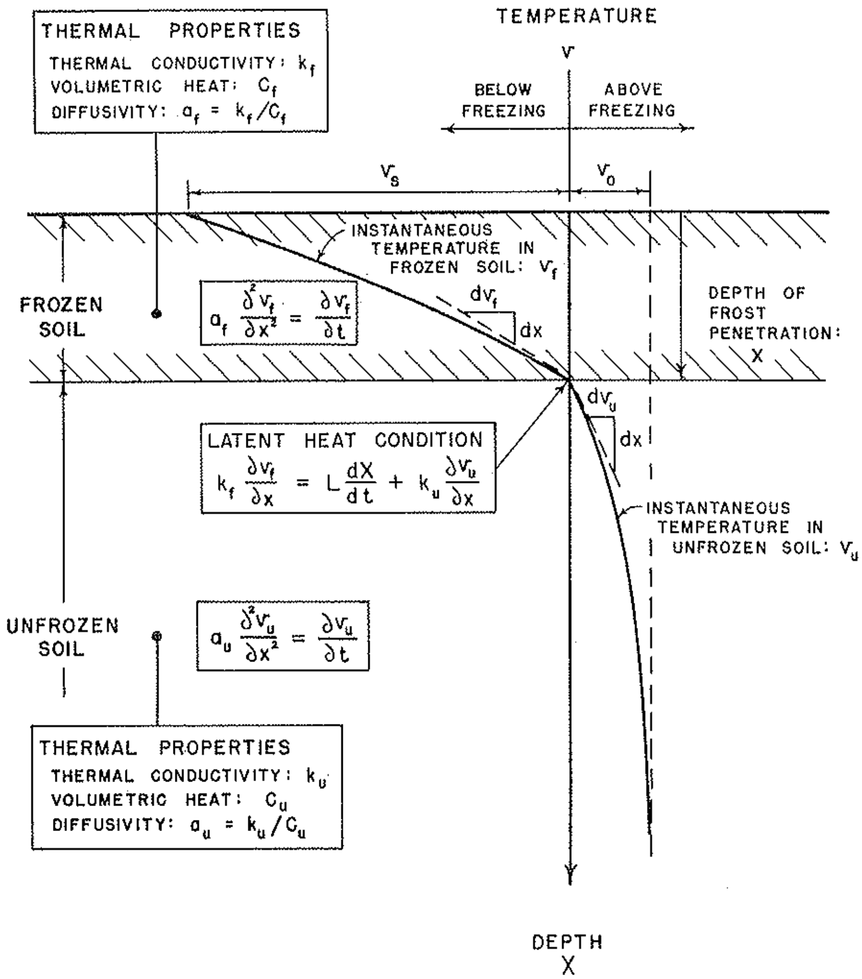


Figure A.

X is equal to the heat evolved in bringing the unfrozen soil from an initial temperature v_o down to the freezing point plus the volumetric heat of the frozen soil plus the total latent heat of fusion given off as the soil moisture freezes to the depth X. The Stefan equation considers the latter only, while other equations bring in the effects of the volumetric heat with varying assumptions.

The equations which must hold for this problem are the diffusion equations in both the frozen and unfrozen soil as indicated in the figure. The latent heat property becomes a continuity condition which states that the rate of flow of heat in the frozen soil at the frost interface must be equal to the sum of the rate of heat flow in the unfrozen soil at the interface and the heat given off at the interface when the soil moisture freezes.

The total heat evolved up to the time the frost has reached a penetration depth

Among others, the following four formulas for the depth of frost penetration X can be written:

$$X = \sqrt{\frac{48knF}{L}} \qquad \text{(Stefan)} \qquad (1)$$

$$X = \sqrt{\frac{48knF}{L + C\left(v_o + \frac{nF}{2t}\right)}} \qquad (2)$$

$$X = \sqrt{\frac{24knF}{L + C\,(v_o + \frac{nF}{2t})}} \qquad (3)$$

$$X = \lambda_4 \sqrt{\frac{48knF}{L}} \qquad (4)$$

where

- X = depth of frost penetration in feet
- k = thermal conductivity in Btu. per foot per degree Fahrenheit per hour
- F = air freezing index in degree days
- n = coefficient used to correct air freezing index to pavement freezing index
- L = latent heat of fusion in Btu. per cubic foot
- C = volumetric heat in Btu. per cubic foot per degree Fahrenheit
- v_o = degrees Fahrenheit by which the mean annual temperature exceeds the freezing point
- t = duration of freezing period in days
- λ_4 = correction coefficient for Equation 4
- $\lambda_4 = f_4(\mu, \alpha)$ given by curves in Figure B

fusion parameter, $\mu = \frac{CnF}{Lt}$

thermal ratio, $\alpha = \frac{v_o t}{nF} = \frac{v_o}{nv_s}$

The first expression is simply the Stefan equation which considers the latent heat of fusion of the soil moisture only. Equations

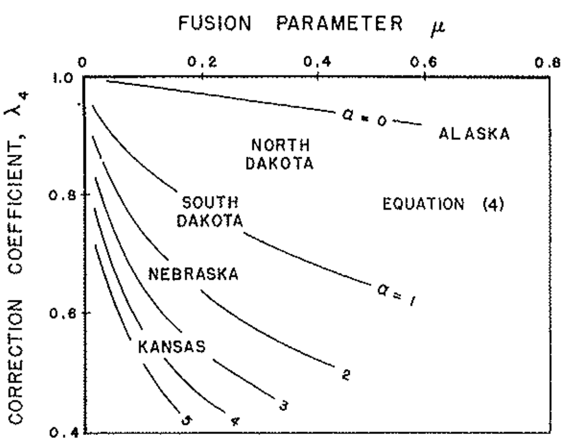
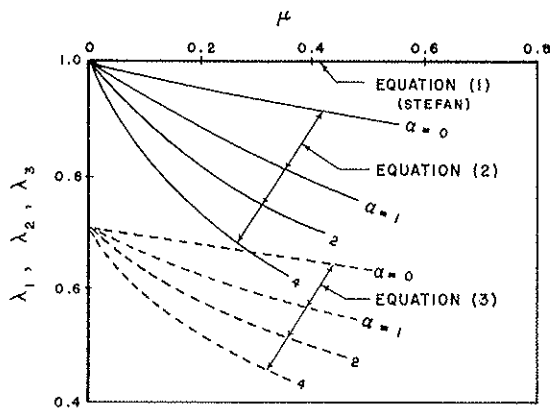


Figure B.

2 and 3 are formulas which have been studied by the Frost Effects Laboratory of the New England Division of the Corps of Engineers. These equations take into consideration the latent heat of fusion and the volumetric heat under various assumptions. Equation 4 was set up in the form shown by the writers. It is similar to a solution obtained by W. P. Berggren[1] in 1943. This formula, a solution of the equations in Figure A, takes into con-



$$\mu = \frac{CnF}{Lt} \qquad \alpha = \frac{v_o t}{nF}$$

$$X = \lambda \sqrt{\frac{48knF}{L}}$$

Figure C.

sideration nearly all of the factors of primary significance to the determination of the depth of frost penetration.

It can be shown that the first three equations can also be written in terms of a correction coefficient λ:

$$X = \lambda\sqrt{\frac{48knF}{L}}\,;\ \lambda = f(\mu, \alpha)$$

where λ is given by the following expression for Formulas 1, 2, and 3:

$$\lambda_1 = 1.0 \qquad (1a)$$

$$\lambda_2 = \frac{1}{\sqrt{1 + \mu(\alpha + 0.5)}} \qquad (2a)$$

$$\lambda_3 = \frac{0.707}{\sqrt{1 + \mu(\alpha + 0.5)}} \qquad (3a)$$

Curves for λ_1, λ_2, and λ_3 are given in Figure C.

The dimensionless parameters μ and α, which are not considered in the Stefan equation, are significant variables af-

[1]Berggren, W. P., "Prediction of Temperature Distribution in Frozen Soils," Transactions, American Geophysical Union, Part III, 1943.

fecting the depth of frost penetration. This can be seen from Figure B. The fusion parameter μ is a measure of the heat removed in the frozen soil below the freezing point as compared to the latent heat of the soil moisture. The thermal ratio α, is a measure of the ratio of the initial ground temperature, or mean annual temperature above the freezing point, to the time-average surface temperature below freezing during the freezing period.

The five localities shown in Figure B demonstrate the effect of varying climatic conditions on values of μ, α and λ. If the mean annual temperature is equal to zero as it is in many parts of Alaska, then α is equal to zero and a correction coefficient of perhaps 0.9 would apply. It can be seen then that the Stefan equation which assumes a correction coefficient of 1.0 for all values of μ would give depth of frost only 10 percent greater than the actual.

On the other hand, in temperate Kansas, for example, the correction coefficient is perhaps 0.55 which means that the actual depth of frost penetration would be about 55 percent of the value determined from the Stefan equation. It is also of interest to note from Figures B and C: in Alaska, Equation 2 gives good results while Equation 3 predicts far too shallow. In Kansas, however, Equation 3 is good while Equation 2 predicts far too deep. The writers' statistical studies of actual frost penetration data[2] have generally confirmed the above statements.

It would seem from the above discussion that the use of the Stefan equation in Alaska would yield depths of frost penetration about 10 percent greater than the actual. The authors found, however, that the equation gave depths about 10 percent too small when a surface correction factor $n = \overline{0.6}$ was used. The following question arises. How reliable are values of n? The authors apparently have data to show that n = 0.6 for bituminous and portland-cement concrete pavements in Alaska. However, if they had used a value of 0.8, their computed results with the Stefan equation would have checked the actual depths very closely on the average.[3] For example, at the Eielson Air Force Base, the actual penetration averaged 16.5 feet while the predicted averaged 14.5 ft. when n = 0.6 was used. In the Stefan equation

$$X \sim \sqrt{n}$$

If n = 0.8 was used in place of 0.6 then

$$\text{computed } X = \sqrt{\frac{0.8}{0.6}} \times 14.5 = 16.7 \text{ feet}$$

which checks the actual reasonably well.

In effect, using n = 0.8 in the Stefan equation is the same as combining n = 1.0 and $\lambda = \sqrt{0.8} = 0.89$ in Equation 4. From Figure B it can be seen that in Alaska $\lambda_4 \approx 0.9$! In other words, it would appear that if the authors had used the air freezing index with the modified Berggren formula (4), which considers all of the significant variables, their computed depths at the Eielson Air Force Base would have checked the actual depths very closely.

The writers are tempted to conclude that while the pavement surface correction factor, n, may not be equal to 1.0 in Alaska, it may be considerably closer to unity than is now believed. Furthermore, it is their general belief that n varies with latitude and may very well be smaller in the more temperate climates.

[2]Corps of Engineers, U. S. Army, New England Division, "Addendum No. 1, 1945-1947, to Report on Frost Investigation, 1944-1945," Frost Effects Laboratory, October, 1949.

[3]In any given locality one could, in fact, find a value of n for use in the Stefan equation which would yield average predicted values equal to the actual. In general, however, the writers would expect more scattering of the results than if a rational value of n was used with equation (4).