COST ECONOMIES IN CONCRETE BRIDGES

A Résumé of Current Structural Research

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SECTION 1. INTRODUCTION

The cost of a concrete bridge is the collective aggregate of several component items For this reason it becomes necessary, in approaching the question of cost reduction, to effect a segregation and to consider each component item of cost separately, and in turn

Neglecting such factors as are not affected by the discussion which follows, the expression representing the annual expense chargeable to the construction of any concrete bridge may be written as follows:

$$\text{Annual Cost} = (r + m + a)C$$

wherein·

r = the annual unit cost for capital (i e , the interest rate)
m = the annual average unit cost for maintenance.
a = the annual amortization cost (i e , the sum, per dollar of first cost, which, if deposited at compound interest, would accumulate a sinking fund sufficient to renew the structure at the end of its service life)
C = the total first cost of the structure in dollars

It is obvious that the total annual cost may be reduced by the reduction of any one or more of the above four factors It is equally obvious that a reduction of the term (r), the annual interest rate, is a problem in finance and administration rather than engineering

Our problem is commencing to narrow down and take shape It comprises the reduction of one or all of the factors (m), (a) and (C) Let us consider each in turn

Reduction in Maintenance Costs

Modern traffic conditions have completely exploded the idea, formerly quite prevalent among concrete construction enthusiasts, that a concrete bridge could be freed from the burden of a maintenance budget. It is true that the maintenance expense for concrete construction is considerably less than that for timber, and in the majority of instances, somewhat less than that for steel Yet, nevertheless this maintenance item, however small, constitutes a *constant and continuing drain* and a consideration of methods by means of which it may be reduced is an undertaking well worth the effort

How may this cost item be reduced? Perhaps this inquiry may be best answered by a consideration of those items of expense falling under this classification which are most commonly encountered throughout the year's work What are the maintenance crews doing to these bridges? Why is it being done? and How may this work be avoided or curtailed? Let us speculate as regards the answer to each of the last three inquiries

What are the maintenance crews doing? It is safe to say some one or more of the following items of work, in the majority of instances at least

(a) Protecting stream banks
(b) Underpinning, riprapping or otherwise safeguarding foundations
(c) Repairing damaged handrailings or ballustrades
(d) Cutting loose a jammed expansion joint somewhere in the structure, re-anchoring a loose expansion floor plate or refilling open expansion joints with tar or asphalt
(e) Chipping out an opened up construction joint and waterproofing the same
(f) Applying a waterproofing to shrinkage cracks in the deck.
(g) Attempting to remedy surface disintegration by patching, or the application of a waterproof blanket of some one of several types

Why are the crews engaged in this work and how may these items be eliminated or curtailed?

Items (a) and (b) oftentimes go hand in hand Sometimes such expense is unavoidable but many times it is the result of a failure to study stream conditions, to properly locate the structure in reference to the waterway or to provide adequate horizontal and vertical span clearances or adequate foundation depths

Item (c) is of course the direct result of a rather lawless and headlong traffic A traffic which sideswipes handrails and crashes through curbs in spite of every precaution to the contrary, and a traffic which, regret it as we may, will always be with us to a certain extent It must be admitted that a portion of this expense seems inevitable, although, it appears possible to curtail this expense to some extent through the employment of the following design precautions.

Adequate roadway widths
Curb protection which is *high* and *wide*.
An intelligent entrance treatment

Careful attention to alignment.

The provision of bullseye, reflector type, traffic lights in the ends of the curbs on the side of approaching traffic

Other traffic safety precautions

Item (d) is the direct result of an *articulated* structure, that is to say, one in which the superstructure is separated from the substructure or from the adjacent superstructure spans (or both) by means of expansion joints In the section of this paper dealing with methods for reducing first costs it will be shown that a considerable first cost economy may be effected through the elimination of expansion joints, converting the structure into one *monolithic continuous elastic frame* and that this method of design is adaptable not only to *single span* structures but also to *multispan structures on elastic supports* The utilization of this principle goes far toward the entire elimination of expansion joints, thus curtailing this item of maintenance expense to a considerable degree With our present knowledge of the art, it has not been possible to do *completely* away with expansion joints but only to reduce *materially* the number needed Multiple, short span, elastic frames up to 250 feet in length (or perhaps longer) may be constructed as continuous monoliths, and it is possibly equally feasible to construct concrete arches of like dimension with arch rib and supported superstructure monolithic and continuous. Above this limit, however, expansion joints appear to be needed, and where they are thus needed and thus used, a further reduction in maintenance expense may be effected through a careful attention to the detail of their design and installation

Items (e), (f) and (g) are the result of inferior construction practices and may be eliminated to a great extent by:

(a) Adequate and proper bonding and waterproofing of construction joints—the employment of copper or galvanized metal *water stops* and similar design and construction precautions.

(b) Proper and adequate curing of all concrete and particularly roadway surfaces

(c) Proper attention to the proportion, mixing, placement and field manipulation of concrete to the end that the material will be *dense* and *waterproof to the maximum possible degree,* and that the surfaces be free from stone or sand pockets, laitance seams or other imperfections

In concluding the consideration of maintenance costs it may be said that notwithstanding the fact that maintenance expense for concrete construction is a comparatively small percentage of its first cost, yet this expense is far from being too small to merit a painstaking attempt looking towards its further reduction Maintenance costs may be reduced below their present existing values, and this reduction, in general, may be effected through an observance of the following basic precautions:

(a) Greater care in the study of stream conditions and of the factors affecting the location of the structure
(b) A more painstaking attention to design detail
(c) Closer attention to field control of materials and workmanship
(d) The employment of *monolithic elastic units* wherever possible, thus cutting down the number of expansion joints to the maximum possible extent

Amortization Costs

The term (a) representing the cost for amortization of capital is generally computed from the expression

$$a = \frac{r}{[(1+r)^n - 1]}$$

wherein

r = the annual interest rate
n = the service life of the structure in years

From the above expression it is apparent that the factor (a) can only be reduced through an increase in the service life (n) of the structure

The same design and construction factors which operate to produce high maintenance expense obviously militate against the life of the structure However, since it is possible to so budget the maintenance as to cure these difficulties as they arise, any concrete bridge may be *kept up to its initial condition as regards strength and serviceability in perpetua*

The above being true there remains but one condition which can curtail the service life of a concrete structure, to wit: *OBSOLESCENCE*

A structure may become obsolete as regards several of its physical attributes It may become obsolete as regards its roadway width, its alignment, its gradient or its general location

An obsolete roadway width, considered alone, will not furnish a sufficient reason for reconstruction inasmuch as a betterment expenditure for widening is generally feasible and practicable If, however, either in connection with an inadequate roadway width or entirely independent of roadway considerations, a structure lies upon an alignment or a gradient which is inadequate for traffic needs, or if, even though it is not in itself inadequate in these particulars, it forms a link in a larger roadway section which is inadequate or obsolete as regards alignment or grade or general location, then the life of the structure is apt to be curtailed and this curtailment is more or less independent of those considerations enumerated in our discussion of maintenance costs

The amortization factor for any concrete bridge therefore hinges largely upon its *location* and the *location* of the *general highway sector* of which it is a part

It was said in connection with our discussion *or maintenance expense* that the factor (m) was a small one in any case where permanent construction was employed This statement does not hold for the amortization factor (a), inadequate location in many instances having rendered structures, which would otherwise have remained in service for fifty years or more, *completely obsolete in ten years time*

Too much emphasis can not be placed upon this phase of structural economics, nor upon the *pressing and urgent need for a more painstaking and thorough study of traffic trend and traffic needs as a condition precedent to the location of any permanent bridge*

First Cost Economics

We now approach the third phase of our discussion,—a consideration of methods whereby the factor (C) representing the first cost may be reduced A consideration of this phase of the problem is, in fact, the principal purpose of this paper, the discussion which has gone before being more or less in the nature of a preamble It must not be understood that the question of reduction in maintenance and amortization costs is unimportant As a matter of fact such questions are extremely and vitally important but a discussion of this character must narrow down else it can not approach thoroughness in treatment in the space available We must therefore turn our attention to a consideration of those methods whereby the first cost of concrete bridge structures may, with no sacrifice in service quality, be reduced

The first cost of a concrete bridge is obviously the product of *yardage* and *unit cost* and the problem of cost reduction is that of reducing one or both of these factors. Unit yardage costs may be reduced to a slight extent by a careful study of the design from the standpoint of the *form carpenter* to the end that the maximum degree of simplicity in form construction, consistent with quality and architectural excellence, may be produced Unit yardage costs may also be reduced by a curtailment of some of the present day specification requirements, but only at a distinct sacrifice in quality and at the expense of additional maintenance later on

It would therefore appear that any material first cost reduction must accrue through the employment of designs having *less yardage,* and that the problem should be approached from this latter direction rather than from the standpoint of unit costs

There are but few methods of approach to this latter problem. The yardage in any concrete bridge may be reduced by a reduction in live load carrying capacity but with present day traffic this is obviously not the solution Highway bridges are in general designed for a traffic loading considerable less than the *maximum individual load* which is likely to desire to cross the structure In nearly every state in the Union there are construction units employing steam shovels or other heavy equipment whose gross weight runs from thirty to forty tons and even higher and these heavy load units, during the course of line revision or widening projects, are moved up and down the highways in such a way as to render their passage over one or more bridge structures nearly imperative It is doubtless not true economy to design for these occasional and extra ordinary loads but their presence and the increasing frequency of their cccurrence certainly dictates against the reduction of present standard loading requirements

A yardage reduction may be effected through a reduction in present roadway widths or through the use of higher unit working stresses. Certainly the former expedient is not sound policy in view of the trend of present traffic development The use of higher unit working stresses appears to offer possibilities Perhaps we are not getting enough out of our materials. The concrete of today is most assuredly an entirely different product from that of a few years ago The introduction of scientific methods of design for concrete mixes, the improvement in methods of field control and manipulation of materials, inundation of aggregates, the employment of field equipment for the accurate proportioning of ingredients by weight,—these and many other developments have operated to produce denser, more uniform and much more dependable concrete than formerly. Should we not take advantage of these conditions in the design of our concrete sections, by employing higher working stresses with a consequent reduction in yardage? In certain cases and to a wertain extent—Yes Yet this is a procedure that must be approached with *extreme caution* An undue trend in this direction will unquestionably result simply in saving a certain percentage of our construction fund only to place it back again in our maintenance budget Several of the national engineering associations have committees at work on this particular problem at the present time for which reason any conclusion as regards the extent to which first cost economy may be effected through the employment of higher unit working stresses is perhaps somewhat premature Certainly some economy is possible—how much is an open question

It would appear that we have just about exhausted the field of inquiry and that as yet we have arrived at no very definite solution We have indicated certain features in design practice and field control of materials and workmanship which will operate to reduce maintenance costs We have shown the need for careful location study in order to prolong service life and decrease amortization costs In the field of first cost reduction we have pointed to the possibility of economies as the result of a fuller utilization of the material (higher unit working stresses) but have drawn no definite conclusion

There appears to be one expedient, however, which has not been considered one in fact which has escaped the attention of many engineers until recent years and the one which is to form the subject matter of the balance of this paper, to wit *A Fuller Utilization of the Elastic Properties of the Structural Frame* This is a rather high sounding phrase and perhaps more or less meaningless without further amplification and explanation

In order to explain just what is meant by this expression let us go back to 1833 when Clapeyron demonstrated the fact that the actual work produced during the deformation of any structure under the action of a system of gradually applied loads was always equal to the sum of one half the product of the final value of each load and the displacement of its point of application projected upon its line of action, and that this value was entirely independent of the order in which these loads were applied, or, stated as an algebraic expression that —

$$W_E = \tfrac{1}{2} \Sigma F \Delta$$

This theorem in connection with the fundamental doctrine of conservation of energy forms the basis of the well known theory of structural work which may be stated thus

$$W_I = W_E = \tfrac{1}{2} \Sigma F \Delta$$

wherein

W_I = the internal work or *resilient* energy stored up in any elastic structure under load

F and Δ represent external loads and the corresponding displacement of their points of application measured along or projected upon their lines of action

Neglecting axial and shearing distortions for the present we may write the expression for the internal work in any solid homogeneous beam or rib as follows

$$W_I = \tfrac{1}{2} \Sigma M^2 \frac{ds}{EI}$$

wherein:

M = the bending moment at any cross section

ds = an increment of length

I = the moment of inertia of any section
E = the modulus of elasticity of the material

With the above general principle in mind let us consider the two types of quandrangular frame indicated in Figure 1

Figure 1a indicates an ordinary simple span slab cut loose from its supporting piers by means of expansion joints

Under the action of a single load F we may write·

$$\tfrac{1}{2} F \Delta = \tfrac{1}{2} \Sigma_c^b M^2 \frac{ds}{EI} \qquad (1)$$

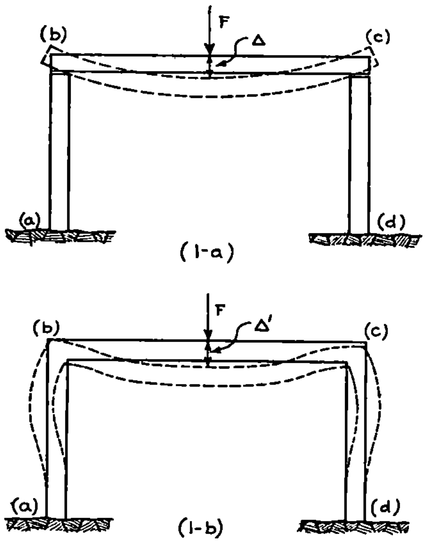

Figure 1

Figure 1b indicates the same frame constructed as a monolithic elastic unit Under the same loading we may, at once, write

$$\tfrac{1}{2} F \Delta' = \tfrac{1}{2} \Sigma_c^b M^2 \frac{ds}{EI} + \tfrac{1}{2} \Sigma_b^a M^2 \frac{ds}{EI} + \tfrac{1}{2} \Sigma_d^c M^2 \frac{ds}{EI} \qquad (2)$$

The left hand terms of these two equations are not materially different The fixity at points (b) and (c) tends to decrease Δ' while the bending of the columns augments it Except for extremely long columns the values will not be materially different As regards the right hand members, however, the *elastic frame furnishes a much longer length for* the storage of resilient energy with the result that the *values of* (M) *required to balance the external work are materially less*

In other words, by adopting a monolithic elastic frame we have put the entire structure to work

This is what was meant by the phase—"*A fuller utilization of the elastic properties of the frame*"

The monolithic frame, viewed in accordance with this concept, is nothing more or less than an *elastic reservoir* whose purpose is to absorb an amount of *resilient energy* sufficient to balance the corresponding loss in load *potential energy* as the structure deflects under load, yielding up this energy to restore the structure to its normal position as the loads are removed For equivalent deflection values under a given loading it is quite obvious that the *larger this reservoir the less will be the value of the unit energy charge,* that is to say the less will be the value of the term M^2/I This carries as a logical and obvious consequence a reduction in section and a saving in yardage

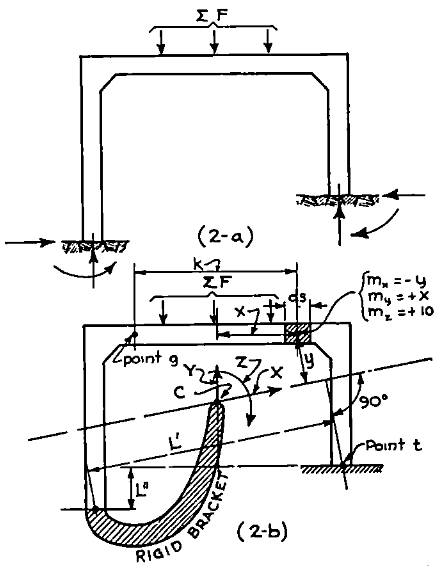

Figure 2

The monolithic elastic frame not only effects a distinct saving in yardage but operates to eliminate expansion joints thus lessening maintenance expense It also presents a wider field for architectural expression

The sections of this paper which follow are simply illustrative of the application of this general basic principle to the analysis of certain concrete bridge types

SECTION 2 ANALYSIS OF SINGLE SPAN ELASTIC FRAMES

Figure 2a represents a single span slab or girder bridge constructed as an elastic unit in accordance with the principles last discussed

Each foundation support is assumed as fixed and unyielding and obviously furnishes three reaction components (two linear and one moment component) If either support (say the left support) were to be removed its action or effect upon the structure could be replaced or reproduced by the attachment of a rigid bracket terminating at some point (c) and acted upon, at this point, by three unknown force components (two linear and one rotary component) as indicated in Figure 2b

The elastic frame has now been replaced by a cantilever under the action of the given load system, plus three redundant forces X Y and Z acting at point (c) Since, by hypothesis, the function of these redundant forces is to completely reproduce the action of the left support, and, since this left support was assumed as rigid, the displacement components Δ_x, Δ_y and Δ_z at point (c) will be zero

Now these three displacement components are each made up of four distinct elements to wit the displacement of point (c) of the *residual cantilever* caused solely by the action of the external loads, and the displacement of this same point due solely to the action of each of the redundant forces X, Y and Z, respectively We may therefore write, for point (c):

$$\left.\begin{aligned} \Delta_{ox} + \Delta_{xx} + \Delta_{yx} + \Delta_{zx} &= 0 \\ \Delta_{oy} + \Delta_{xy} + \Delta_{yy} + \Delta_{zy} &= 0 \\ \Delta_{oz} + \Delta_{xz} + \Delta_{yz} + \Delta_{zz} &= 0 \end{aligned}\right\} \qquad \text{(Group 3)}$$

Wherein

Δ_{ox} = that portion of the displacement component of the residual cantilever at point (c) measured in the direction chosen for the redundant X, which is due solely to the action of the external loading ΣF

Δ_{xx} = the portion of this same displacement component caused solely by the redundant force X acting at point (c) on the residual cantilever

Δ_{yx} = the portion of this same displacement component caused solely by the redundant force Y

Δ_{zx} = the portion of this same displacement component caused solely by the redundant moment Z

Since the displacements occurring at point (c) are directly proportional to the load which causes them we may write

$$\Delta_{xx} = X\,\delta_{xx} \qquad (4)$$

$$\Delta_{yx} = Y\,\delta_{yx} \qquad (5)$$

$$\Delta_{zx} = Z\,\delta_{zx} \qquad (6)$$

Where δ_{xx}, δ_{yx} and δ_{zx} represent the displacement components at point (c) of the residual cantilever caused by the forces X = unity, Y = unity and Z = unity respectively said displacement components being of course, measured in the direction chosen for the line of action of the redundant X as noted We may therefore write the first equation in group 3 as follows

$$\Delta_{ox} + X\,\delta_{xx} + Y\,\delta_{yx} + Z\,\delta_{zx} = 0 \qquad (7)$$

In a similar manner the equations representing displacement components measured in the direction chosen for the other two redundant forces may be expanded into the following form·

$$\Delta_{oy} + X\,\delta_{xy} + Y\,\delta_{yy} + Z\,\delta_{zy} = 0 \qquad (8)$$

$$\Delta_{oz} + X\,\delta_{xz} + Y\,\delta_{yz} + Z\,\delta_{zz} = 0 \qquad (9)$$

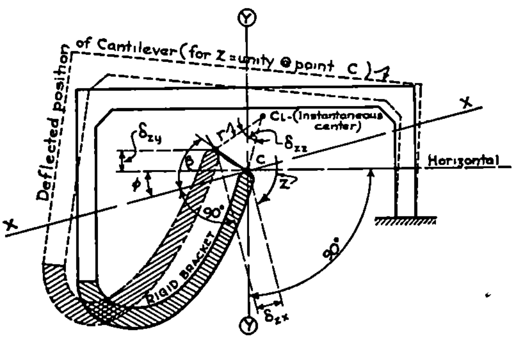

Figure 3

Where Δ_{oy}, δ_{xy}, δ_{yy} and δ_{zy} are displacement components measured along the line of action chosen for the redundant Y and Δ_{oz}, δ_{xz}, δ_{yz} and δ_{zz} are *angular* displacement components measured in the direction chosen for the redundant Z

If the redundant force Z = unity (a unit moment couple) be applied alone at point (c), the residual cantilever will obviously rotate about some point (c_i) as an instantaneous center From Figure 3 it is apparent that

$$\delta_{zx} = r\,\delta_{zz}\cos\beta \qquad (10)$$

$$\delta_{zy} = r\,\delta_{zz}\sin(\beta - \phi) \qquad (11)$$

If therefore point (c) be chosen coincident with the instantaneous center (c_i), r = o and the terms δ_{zx} and δ_{zy} both vanish

In regard to the direction of the linear redundant forces, if the redundant Y be assumed to act in any given direction (say vertically upward) it is apparent that under the action of this force there occurs a displacement of point (c) to some position (c″) as shown in Figure 4 Now if the redundant axis $X - X$ be assumed as lying at right angles to the displacement line c — c″ it is at once apparent that the component of this displacement measured along the axis $X - X$ is zero, hence the term δ_{yx} vanishes and likewise the term δ_{xy} (since from Maxwells theorem $\delta_{xy} = \delta_{yx}$)

If, therefore, the terminal point (c) be located at the instantaneous center of rotation (c_1) and if the X redundant axis be so chosen as to cause the terms δ_{xy} and δ_{yx} to vanish as indicated above, the expres-

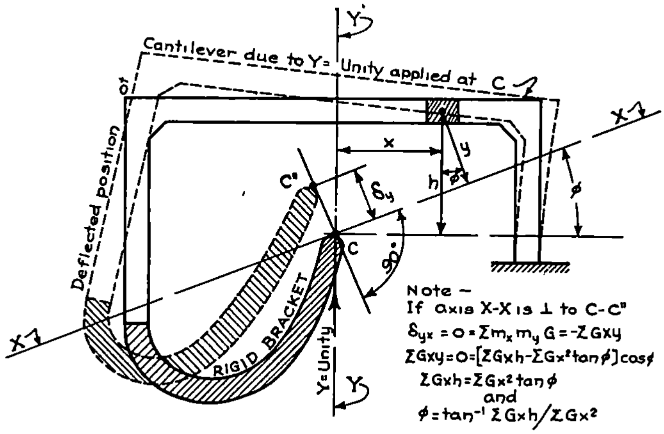

Figure 4

sions representing the redundant forces may be written in the following simplified form

$$X = \left[\frac{\Delta_{ox}}{-\ \delta_{xx}}\right] \tag{12}$$

$$Y = \left[\frac{\Delta_{oy}}{-\ \delta_{yy}}\right] \tag{13}$$

$$Z = \left[\frac{\Delta_{oz}}{-\ \delta_{zz}}\right] \tag{14}$$

These are the fundamental elastic equations universally applicable to any elastic single span frame, and are entirely generally and quite logical In fact these formulas may be derived in a manner more or less independent of mathematics, as a matter of *pure logic* as follows

(a) Since the span is fixed at each support either of these supports may be replaced by a rigid bracket terminating at *any point* in the plane, its terminal point being held in a rigid and fixed position by three *unknown* forces X Y and Z

(b) Since *any point* in the plane may be chosen such a point may be selected as will eliminate the displacement tendency of each force along the line of action of the other two forces, that is to say a point such that the force X can cause no rotary displacement nor any linear displacement along the line of action of Y, and similarly for the other forces

(c) The above being true, it follows that if, with all three forces removed, the residual frame deflects a certain distance Δ_{ox} *measured along the line of action of the force* X, then the only one of the three forces which has any tendency to bring it *back to its original position is the force* X (reversed in direction) since neither of the other two forces Y and Z produce a displacement component in this direction

(d) Since a unit value of X produces a displacement component equal to δ_{xx} then the actual value of X necessary to *counteract* the displacement Δ_{ox} caused by the external loading, and thus *maintain fixity* is obviously equal to $\Delta_{ox} \div (-\ \delta_{xx})$ that is to say, from pure logic only

$$X \text{ must equal } \Delta_{ox}/-\ \delta_{xx}$$

The expressions representing the other two redundants may be *reasoned out* independently of mathematics in a similar manner

Equations 12, 13 and 14 are obviously not as yet in proper form for convenient application but may be rendered so by means of the following simple transformations

Neglecting axial and shearing distortions, for the present, the general expression for deflections may be expanded by means of the well-known standard formula

$$\Delta_a = \Sigma\, Mm_a \frac{ds}{EI} \text{ or } \Sigma\, Mm_a\, G \left(\text{where } G = \frac{ds}{EI}\right) \qquad (15)$$

Assuming the redundant forces to act as indicated in Figure 2b (and designating as positive all moments causing compression in the outer fibres of the frame and vice versa) we may write.

$$m_y = +\ x$$
$$m_x = -\ y \qquad .\ (16)$$
$$m_z = +\ 1\ 0$$

whence

$$X = \frac{\Sigma\, M_o\, Gy}{\Sigma\, Gy^2} \qquad (17)$$

$$Y = -\frac{\Sigma M_o Gx}{\Sigma Gx^2} \qquad (18)$$

$$Z = -\frac{\Sigma M_o G}{\Sigma G} \qquad (19)$$

wherein:

M_o = the moment at any point on the residual frame due to the given external loading, and

$G = \frac{ds}{EI}$ (a term hereinafter referred to as the "elastic weight" of the linear increment (ds))

For a *unit* load at any point (g) (Fig 2b) M_o is equal to the term $-k$ for any point to the right of (g) and zero for all points to the left whence

$$X_g = -\Sigma_g^t kGy/\Sigma Gy^2 \qquad (20)$$

$$Y_g = \Sigma_g^t kGx/\Sigma Gx^2 \qquad (21)$$

$$Z_g = \Sigma_g^t kG/\Sigma G \qquad (22)$$

These last equations enable the plotting of *influence lines* for the redundant forces X Y and Z for a *unit load moving across the span* and from these influence lines the necessary influence lines for moment and stress at any point in the frame may be readily developed from statics

The location of the point (c) (generally known as the "elastic center" of the system) is obviously determined from the criterion.

$$\delta_{xz} = \Sigma\, m_x m_z\, G = \Sigma\, Gy = 0$$

and (23)

$$\delta_{yz} = \Sigma\, m_y m_z\, G = \Sigma\, Gx = 0$$

In other words the point (c) must lie at the *center of gravity of* the elastic load system ΣG.

The direction of the X redundant axis is similarly determined from the criterion

$$\delta_{xy} = \Sigma\, m_x m_y\, G = \Sigma\, Gxy = 0 \qquad \text{(See Fig 4)}$$

The expressions representing the redundant forces induced by *temperature* and *shrinkage* effects may be derived in much the same manner The derivation need not be given here; the formulas are as follows:

For temperature effects

$$\left.\begin{aligned} X_t &= \pm\, ctL'/\Sigma\, Gy^2 \\ Y_t &= \pm\, ctL''/\Sigma\, Gx^2 \\ Z_t &= \text{zero.} \end{aligned}\right\} \qquad (24)$$

For shrinkage effects:

$$\left.\begin{aligned} X_s &= \beta\, L'/\Sigma\, Gy^2 \\ Y_s &= \beta\, L''/\Sigma\, Gx^2 \\ Z_s &= \text{zero} \end{aligned}\right\} \qquad (25)$$

Wherein
c = the thermal coefficient
t = the temperature change
β = the coefficient of shrinkage
L' and L'' are as indicated in figure 2b

So far no attempt has been made to consider the effect of axial or shearing strains, equations 15 to 22, including the effect of bending strain only Shearing distortions are always of negligible significance and for ordinary structures of this type the effect of axial distortions may likewise be neglected without material error

If it is desired to include the effect of axial stress, it is, of course, only necessary to include in equation 15 the term.

$$\Sigma\, Nn_a \frac{ds}{AE}$$

Wherein (N) and (n_a) represent *axial* forces and (A) crosssectional areas

With the above modification the basic formulas may be derived in a manner similar to that already outlined In general, however, unless the supporting columns are very long and of small crossection the effect of axial strain is very small

The above is a very brief synopsis of the method of analysis applicable to the single span elastic frame No attempt has been made to go into any great amount of detail but rather to indicate as clearly as possible in the limited space available the general method of approach. While the formulas given may, at first glance, appear rather complex the employment of influence lines together with several very convenient graphical aids renders the application comparatively short and simple. These methods in detail as applied to the fixed arch bridge were rather completely discussed by the writer several years ago (see Hool and

Kinne's "Reinforced Concrete and Masonry Structures," page 452 et seq) and the same general method of procedure may be employed for the fixed elastic frame as discussed therein for the elastic arch, in fact these two types are essentially identical except for geometric form

The single span elastic frame is entirely suitable for span units up to 70 feet and in certain cases up to 80 or 90 feet in length Its principal advantages over the articulated or *simple girder type* are economy in yardage, decreased depth at the center of the span thus affording greater clearance possibilities, increased opportunity for architectural expression and freedom from troublesome and unsightly expansion joints As compared with the *elastic arch,* it presents the advantage of freedom from lateral thrust components on foundations and in general a greater waterway area per unit span

Where the vertical supporting columns are long, enough flexibility exists to render it possible to take care of the resulting column stresses and to secure proper anchorage at the supports without undue design or construction difficulties On the other hand, the longer and more flexible the column system the greater will be the angular distortion at the haunches which distortion, in turn, increases the central distance between points of contraflexure and therefore increases the stress at the center of the span

For short, stubby columns the span economies are more marked but the matter of caring for the bending stresses in the columns and at the supports becomes increasingly difficult For such conditions, it, many times, becomes necessary to introduce column hinges as described in the next section

Where this type of construction is employed for a single span bridge without flanking spans, the earth pressure against the columns introduces an added complication Space will not permit us to go into this except to state that the general method of treatment is fundamentally the same as previously described

SECTION 3 HINGED ELASTIC FRAMES

As stated in the last section, one of the outstanding difficulties in *hingeless* elastic frame construction lies in the exceptionally large bending moments introduced at the supports in all cases where the columns are comparatively short and rigid Where the foundations are on piling the bending at the supports is reduced by virtue of the elastic yield of the piles and the stresses are transferred through this flexible medium to a point further down, thus cutting down stress to a marked degree, (this feature of elastic frame analysis is reserved for discussion later on) For solid rock foundations it is possible, though not always practicable, to take care of the bending stresses at the supports through a system of *anchor dowelling* but for *gravel* and *boulder foundations* it becomes almost essential that hinges be introduced in columns which are not sufficiently long and flexible to carry the necessary frame distortions without serious foundation stress

There are, of course, many possible types of hinge arrangements, and space will not permit a consideration of them all The paragraphs which

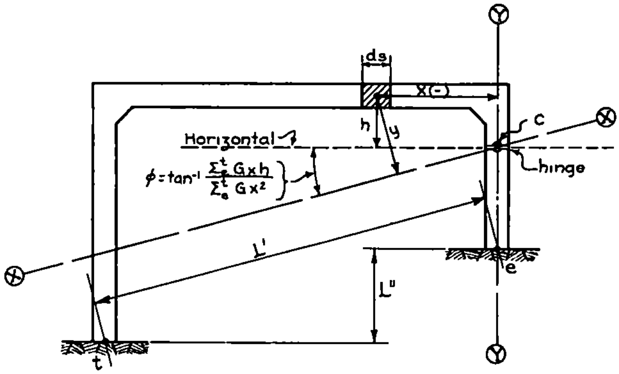

Figure 5

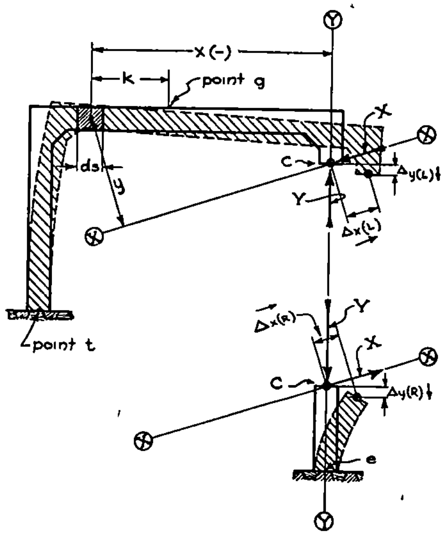

Figure 6

follow are an attempt to cover the salient features involved in the analysis of both *single hinged and double hinged column* supports for a single span elastic frame.

Let us first consider the general case of an unsymmetrical single span frame with a single hinge located as indicated in Figure 5 If the frame segments were to be separated at the hinge, as indicated in Figure 6, it is apparent that each segment would become a residual cantilever under the action of the external load system and two unknown redundant forces X and Y (There would be no (Z) redundant component since the hinge permits rotation)

Since the two segments are connected by the hinge the corresponding *linear displacement components must be equal,* that is to say:

$$\left.\begin{aligned} \Delta_{x(R)} - \Delta_{x(L)} &= 0 \\ \Delta_{y(R)} - \Delta_{y(L)} &= 0 \end{aligned}\right\} \quad . \qquad (1)$$

Expanding these expressions as indicated in Section 2, and assuming the redundant forces to act as indicated in Figure 6, we may write

$$\left.\begin{aligned} X\,(\delta_{xx(R)} + \delta_{xx(L)}) + Y\,(\delta_{yx(R)} + \delta_{yx(L)}) + (\Delta_{ox(R)} + \Delta_{ox(L)}) &= 0 \\ X\,(\delta_{xy(R)} + \delta_{xy(L)}) + Y\,(\delta_{yy(R)} + \delta_{yy(L)}) + (\Delta_{oy(R)} + \Delta_{oy(L)}) &= 0 \end{aligned}\right\} \qquad (2)$$

Now, by taking the direction for the redundant axis $X - X$ such that the terms $(\delta_{xy(R)} + \delta_{xy(L)})$ and $(\delta_{yx(R)} + \delta_{yx(L)})$ vanish (which is accomplished in the same manner as indicated in section 2) and designating the terms $(\delta_{xx(R)}) + \delta_{xx(L)})$ and $(\delta_{yy(R)} + \delta_{yy(L)})$ by the symbols δ_{xx} and δ_{yy} without further subscript, we may write the expression for these redundant forces as follows.

$$X = -\left[\frac{\Delta_{ox(R)} + \Delta_{ox(L)}}{\delta_{xx}}\right] \qquad (3)$$

$$Y = -\left[\frac{\Delta_{oy(R)} + \Delta_{oy(L)}}{\delta_{yy}}\right] \qquad (4)$$

These expressions may be developed for convenient use in exactly the same manner as discussed in Section 2. For example, for a unit load at any point (g) on the left hand segment $\Delta_{ox(R)}$ and $\Delta_{oy(R)}$ have zero values, and

$$\left.\begin{aligned} \Delta_{ox(L)} &= \Sigma\, m_o m_x\, G = \Sigma_g^t\, kGy \\ \Delta_{oy(L)} &= \Sigma\, m_o m_y\, G = -\,\Sigma_g^t\, kGx \end{aligned}\right\} \quad . \qquad (5)$$

whence

$$\left.\begin{aligned} X_g &= -\,\Sigma_g^t\, kGy/\delta_{xx} = -\,\Sigma_g^t\, kGy/\Sigma\, Gy^2 \\ Y_g &= \Sigma_g^t\, kGx/\delta_{yy} = \Sigma_g^t\, kGx/\Sigma\, Gx^2 \end{aligned}\right\} \qquad (6)$$

The formulas for temperature and shrinkage effects are exactly the same as those for the *hingless* frame, to wit:

$$\left.\begin{array}{lll} X_t = \pm\, ctL'/\Sigma\, Gy^2 & \text{and} & Y_t = \pm\, ctL''/\Sigma\, Gx^2 \\ X_s = \beta\, L'/\Sigma\, Gy^2 & \text{and} & Y_s = \beta\, L''/\Sigma\, Gx^2 \end{array}\right\} \ldots\ldots (7)$$

The balance of the analysis is exactly the same as for the hingless type of design.

Case II. Two hinged Column Supports

If the right hand column of the design illustrated in Figure 6 were to be hinged top and bottom, it is apparent that the left hand segment could be regarded as a residual cantilever under the action of the external load system *plus a single redundant force Y*. The X component has vanished due to the fact that the double hinged column permits free lateral movement at point (*c*) acting exactly the same as a roller rest at this point.

The expression for the redundant Y is obviously as follows:

$$Y = -\left[\frac{\Delta_{oy(R)} + \Delta_{oy(L)}}{\delta_{yy}}\right] \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Also, for shrinkage and temperature effects:

$$Y_t \pm \frac{ct\, L''}{\Sigma\, Gx^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$Y_s\, \frac{\beta\, L''}{\Sigma\, Gx^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

As in the case of the hingless frame, no attempt has been made to discuss detailed design methods but only to indicate basic principles in their broadest terms.

It will be observed that the interposition of a *single* hinge operates to eliminate the moment redundant, while the use of a *second* hinge eliminates both the moment redundant and also one linear redundant component inasmuch as the resultant at the upper column hinge point must be on a line passing through both hinges and is thus determinate both in direction and point of application.

It is well to observe that the basic expression representing any redundant force component is essentially the same for both fixed and hinged types and may be written for the general case as follows:

$$R = \frac{\Delta}{-\,\delta_R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

wherein

R = any redundant force.

Δ = the displacement of the point of application of this redundant measured along its line of action and caused by the external loading (or by temperature or shrinkage effects) on the residual frame.

δ_R = the corresponding displacement caused solely by a unit redundant (R = 1.0).

Several types of hinges are possible and feasible for this type of construction. Figure 7 is a rather simple hinge type consisting of a mortised joint and a central dowel pin, the main column steel being cut at the hinge point and the concrete surfaces separated by a layer of tar paper and asphalt over the *bearing* area and a one-fourth inch layer of asphaltic felt at the edges.

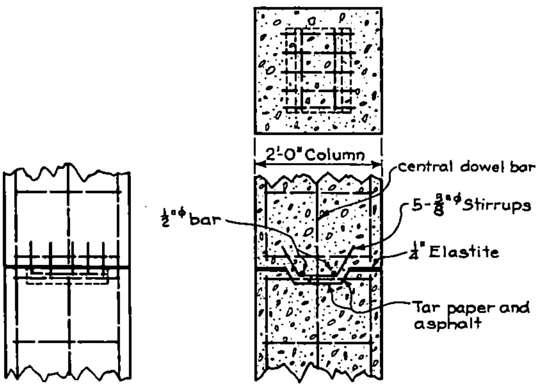

Figure 7. Typical Hinge Detail

Figures 14 and 20 indicate rather more elaborate installations. The hinge indicated in Figure 14 consists of a *complete articulation* the upper segment of the column being floated on a strip of elastite. The hinge reaction is carried by a group of 5—1⅛ bars passing through a corresponding group of 2½ wrought iron pipes filled with asphalt or tar mastic. These five bars *acting as 12 inch free columns* support the load and the plastic fill between the bars and the pipe encasement permits free hinge movement under load, temperature and shrinkage strain.

The hinge type indicated at (*A*) in Figure 20 permits lateral as well as rotary movement, thus combining the functions of a hinge and a *sliding expansion joint*. It will be observed that the hinge action is not as clear cut as the type indicated in Figure 14. Hinge (*B*) of Figure 20 is a *tension hinge*. This device was necessary because of the fact that loading on the outboard span (the 60-foot span) produces a negative reaction or uplift at the shore support under certain conditions. The hinges shown in Figure 20 are protected by a soldered copper dust guard and water stop.

Figure 8 is a detail drawing of a circular hinge, the articulation over the bearing area being accomplished by means of a layer of No. 12 gage brass coated with graphite and grease.

The above are only a few of many feasible and practicable hinge types the principal essentials, in any case, being:

(a) Freedom of movement
(b) Positive transferrence of hinge reactions
(c) Adequate moisture and weather proofing
(d) Simplicity and low first cost

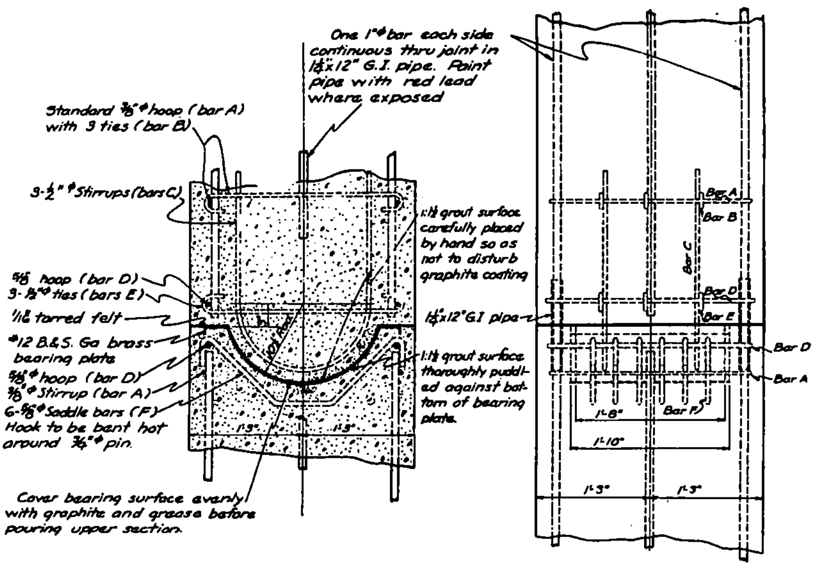

Figure 8. Assembly View—Hinge for 30-inch Square Column

SECTION 4. MULTI-SPAN ELASTIC FRAMES

Thus far our discussion has been of *single* span frames. We are now to consider a rather more complex phase of elastic frame analysis to wit: the determination of stress in multi-span frames of the type indicated in Figure 15.

It is in this field that the elastic frame presents its greatest utility, multi-span construction without expansion joints being entirely feasible and practicable for structures up to 250 or 300 feet in total length and perhaps even longer.

The fundamental theory underlying the analysis of this structural type, by pure mathematics, is somewhat complex but its application, due to the employment of influence lines and certain graphical short cuts, is really not as tedious a matter as might at first be imagined. We will now direct our attention to a derivation of those fundamental relationships which form the basis of this theory.

In Figure 9a is indicated an ordinary fixed elastic frame to the analysis of which the method discussed in Section 2 obviously applies

If the right hand column were to be split up into two separate segments as indicated at Figure 9b the same method of analysis would still apply Proceeding a step further, the fundamental method of treatment would still remain unaltered if the outside segment were extended and developed into *an additional span and column support* as indicated

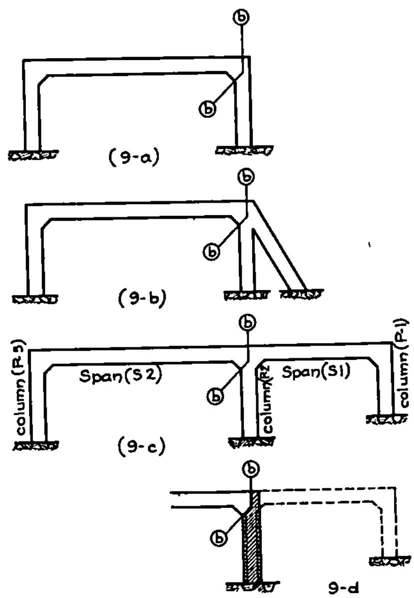

Figure 9

at 9c It should now be apparent that if the *elastic displacement* of the additional span and pier (dotted in Figure 9d) under any given load condition is the same as the column or pier from which it was developed, the elasticity of the original span segment (to the left of *b–b*) is in no wise altered and the stresses under equivalent loading are identical in both cases

The method of analysis hereinafter discussed is nothing more or less than the above development *in reverse* In other words our method consists essentially in the determination of the *elastic displacement of each span and column combination* (beginning at either end) and the replacement of such combination by an "ideal" or "substitute" support section (as indicated by the crosshatched pier section in Figure 9d) This substitute section must obviously have an elastic displacement under any given load condition exactly the same as the original structure which it has replaced

In Figure 10 is indicated the right hand span of the two span structure shown in Figure 9c The right hand support has been removed and replaced by a rigid bracket terminating at point *o*, the elastic center of the system *P1* - *S1* - *P2* (*a–b–c*, Figure 10) This elastic center is obviously located by means of the criterion $\Sigma_c^a\, Gx = O$ and $\Sigma_c^a\, Gy = O$ as discussed in section 2, and the direction of the redundant axis $X—X$ is determined from the criterion $\delta_{xy} = \Sigma_c^a\, Gxy = O$ as set forth in that section.

If a *second* rigid bracket were to be attached to the elastic system *a–b–c* at point *b*, and if at any point (O') three new forces X' Y' and Z'

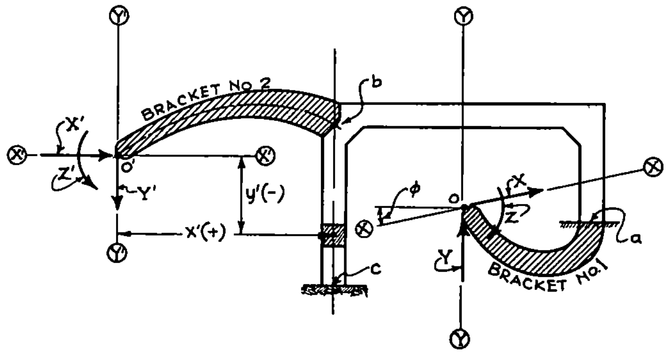

Figure 10

were to be introduced, the angular distortion of point O' may be obtained at once, by analyzing the elastic system *a–b–c*, as follows

$$\Delta'_{zz} = \Sigma_c^b\, M_z m_z\, G = \Sigma_c^b\, M_z G \ldots \quad (1)$$

$$\Delta'_{xz} = \Delta'_{zx} = \Sigma_c^b\, M_z m_x\, G = \Sigma_c^b\, M_z G y'. \quad (2)$$

$$\Delta'_{yz} = \Delta'_{zy} = \Sigma_c^b\, M_z m_y\, G = \Sigma_c^b\, M_z G x' \quad (3)$$

Now suppose that the substitution indicated in Figure 9d were to be made so that, instead of the quadrangular frame *a–b–c*, we have the "ideal" section indicated in Figure 11 Applying the *same* rigid bracket terminating at the *same* point O' in the plane, and introducing at said terminal point the *same* three forces X' Y' and Z' we may obtain the angular distortion of this new or "substitute" elastic system from the following expressions:

$$\Delta'' = \Sigma_c^b\, m_z m_z\, G = \Sigma_c^b\, G' = W_1 \quad (4)$$

$$\Delta''_{zz} = \Sigma_c^b\, m_z m_z\, G = \Sigma_c^b\, G'y' = \bar{y}'\, W_1 \qquad .(5)$$

$$\Delta''_{yz} = \Sigma_c^b\, m_y m_z\, G = \Sigma_c^b\, G'x' = \bar{x}'\, W_1 \qquad (6)$$

Wherein

$W_1 = \Sigma_c^b\, G'$ represents the elastic weight of the entire "*substitute*" or "*ideal*" section, and

$\bar{y}'$ and $\bar{x}'$ represent the coordinates of W_1 measured from point O' and at right angles to the axes $X' - X'$ and $Y' - Y'$

Now since by hypothesis this new "ideal" or substitute section must be such as to *exactly reproduce the elastic effect* of the quadrangular frame which it has replaced, the angular displacement of the two sys-

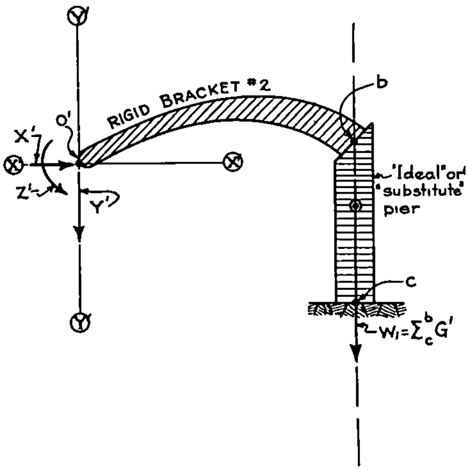

Figure 11

tems must be identical for equal loading (that is to say $\Delta'_{zz} = \Delta''_{zz}$; $\Delta'_{xz} = \Delta''_{xz}$ and $\Delta'_{yz} = \Delta''_{yz}$) whence

$$W_1 = \Sigma_c^b\, M_z G \qquad (7)$$

$$\bar{y}' = \Sigma_c^b\, M_z G y'/W_1 \qquad (8)$$

$$\bar{x}' = \Sigma_c^b\, M_z G x'/W_1 \qquad (9)$$

It may be well to observe at this point that, in the above expressions, the term (M_z) represents a moment at any section of the column b–c *regarded as an element of the elastic frame a–b–c*, while the unit moments m_x, m_y and m_z are corresponding moments on the section *regarded as a cantilever* To illustrate,—for any given point m_z = unity, while

$$M_z = [X_z y - Y_z x - Z_z + 1\,0]$$

X_z, Y_z, etc, being the redundant forces induced *at point O* (the elastic center of system *a–b–c*) by a unit moment couple

That this is true must be apparent from the fact that we are attempting to determine the distortion not of the *free pier or column b–c* but of the *composite elastic system a–b–c*

We have now determined the value and also the coordinates of the point of application of an "ideal" elastic weight W_1 which will completely

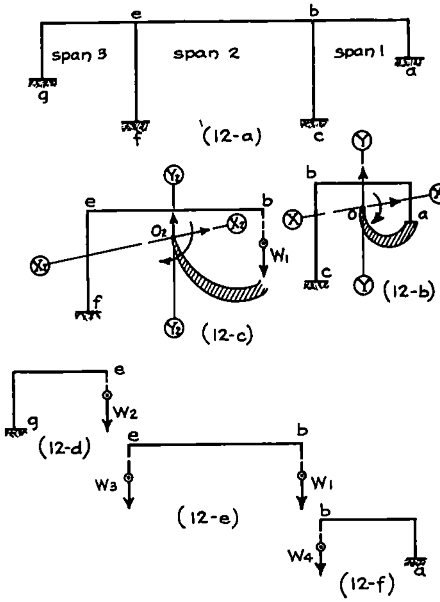

Figure 12

reproduce the effect of system *a–b–c* upon the rest of the structure as far as *angular* distortion is concerned [1]

If we neglect the refinement in analysis discussed in the foot note, we may assume that the substitute system as hereinabove determined

[1] If we proceed to a consideration of *linear* distortions at point O' under the action of the same load system to wit X' Y' and Z' we would develop six more equations representing Δ'_{xx}, Δ'_{yy} and Δ'_{xy} for the original system and Δ''_{xx}, Δ''_{yy} and Δ''_{xy} for the "ideal" or substitute system Equating these expressions yields three additional independent conditions which constitute a further complication and a further refinement in analysis In the case of long span structures such as a system of multiple span arches on elastic piers it is sometimes necessary to consider this refinement in analysis Since the mathematical derivation is long, and since the error introduced in the case of elastic frame bridges of comparatively short individual span lengths is small, we need not go into this matter at this point, but may, without material error, make the assumption given hereinabove (This matter is discussed in greater detail by Mr Thayer and the writer in their volume "Elastic Arch Bridges" (John Wiley and Sons))

completely and *faithfully* reproduces the effect of the original quadrangular frame *a–b–c* as regards the elastic distortion of the rest of the structure under *all conditions of loading*

To recapitulate and illustrate the application of this method of analysis, let us consider the three span elastic frame indicated in Figure 12

Considering first the elastic system *a–b–c* as an independent unit, the right hand support may be removed and replaced by a rigid bracket as indicated (Figure 12) The elastic center of this system is next determined, and also the direction of the redundant axis $X - X$ (such that $\delta_{xy} = O$)

Loading this elastic system with a unit moment couple (Z = Unity) the redundant forces X_z, Y_z and Z_z (active as the elastic center O) are readily obtained and from these values the term

$$M_z = X_z y - Y_z x - Z_z + 1\ 0$$

is evaluated

Having the values of M_z, the value and location of the elastic weight W_1 of the "ideal" section is readily obtained from equations 7, 8 and 9

The elastic substitute W_1 completely replaces the elastic effect of span No 1 and its supporting columns so that spans 1 and 2 may be converted into the *substitute system* indicated in Figure 12c Proceeding in exactly the same manner the elastic weight W_2 representing the effect of spans 1 and 2 and their supporting columns upon span 3 may be determined

In a similar manner the elastic weight W_3 representing span 3 and its supporting columns (system *g–e–f*) and also the elastic weight W_4 representing system *g–e–f–b–c* may be determined

The original composite structure has now been split up and replaced with *three single span elastic equivalents* which may obviously be analyzed for gravity loading in the manner outlined for single spans in Section 2 of this paper (see Figures 12d, 12e, and 12f)

The above method suffices for a complete determination of all stresses due to loads on the *particular span under consideration* To determine the effect upon any span (say span 2) of loading upon an *adjacent span* (say span 1) it is only necessary to determine for any load on span 1 the thrust, moment and shear at point (b) regarded as a point on this span (elastic system a–b–W_4), and next to analyze span 2 (system W_3–e–b–c) for these same thrusts, moments and shears at point (b) regarded as a point *on this second elastic system*

A certain amount of confusion may exist as regards the selection of those particular elastic systems which are operative for transferred or "*junction point*" reactions, for which reason a few words of explanation may be in order

A load on span 1 is carried to point (b) by virtue of elastic system *a–b–*W_4 and an analysis of this system in the regular manner will suffice to determine the junction point reactions at point (*b*) for this load condition Now these same junction point forces (with signs reversed) react against system *c–b–e–*W_3 (not W_1*–b–e–*W_3) and an analysis of this system will suffice to determine the effect of the loading in span 1 upon the pier (*b–c*) and the span (*e–b*) The junction point reactions at point (*e*) are also determined by an analysis of this system and these junction point forces (with signs reversed) react against system *f–e–g* An analysis of this last system determines the effect of loading in span 1 upon pier *e–f* and the end span and support (*e–g*) Thus the *entire structure* may be analyzed for loading on any portion of any of the three spans

The detailed method for calculating and handling "junction point" reactions and also the question of temperature and shrinkage effects in multispan frames is rather long for presentation at this point This entire matter is considered in detail in the volume "Elastic Arch Bridges,"[2] being published by Mr. E S Thayer and the writer at the present time, to which reference is made for a more complete discussion than is possible in the limited space available in this connection.

All in all the method of stress determination above described is somewhat tedious but is probably as simple a method for rigorous analysis as can be devised Its principle utility is in furnishing a valuable *mathematical check upon mechanical methods* of stress determination such as the Beggs Deformeter. In fact it is only because of the development of *mechanical methods* for stress determination that an analytical method as complex as this finds a field of utility at all Unaided and unchecked by mechanical means, a complex mathematical derivation affords so many opportunities for error as to be almost unworkable Purely mechanical methods on the other hand, if unchecked by an independent method of approach, are open to the objection that errors in observation may lead to rather erratic results. The two methods of attack are, therefore, mutually beneficial and complementary and, together, they have resulted in rendering feasible the analysis of complex structural types heretofore of rather formidable aspect Such types, in general, have shown distinct economy as compared with the simpler structural types as has been previously pointed out.

Figure 13 is a graphic comparison of moment influence lines for a two span continuous elastic group as determined by mathematical vs. mechanical methods. The agreement in results is sufficiently close to inspire confidence in either method

The writer has built a number of these monolithic elastic frames during the past few years, each time with marked economy (from 10 to 20 per cent) in yardage and with results which were rather pleasing architecturally Several of these structures have been in service for several

[2] John Wiley & Sons, New York

seasons and, as yet, no tendency to form stress cracks has been observed, in fact these structures are bidding fair to stand up under service *much better than the articulated or expansion joint type*

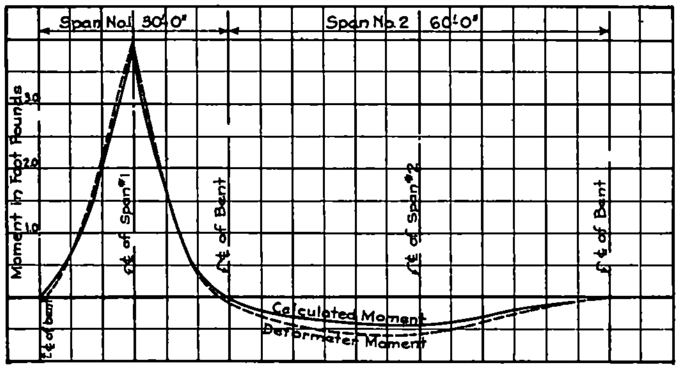

Figure 13. Influence Line for Moment at Center of Span No. 1. Comparison between Analytic and Mechanical Results

——— Calculated from multi-span theory.

--------- Mechanical results from use of Beggs Deformeter.

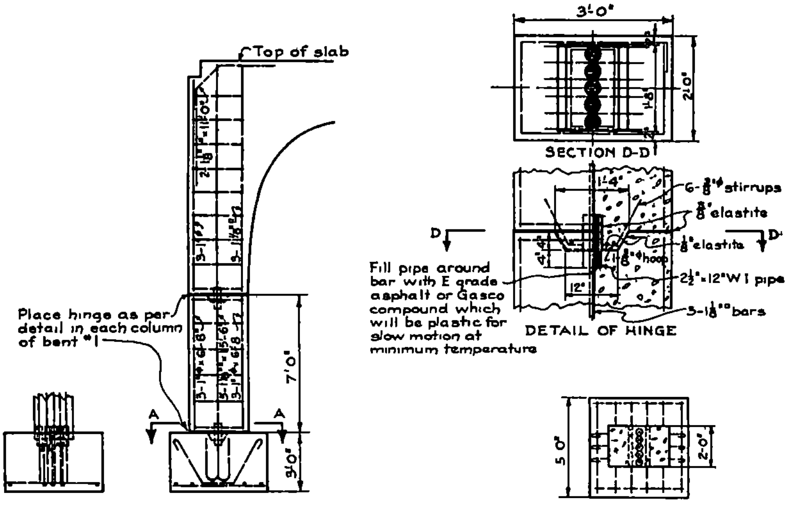

Figure 14. Hinge Details for a Double Hinged Column at One End of a Seven Span Elastic Unit, as Shown in Figure 15

Figure 14 shows the hinge details at Bent No 1 of the bridge pictured in Figure 15, which is a seven span elastic unit with a double hinged column at one end, the other columns being of sufficient length and flexibility to preclude serious bending stress at the foundations

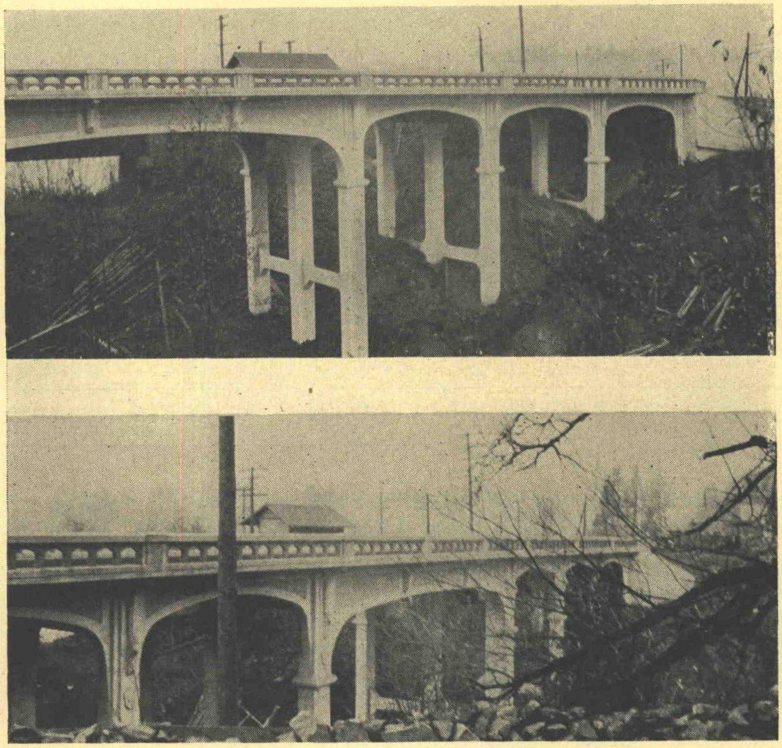

Figure 15. Two Partial Views. A Monolithic Elastic Seven Span Unit

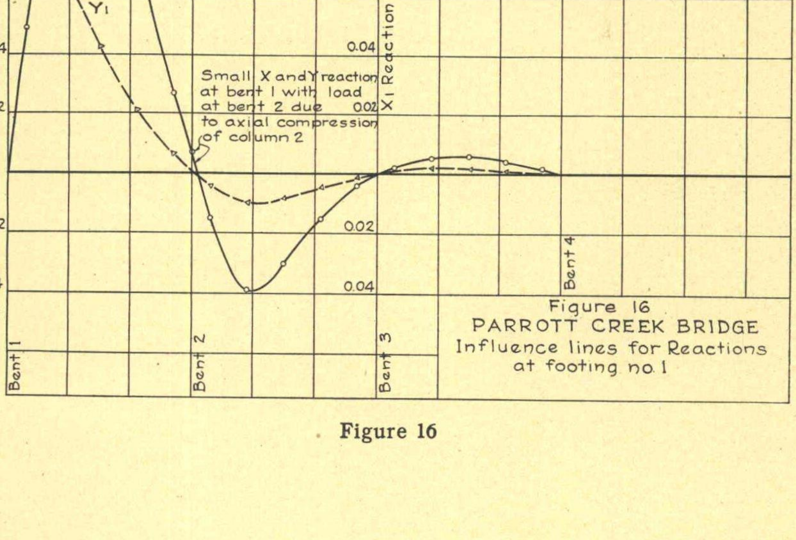

Figure 16

Figures 16 to 19 are influence diagrams for this structure which was originally assumed as hinged at both end bents, the right hand column

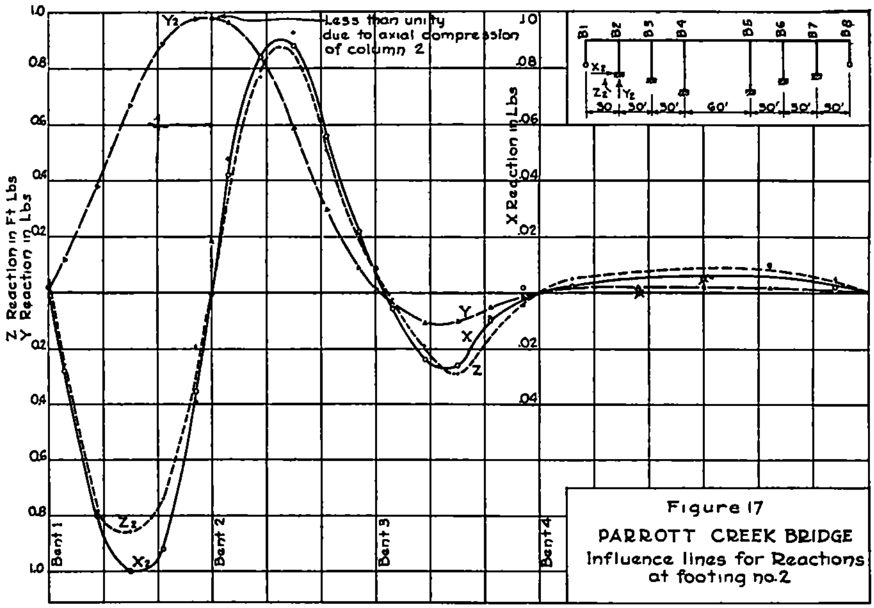

Figure 17

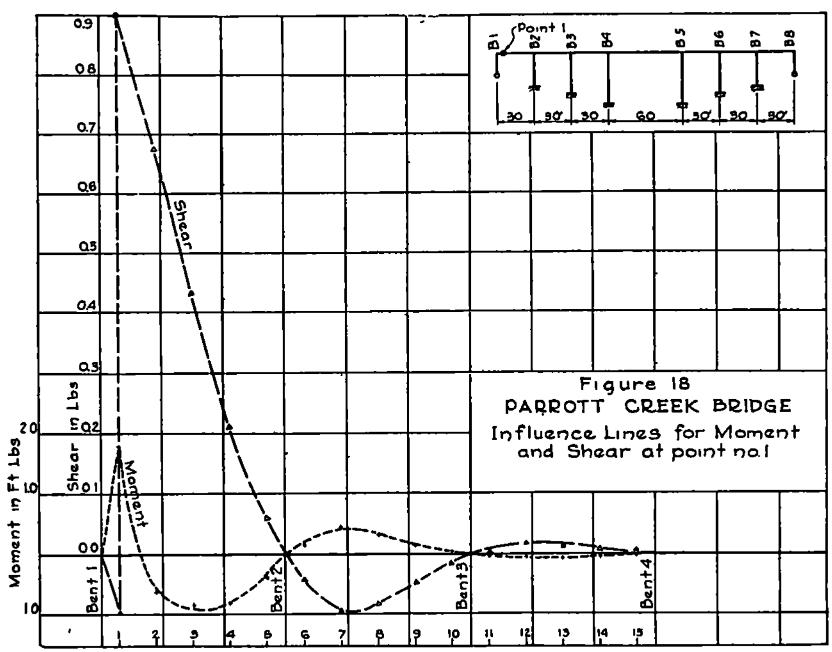

Figure 18

being carried down as a long, unhinged column at a later date due to certain unexpected foundation difficulties It will be observed that, in general, the effect of live load is not transmitted beyond the third span over to any appreciable extent

Figure 20 is an elevational view and Figures 21 and 22 are photographic views of another structure of this type recently completed by the writers' organization This structure, which is on a curve, consists of a group of three sixty foot central spans flanked by a short abutment span at either end The short stubby abutment spans rendered it necessary to provide, at point (*A*), a combination hinge and

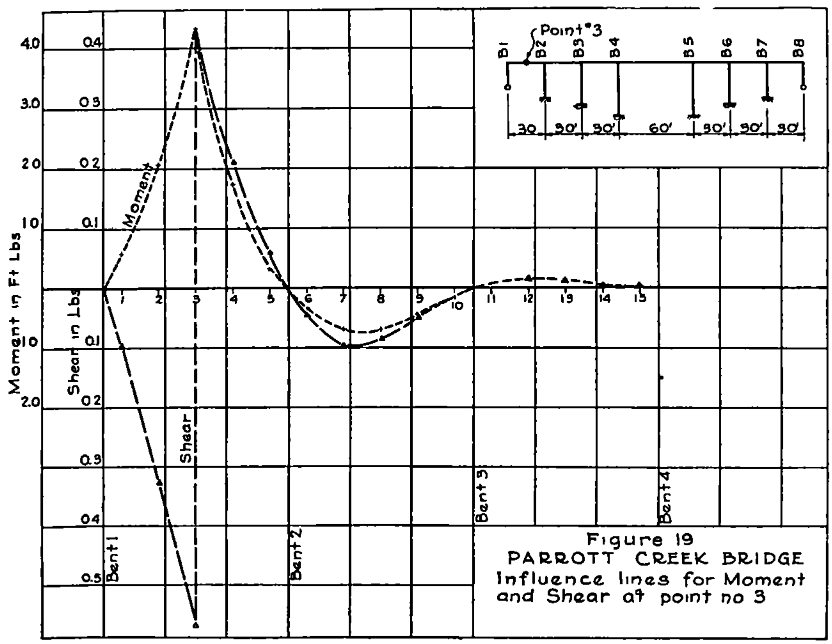

Figure 19

sliding expansion plate, while at point (*B*) the hinge detail was designed to take a certain amount of negative reaction or uplift

Figure 23 is still another design of this type, this being a three span group—a complete elastic monolith

SECTION 5 MONOLITHIC FRAMES ON ELASTIC SUPPORTS

The elastic frames discussed in the foregoing sections have all been assumed as resting upon *rigid* and *unyielding* supports Let us now turn our attention to the case of *elastic support displacement* and to a consideration of methods by means of which this factor may be taken into account in the analysis.

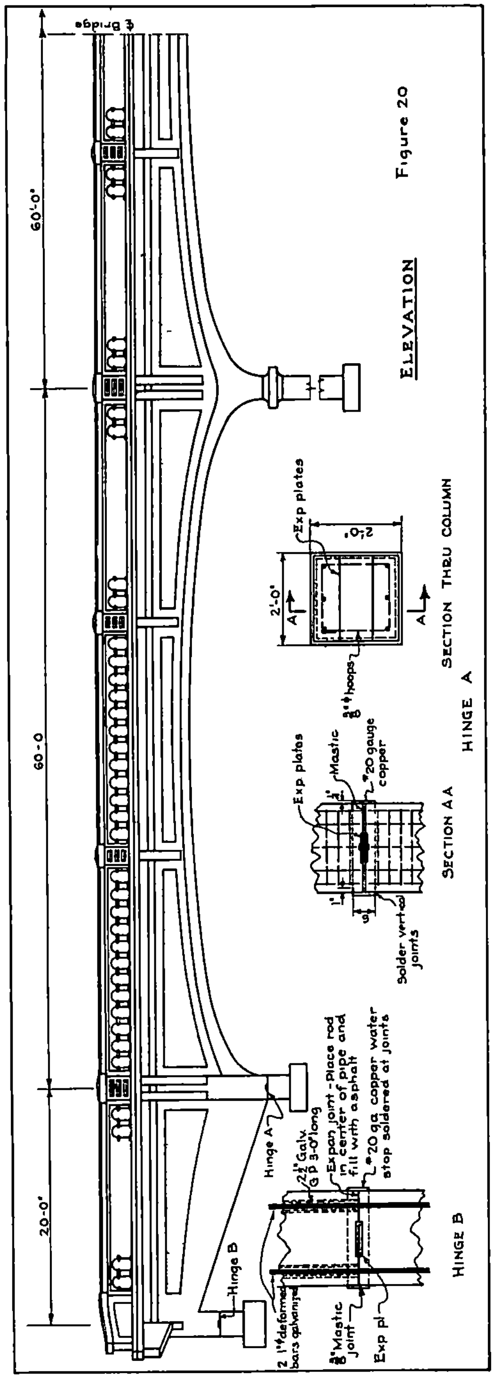

Figure 20 A Multi-Span Elastic Frame Bridge on a Curve. (3–60 Center Spans and One 20-Foot Flanking Span at Either End)

Figure 21. A Photograph of the Bridge Shown in Figure 20

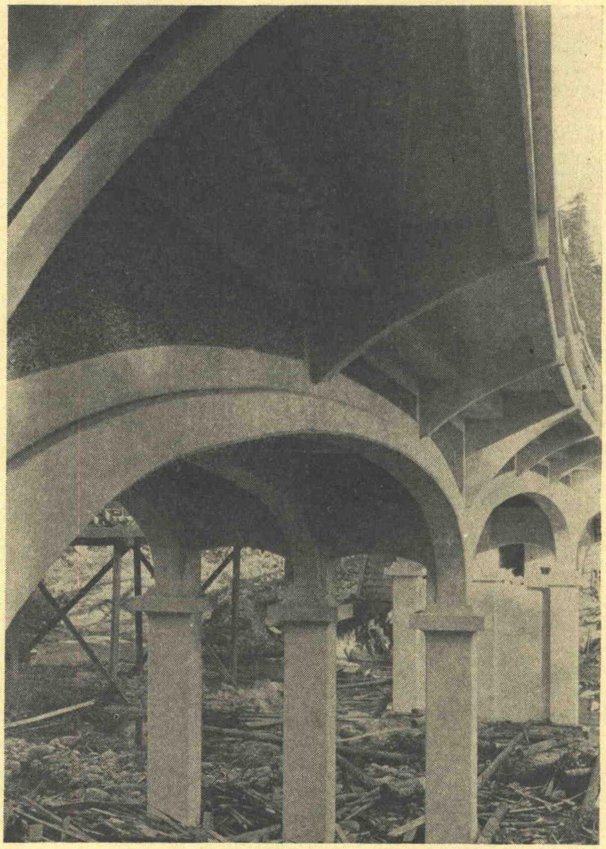

Figure 22. A Close Up of the Underside of the Structure Shown in Figures 20 and 21

There seems to be a general feeling among engineers that monolithic elastic units, whether they be rectangular frames or arches, are only adapted to *rigid support conditions.* This is not strictly true. *Plastic* foundation displacement is to be avoided at all costs as the stresses induced thereby are *indeterminate* and likely to be large. *Elastic* displacement on the other hand can be taken care of in the design *if there can be found a basis upon which it may be evaluated.* As a matter of fact a *determinate* elastic footing yield is oftimes (though not always)

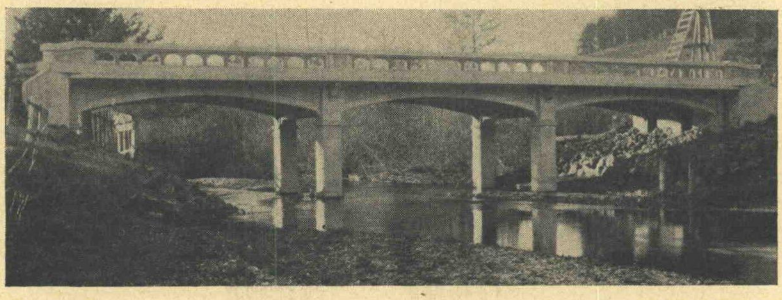

Figure 23. A Three Span Elastic Frame Bridge

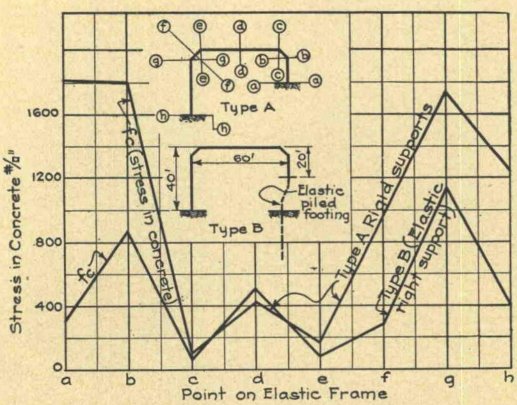

Figure 24. Showing Effect of Foundation Elasticity on Stresses in Quadrangular Elastic Frame

a stress *reducing,* rather than a stress *increasing* factor. For example, the single span, quadrangular frame shown in Figure 24 was built by the writer several years ago as an approach to a series of steel truss spans. The 40-foot leg constitutes the river pier and rests upon *solid* footing. The rear leg rests upon an *elastic piled footing* (as indicated by "Type B" in Figure 24). A stress comparison (as indicated in Figure 24) was made as a matter of general interest and disclosed the fact that the elastic yield of the piled footing at section *a–a* reduces the unit stress in the concrete at this point from 1800 to less than 400 pounds per square inch This stress relief seems to be carried clear around the frame except at the center of the span where, as would naturally be expected, the increased flexibility of the short column leg due to the elastic distortion of the piling increases the distance between contraflexure points and results in a slight stress *increase*

The above is an instance (and there are numerous other instances encountered throughout each seasons work) wherein flexibility operates to the advantage of the structure as regards stress, although, of course, the rigidity is correspondingly decreased

The problem which presents itself in all cases of this kind, is that of *an approximation of the elastic properties of any footing* sufficiently close for practical design purposes This is a problem which has not received

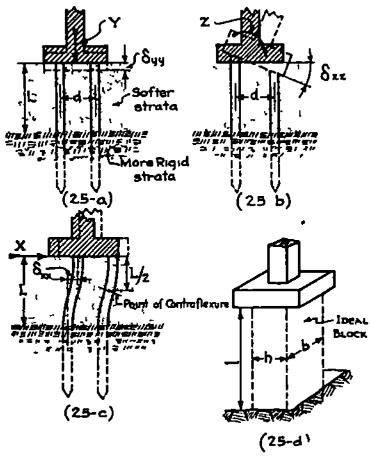

Figure 25

the attention from engineers which its importance deserves and further research in this particular field is very much needed. The following method of approximating the elastic effect of a piled footing has been used by the writer in several instances This method is particularly applicable to piling driven through comparatively soft upper strata into a rigid stratum below

Let Figure 25 represent a footing of this character (which, in the interest of simplicity, has been assumed as supported on only two rows of piling)[3] Let

n = the number of piling in each row

A_p = the crossectional area of one pile.

[3] If there were more than two rows of piling, the same method of analysis would apply but the expressions involved would be somewhat more complex

L = the distance from the bottom of the footing to the assumed plane of fixity

d = the distance between the two rows of piling.

I_p = the moment of inertia of one pile

E_p = the elastic modulus of the material used for piling.

In order to effect even an approximate solution of this problem it is necessary that certain assumptions be made as regards the distribution of load between piles and footing material

As a basis for calculating vertical footing displacements it has been assumed that the piles take the *entire load* at the base of the footing Such an assumption is, of course, not in strict accordance with fact inasmuch as the material into which the piles are driven undoubtedly has a certain bearing resistance The *amount* of this resistance, however, is a factor which varies between wide limits, depending upon the physical properties of the foundation material, its moisture content, the spacing of the piles, and other like conditions The usual design specification for piled footings provides that this factor be disregarded in the design, and until further data are collected in reference to this matter, the only way in which this particular problem can be approached at all is by assuming that the vertical resistance of the foundation material is negligible at least for a certain distance below the footing level As the depth of penetration increases, the axial compression in the piling decreases owing to the transference of load to the surrounding material through skin friction For certain foundation conditions there is probably a certain amount of residual column stress in the piling at the tip, but for conditions ordinarily encountered, the greater portion of the stress has been completely transfered to the surrounding foundation material before reaching the bottom of the piling. These facts being true, it follows that the vertical distortion may, with reasonable accuracy, be assumed as equivalent to that of a free column of some length (L) (as yet undetermined) over which length the pile stress is uniform

The same assumptions which are made for vertical distortion obviously hold also for angular distortion inasmuch as such distortion is purely a function of the axial stress in the piles

For *lateral* movement it has been assumed that the piling are thrown until *double bending* above a certain point of fixity, and the distance of this plane of fixity below the base of the footing has been assumed as equal to the equivalent free column length of the pile

If, at the base of the footing three unit forces (X = unity, Y = unity and Z = unity) as indicated in Figure 25, be successively applied, the stress in each pile, on the basis of the assumptions hereinabove made, will be given by the expressions

$$s_z = 1/nd, \; s_y = 1/2n \quad \text{and} \quad s_x = 1/2n$$

Also the displacements under these unit loadings are given by the following expressions.

$$\delta_{zz} = s_z L/A_p E_p \frac{d}{2} = \frac{2\,L}{nd^2\,A_p E_p} \quad (1)$$

$$\delta_{yy} = s_y L/A_p E_p = \frac{L}{2\,n\,A_p E_p} \quad (2)$$

$$\delta_{xx} = 2(s_x)\left(\frac{L}{2}\right)^3 \Big/ 3\;E_p I_p = \frac{L^3}{24\,n\,E_p I_p} \quad (3)$$

From the above expressions we may also write

$$\frac{\delta_{zz}}{\delta_{xx}} = \frac{48\,I_p}{A_p\,L^2\,d^2} \quad (4)$$

$$\frac{\delta_{zz}}{\delta_{yy}} = 4/d^2 \quad (5)$$

Now let us assume the piled footing as replaced by a solid block of concrete of the dimensions indicated in Figure 25

From the ordinary deflection formulas

$$\delta_{zz} = \Sigma \frac{m_z^2\,ds}{EI} = \frac{12\,l}{E_c b h^3} \quad (6)$$

$$\delta_{yy} = \Sigma \frac{s_y^2\,l}{AE} = \frac{l}{E_c b h} \quad (7)$$

$$\delta_{xx} = \Sigma \frac{m_x^2\,ds}{EI} = \frac{4\,l^3}{E_c b h^3} \quad (8)$$

Also:

$$\frac{\delta_{zz}}{\delta_{xx}} = 3/l^2 \quad (9)$$

$$\frac{\delta_{zz}}{\delta_{yy}} = \frac{12}{h^2} \quad (10)$$

If this substitute or "ideal" concrete section is to replace the elastic effect of the piled footing, then the dimension of the block must obviously be such that the displacements under equivalent loading are identical

Equating the right hand terms of equations 4 and 9, 5 and 10, and 3 and 8, and solving for the dimensions of the "ideal" section, we obtain:

$$l = \sqrt{\frac{A_p L^2 d^2}{16 I_p}}$$

$$h = d\sqrt{3}$$

$$b = \left(\frac{96\, nl^3 I_p}{L^3 h^3}\right)\left(\frac{E_p}{E_c}\right)$$

This "ideal" section, determined as above, is now substituted for the piled footing and the analysis carried out in the usual manner (see Section 2). The "ideal" concrete block is, of course, assumed as resting upon an unyielding footing at its base as this was the assumption under which the equivalent dimensions were determined

The above method of analysis is, of course, nothing more or less than a very crude approximation and leaves much to be desired in the way of refinement It has already been pointed out that the assumptions which must necessarily be made are not in strict accordance with fact

In the first place, the vertical resistance of the material into which the piling have been driven has been neglected entirely while as a matter of common observation, the resistance of even the softest upper strata will be sufficient to restrain the piles somewhat

In the second place, it has been assumed that the lateral distortion is a free double bending above a plane of fixity whereas the passive resistance of the surrounding material will undoubtedly restrain this bending to a certain extent, depending, of course, upon the characteristics of the soil into which the piles are driven

In the third place, it has been assumed that the plane of lateral fixity lies at a distance below the bottom of the footing equal to the equivalent free column length of the piling

Another factor which has been neglected is the *plastic lateral yield* of the surrounding material below the assumed plane of fixity. We have assumed this plastic yield as being negligible below this arbitrary plane, and yet so great at a point directly above this plane as to completely remove all lateral restraint. Such a condition may be approximated for certain types of foundations (soft silt over a stiff clay or a heavy gravel sub-strata) but for ordinary conditions, the plastic lateral yield will *gradually* diminish with the penetration rather than suddenly wipe out.

It will be observed that the neglected factors mentioned are, to a certain extent, compensating, and probably so small as to render the assumptions made sufficiently close to the truth as to provide a *reasonably accurate* basis for an evaluation of the elastic properties of a pile footing.

In order to determine this fact, the writer is now conducting a series of tests, looking toward the determination of the lateral stiffness of pile footings under different soil and load conditions

Figure 26 is a stress deflection curve for a group of piles recently tested In reference to the assumptions made as regards lateral dis-

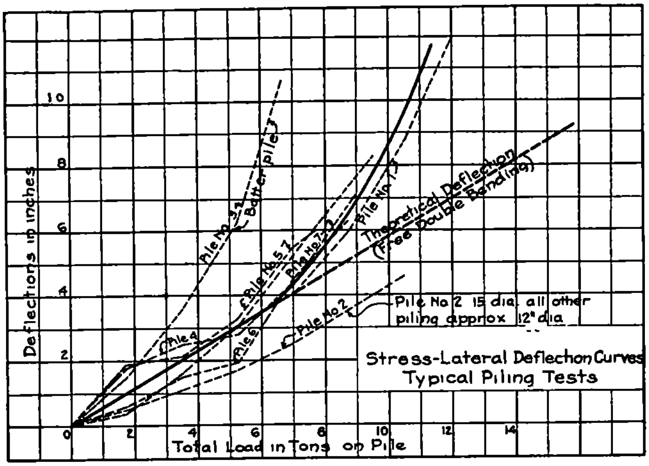

Figure 26

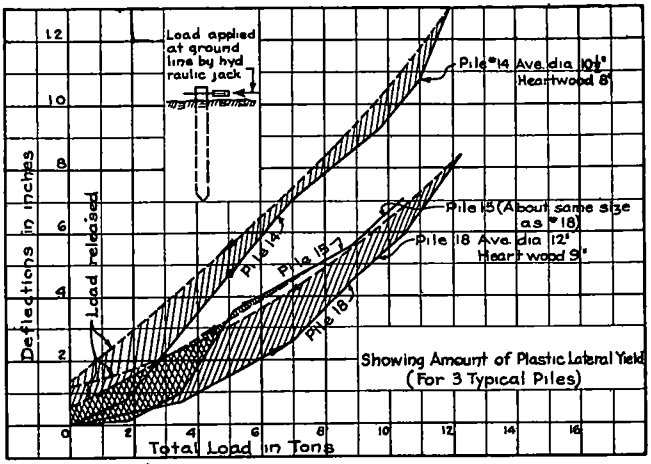

Figure 27

tortion, it will be observed that for loads up to about six tons per pile (which is probably a more severe lateral load than that which will ever be imposed upon the structure in actual service) the piles very closely approximate the action of a free beam in double bending As regards the question of *plastic lateral yield,* the curves of Figure 27 (which indicate the behavior of a few typical piles under release of lateral load) show a small amount of plastic deformation, but indicate that most of the yield is *elastic* The above tests, are, of course, not conclusive, and the length of "equivalent free bending" as determined by these tests obviously holds true only for the particular soil conditions encountered

Furthermore, even assuming the lateral yield to follow the law of elastic free bending, there is yet to be settled the question of vertical displacement and the determination of equivalent free column lengths Until these or other similar data are assembled and digested, it is therefore rather difficult to make an estimate of the value of this method of analysis It has been submitted at this point as merely illustrative of the possibilities in elastic footing analysis and as pointing to the need for further research along this line.

CONCLUSION

In the discussion which forms the subject matter of this paper the writer has attempted to point out certain methods by which cost economies in concrete bridge construction may be effected

In Section 1 are indicated certain features of design practice and of the field control of materials and workmanship whereby future maintenance costs may be reduced The need for careful location studies in order to lessen the danger of obsolescence and thus prolong service life and decrease amortization costs has also been emphasized

In the field of first cost reduction the writer has attempted to show the importance of what he has termed *a fuller utilization of the elastic properties of the structural frame* and has pointed out the fact that this particular phase of structural endeavor is apt to prove one of the most fruitful fields of investigation and inquiry

The sections which follow Section 1 have illustrated the application of this principle to the design of several structural types No attempt has been made at detailed mathematical derivation as the limited available space precludes such a treatment It is hoped, however, that the discussion has been sufficiently thorough to indicate the application of these basic principles and to illustrate the importance of investigations of this character.

If this paper does this, and stimulates further discussion and research the writer will feel that his effort has not been entirely in vain

DISCUSSION

ON

COST ECONOMIES IN CONCRETE BRIDGES

MR A. L GEMENY, *U S Bureau of Public Roads* I should like to add one idea to Mr McCullough's paper I feel that in bridge building there is a gap between design and construction which has never been completely filled We design a structure, we assume certain ideal conditions, and when we build the structure we are not so certain we have gotten it in conformance with the assumptions In this particular type of construction—the continuous structure—I think that is particularly true, and the importance of actually achieving the conditions which have been assumed is of very great importance, and I am inclined to think that there can be developed a method by which counter-stresses can be introduced into a structure so as to place it in the predetermined elastic condition which we have assumed in the design This has been done in the special case of the elastic arch and some very remarkable results have been obtained in France The Bureau of Public Roads and the State of Oregon are now cooperating in a project which has for its aim further study of this problem, and the French idea is being extended to some extent to the application of the principle of compensation or adjustment to a predetermined elastic condition by applying it to a number of spans on elastic piers, and we hope to get a good deal of information from that experiment

MR. P J FREEMAN, *Chief Engineer Bureau of Tests and specifications Allegheny County, Pa* As a matter of information—perhaps it does not belong exactly in the discussion of "Cost Economies in Concrete Bridges"—, I heard a paper presented by Mr T W Dodd, President, St Joseph Structural Steel Co , St Joseph, Mo , before the American Institute of Steel Construction, in which he showed a very careful analysis of various types of bridges under different conditions

It seems to me that if this organization could have a paper along similar lines that it would be well worth while For example, in some cases it was pointed out that the best economy would be gained by having some spans of concrete, others of steel Of course we pre-suppose this paper was largely for the interest of the Steel Industry On the other hand, it was a very carefully prepared paper on which the engineer had worked for nearly a year, submitting it to a number of people for criticism He pointed out by means of slides and drawings a number of examples of bridges where gross carelessness had been exercised in the original selection of the length of spans We saw a picture of a span with a pier in the middle of a little stream perhaps 25 or 30 feet wide It would seem to me that it is within the province of the Highway Research Board to bring out such things There are a good many highway departments where they do not have good bridge organizations and they select a more or less standard type of bridge and stick it around any place without thought of the economy for the particular location. This may not happen where there is a large organization, but I am simply calling attention to the possibility of this topic for discussion at some future meeting of the highway Research Board

MR A G LIVINGSTON; *Bridge Engineer, Delaware.* It is quite evident, that maintenance is necessary for all bridges The highway law of Delaware provides for $300 or somewhat less to be appropriated for each mile I have just been figuring on about eight bridges which I have in mind and I find that on a cost basis of probably $30,000 a mile I require for the maintenance of my bridges somewhat more than this sum of $300 per mile. We have heard a number of discussions on accounting and I have been trying to dig out just how much the maintenance on these different bridges really costs

On these bascule spans we have the various operators there primarily to operate the bridge, but at the same time they put in considerable time in maintaining all equipment and making minor changes in electrical apparatus They are qualified to do this under proper supervision

The previous speaker said that there is a gap between our ideal designing provisions and probably some of our construction provisions I have in mind just now one bridge that probably was constructed in a hasty manner, and I know we have put at least a $5000 maintenance item on it. We have said and I think we all believe that there are considerable economies in the rigid frame I have thought for a long time probably I was using too much concrete in many of the bridges which I designed for Delaware and have tried to reduce it I thought I would like to get the view of others on approximately how much concrete could be used and I talked to one, who is of considerable intelligence and a contractor as well as engineer He said "Livingston you get your bridges all right, and you are getting a fair unit price Now we are going to get the same money out of the bridge as we did before, whether you reduce the quantities or not" Now whether that is so or not I do not know, but I have met contractors who said "What are you doing You are not giving us concrete—you are giving us steel covered with concrete" I saw something like that in the Engineering News Record about a wall I like to draw up the structure and look at it If it does not look right I want to put in more concrete, whether the design is right or not and probably the appearance is as good an aid to judgment as anything else.

The question of increased loads and special loads being brought over the bridge is very vital It has come up a number of times in Delaware, and in one case I was strongly against allowing this type of load to go over a particular bridge because I felt that once the start was made, it would occur frequently and while I felt no particular horror, if these particular loads went over several times, I knew that there would be interference to traffic and eventually some harm might be done.