EVALUATION OF WHEEL-LOAD DISTRIBUTION FOR THE PURPOSE OF COMPUTING STRESSES IN CONCRETE PAVEMENTS

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SYNOPSIS

In the Westergaard analysis of stresses in concrete road slabs the radius of load distribution "a" is an important factor Since the computed stress in the slab may have a very appreciable range of variation depending upon the value of "a" it is necessary that this factor for various wheel loads as affected by different types and arrangements of tires be determined before rational stress analysis can be applied

The size of contact area per tire largely determines the degree of load distribution effect In the case of a wheel load applied through the medium of dual tires the spacing of the tires may also have a pronounced influence upon the value of the effective radius of distribution

The objective of this analysis is the evaluation of proper allowable or working distribution factors suitable for general use in stress computations for given wheel loads with the tire equipment most likely to be used in connection with that loading

The load distribution factor for solid and cushion tires must be obtained experimentally, but for pneumatic tires the load-inflation quotient closely approximates the contact area This discussion is confined to pneumatic tires

Where dual tires are involved an equivalent radius of distribution for the wheel load is readily obtained from a proper combination of the separate slab stresses produced by each tire

By charting values of the distribution radius it has been found possible to express by an empirical formula the virtual radius "a" in terms of the wheel load for each load position

Examination of the recommendations of the Tire and Rim Association show that for wheel loads in excess of 3000 lb the cost per wheel is less with duals than with single tires, which largely accounts for the prevalence of dual tires on comparatively light trucks and busses

Analysis of relative costs indicates that one may assume, for design purposes, that the most probable type of tire equipment, exclusive of passenger car balloons, may be, high pressure singles for wheel loads up to 3000 lb , high pressure duals from 3000 to 8000 lb , and balloon duals for wheel loads over 8000 lb

It is assumed that the great majority of operators will adhere rather closely to the load inflation recommendations of the makers

Consistent working values for the equivalent radius of load dis-

tribution for single tire contacts may be computed from $a' = 0.250\sqrt[3]{P}$ for circular distribution and $a' = 0.354\sqrt[3]{P}$ for semi-circular distribution P is the load on the tire

Representative and consistent values of the distribution radius for dual tires may be computed from the following formulas for corner loading $a_c = 0.29\sqrt[3]{W}$, for interior loading $a_i = 0.073\sqrt{W}$, for transverse edge loading $a_e = 0.17W^{0.42}$, for longitudinal edge loading $a_c \times 0.16W^{0.45}$ W is the total load on the wheel

GENERAL CONSIDERATIONS

A pavement, under the action of vehicular traffic, is subjected to a series of applied loads having the general characteristics of concentrated forces It is, however, obvious that a traffic load can never be actually concentrated on the pavement surface at a single point of application Wheels of modern vehicles are always equipped with some kind of rubber tire and, regardless of whether that equipment be a pneumatic tire under very high inflation pressure or even solid rubber, the compressibility of the rubber alone will result in some appreciable area of contact between tire and pavement, no matter how small the applied load may be Any wheel load, therefore, is always distributed over some appreciable area of the pavement surface, the extent and character of the distribution being dependent upon wheel-load magnitude and also upon the type, condition and arrangement of tire equipment

Recognition of this load-distribution feature as an important factor in the procedure of stress analysis as applied to concrete pavements is evidenced by the fact that the computation of stresses induced by traffic loads, in accordance with the well-known stress analysis by Dr H M Westergaard (1),[1] requires as one of the necessary known conditions a linear dimension "a" designated as "Radius of Load Distribution" and intended to represent the distribution effect of the wheel load as applied on the pavement surface In the mathematics of stress computation, a wheel load is thus considered as being uniformly distributed over some comparatively small area which, for mathematical expediency, is assumed to be a circle for the cases of corner and interior loading, and a semi-circle for the case of edge loading

As an indication of the degree of influence of the load-distribution factor upon induced slab stress, it is observed that an increase in the value of radius of load distribution from a = 0 to a = 8 in may account for a reduction of as much as 50 per cent or more in the value of computed stress, depending upon position of the load, slab thickness, and subgrade stiffness Since, for any given set of conditions, the maximum intensity of computed stress in the slab may thus have a very appreciable range of variation, depending upon the value of "a" used in the computation, it is obvious that appropriate and consistent evaluation of the equivalent radius of load distribution for any wheel load of given

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[1] Numbers in ( ) refer to list of references

magnitude is a necessary and important prerequisite to rational stress analysis as applied to concrete pavements

The objective of the analysis herewith presented is not a mere compilation of equivalent distribution radii for all possible combinations of load, inflation pressure, tire type, and tire size, but rather the evaluation of proper allowable or "working" distribution factors suitable for general use in the making of stress computations when merely wheel load, and not some specific tire equipment, is one of the known or given conditions However, the practical problem of numerically evaluating the distribution factor for a wheel load of given magnitude necessarily involves analysis of the area of tire-pavement contact under the given load which in turn involves type and arrangement of tire equipment Therefore any attempt to evaluate the distribution radius in terms of wheel load thus requires prediction as to certain general features of the tire equipment that would most probably be used with a wheel load of given magnitude

Having predicted for a certain wheel-load some specific type and arrangement of tire equipment, the next step is to determine the effective distribution of the wheel load consistent with the assumed tire equipment This requires: first, a study of the distribution properties of single-tire contacts, and second, determination of the additional distribution effect due to companion-tire spacing when dual tires are involved

A wheel load applied to a smooth rigid pavement through the medium of a rubber tire will develop a certain contact area, the size and shape of which are dependent upon the magnitude of applied force, the compressible properties of the rubber, the type and condition of tread, and also, in the case of pneumatic tires, upon the inflation pressure, and the casing stiffness of the tire Size of the contact area per tire is of prime importance, since this feature largely determines the degree of load-distribution effect The exact shape of the contact is relatively unimportant to the discussion at hand, since, in the final evaluation of load distribution, the actual contact area must necessarily be converted into some definite geometrical shape, such as a circle or semi-circle, the properties of which can be expressed algebraically with reasonable simplicity

In considering the general subject of tire-pavement contact, the question thus arises at the outset as to whether or not any practical significance should be attributed to certain known differences between the characteristics of actual contacts and those assumed for mathematical expediency For instance, uniform pressure and circular distribution are assumed in the mathematics of stress analysis, but research has shown that the configuration of an individual tire-pavement contact actually resembles more closely a rectangle with bulging sides and slightly rounded corners, rather than a circle, also, that the intensity of pressure is not uniformly distributed but is somewhat greater directly under the axle center than at other points of the contact area (2)

The quantitative effect of these differences between assumed and actual contact conditions is no doubt comparatively small But to attempt any refinement which would take into account the existence of non-uniform pressure and a complicated configuration of the contact area would at best be largely speculative owing to the mathematical complexity of stress derivation. Hence, for the sake of expediency, if for no other reason, it becomes necessary to utilize in our stress formulas the geometrical simplicity of the circle in conjunction with the assumed condition of uniform pressure intensity if we are to escape the derivation of stress formulas which otherwise would become so cumbersome mathematically that their practical utility would be largely destroyed

Dual-tire equipment introduces into the problem the feature of spacing of companion tires in addition to the contact properties of each individual tire A wheel load applied through the medium of dual tires constitutes, in effect, two separate loads, each equal to half the total wheel load and each having its own separate contact area Each contact area, in turn, creates a distinct centroid of load application on the pavement surface, the distance between the two centroids being, of course, equal to the center-to-center spacing of the two tires constituting the particular dual combination

Except for the case of corner loading, the maximum intensity of stress in a concrete pavement slab caused by a load concentrated on a very small area of the pavement surface occurs directly under the load and is highly localized in the sense that the stress intensity diminishes very rapidly in all directions as the distance from the load is increased. Moment diagrams developed by Westergaard indicate that at distances from the load of only 9 to 15 inches, depending on conditions, the stress intensity may be no more than about half the intensity under the center of load application When it is realized that standard spacings of dual tires vary approximately from 8 to 16 inches, depending upon type and size of tire, it is obvious that the feature of dual-tire spacing alone may have a very pronounced influence upon the value of the effective or "virtual" radius of distribution for the wheel load as a whole

For those cases where dual-tire equipment is involved, the additional distribution effect due to companion-tire spacing has, in this analysis, been taken into account by actually analyzing the separate slab stresses produced by each individual tire load, a proper combination of which then represents the stress produced by the wheel load as a whole From the maximum combined stress thus determined, an equivalent or "virtual" radius of distribution for the wheel load is readily obtained by mere substitution in the proper stress formula. Similar computations are then made for a number of different cases in order to ascertain the degree of variation in the value of the distribution radius resulting from a wide range in wheel load, slab thickness, and subgrade stiffness, and for each of the critical load positions—corner, interior, and edge loading

This method of evaluating the distribution factor for dual-tires by means of stress analysis leads to the conclusion that, for the same wheel load, a somewhat larger distribution radius is permissible for interior loading than for corner loading, also, for the case of edge loading, a somewhat larger distribution radius is permissible for a longitudinal edge than for a transverse edge. These distinctions arise as a result of the influence of the spacing of companion tires and constitute a phase of wheel-load distribution which is not disclosed by the commonly-used method of evaluation whereby an equivalent radius is deduced by merely using the numerical sum of the contact areas of the two tires comprising the dual combination

Owing to the relative importance of pneumatic as compared with solid and cushion tires, as judged from the viewpoint of prevalence of use, the discussion is confined to a consideration of pneumatic equipment

Furthermore, the analysis is limited to static loads, although it is fully realized that impact may have a material influence upon the distribution factor However, any tendency for impact to increase the distribution radius would be reflected wholly in its effect upon the contact of each individual tire It would not, of course, change the fixed distance between tire centers where dual tires are involved; and dual-tires would undoubtedly be the prevailing type of equipment with any wheel load of magnitude sufficient to control the details of slab design In any event, the general effect of impact would probably be to increase slightly the virtual radius of distribution, but it is quite probable that future research may develop the fact that a satisfactory practical "working" value of the distribution radius under impact may not differ greatly from a value derived from the static relationship by merely utilizing a certain percentage increase in static load

PNEUMATIC TIRE EQUIPMENT

Relative Importance of Tire Types Rubber tires may be grouped into three general classes, solid, cushion, and pneumatic, of which the pneumatic tire is by far the type most prevalently used at the present time Although solid and cushion tires are still used to some extent, their use is almost exclusively limited to certain localized industrial or congested centers where operating conditions permit slow speed and comparatively short haul That both solid and cushion tires have become practically obsolete as equipment for nation-wide vehicular traffic is evident not only from the observations of every recent traffic survey but is substantiated by statistics published by the National Automobile Chamber of Commerce (3) showing that practically all buses, and 97 per cent of all trucks, produced during the past seven years have been equipped with pneumatic tires This definite trend as to predominant type of equipment accounts for the general agreement among highway engineers that, except in certain restricted areas, pneumatic equipment may always be assumed in determining pavement stresses

In view of the outstanding relative importance of the pneumatic type, the discussion herewith presented is confined to a detailed consideration of pneumatic equipment, although brief reference is occasionally made to certain comparative load-distribution properties of solid and cushion tires

Sizes and Load-Carrying Capacities The recommendations of The Tire and Rim Association undoubtedly constitute the best source of information concerning the rated load-carrying capacities of tires of various types and sizes These recommendations, it is understood, are based upon extensive tests by the various tire manufacturers and thus represent the consensus of opinion among automotive engineers as to the proper "working" load limits for different types and sizes of tires These recommendations (4) are compiled in Table I, which lists the maximum recommended load and corresponding inflation pressure for various sizes of pneumatic truck-and-bus tires of the two so-called "high-pressure" and "balloon" types, also standard dual spacings for different tire sizes

Reference to Table I shows that the maximum single-tire load that should be carried by a pneumatic tire is 6,000 lb for the high-pressure type, and 9,100 lb for the balloon type It is thus seen that, even by utilizing dual tires, the greatest wheel load recommended for pneumatic equipment under any condition is approximately 18,000 lb , or a maximum axle load of approximately 36,000 lb Although, tires having these high load-carrying capacities may be obtained, still maximum possible wheel loads as indicated are in reality more theoretical than practical, since, in practice, legal operating restrictions, rather than available tire size, will usually limit the permissible magnitude of wheel-load concentration

Detailed description of permissible alternate tire equipment for various wheel loads is given in Table II For the purpose of cost comparison, tire-and tube cost per wheel is also shown in all cases where two or more different types of equipment may be used for the same wheel load These costs are based upon published list prices in effect during October, 1934

Equipment Trends Since, from the standpoint of load-carrying requirement, any one of several types of tire equipment may be used for a given wheel load, the question thus arises as to which one of the several alternate types permissible for a certain wheel load may be considered as the most representative of general average use There

TABLE I

SERVICE LOAD, INFLATION PRESSURE, AND DUAL SPACING FOR VARIOUS TYPES AND SIZES OF PNEUMATIC TIRES

Recommendations of the Tire and Rim Association

General Type	Tire Size	Inflation Pressure lb per sq in	Maximum Load Per Tire lb	Standard Dual Spacing in c to c
Truck and Bus Balloon	5 50 - 20	40	1225	7 25
	6 00 - 20	45	1400	7 75
	6 50 - 20	50	1650	8 25
	7 00 - 20	55	1900	9 00
	7 50 - 20	55	2100	10 00
	7 50 - 24	55	2400	10 00
	8 25 - 20	60	2550	10 50
	8 25 - 24	60	2950	10 50
	9 00 - 20	65	3250	11 50
	9 00 - 24	65	3650	11 50
	9 75 - 20	70	3900	12 00
	9 75 - 24	70	4400	12 00
	10 50 - 20	75	4700	13 25
	10 50 - 24	75	5200	13 25
	11 25 - 20	80	5450	14 00
	11 25 - 24	80	6050	14 00
	12 00 - 20	85	6250	15 25
	12 00 - 24	85	6950	15 25
	12 75 - 20	90	7200	16 00
	12 75 - 24	90	8000	16 00
	13 50 - 20	95	8200	16 00
	13 50 - 24	95	9100	16 00
Truck and Bus High Pressure	30 x 5 (6 ply)	70	1600	7 75
	30 x 5	75	1700	7 75
	34 x 5	75	1950	7 75
	32 x 6	80	2200	9 00
	36 x 6	80	2500	9 00
	34 x 7	85	2800	10 00
	38 x 7	85	3200	10 00
	36 x 8	90	3600	11 50
	40 x 8	90	4000	11 50
	38 x 9	95	4500	12 75
	42 x 9	95	5000	12 75
	40 x 10	100	5500	12 75
	44 x 10	100	6000	12 75

Note For truck and bus balloons of 7 50 to 13 50 in cross section, maximum tire load with 22-in rim may be taken as intermediate between loads shown for 20- and 24-in rim diameters

is, of course, no definite assurance that a certain specific type of tire equipment will always be used with a wheel load of given magnitude Although practice will no doubt vary, it is nevertheless known that certain definite trends do exist For instance there is an ample statisti-

TABLE II

Alternate Types of Tire Equipment for Various Wheel Loads

Wheel Load lb	General Type	Tire Size	Number of Tires per Wheel	Tire-and-Tube Cost per Wheel Dollars
1,000	H P	30 x 5	Single	21*
	Bal	5 50 – 20	Single	20
2,000	H P	34 x 5	Single	29
	Bal	7 00 – 20	Single	36
	H P	30 x 5	Dual	43*
	Bal	5 50 – 20	Dual	40
3,000	H P	38 x 7	Single	64
	Bal	8 25 – 24	Single	69
	H P	30 x 5	Dual	52
	Bal	6 50 – 20	Dual	55
4,000	H P	40 x 8	Single	90
	Bal	9 75 – 20	Single	96
	H P	34 x 5	Dual	59
	Bal	7 00 – 20	Dual	71
5,000	H P	42 x 9	Single	155
	Bal	10 50 – 22	Single	119
	H P	36 x 6	Dual	96
	Bal	8 25 – 20	Dual	123
6,000	H P	44 x 10	Single	208
	Bal	11 25 – 24	Single	166
	H P	38 x 7	Dual	128
	Bal	8 25 – 24	Dual	137
7,000	Bal	12 00 – 24	Single	192
	H P	36 x 8	Dual	165
	Bal	9 00 – 22	Dual	157
8,000	Bal	12 75 – 24	Single	232
	H P	40 x 8	Dual	180
	Bal	9 75 – 20	Dual	191
9,000	Bal	13 50 – 24	Single	265
	H P	38 x 9	Dual	279
	Bal	9 75 – 24	Dual	205
10,000	H P	42 x 9	Dual	309
	Bal	10 52 – 22	Dual	237
11,000	H P	40 x 10	Dual	393
	Bal	11 25 – 20	Dual	306
12,000	H P	44 x 10	Dual	416
	Bal	11 25 – 24	Dual	331

* Based on 6-ply Truck Type

TABLE II—*Concluded*

Wheel Load lb	General Type	Tire Size	Number of Tires per Wheel	Tire-and-Tube Cost per Wheel Dollars
13,000	Bal	12 00 - 22	Dual	370
14,000	Bal	12 00 - 24	Dual	385
15,000	Bal	12 75 - 22	Dual	413
16,000	Bal	12 75 - 24	Dual	464

cal basis for assuming that the general class of tire equipment will almost invariably be pneumatic, except possibly in certain restricted localities where solid tires may be used to a very limited extent Furthermore, the most casual observation of general vehicular traffic will reveal the pronounced predominance of dual tires on the rear wheels of trucks and buses of even comparatively light load capacity—a situation which, no doubt, is largely influenced by the fact that, for either high-pressure or balloon equipment, the tire cost per wheel is less with duals than with single tires for wheel loads in excess of about 3,000 lb Another consideration tending to influence a preference for dual-tire equipment, even with wheel loads for which single tires could be used, is the fact that such equipment frequently permits the interchangeable use of front and rear tires—a feature generally considered distinctly advantageous from an operating standpoint

As to general tire type, recent manufacturing statistics, as well as general observation, indicate a rather definite trend toward the use of balloon-type tires on practically all weights and classes of vehicles This is evident from the data shown in Figure 1, which graphically charts the results of a tire-type analysis of some 1260 makes and models of commercial vehicles as reported in the October 1933 issue of The Commercial Car Journal Although these statistics reveal the fact that current practice definitely favors balloon equipment, for practically all wheel loads, it is nevertheless observed that high-pressure tires are still being used to an appreciable extent within certain load limits For instance, of the 720 vehicles included between the vehicle-capacity limits of 1 to 4½ tons, which represent rear-wheel loads from about 2,000 to 8,000 lb , approximately 145 vehicles, or about 20 per cent, were equipped with high-pressure tires, while more than 30 per cent were high-pressure-equipped in the group having a rated rear-wheel load of approximately 4,000 lb.

Predicted Equipment for Various Wheel Loads Any attempt to evaluate consistently the equivalent or "effective" radius of distribution for a given wheel load merely in terms of load magnitude obviously requires prediction as to certain general features of the tire equipment that would most probably be used with the given load This is necessary to the analysis herewith presented in order to provide a basis for estimating the area of tire-pavement contact and also the spacing of companion tires in case dual-tire equipment is involved

In thus predicting what might be termed the "most probable" tire equipment for a given wheel load there is no intention to imply literally that a certain specific equipment will always be used with a particular load The object is merely to set up reasonable and consistent hypotheses as to inflation pressures and dual-tire spacings that may be considered as being representative of general average operating conditions It is frankly admitted that the predictions, herewith made, are based entirely upon an analysis of equipment trends But it is believed that these trends are sufficiently indicative so that certain typical details of tire equipment, that might reasonably be expected to be used with wheel loads of various magnitudes, can be approximated with sufficient accuracy to serve the practical purposes of stress computation

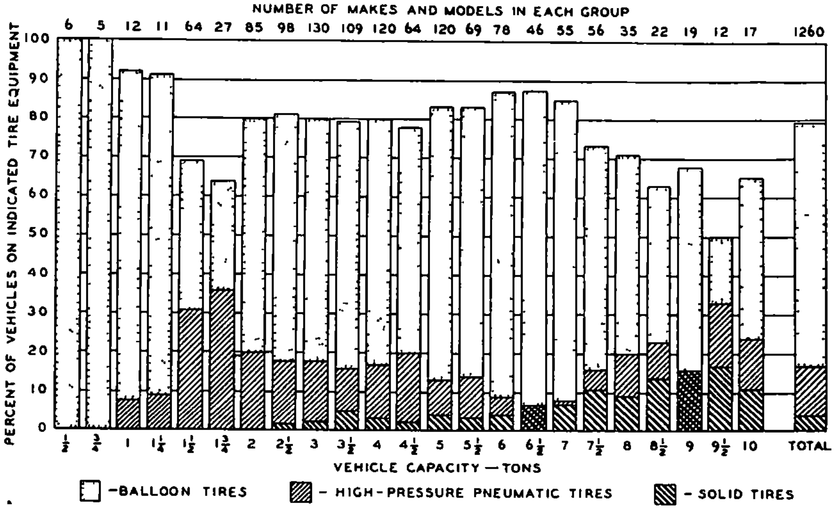


Figure 1 Tire Equipment vs. Rated Vehicle Capacity (from Commercial Car Journal, October, 1933).

There are, of course, numerous factors that influence the selection of tire equipment Initial cost, operating cost as measured by mileage return, traction and cushioning properties, and appearance—all influence the manufacturer, to greater or lesser degree, in his adoption of specifications for the tire equipment to be used on a given vehicle In the case of private automobiles, tire selection is undoubtedly based almost entirely upon appearance and riding properties However, in the case of commercial vehicles, tire cost, although not the sole criterion, is probably the dominant factor influencing the selection of tire equipment And it is the commercial truck or bus, rather than the private automobile, that usually dominates the details of pavement design

Except in the case of private automobiles, or, in other words, for wheel loads in excess of about 1,500 lb, relative tire costs for a given load therefore appear to provide a satisfactory basis for predicting hypothetical pneumatic equipment for design purposes Predictions made by thus comparing the relative tire costs listed in Table II and selecting for each wheel load, from among the alternates permissible for that load, the equipment representing the least tire-and-tube cost per wheel, indicate that representative types of tire equipment may reasonably be assumed to be high-pressure singles for wheel loads up to about 3,000 lb, high-pressure duals from 3,000 to approximately 8,000 lb, and balloon duals for wheel loads in excess thereof

While these assumptions as to probable tire equipment appear to be thoroughly consistent with observed trends in predicting the approximate wheel load at which average equipment may be expected to change from single to dual tires, still the validity of assuming high-pressure tires for wheel loads up to about 8,000 lb might well be questioned in view of the pronounced trend toward use of balloon equipment for practically all loads However, statistics as well as general observation indicate quite clearly that, within the wheel load limits for which high-pressure equipment is thus predicted, high-pressure tires are being used to an appreciable extent Bearing in mind the fact that the ultimate objective of the analysis herewith presented is evaluation of the load-distribution factor for design purposes, the probable use of high-pressure equipment can not, therefore, be entirely ignored

It appears therefore that the above predictions, involving as they do the assumption of high-pressure equipment for wheel loads up to 8,000 lb or tire loads up to 4,000 lb, are not grossly inappropriate, since they merely introduce some conservatism which the uncertainties of the situation apparently demand. Incidentally, assumption of high-pressure instead of balloon equipment is not only on the conservative side, from the viewpoint of induced stress in the pavement slab, but quantitatively has relatively minor effect upon the final deduced values of equivalent distribution radii, especially after application of the slight correction for tire type as indicated in Table V

High-pressure equipment has also been assumed even for the 1,000 lb wheel load, notwithstanding the fact that, for loads up to about 1,500 lb, passenger balloons would invariably be used This assumption, which is purely arbitrary, is made for the purpose of maintaining uniformity of increment variation and appears to be justified by the fact that, for this light load, the difference in computed slab stress is of very little if any practical significance regardless of whether the distribution radius be evaluated on the basis of a 30 or even a 60-lb pressure Furthermore, freedom in selection of rim diameter has been assumed in compiling Tables II and III, as no logical basis can be established for predicting the most probable rim diameter for a given tire load This, however, is apparently a feature of minor influence, as is indicated by the fact that, if predictions were based on a constant 20-in rim diameter, which is a very popular rim size for trucks and buses, practically the same final distribution radii would result, being slightly less only for wheel loads in excess of 12,000 lb (6,000-lb tire loads), and, even in those cases, the single-tire radius is reduced not more than about 0 15 in

TABLE III

PREDICTED TIRE EQUIPMENT FOR VARIOUS WHEEL LOADS

Wheel Load lb	Type	Tire Equipment			Load per Tire lb
		Diameter of Tire Section in	Number of Tires per Wheel	Inflation Pressure lb per sq in	
1000	H P	5*	Single	60	1000
2000	H P	5	Single	75	2000
3000	H P	5	Dual	75	1500
4000	H P	5	Dual	75	2000
5000	H P	6	Dual	80	2500
6000	H P	7	Dual	85	3000
7000	H P	8	Dual	90	3500
8000	H P	8	Dual	90	4000
9000	Balloon	10 50	Dual	75	4500
10000	Balloon	10 50	Dual	75	5000
11000	Balloon	11 25	Dual	80	5500
12000	Balloon	11 25	Dual	80	6000
13000	Balloon	12 00	Dual	85	6500
14000	Balloon	12 00	Dual	85	7000
15000	Balloon	12 75	Dual	90	7500
16000	Balloon	12 75	Dual	90	8000

* Based on 6-ply Truck Type

SINGLE-TIRE DISTRIBUTION

Factors Affecting Area of Tire-Pavement Contact The actual area of a pneumatic tire-pavement contact is dependent upon the magnitude of applied load, intensity of inflation pressure, and various physical properties of the tire itself Applied load and inflation pressure are the factors of major influence, but tire-contact tests have shown that the size of the contact area may also be affected to a certain extent by such tire properties as compressibility of the rubber, type of design and worn condition of the tread, stiffness of the tread and side walls, and apparently tire size as affecting transverse and longitudinal curvature of the tire surface The definite degree to which any one or any combination of these factors affects the size of contact area can, of course, be determined only by imprint test upon various types and sizes of tires and in different states of wear Although the published bibliography of research on this particular subject is rather limited, still there are available certain test data which throw considerable light upon the relative, if not the actual quantitative, effect of many of these influencing factors

Imprint Test Data Tire-contact tests clearly indicate that, for all types and sizes of pneumatic tires, the actual contact area tends to vary in general accordance with the value of the load-inflation quotient That this general tendency holds to a rather marked degree is indicated

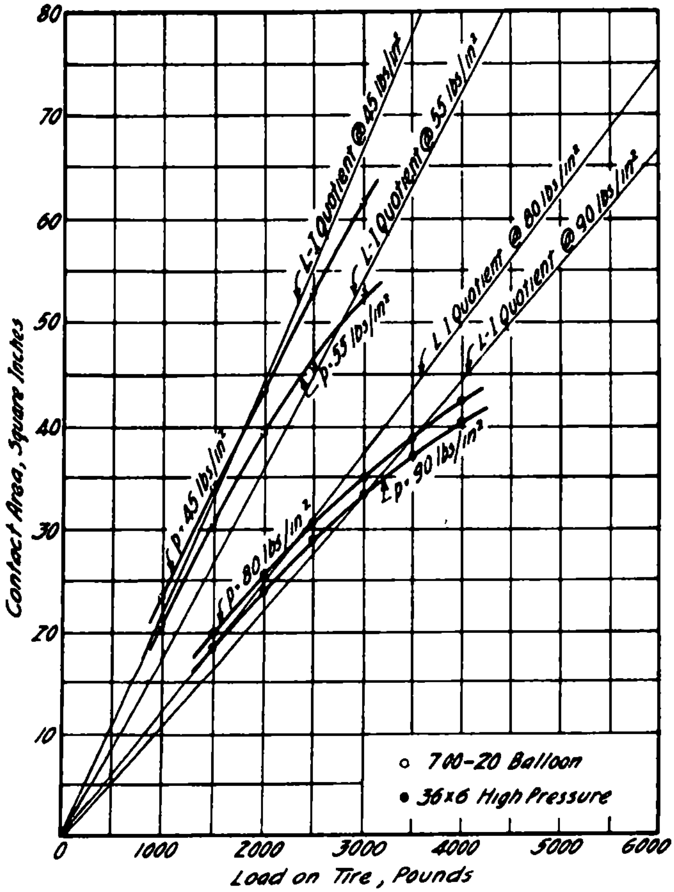


Figure 2. Contact Areas Determined by Spangler Tests

by imprint tests reported by M G Spangler (5), certain results of which are plotted in Figure 2 This chart shows the observed or actual contact areas developed by two different types and sizes of tires under various applied loads and at different inflation pressures It is observed that, in each case, the variation in actual contact area follows very closely the load-inflation quotient, thus indicating that, although the law of fluid pressure does not exactly apply, the degree of deviation therefrom apparently is not seriously important, at least from the viewpoint of the ultimate objective of this discussion.

The same general trends are observed in the test data presented in Table IV. Although slightly greater relative variation is here noted, still actual contact areas for the truck-and-bus balloon group are only from 2 to 8 per cent less than the load-inflation quotient, and, for the truck and bus high-pressure group, from 10 to 17 per cent more. According to these data, use of the load-inflation quotient as the assumed contact area would therefore be slightly conservative in the case of high-pressure tires, and, even in the case of balloons, would represent an adverse error of not more than 8 per cent in contact area, or an error of only about 4 per cent in the equivalent radius of load distribution. True, the passenger balloon group apparently shows an adverse error of about 20 per cent if the contact area be based on the load-inflation quotient, but even this would represent an error of not more than about 10 per cent in equivalent radius, and for a relatively unimportant tire type as judged from the standpoint of slab stress, since the use of passenger balloons would probably be confined to tire loads of not more than about 1,200 lb.

TABLE IV

COMPARISON OF THEORETICAL AND ACTUAL TIRE-PAVEMENT CONTACT AREAS

Tire Type	Size of Tire	Load lb	Inflation Pressure lb per sq in	Contact Area		
				Actual sq in	Load-Inflation Quotient sq in	Ratio L-I Quotient to Actual
Passenger Balloon	4.40 x 21	750	32	18.9	23.5	1.24
	4.75 x 19	840	32	22.6	26.3	1.16
	5.00 x 19	895	32	23.8	28.0	1.18
	5.25 x 17	885	32	26.3	27.7	1.10
	5.25 x 21	1000	32	26.7	31.3	1.17
	5.50 x 18	1000	32	26.2	31.3	1.19
	6.00 x 20	1190	32	31.1	37.3	1.20
	6.00 x 21	1190	32	33.5	37.3	1.11
Truck and Bus High Pressure	30 x 5	1700	75	25.3	22.7	0.90
	32 x 6	2200	80	31.4	27.5	0.88
	34 x 7	2800	85	38.7	33.0	0.85
	36 x 8	3600	90	48.1	40.0	0.83
	38 x 9	4500	95	55.8	47.4	0.85
Truck and Bus Balloon	6.00 x 20	1400	45	28.8	31.1	1.08
	7.00 x 20	1900	55	33.8	34.6	1.02
	7.50 x 20	2100	55	35.9	38.2	1.06
	8.25 x 20	2550	60	39.4	42.5	1.08
	9.00 x 20	3250	65	47.9	50.0	1.04

In Table IV are presented certain heretofore unpublished data on actual contact areas obtained through the courtesy of Mr Warner Tufts, Special Investigator for the Federal Coordinator of Transportation, and which it is understood have been compiled from certain records of tire-contact tests made by various tire manufacturers These independent test data afford several direct comparisons with the results of the Spangler tests as shown in Figure 2 For instance, according to the data of Table IV, a 7 00 x 20 balloon tire at 55 lb inflation pressure developed a contact area of 33 8 sq in under a load of 1,900 lb Spangler found a contact area of approximately 37 5 sq in with a tire of the same type and size, and under the same inflation pressure and load Thus, one investigator finds the actual contact area for this case to be 2 3 per cent less than the load-inflation quotient, while the other finds it to be 8 per cent more, a difference in recorded test results of about 10 per cent This difference may have been the result of some one or a combination of several causes The tires, although of the same type and size, may have been manufactured by different concerns and may accordingly have had slightly different stiffness properties They may have had different tread designs, for Moyer (6) found, with new 4 75 x 19 passenger balloons under 815 lb at 35-lb pressure, a difference in contact area of about 10 per cent caused by two different tread designs which he distinguishes as "open" or "closed" types

Furthermore, there may have been a difference in the procedure of recording the inflation pressure, i e whether recorded before the load was applied or at the time the imprint was taken If the former procedure was followed, the actual internal pressure at the time of imprint would have been somewhat more than 55 lb , which in turn would account for the somewhat smaller contact area Spangler observed that the internal pressure increased as the load increased, and, for that reason, he released the pressure to a predetermined amount at the time the imprint was taken Although the exact procedure followed in making the test of Table IV is not known, this difference, if it existed, may easily account for the smaller recorded imprint as compared with Spangler's results Similar differences are quite likely to occur in practice, depending upon whether a tire is inflated to its rated pressure before or after the vehicle is loaded to its maximum capacity

Imprint test data, furnished through the courtesy of the U S Bureau of Public Roads, are presented in Figure 3 Recorded contact areas are shown for both high-pressure and balloon tires of various sizes and under application of over-load as well as rated-load It is observed that, within a load range of from 2,000 to 8,000 lb , high-pressure tires developed from 18 to 25 per cent less contact area than balloon tires as based upon comparison of the average-curve values indicated in Figure 3 This difference is just about equal to the difference in standard inflation pressures as recommended for the two types of tires.

Furthermore, it is significant to note that, for tire loads of 2,000, 4,000, and 6,000 lb, the imprint area corresponding to the average-curve value is in each case practically equal to the load-inflation quotient as based upon the standard recommended inflation pressure for the given load This holds true for both types of tires and appears to furnish further investigational evidence of the close relationship between contact area and load-inflation quotient, presuming, of course, that, in these particular tests, the standard recommended inflation pressure was used for each size of tire tested.

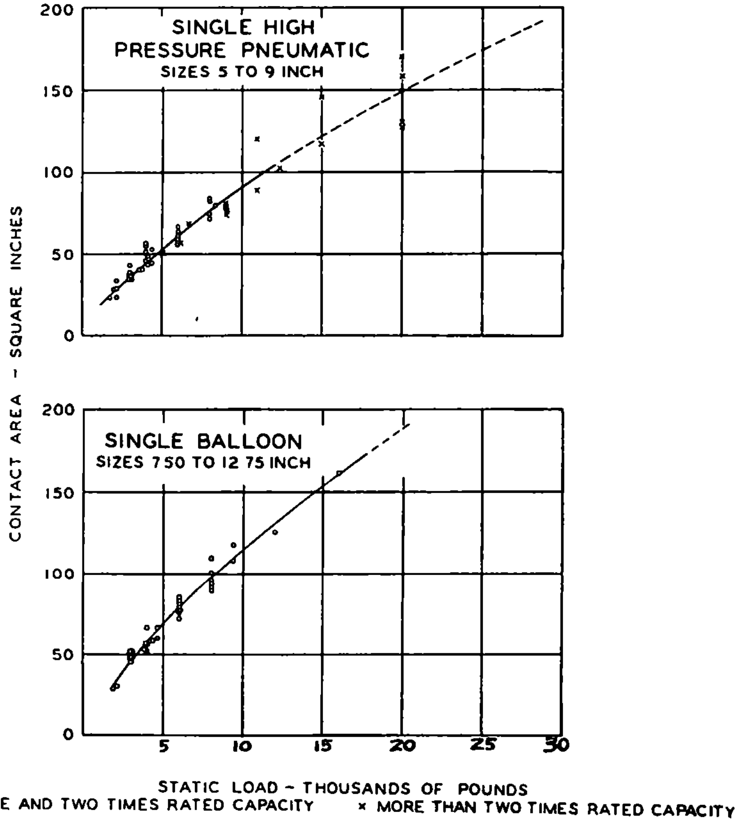


Figure 3. Single-Tire Imprint Tests. (By U. S. Bureau of Public Roads)

Effect of Tire Wear. Tests show that, after the tread of a pneumatic tire has worn smooth, the contact area is somewhat greater than that produced by the same tire when new. Comparative tests by Spangler (5) on 4 75 x 19 passenger balloons at 35-lb pressure indicate that the difference in contact areas is practically constant, under a fixed inflation pressure, for a rather wide range in applied load, the contact area being about 25 per cent more with the worn tread than with the new tread at a load approximately equal to the maximum rated capacity for the tire tested. Moyer (6) also found practically the same difference with the same size and type of tires under the same load and pressure.

Whether or not approximately the same percentage difference would hold also in the case of the larger and more heavily loaded tires of the truck-and-bus class is not evident from the test data available. However, regardless of the actual percentage difference, there is, no doubt, a general tendency for increase in contact area as a tire wears. This is probably due, not only to the smoothing out of the tread projections, but also to a certain decrease in casing stiffness due to loss of rubber thickness and the effect of repeated flexure of the tire section under continued use. It is, therefore, evident that the general effect of wear on any pneumatic tire is a tendency toward a gradual increase in contact area which, in turn means the gradual development under continued use of a more favorable condition with respect to the intensity of stress induced in the pavement slab.

TABLE V

General Values of Distribution Radii for Single Tire Loads

Tire Type	Tire Load lb	Inflation Pressure lb. per sq. in	Load-Inflation Quotient	Correction for Tire Type	Contact Area sq. in	Radius of Equivalent Circle in	Radius of Equivalent Semi-Circle in
High Pressure	1000	60	16.7	+10%	18.4	2.42	3.42
	2000	75	26.7	+10%	29.4	3.06	4.33
	3000	85	35.3	+10%	38.8	3.52	4.97
	4000	90	44.4	+10%	48.8	3.96	5.60
Balloon	5000	75	66.7	−8%	61.3	4.42	6.25
	6000	80	75.0	−8%	69.0	4.70	6.64
	7000	85	82.4	−8%	75.7	4.91	6.94
	8000	90	88.9	−8%	81.7	5.11	7.23

With reference to the effect of tire wear, a pertinent comparison may be made between pneumatic and solid tires. In the case of a pneumatic tire, continued use of the tire is conducive to a gradual reduction in slab stress, whereas, in the case of a solid tire, wear tends to produce the opposite effect, by reason of a gradually decreasing contact area due to loss in rubber thickness and resiliency, and the aggravation of impact effect resulting from the development of a wavy or lumpy condition of the tread.

Equivalent Radius of Load Distribution. The load-distribution factor as utilized in the mathematics of stress computation is a linear dimension representing the radius of a circle when the stress is computed for the case of either corner or interior loading, or the radius of a semicircle when the stress is computed for the case of edge loading. In order that this assumed distribution may be closely approximate in stress effect to that caused by the actual load distribution, it is obvious that the area of the assumed circle or semi-circle must be substantially equal to the actual tire-pavement contact Although there are numerous variables tending to affect the actual size and character of tire-pavement contacts, it is nevertheless clearly evident from all available test data that the major factor influencing the size of contact for pneumatic tires is the load-inflation quotient, all others having decidedly minor effect in comparison therewith Therefore, the question of whether or not it is permissible to generalize equivalent distribution radii for single-tire contacts in terms of wheel load depends largely upon the reasonableness with which anticipated inflation pressure for a tire load of given magnitude can be predicted as being representative of general average operating conditions

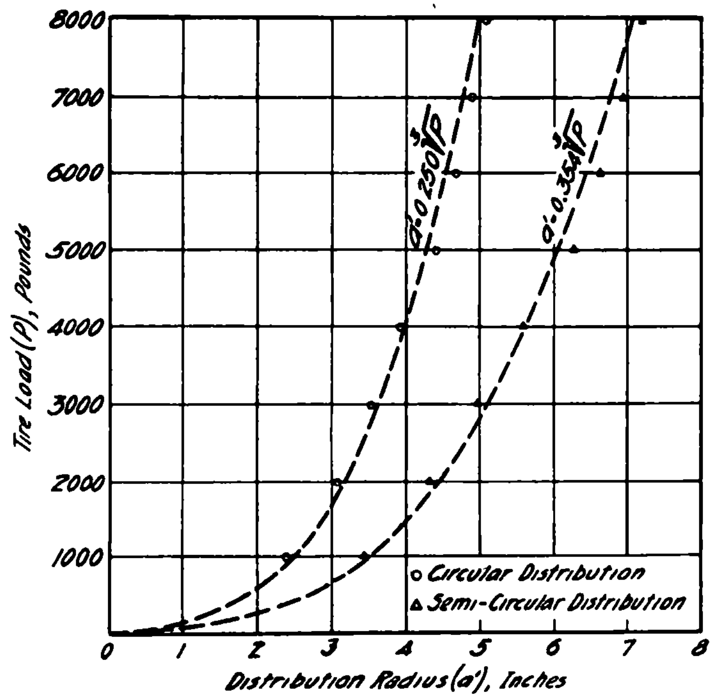


Figure 4. Distribution Radii for Single-Tire Equipment

It would seem that a very satisfactory basis for making such a prediction, either for a high-pressure or for a balloon tire, is to be found in the recommendations of the Tire and Rim Association—recommendations which are generally recognized as representing the best practice in load-inflation relationship, and which, accordingly, is undoubtedly adhered to very closely by the rank and file of vehicle operators These recommendations, together with predictions as to tire type as listed in Table III, thus provide a reasonable basis for estimating equivalent distribution radii for various tire loads under average operating conditions

If the numerical value of the contact area were assumed to be always equal to the load-inflation quotient, then the equivalent radius of load distribution "a" for a single-tire contact could be expressed in terms of the tire load "P" and the inflation pressure "p" by merely equating the load-inflation quotient to the area of a circle or semi-circle, as the case may be, having a radius "a " Use of the load-inflation quotient as thus representing the contact area would no doubt be sufficiently accurate for practical design purposes But since, both the Spangler tests and the data of Table IV indicate that the load-inflation quotient may be slightly less in some cases and in others slightly more than the actual contact area, some recognition might appropriately be given to these indicated differences. Thus, if one accepts the trends indicated by the test data of Table IV as being reasonably representative of the general relationship between theoretical and actual contact areas, it would appear permissible to apply to the load-inflation quotients of Table V certain corrections based upon tire type Accordingly, an increase of at least 10 per cent for the high-pressure type, and a decrease of as much as 8 per cent for the balloon type would appear to be justified, thus obtaining a closely approximate contact area and the radius of its equivalent circle or semi-circle appropriate for general use with any tire load of known magnitude These equivalent radii for the cases of circular and semi-circular distribution, when plotted as in Figure 4, are found to lie very closely to curves having equations as shown

It thus appears that consistent "working" values for the equivalent radius of load distribution, appropriate for single-tire contacts, may be computed by the formulas,
for circular distribution,

$$a' = 0\ 250 \sqrt[3]{P} \qquad (1)$$

for semi-circular distribution,

$$a' = 0\ 354 \sqrt[3]{P} \qquad (2)$$

in which, a' = radius of load distribution, in inches
P = load on tire, in pounds

DUAL-TIRE DISTRIBUTION

Basis of Evaluation The procedure herein suggested as a logical method of analyzing the additional distribution effect due to dual-tire spacing consists briefly in first determining the greatest slab stress produced by one tire load consistent with its particular distribution, and the simultaneous stress produced by the other tire load The maximum consistent combination of these stresses then represents the maximum stress produced by the combined action of the two companion tires, and, by substituting this stress in the proper stress formula, one may solve for the value of "a" which will thus represent an equivalent or "virtual" radius of distribution for the wheel load as a whole.

In thus utilizing stress analysis as a means of evaluating the distribution radius for dual tires, general use has been made of the stress theory developed by Westergaard, but in so doing, it has been expedient to introduce certain assumptions and approximations. In the case of corner loading, certain assumptions are made as to the relative effect of the P_2-load, from which there are computed what appear to be minimum and maximum limiting values of the equivalent distribution radius, using finally in each case the minimum values for the sake of conservatism. Both in the cases of interior and edge loadings, use has been made of the convergent point-moment curves developed by Westergaard, but introducing certain approximations involved in "estimating" the shape and location of the moment curve in the vicinity of each applied load in order to establish the necessary "closing curve" consistent with a specific combination of tire distribution, slab thickness, and subgrade modulus for each case analyzed.

This basis of evaluation obviously requires separate analysis and stress computation for each of the three critical load positions, corner, interior and edge loading. According to Westergaard (1), maximum slab stresses for these different load positions are represented by the following formulas:

Case I, Corner Loading

$$S_c = \frac{3\,W}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right] \qquad (3)$$

Case II, Interior Loading

$$S_i = \frac{0.3162\,W}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 1.069\right] \qquad (4)$$

Case III, Edge Loading

$$S_e = \frac{0.572\,W}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right) + 0.359\right] \qquad (5)$$

These formulas are based upon the following notation:

S_c, S_i, S_e = maximum stress, in pounds per sq. in., for corner, interior, and edge loading respectively.
W = wheel load, in pounds.
h = slab thickness, in inches.
k = modulus of subgrade reaction, in lb. per in.^{3}
a = equivalent radius of load distribution, in inches.
b = radius of equivalent resisting slab section, in inches, or

$$b = \sqrt{1.6\,a^2 + h^2} - 0.675\,h \qquad (6)$$

l = radius of relative stiffness of slab to subgrade, in inches, or

$$l = 22.49\sqrt[4]{\frac{h^3}{k}} \qquad (7)$$

Formulas (3) to (7) inclusive are based upon concrete having a modulus of elasticity E = 3,000,000, and Poisson's ratio u = 0.15.

It is thus evident that the effect of dual-tire spacing upon the virtual radius of wheel load distribution is influenced, not only by magnitude of wheel load and its position on the slab, but also to some extent by slab thickness and subgrade modulus, since a given value for "W" fixes the probable spacing of companion tires, while "h" and "k" determine the value of "l." Hence, various combinations of "W," and "l" must be analyzed for each load position in order to ascertain the nature of variation in the load distribution factor for various magnitudes of wheel load when applied either on a corner, on the interior, or on an edge of the slab.

As a means of illustrating the method of evaluation by stress analysis, the specific case, represented by W = 8,000, h = 8, and k = 100, will be analyzed for each of the three critical load positions. For the given values of "h" and "k," Formula (7) gives,

$$l = 22.49\sqrt[4]{\frac{8\times 8\times 8}{100}} = 33.83 \text{ in.}$$

According to Table III, the predicted tire equipment for an 8,000-pound wheel load would be 8-in. high-pressure duals, the standard spacing of which (Table I) is B = 11.5 inches. Each tire would carry a load P_t = 4,000 pounds, and, from Formula (1), the radius of load distribution for each tire, on the basis of circular distribution, would be,

$$a' = 0.250\sqrt[3]{4000} = 3.97 \text{ in.}$$

and on the basis of semi-circular distribution,

$$a' = 0.354\sqrt[3]{4000} = 5.62 \text{ in.}$$

Case I, Corner Loading. The slab corner, under the most adverse wheel-load position for corner loading, would be acted upon by two loads $P_1 = P_2 = 4{,}000$, located as in Fig. 5, and each distributed over a circular area of the pavement surface having a radius of 3.97 inches. Calling S_c' the stress produced by P_1 alone, Formula (3) gives,

$$S_c' = \frac{3\times 4000}{8\times 8}\left[1 - \left(\frac{1.414\times 3.97}{33.83}\right)^{0.6}\right] = 124 \text{ lb per sq in.}$$

According to Westergaard, this maximum stress for the load P_1 occurs on a diagonal section located at a distance from the slab corner,

$$X = 2.38\sqrt{a'l} = 2.38\sqrt{3.97\times 33.83} = 27.6 \text{ in.}$$

It is no doubt sufficiently accurate for the purpose at hand to assume that the moment on a diagonal corner section caused by a load applied in the vicinity of the corner is directly proportional to the distance of the load from the section. With this assumption and calling S''_c the stress due to P_2 on the section where P_1 causes its maximum stress, one may then write, from the geometry of Fig 5,

$$S_c'' = S_c' \frac{x_2}{x_1} = 124 \frac{27\,6 - (1\,414 \times 3\,97) - (\,707 \times 11\,5)}{27\,6 - (1\,414 \times 3\,97)}$$

$$= 78 \text{ lb per sq in}$$

Hence, the combined stress due to P_1 and P_2, or, in other words, the stress due to the wheel load W is,

$$S_c = S_c' + S_c'' = 124 + 78 = 202 \text{ lbs per sq in}$$

Substituting this stress and the full wheel load in Formula (3),

$$202 = \frac{3 \times 8000}{8 \times 8}\left[1 - \left(\frac{1\,414\text{ a}}{33\,83}\right)^{0\,6}\right]$$

whence, a = 6 59 in

This value of the virtual radius for the wheel load as a whole is in the nature of a first approximation, since its determination has been made upon the assumption that the maximum stress due to the entire wheel load occurs at the same section where P_1 alone produces its maximum stress This obviously is inconsistent, since the effect of applying P_2 simultaneously with P_1 is to make the effective radius of distribution for the combination somewhat greater than the 3 97-in radius for P_1 alone, which in turn, means that the section of maximum stress, due to the combined effect of P_1 and P_2, occurs at a somewhat greater distance from the corner than 27 6 in as limited by a′ = 3 97 If, for instance, the effective radius for the combined action of P_1 and P_2 were as much as 6 59 in , as indicated by the first approximation, the section of maximum stress due to the wheel load would be located 35 6 in from the corner instead of 27 6 in as first assumed

Although this would tend to increase S_c'' by reason of an increase in the ratio $x_2{:}x_1$, still this greater ratio would be applied against a smaller value for S_c' than its maximum 124 lb per sq in Therefore, it is safe to assume that the section of maximum stress for the wheel load is located somewhere between the limits 27 6 and 35 6 in from the extreme corner of the slab Other trials could thus be made until a value of "a" is found which is consistent with the position of the maximum-stressed section However, it is observed that, with the wheel positioned as in Figure 5, the resultant of the two tire loads is eccentric with respect to the line bisecting the corner angle and thereby tends to produce a slight twisting moment on the section of maximum stress As this is not usually taken into account in stress computations, it seems advisable to be somewhat conservative in the evaluation of "a" for the case of corner loading by basing the ratio $x_2{:}x_1$ on the upper limit for location of the maximum-stressed section (in this case 35 6 in), and also applying this ratio against the maximum possible value of S_c' (in this case 124 lb per sq in)

Recomputation on this basis gives,

$$S_c'' = 124 \frac{35\ 6 - (1\ 414 \times 3\ 97) - (\ 707 \times 11\ 5)}{35\ 6 - (1\ 414 \times 3\ 97)} = 90 \text{ lb per sq in}$$

and, $S_c = S_c' + S_c'' = 124 + 90 = 214$ lbs per sq in Substituting $S_c = 214$ in Formula (3), gives, for the wheel load, a virtual radius $a = 5\ 84$ in compared with 6 59 in as obtained by the first approximation Use of 5 84 in as the virtual radius of load distribution for this case is therefore conservative, since the actual value would be somewhere between 5 84 and 6 59 in

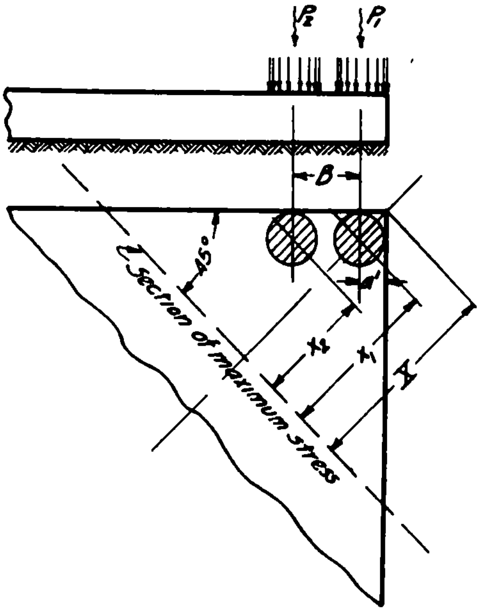


Figure 5 Relative Position of Tire Loads for Corner Loading

Case II, Interior Loading For the illustrative case assumed above, positioning of the wheel load on the interior portion of the slab is equivalent to the application of two loads P_1 and P_2, spaced 11 5 in apart, each equal to 4,000 lb , and each distributed over a circular area of the pavement surface having a radius of 3 97 in

Considering first the stress produced by P_1 alone and calling b′ the b-factor for this single load, Formula (6) gives,

$$b' = \sqrt{1\ 6(3\ 97)^2 + 64} - 0\ 675 \times 8 = 4\ 05 \text{ in}$$

Calling S_i' the stress produced by P_1 alone, Formula (4) gives,

$$S_i' = \frac{3162 \times 4000}{64}\left[4 \log\left(\frac{33\ 83}{4\ 05}\right) + 1\ 069\right] = 94 \text{ lb per sq in.}$$

According to Westergaard (1), a load applied on the interior of a road slab at a considerable distance from all edges produces both radial and circumferential stress, which are of equal intensity under the center of the load. Furthermore, the intensity of each decreases very rapidly as the distance from the load increases, the radial intensity decreasing more rapidly than the circumferential. Hence, the intensity of stress under the load is equal in all directions, but, at some distance from the load is greater in one direction than in a direction perpendicular thereto. These are principal stresses, one being the maximum, the other the minimum. Since we are here concerned with maximum intensity, circumferential rather than radial stress is considered. Accordingly, the stress diagrams shown in Fig. 6 for the two tire loads are each based on circumferential stress intensity.

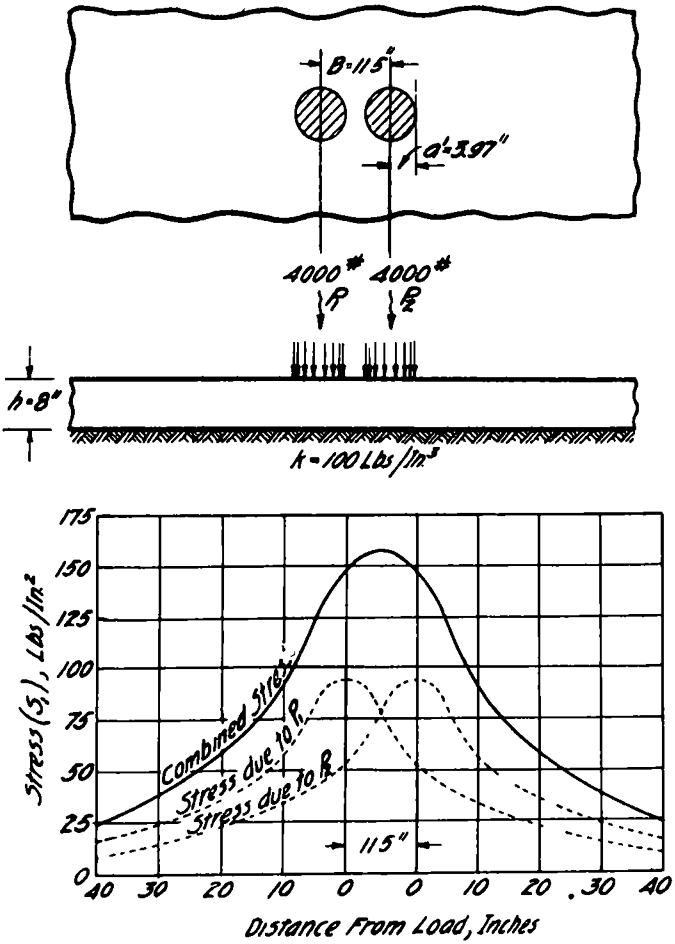


Figure 6. Stress Diagram for Interior Loading with Dual-Tire Equipment

By the principle of superposition, the maximum intensity of combined stress, that is the maximum stress due to the wheel load as a whole, is found to occur midway between the two tire loads and to be equal to 158 lb per sq in Substituting this stress and the full wheel load in Formula (4), we have,

$$158 = \frac{3162 \times 8000}{64}\left[4 \log\left(\frac{33\ 83}{b}\right) + 1\ 069\right]$$

whence, b = 6 26 in Substituting this value of "b" in Formula (6),

$$6\ 26 = \sqrt{1\ 6\ a^2 + 64} - 0\ 675 \times 8$$

whence, a = 6 71 in.

Case III, Edge Loading For the condition of edge loading, two cases require consideration, since the relative position of the two tire loads with respect to the slab edge depends upon whether the edge in question is parallel or normal to the line of vehicular travel Although, in either case P_2 would act a fixed distance (in this case 11 5 in) from P_1, still, with P_1 applied on a longitudinal edge of the pavement, P_2 would act at a distance of 11 5 in from the edge, whereas, in the case of an edge formed by a transverse joint or crack, the wheel in crossing such a joint would cause both P_1 and P_2 to be simultaneously applied on a slab edge

With a uniform slab thickness, the stress produced by P_1 alone would, of course, be the same when applied either on a longitudinal or a transverse edge, and, since stress computations for edge loading are based on semi-circular load distribution, the distribution radius for a single tire load of 4,000 lbs would be a′ = 5 62 in as computed above for semi-circular distribution Calling b′ the b-factor for this single load, Formula (6) gives,

$$b' = \sqrt{1\ 6(5\ 62)^2 + 64} - 0\ 675 \times 8 = 5\ 30 \text{ in}$$

Calling S_e' the edge-stress produced by P, alone, Formula (5) gives,

$$S_e' = \frac{572 \times 4000}{64}\left[4 \log\left(\frac{33\ 83}{5\ 30}\right) + 0\ 359\right] = 128 \text{ lb per sq in}$$

Considering first the case of transverse-edge loading, the tire loads would be positioned as in Figure 7 and the stress diagrams for the separate tire loads overlap as indicated by dotted lines By superposition, the maximum stress for the wheel load as a whole is found to occur midway between the loads and to be equal to 225 lb per sq. in Substituting this stress and the full wheel load in Formula (5), we have

$$225 = \frac{572 \times 8000}{64}\left[4 \log\left(\frac{33\ 83}{b}\right) + 0\ 359\right]$$

whence, b = 6 80 in

Substituting this value of "b" in Formula (6),

$$6\ 80 = \sqrt{1\ 6\ a^2 + 64} - 0\ 675 \times 8$$

whence, $$a = 7\ 28 \text{ in}$$

For the case of longitudinal-edge loading the tire loads would be positioned as indicated in Figure 7 The maximum stress due to P_1 is 128 lb per sq in as before, but the longitudinal stress under P_1 due to P_2 is somewhat less owing to its being applied an appreciable distance from the pavement edge According to deflection and moment curves developed by Westergaard (1), it is estimated that the maximum longitudinal stress produced under P_1 by P_2 is, for this particular case, approximately equal to 65 lb per sq in This determination is based on the assumption that the influence diagram for maximum edge stress drops off in a direction normal to the edge at the same rate as in a direction along the edge This assumption, while not strictly in accord with the Westergaard theory, is approximately correct and is conservative for the purpose herein utilized Accordingly, in the case of longitudinal-edge loading, we obtain, for the wheel load as a whole,

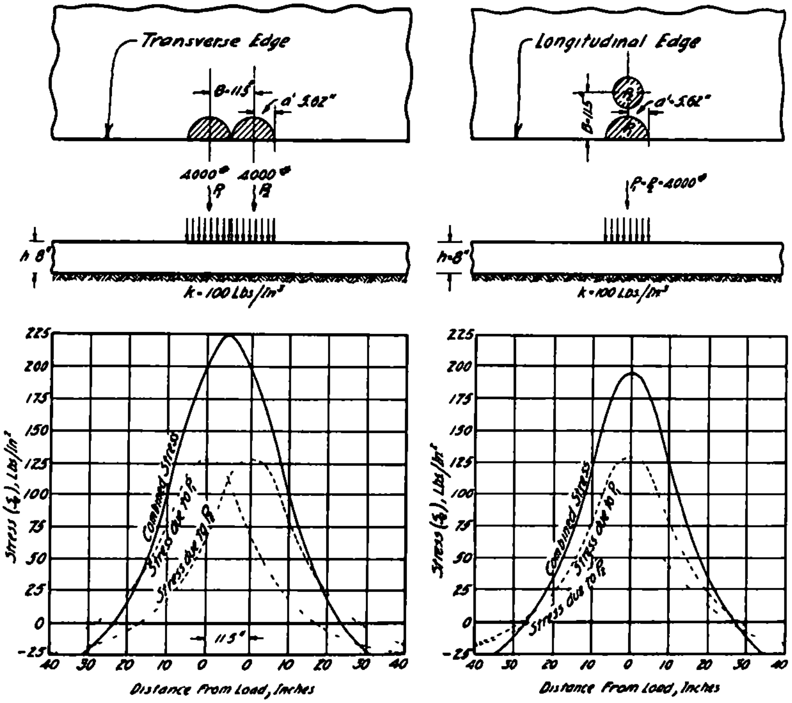


Figure 7 Stress Diagram for Transverse and Longitudinal-Edge Loading with Dual-Tire Equipment

$$S_e = S_e' + S_e'' = 128 + 65 = 193 \text{ lb per sq in}$$

Substituting this stress and the full wheel load in Formula (5) and solving for "b", gives b = 8 79 in which, according to Formula (6), corresponds to a = 9 27 in

Evaluation of the load-distribution factor by the method of stress analysis thus indicates that, for this particular combination of load, slab and subgrade, the distribution radius may be taken as 7 28 in for transverse-edge loading and 9 27 in for longitudinal-edge loading

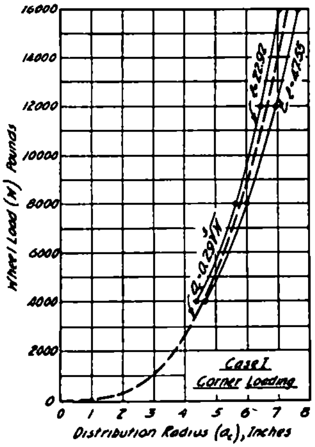


Figure 8

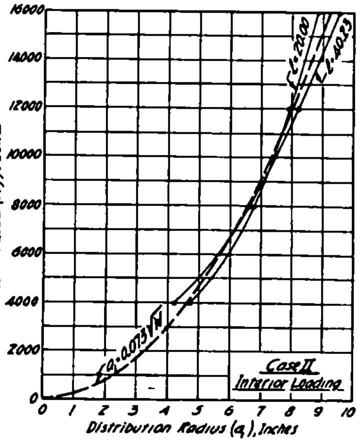
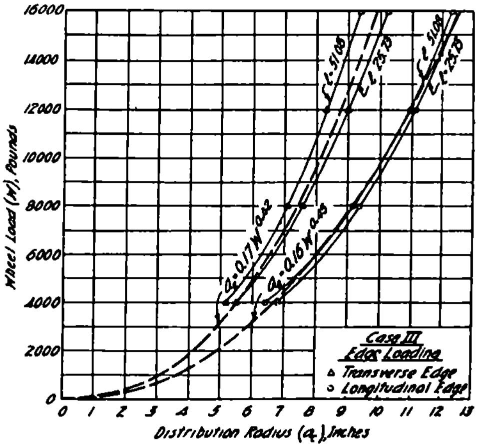


Figure 9



Figures 8, 9 and 10. Distribution Radii for Dual Tire Equipment

Effect of Variable Factors Since for each load position, wheel load, slab thickness, and subgrade modulus, are all involved in evaluation of the load-distribution factor by the method of stress analysis, the question arises as to what influence anticipated variation in the values of these several factors may have upon computed values of the distribution radius. As judged by current practice, concrete pavements may be expected to vary in thickness from about a 7-5-7 to an 11-8-11 section. This indicates a reasonable anticipation of probable range in slab thickness of from 6 to 10 in. for diagonal corner sections, 5 to 8 in. for interior thicknesses, and 7 to 11 in. for edge thicknesses.

With an assumed range in the value of subgrade modulus of from $k = 50$ to $k = 200$, anticipated variation in the value of relative slab-subgrade stiffness "l" would accordingly range from $l = 22.92$ to $l = 47.55$ for corner loading, $l = 20.00$ to $l = 40.23$ for interior loading, and $l = 25.73$ to $l = 51.08$ for edge loading. For these limiting values of "l", which represent a range in the value of the stiffness factor of approximately 100 per cent in each case, and for wheel loads ranging from 4,000 to 16,000 lb., virtual radii have been computed for each critical load position by the method of stress analysis as illustrated above. The limits of variation in the value of distribution radius thus determined are shown in Figures 8, 9, and 10, from which it is apparent that the maximum variation in the computed value of "a" due to relative stiffness of slab to subgrade is comparatively small for each load position and no doubt negligible for all practical purposes.

General Values for Dual-Tire Equipment. In view of the comparatively minor effect of slab thickness and subgrade modulus upon the distribution radius, as shown by values computed with variations of approximately 100 per cent in the slab-subgrade stiffness factor, it is apparently permissible for all practical purposes to express the distribution radius, for each critical load position, merely as a function of the wheel load. Accordingly, in Figures 8 to 10, curves indicating satisfactory general values have been plotted for each load position. The respective equations of these curves lead to the conclusion that representative and consistent values of the distribution radius for dual-tire equipment may be computed by use of the following formulas:

Case I, Corner Loading,

$$a_c = 0.29 \sqrt[3]{W} \tag{8}$$

Case II, Interior Loading,

$$a_i = 0.073 \sqrt{W} \tag{9}$$

Case III-a, Transverse-Edge Loading,

$$a_e = 0.17 W^{0.42} \tag{10}$$

Case III-b, Longitudinal-Edge Loading,

$$a_e = 0.16 W^{0.45} \tag{11}$$

SUMMARY

Prediction of the most probable type of tire equipment for a given wheel load in accordance with the relative cost trends of Table II leads to the conclusion that wheels carrying loads up to about 3,000 lb. will, within reasonable probability be equipped with single tires, whereas dual-tire equipment will prevail with wheel loads in excess thereof Therefore, any attempt to develop empirical formulas expressing, in general, the distribution radius merely in terms of the wheel load imposes the requirement that such formulas will give values for the distribution radius reasonably consistent with single-tire distribution for loads less than about 3,000 lb , and consistent with dual-tire distribution for loads greater than about 3,000 lb

Comparison of radii for single tires (Fig 4) with those for dual tires (Figs 8, 9 and 10) discloses the fact that, except for the case of corner loading, the general formulas for dual-tire distribution give values for the distribution radius which are not seriously inconsistent with single-tire distribution when utilized with wheel-load values of 3,000 lb. or less In the case of corner loading, values of the radius if computed by the dual-tire formula are about 16 per cent greater than if computed by the single-tire formula, thus resulting in an appreciable adverse error for light wheel loads if utilized as a general formula for any load magnitude Similarly, for the case of longitudinal-edge loading, use of the dual-tire formula for loads of about 3,000 lb or less will give values which are adversely inconsistent with single-tire distribution to the extent of about 9 per cent

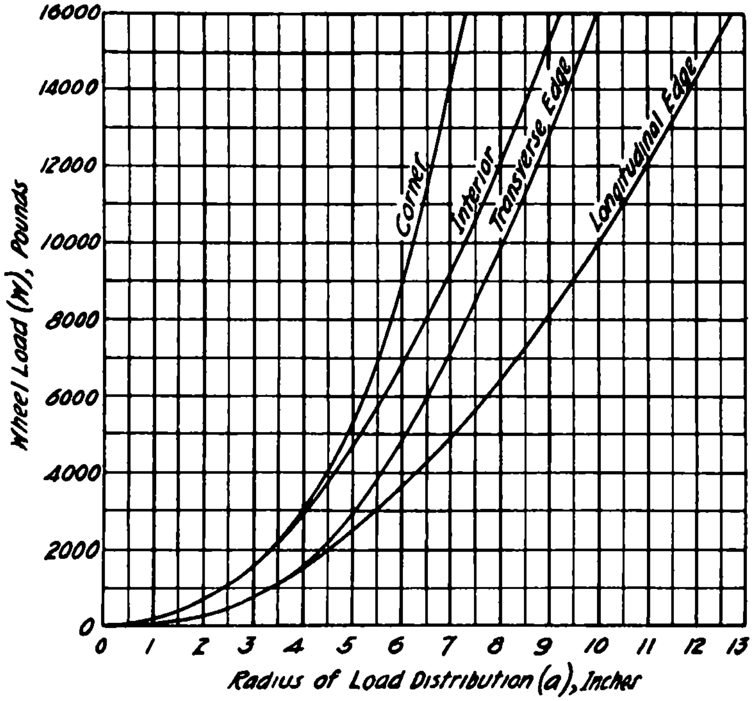


Figure 11. Relative Values of the Load-Distribution Factor for Different Load Positions.

There is, however, neither justification nor necessity for attempting to fix some exact wheel-load value at which single-tire equipment ceases and dual-tire equipment begins, since obviously no such prediction of definite numerical value can be made It is, however, certain that, at some approximate magnitude of wheel load, dual-tire equipment becomes more economical than single-tire equipment Although comparative costs indicate that such a point occurs at a wheel load of approximately 3,000 lb , still, under average operating conditions, a mixed usage of types undoubtedly occurs within the wheel-load range of from about 2,000 to 4,000 lb It therefore appears consistent with indicated probability to base any general evaluation on single-tire equipment for wheel loads up to and including 2,000 lb and upon dual-tire equipment for loads of 4,000 lb and greater, also, to utilize arbitrarily intermediate values for the 3,000-lb load so that the increment for this particular load will be representative of a gradual transition from one type of equipment to the other Final summarized values of the distribution radius for various wheel loads and for different load positions, as plotted in Figure 11, have been computed on this basis

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