PRINCIPLES OF SOIL MECHANICS INVOLVED IN FILL CONSTRUCTION

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SYNOPSIS

The stability of the fill and the supporting power of the undersoil are important factors in design The stresses in the undersoil caused by the weight of the fill are independent of elastic constants since the problem is one of plane deformation The greatest shearing stress at any point in the undersoil is $\frac{p}{\pi}$ where p is the unit load, and if this does not exceed the cohesion corresponding to the maximum allowable deformation, the supporting power of the undersoil is ample If the cohesion of the undersoil is less than $\frac{p}{\pi}$, it does not follow that failure will result Further study of the supporting power of the undersoil is necessary in this case and for this purpose Prandtl's method of plastic equilibrium may be used The ϕ circle method is applied to the problem of the stability of the fill itself

The design of a fill requires consideration of the stability of the fill itself and the supporting power of the undersoil Seepage, hydrostatic uplift, and capillary saturation are factors which affect the stability of fills, but do not prevent its approximate determination by analytical methods Gradual settlement of the fill, due to consolidation of the fill materials, the undersoil, or both, is also a very important consideration, but this is not necessarily associated with the resistance to shear, upon which stability depends The limitations in design of cross section and the possible factors of safety should be known before construction of the fill, and in this respect soils must be considered in the same light as any other engineering material

STATEMENT OF THE PROBLEM

Fills fail when deformations in the fills or their foundations exceed those permissible[1] in the design of the particular structure

The first task, then, is to make a systematic analysis of stress distribution Next must be determined the resistance of the earth materials to these stresses

Observations of numerous slides have led engineers generally to conclude that the most probable surface of failure in an embankment of fairly homogeneous earth is cylindrical in shape, and the slide is approximately circular in cross section A method of determining the position of the "most dangerous sliding circle" by a laborious graphical procedure, termed the "method of slices," has been published in *Public Roads* (1)[2] Donald W Taylor (2) has suggested application of the so-called ϕ circle method, used by Krey (3), in the consideration of safe bearing loads in foundation problems

Determination of stress distributions in the undersoil begins with the point load formula of Boussinesq, which assumes the stressed material to be isotropic and elastic Integrating the point load formula for all the points located on a straight line furnishes an expression of the stress produced at a point in the earth

[1] The safe working strength of soils, it is emphasized, is not necessarily based on their ultimate strengths For cohesive soils especially, it is more often determined from the stress-deformation relations of the particular soil involved, and is based on the allowable settlements, deformations, or other movements of the structure being designed See Figure 7 Report of Department of Soils Investigations, page 474 this volume

[2] Figures in parentheses refer to list of references at end

by a uniformly loaded surface strip of infinite length and of infinitesimal width Integration of the expression for line loads over a given width furnishes the quantitative expressions for the normal and shearing stresses at a given point beneath the loaded area Such working formulas have been developed by S D Carothers (4), (5), (6)

Coulomb's formula for the ultimate shearing resistance is

$$s = c + p_n \tan\phi$$

where s = shearing resistance, c = cohesion and p_n = normal pressure, all with reference to a unit of area The pressure, p_n, is normal to the plane of shear and ϕ is the angle of internal friction The laboratory procedures for obtaining the values of c and ϕ, are not considered in this paper and p_n is obtained analytically from the known stress distribution and varies over a wide range throughout the supporting earth below the fill In general, its value is much greater than zero and for this reason it is on the side of safety and expediency to be guided by the simple rule

A cohesive supporting soil is considered as safe if its greatest shearing stress does not exceed the cohesion corresponding to the maximum allowable deformation In such a case no further consideration need be given to the problem of bearing capacity of the supporting soil

If, however, the greatest shearing stress beneath the loaded area exceeds the cohesion of the undersoil, further study of its bearing capacity is required and for this purpose Prandtl's method of plastic equilibrium may be used

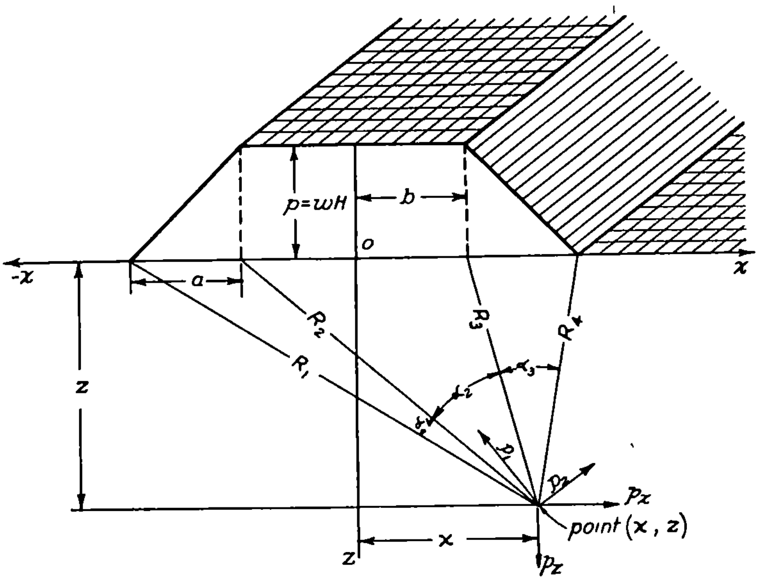
CASE 1 FILL ON GOOD UNDERSOIL

Figure 1 is a pressure diagram of the vertical cross section of a fill The weight of fill material on a square foot at the surface of the supporting soil (excluding the slopes) is wH = p, where w = average weight per cubic foot of fill material and H = height of fill If there is hydrostatic uplift, then w = the buoyed weight of a cubic foot of soil mass Horizontal distance is denoted by x and vertical distance by z The "y" direction is along the length of the fill The problem is one of plane deformation and Figure 1 is in the xz plane The x and z axes and the origin are as shown in the diagram The angles α_1, α_2, and α_3 between the radial lines R_1, R_2, R_3, and R_4, drawn to any point (x, z) in the undersoil, are measured in radians The major principal stress, p_1, is in the direction of the bisector of angle α_2 and the minor principal stress p_2 is perpendicular in direction at any point to p_1 The vertical normal stress at any point (x, z) is p_z and the horizontal normal stress is p_x, the subscripts referring to the direction of these two normal stresses The shearing stress corresponding to p_z and p_x is s_{xz} The maximum shearing stress at a point (x, z) is s max and this is equal to one-half the difference of the two principal stresses at that point, that is $s\ max = \frac{p_1 - p_2}{2}$. The loci of points of constant s max are shown in Figure 2 for the distance "a" equal to the distance "b" (Fig 1) The greatest value of s max for this case is 0 31p It is on the oz axis at a depth z = 3/2a For a = 2b, the greatest value of s max is approximately 0 3p and is on the oz axis, a depth equal to 0 96a, below the bottom of the fill

For any type of fill, within the range of a = b to a = 2b, the greatest value of s max is approximately 0 3p Undersoils which have cohesions equal to or greater than 0 3p furnish ample support for the fill

This quantity, 0 3p, is the first to compare with the shearing strength of the undersoil If this stress is lower than the strength, the answer is evident, if higher, it does not necessarily follow that danger of failure exists If the undersoil has a uniform strength of 0 3p, it is seen that only inside the $s=0.3p$ curve, Figure 2, is the strength exceeded by the stresses This danger zone is confined and is surrounded on all sides by material having these conditions, a plastic zone having been developed, the theoretical stress diagram is no longer correct

For example, let it be assumed that a



$$p_z = \frac{p}{\pi a}\left[a(\alpha_1+\alpha_2+\alpha_3)+b(\alpha_1+\alpha_3)+x(\alpha_1-\alpha_3)\right]$$

$$p_x = \frac{p}{\pi a}\left[a(\alpha_1+\alpha_2+\alpha_3)+b(\alpha_1+\alpha_3)+x(\alpha_1-\alpha_3)-2z\log_e\frac{R_1R_4}{R_2R_3}\right]$$

$$s_{xz} = -\frac{zp}{\pi a}(\alpha_1-\alpha_3)$$

$$p_1 = \frac{p}{\pi a}\left[a(\alpha_1+\alpha_2+\alpha_3)+b(\alpha_1+\alpha_3)+x(\alpha_1-\alpha_3)-z\log_e\frac{R_1R_4}{R_2R_3}\right]+\frac{zp}{\pi a}\sqrt{\log_e^2\frac{R_1R_4}{R_2R_3}+(\alpha_1-\alpha_3)^2}$$

$$p_2 = \frac{p}{\pi a}\left[a(\alpha_1+\alpha_2+\alpha_3)+b(\alpha_1+\alpha_3)+x(\alpha_1-\alpha_3)-z\log_e\frac{R_1R_4}{R_2R_3}\right]-\frac{zp}{\pi a}\sqrt{\log_e^2\frac{R_1R_4}{R_2R_3}+(\alpha_1-\alpha_3)^2}$$

$$s\ max = \frac{zp}{\pi a}\sqrt{\log_e^2\frac{R_1R_4}{R_2R_3}+(\alpha_1-\alpha_3)^2}$$

Figure 1 Stresses in Earth Below Fill

a reserve of resisting capacity The material in the plastic zone may yield to some indeterminate extent and transmit to the adjoining material that part of the load which it cannot resist itself Under fill 20 feet high is to be constructed on the undersoil, Table 1

The value p is equal to $wH = 90\times 20 = 1800$ lb sq ft The greatest shearing stress at any point in the undersoil is

TABLE 1
Assumed Properties of Soils

Material	c = cohesion	ϕ = angle of internal friction	w = weight per cubic foot
	lb per sq ft	*Deg*	*lb*
Fill	200	5	90
Undersoil	800	15	110

0 3p = 0 3 × 1800 = 540 lb per sq ft, which is less than 800 lb per sq ft, the cohesion of the undersoil Furthermore, assumed that the fill material is disturbed and not consolidated It is to be designed safe with respect to the cohesion of the fill material (see Table 1) alone This is explained in the following paragraph

In the older method of "slices" the equation for equilibrium is

$$R\Sigma T = R\Sigma N \tan\phi + RLc$$

where R is the radius of the most dangerous circle, T is the tangential stress

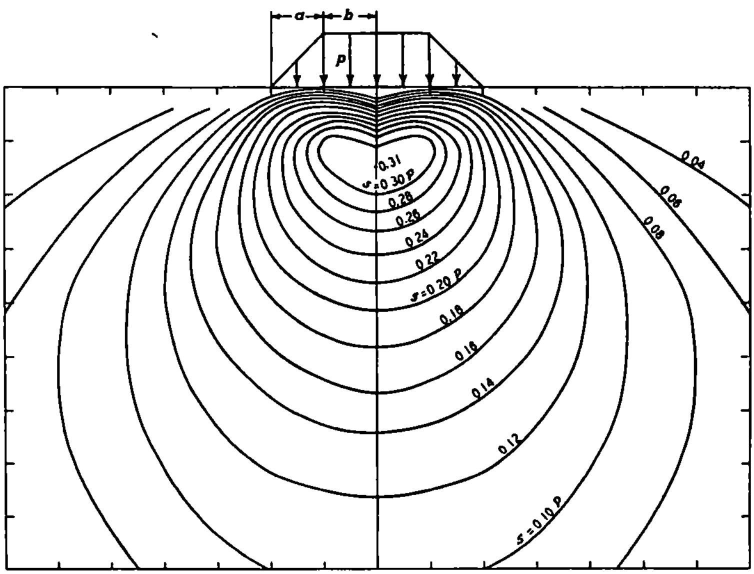


Figure 2 Isoshear Lines Under Typical Linear Fill, a=b

the total shearing resistance at a point on a plane of maximum shearing stress is

$$s = 800 + p_n \tan 15°,$$

where p_n is the pressure that is normal to the plane By inspection it can be seen that p_n is greater than zero and therefore the shear strength furnished by p_n tan 15° is additive to the shear strength of 800 lb per sq ft furnished by the cohesion alone

In the design of the fill by the "method of slices" it is assumed at the outset that the fill slopes at an angle ι = 45° with the horizontal See Figure 3 It is further

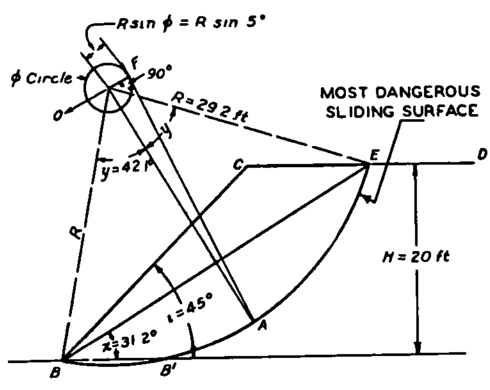


Figure 3 Illustration of the Most Dangerous Sliding Circle for a Given Slope

(tangent to the circle at the midpoint of a given slice), N is the stress normal to the tangential direction, L is the length of the entire arc (of the circle) through the earth, and c is the unit cohesion The expression on the left of the equation of equilibrium, RΣT, is the shearing stress moment That on the right, RΣN tan φ +RLc, is the resisting moment Solving for c,

$$c\,(\text{computed}) = \frac{\Sigma T - \tan\phi\Sigma N}{L}$$

and

$$\frac{c\,(\text{as determined from tests})}{c\,(\text{computed})} = \text{factor of safety}$$

with respect to cohesion alone This procedure assumes utilization of all of the

TABLE 2

DATA ON CRITICAL CIRCLES BY φ CIRCLE METHOD

(Taken from Table 1, Stability of earth slopes, by D W Taylor)

i Deg	φ Deg	x Deg	y Deg	$\frac{c}{FwH}$
90	5	50	14	0 239
45	0			
	5	31 2	42 1	0 136
	10	34	39 7	0 108
	15	36 1	37 2	0 083
30	10	25	44	0 075
15	5	11	47 5	0 070

available friction, φ in the equation being the limiting value of the angle of obliquity

The values for the angles x and y are obtained from Table 2 (Table 1 of Taylor's article) The fill section, BCD, (Fig 3) is drawn to scale For $i=45°$ and $\phi=5°$ (Table 1), we find in Table 2 that $x=31\ 2°$ and $y=42\ 1°$ This enables us to draw the chord BE (Fig 3) inclined at an angle $x=31\ 2°$ with the horizontal From the points, B and E, draw the radial lines, R, R, to intersect at an angle, 2y, at the center O, of the φ circle, which is on the perpendicular bisector of BE

With R as radius and O as center (see Fig 3), describe the arc, BAE, of the most dangerous sliding circle OA bisects the angle, 2y With O as center and $OF = R \sin\phi = R \sin 5°$ as radius, draw the φ circle Draw AF tangent to the φ circle

The result of the earlier graphical solutions of the slope problem was expressed as a vector quantity, $\frac{2c}{w}$, c being the unit cohesion and w the unit weight of the material If the length of this vector is divided by any linear dimension such as H, the vertical height, the result, $\frac{2c}{wH}$ is a dimensionless abstract number which, when determined for the most dangerous circle, describes the requirements for stability The form used by Taylor for this abstract number ("stability" number) is $\frac{c}{FwH}$ wherein F is the factor of safety with respect to cohesion alone

It will be observed in Figure 3, that the most dangerous (critical) circle as drawn cuts into the relatively cohesive undersoil below the plane, BB′ In the various publications dealing with the theory of the sliding circle it is always assumed that the soil throughout the entire depth traversed by the most dangerous sliding circle is uniform with respect to both cohesion and friction This assumption serves to simplify an otherwise most complicated problem In the present case this assumption is made only with respect to the fill material, but it is also assumed that the resistance to sliding of the fill material over the surface of the more cohesive supporting soil at the plane boundary, BB′, is approximately the same as that which is assumed for the arc BB′ (Fig 3) Actually, the fill would tend to shear along B′AE and the slide over the surface BB′ would be in the nature of a detritus slide If the value of ϕ for the fill material were much greater than 5°, the value of the angle x would be greater than 31 2°, the value of the angle y would be less than 42 1° and the dangerous circle would not cut into the supporting soil below the plane, BB′

The above construction is permissible when the most dangerous circle passes through the toe of the slope, which is the case when $n=\frac{1}{2}(\cot x-\cot y-\cot i+\sin\phi \csc x \csc y)$ is a negative quantity or zero Here n is the ratio of the distance from the toe of the slope to the intersection of the dangerous arc with the ground surface to the vertical height, H On substituting values from Table 2, $x=31\ 2°$, $y=42\ 1°$, $i=45°$, and $\phi=5°$, it is found that n is negative

The quantity, $\frac{c}{FwH}$, for $i=45°$ and $\phi=5°$ is found from Table 2 to be 0 136 In this case, c = cohesion = 200 lb per sq ft, w = unit weight of fill material = 90 lb per cu ft and H = 20 ft F, the factor of safety, is then found as follows

$$\frac{c}{FwH}=0\ 136=\frac{200}{F\times 90\times 20}$$

and

$$F=\frac{200}{90\times 20\times 0\ 136}=0\ 8$$

With respect to cohesion alone, therefore, the fill is unstable for $i=45°$ and H = 20 ft The value of F may be increased by decreasing either i or H Suppose that a safety factor of 2 with respect to cohesion is desired and that the height, H, must be maintained at 20 ft Then the value $\frac{c}{FwH}$ becomes

$$\frac{c}{FwH}=\frac{200}{2\times 90\times 20}=\frac{1}{18}=0\ 056$$

From curves such as shown in Figure 4, it can be found that for $\phi=5°$ and $\frac{c}{FwH}=0\ 056$, i equals 12°

CASE 2 FILL ON QUESTIONABLE UNDERSOIL

The soil data for this case are given in Table 3 The fill is to be constructed with a 1 1 slope and with dimensions as shown in Figure 5 The factor $\frac{c}{FwH}$ is equal to 0 083 when $\phi=15°$ and $i=45°$ (Table 2)

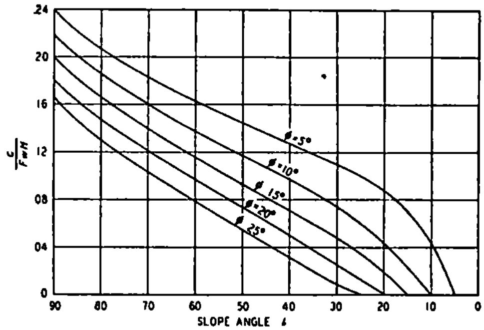


Figure 4 Chart for Stability Number, $\frac{C}{FwH}$

TABLE 3

Assumed Properties of Soils

Material	c = cohesion	ϕ = angle of internal friction	w = weight per cubic foot
	lb per sq ft	*Deg*	*lb*
Fill	800	15	110
Undersoil	200	5	100

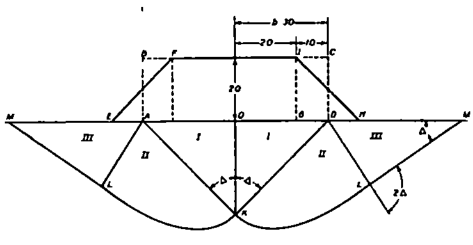


Figure 5 The Surface of Failure in the Supporting Soil

In this case,

$$\frac{c}{FwH}=\frac{800}{F\times 110\times 20}=0\ 083$$

$$F=\frac{800}{110\times 20\times 0\ 083}=4\ 4$$

The fill of itself is, therefore, stable as constructed

The above computation assumes that *all* of the available friction along the arc of the dangerous circle is utilized in holding the earth in place If a part of the available friction is not developed or mobilized, there remains a certain unused strength which represents a factor of safety *with respect to friction* The angle ϕ is the limiting value of the angle of obliquity and if at any point of the circular arc the obliquity of stress is ϕ' which is less than ϕ, then

$$\tan\phi = F_F \tan\phi'$$

where F_F is the factor of safety with respect to friction alone and

$$F_F = \frac{\tan\phi}{\tan\phi'}.$$

Thus for $\phi'=\phi$, F_F becomes unity and for ϕ' less than ϕ, F_F exceeds unity

The value, 4 4, as above computed, is the factor of safety as related to cohesion alone Shearing resistance is determined by both cohesion and friction The true factor of safety, F_t, takes into account both c and ϕ If the true factor of safety is 4 4, then the cohesion corresponding to this value may be computed and its magnitude compared to the actual cohesion From the preceding formula, $\tan\phi' = \frac{1}{4\ 4}\tan 15°$, therefore $\phi' = 3\ 5°$, the average developed obliquity, or the obliquity which (if constant) yields the same total frictional shearing stress as actually is developed Taylor has computed the value for $\frac{c}{FwH}$ corresponding to $i=45°$ and $\phi = 3\ 5°$ to be equal to 0 147

Hence

$$\frac{c}{FwH} = \frac{c}{4\ 4 \times 110 \times 20} = 0\ 147$$

or c=1,423 lb per sq ft

This means that to have a *true* factor of safety of as much as 4 4, the cohesion of the fill material must be 1,423 rather than 800 lb per sq ft, with only about 23 per cent of the total available friction being utilized

Next consider the bearing capacity of the supporting soil With reference to Figure 5, if the supporting power, q, of the undersoil is less than the unit load to which it is subjected, failure takes place over the surfaces MLK, AK, and DK The diagram represents a section of unit thickness in the direction perpendicular to the plane of the paper Prandtl's (7) method of plastic equilibrium is applied in this case and in its application we must assume perpendicular slopes It is on the side of safety to reconstruct graphically the section, EFIH, Figure 5, to have the form of ABCD without changing the volume or mass of fill material The dimensions of the fill as reconstructed are shown in the diagram (Fig 5) The width, 2b, is 60 ft, half the width=b=30 ft and the height is 20 ft When the supporting earth fails in shear, zone 1 moves down bodily, shearing at the planes AK and KD Zone II undergoes a combination of rotation and sliding along the log spiral, KL Zone III moves outward and upward, shearing along the plane, LM

The radial line drawn from either of the points, A or D, to any point on the spiral, LK, makes a constant angle 2Δ, with this curve The value of Δ is $45° - \phi/2$.

It is assumed that the ground water level is at the surface, MM (Fig 5), of the supporting soil Taking the weight of a cubic foot of water as 62 5 lb, the buoyed weight of a cubic foot of the supporting soil is 100−62 5 or 37 5 lb =w'=effective unit weight The formula used in computing q, the bearing capacity of the undersoil, is

$$q = (c\cot\phi + w'b\cot\Delta)\left[\frac{1+\sin\phi}{1-\sin\phi}e^{\pi\tan\phi} - 1\right]$$

and on substitution of 200 for c, 30 for b, 5° for ϕ and 42 5° for Δ,

$$q = (200 \times 11\ 43 + 37\ 5 \times 30 \times 1\ 091)\left[\frac{1+0\ 087}{1-0\ 087} e^{3\ 1416 \times 0\ 0875} - 1\right]$$

or $q = 1{,}990$ lb per sq ft

Here e is the Naperian base (natural logarithms) The bearing load per unit area $= 20 \times 110 = 2{,}200$ lb per sq ft The factor of safety, F, is

$$F = \frac{1{,}990}{2{,}200} = 0\ 9$$

This factor of safety may be increased by increasing the width of the fill or by decreasing its height The height of the fill required for a factor of safety of 2 is determined as follows

$$F = \frac{q}{w \times H} = \frac{1{,}990}{110 \times H} = 2,$$

$$H = \frac{1{,}990}{220} = 9 \text{ ft}$$

Supporting Power Is Related to Compressive Strength

It will be of help to the reader of the current literature pertaining to the bearing capacity of soils to bear in mind that the essential features of all bearing capacity formulas may be expressed by the one very simple formula,

$$q = Kp_c,$$

where q = bearing capacity, K is some multiplier, and p_c is the compressive strength of the soil as determined by a laterally unconfined compression test In the derivation of his formula for q, the supporting power, Prandtl used the expression,

$$p_c = \frac{2c \cos \phi}{1 - \sin \phi}$$

which is obtained from the Mohr diagram as shown in Figure 6

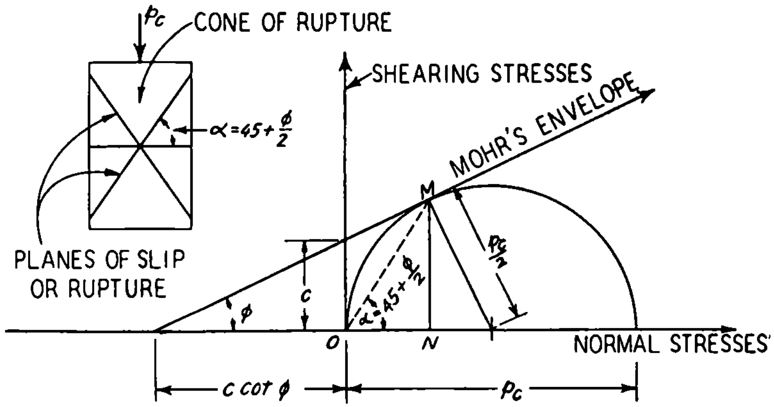


Figure 6 Mohr's Diagram Illustrating Compressive Strength, p_c of a Material *OM* = Plane of Shear, c = Unit Cohesion and ϕ = angle of Internal Friction Shearing Stress on Plane *OM* under Vertical Pressure p_c is *mn*

With a radius equal to $\frac{2}{p_c}$, a circle having its center on the horizontal axis and at a distance $\frac{2}{p_c}$ from the origin O, is drawn For the ideal case the angle α which the planes of slip or fracture of the specimen tested, make with the horizontal, being determined, the line OM is constructed through the origin and at the angle α with the horizontal The envelope line is then drawn tangent to the circle at the point M where the line OM intersects the circle

The intersection of the envelope line with the shearing stress axis discloses, according to Mohr's theory, the cohesion, c, of the material The angle of the envelope line with the horizontal, equals the angle of internal friction of the material, and $a=45°+\frac{\phi}{2}$

The use of Mohr's diagram in connection with analyses of test data of plastic soils, which deform considerably before failure or have no readily distinguishable planes of slip or fracture, is subject to special considerations not discussed herein

For purely cohesive soil, $\phi=0$, the values for q in terms of unit cohesion, c, as obtained by the methods of Terzaghi, Krey, and Prandtl, assuming an infinite strip, uniformly loaded, are as follows

Method of Terzaghi, q=4c
Method of Krey, q=6 6c
Method of Krey (simplified), q=6 0c
Method of Prandtl, q=5 14c

The authors tentatively prefer the method of Prandtl, which is based on well established principles of mechanics as they pertain to the condition of plastic equilibrium For a cohesive soil the method of Terzaghi is more conservative than the others and like Prandtl's method has the advantage of being finally expressed in a single formula The graphical method of Krey does not have this advantage and its use involves considerable time and labor As already mentioned, however, the adaptation of his method to the problem of stability of slopes has simplified the older solution of this problem as presented by Petterson, Fellenius, and others [7]

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DISCUSSION ON SOIL MECHANICS IN FILL CONSTRUCTION

PROF D P KRYNINE, *Yale University* · The authors have applied the general method which is used in such cases, and I wish to say a few words about the method itself The embankment and its foundation are considered in this method as separate bodies whereas in reality they form a whole, and have a common set of principal stresses, although, if the embankment and the foundation are of different materials, there may be some break in principal stresses at the surface of contact In a research project committee of the Highway Research Board, this problem is being considered in some detail

Furthermore, I wish to call attention to the following inconsistency of the so-called "slice method" Vertical pressure at a point located at a certain depth below a slope is measured by the weight of the corresponding earth column If the height of the embankment is increased by placing some additional earth at its top, vertical pressure at the given point computed according to the "slice method" would not change, and this seems illogical

The designer of an embankment should keep in mind that trajectories of maximum shearing stress under the center of the embankment are purely theoretical in character In reality, earth material at that section is confined by the principal stresses and shear action may take place only close to the edges of the foundation

Finally, I believe that the safety factor, 2, which Mr Palmer has assumed, could be somewhat decreased without affecting the stability of the embankment