LATERAL PRESSURES ON RETAINING WALLS CAUSED BY SUPERIMPOSED LOADS

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The process of designing a retaining wall or abutment involves not only the determination of the overturning and translating forces attributable to the backfilling material, but to those similar forces caused by the superimposition of loads on the surface of the backfill as well. These superimposed surface loads, for the purpose of this discussion, may be any loads which rest on the surface of the backfill at or above the level of the top of the retaining wall. They may consist of concentrated loads such as truck wheels, or distributed loads such as foundations of adjacent buildings, railroad tracks, material piles, or simply additional or surcharged backfill material.

It is not uncommon practice to assume that superimposed loads of these types produce lateral pressures on retaining walls which are uniformly distributed over the backface of the wall throughout its entire height. Recent researches by Gerber[1] at the Erdgenossichen Technischen Hochschule at Zurich and by the author[2] at the Iowa Engineering Experiment Station at Ames, Iowa, have indicated that this assumption may be incorrect. Rather, the pressure due to surface loads is non-uniform and has its maximum value at some distance below the top of the wall which is a function of the distance from the load to the wall and is independent of the height of the wall. Roughly, the maximum pressure occurs at a depth below the top of the wall equal to $\frac{1}{2}$ to $\frac{2}{3}$ of the distance from the wall to the load.

Gerber's studies were made by loading the surface of a mass of sand contained in a concrete bin 80 cm. (2.62 ft.) wide, 80 cm. deep, and 4.5 m. (14.7 ft.) long, through cast iron plates of various shapes but having a uniform area of 1000 sq. cm. (155 sq. in.). The normal components of pressures exerted against one side of the bin were measured by means of a series of Amsler pressure cells.

The author's studies were made on a series of actual retaining walls, with truck wheel loads and a uniformly distributed parallel line or strip load applied at the surface of the gravel backfill and at various distances from the backface of the wall. The normal pressures on the walls were measured by means of Goldbeck pressure cells and by means of stainless steel friction ribbons made to slide between two stainless steel surfaces and to pass over small rollers through the wall so that the ends of the ribbons were available for pulling from the front side of the wall. These ribbons were covered with suitable flexible waterproof covering and were calibrated by means of air pressure to obtain the relationship between normal pressure on the ribbon and the pull required to start it in motion. Then with the backfill in place, the loads were applied in various positions and the ribbons pulled to obtain the magnitude and distribution of pressures on the wall. Typical results of these experiments are shown in Figures 1, 2 and 3 for a concentrated load. A photograph of the loaded truck in place adjacent to one of the walls is shown in Figure 4.

[1] Gerber, Emil. "Untersuchungen über die Druckverteilung im örtlich belasteten Sand." Diss. A.-G. Gerb. Leemann, Zurich, 1929.

[2] Spangler, M. G. "Horizontal pressures on retaining walls due to concentrated surface loads." Bul. 140, Iowa Engineering Experiment Station, Iowa State College, Ames, Iowa, 1938.

Similar pressure measurements were made for a uniformly distributed line load parallel to the wall during the studies made at Ames. The results of this load situation are shown in Figure 5. An isometric representation of the pressure distribution for a concentrated load is shown in Figure 6.

A study of the data from the Ames experiments indicates clearly that the lateral pressure on a retaining wall due to a concentrated surface load is distributed in substantially the same manner as that indicated by the Boussinesq formula for horizontal stress in an elastic solid due to a point load on the surface. The mag-

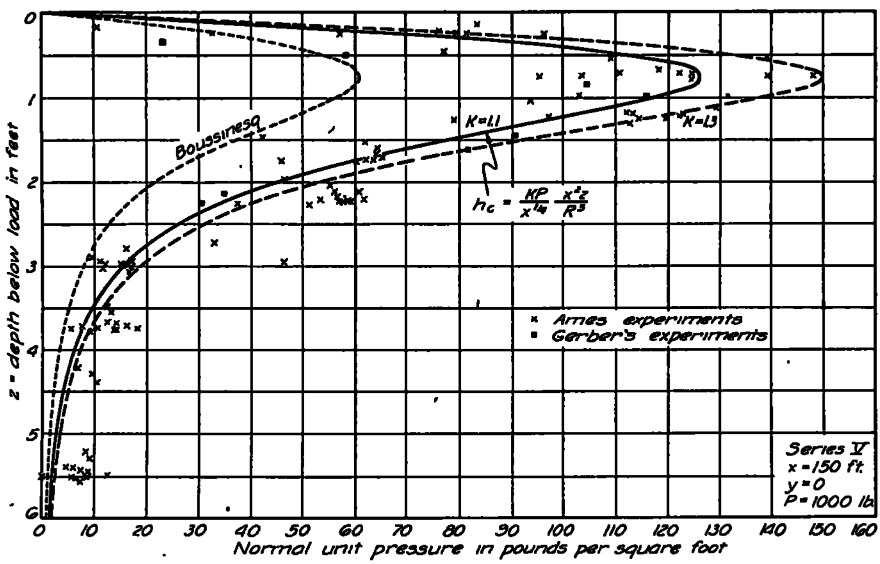


Figure 1. Measured Pressures on a Retaining Wall with a Wheél Load Placed 1.5 Feet from the Wall

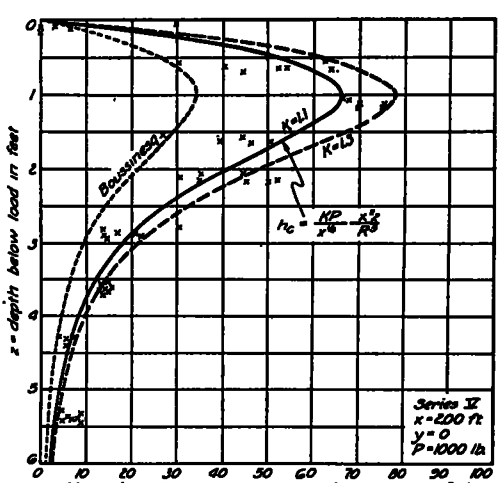


Figure 2. Measured Pressures on a Retaining Wall with a Wheel Load Placed 2.0 Feet from the Wall.

nitude of the experimental pressures was considerably greater than the calculated Boussinesq pressures, however, and this was attributed to the fact that the relatively rigid retaining wall suddenly interrupted the lateral strains in the backfill mass at the plane of the backface of the wall, causing an accumulation of stress which was greater than would have existed on the same vertical plane if the wall had not been present and the gravel mass had been indefinite in extent. The divergence between the experimental and the Boussinesq pressures was a varying quantity being greater when the load was close to the wall.

Boussinesq, a famous elastician of the nineteenth century, solved the problem of stress distribution in a semi-infinite elastic solid due to a point load applied at the boundary plane, but made no suggestion as to the applicability of the solution to stresses in earth masses. The late Professor John H. Griffith suggested the use

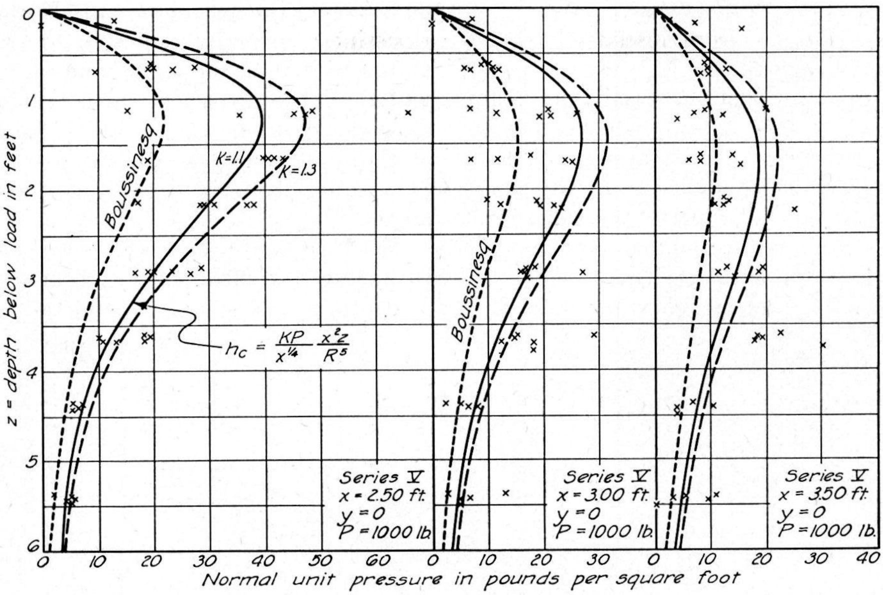


Figure 3. Measured Pressures on a Retaining Wall with a Wheel Load Placed 2.5 Feet, 3.0 Feet, and 3.5 Feet from the Wall

of the Boussinesq theory in the field of soil mechanics in a report prepared for the subcommittee on bearing values of soils of the American Society of Civil Engineers[3] of which he was chairman. Since 1920, many investigators have used the Boussinesq type of formula to express the vertical pressure distribution in soils due to concentrated loads or foundation pressures, and a number of modifications of the basic formulas have been developed.

An empirical equation which defines a surface approximately passing through the points representing the measured normal pressures is

$$h_c = \frac{KP}{x^n} \frac{x^2 z}{R^5} \qquad (1)$$

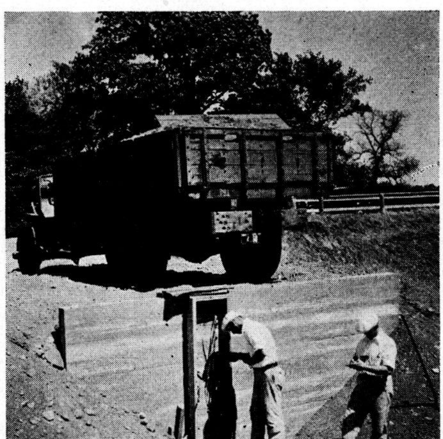


Figure 4. Pulling the Friction Ribbons with the Loaded Truck in Place on the Backfill

[3] Revised report of subcommittee on soils, U. S. Bureau of Standards. Proc. Am. Soc. C. E. 46:916–141. 1920.

in which

h_x = normal unit pressure on the wall at any point

P = applied wheel load

x = distance from load to backface of wall

y = lateral distance from any point on the wall to the normal vertical plane containing the load

z = vertical distance from any point on the wall to the horizontal plane containing the load

$R = \sqrt{x^2 + y^2 + z^2}$

K and n = empirical constants

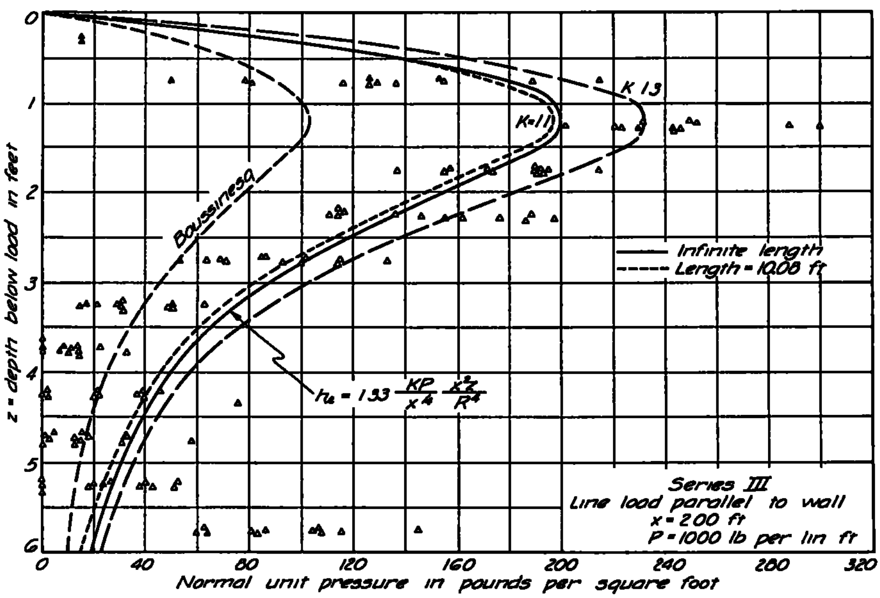


Figure 5 Measured Pressures on a Retaining Wall with a Line Load 10 ft 1 in Long Placed Parallel to and 2 0 Feet Back of the Wall

The second factor of the right hand member of this equation is identical with that of the Boussinesq formula for stress in the x-direction The first factor corresponds to the constant in the Boussinesq formula, but is considerably larger in magnitude in these experiments because of the sudden strain interruption caused by the wall, as mentioned above This factor involves the reciprocal of x^n, because the magnitude of restraint offered by the wall to the normal strains within the gravel mass is relatively greater when the load is near the wall, making the deviation of the actual pressures from the Boussinesq pressures greater for small values of x than for larger values The value of the exponent n is probably dependent upon the relative rigidity of the wall and the backfill material It was found to be $\frac{1}{4}$ in these experiments

The disposable, K, may be considered to include the effect of the interruption of continuity of strains within the backfill mass by the retaining wall, the characteristics of the backfill material, the area of application of the wheel load and its distribution over the area, and other factors It is also a dimensional constant and varies with the units of length in which x, y, and z are expressed, since the equation is dimensionally incorrect if K is introduced as an abstract number It may be written $K(C)^n$, in which C is the number of units of length in one foot

The average values of K in these experiments was about 1 1 in foot units

A uniformly distributed line load parallel to the back of the wall may be considered to be a series of closely spaced

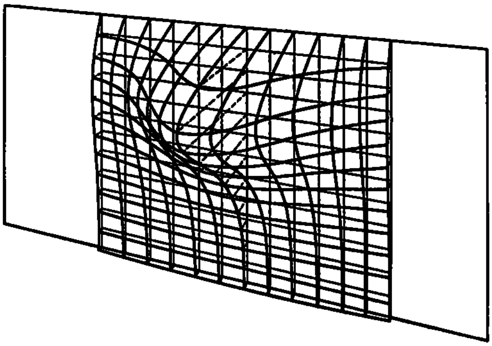
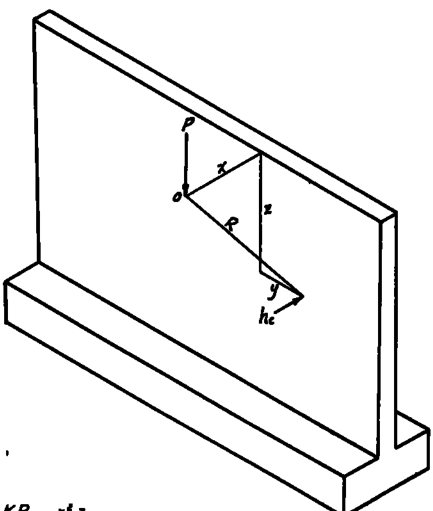


Figure 6 Isometric Representation of Distribution of Lateral Pressure on a Retaining Wall Caused by a Concentrated Surface Load



$$h_c = \frac{KP}{x^n}\,\frac{x^2 z}{R^4}$$

$$h_c = \frac{KP}{x^n}\,\frac{x^2 z}{(x^2+by^2+z^2)^2} \quad \text{(Alternate formula)}$$

Figure 7 Diagrammatic Sketch of a Concentrated Load Adjacent to a Retaining Wall

equal concentrated loads Therefore y in equation (1) may be considered a variable and the expression integrated between appropriate limits to obtain the normal pressure at any point on a wall due to a uniformly distributed parallel line load, as

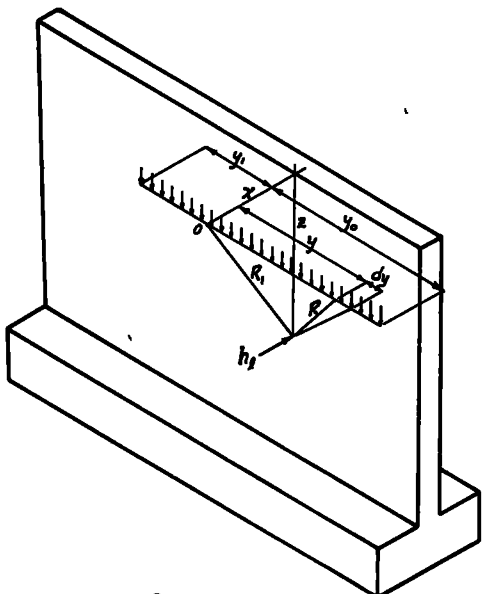
$$h_l = KP \int_{-y_1}^{y_0} \frac{x^{(2-n)} z}{R^5}\, dy \qquad (2)$$

in which

h_l = normal unit pressure on the wall at any point, due to a line load parallel to the wall

P = load per unit length of line

Equation (2) may be written



$$h_l = 1.33\,\frac{KP}{x^n}\,\frac{x^2 z}{R_1^4}$$

$$h_l = \frac{1.33}{\sqrt{b}}\,\frac{KP}{x^n}\,\frac{x^2 z}{R_1^4} \quad \text{(Alternate formula)}$$

Figure 8 Diagrammatic Sketch of a Parallel Line Load Adjacent to a Retaining Wall

$$h_l = KP\,\frac{x^{(2-n)} z}{R_1^4} \int_{\arctan \frac{-y_1}{R_1}}^{\arctan \frac{y_0}{R_1}} \cos^3\theta \, d\theta \qquad (3)$$

in which

$$R_1 = \sqrt{x^2 + z^2}$$

$$\theta = \arctan \frac{y}{R_1}$$

Integration of equation (3) gives

$$h_l = KP\frac{x^{(2-n)}z}{R_1^4}\left[\frac{R_1^2 y_0}{3(R_1^2 + y_0^2)^{1.5}} + \frac{2y_0}{3\sqrt{R_1^2 + y_0^2}} - \frac{R_1^2 y_1}{3(R_1^2 + y_1^2)^{1.5}} - \frac{2y_1}{3\sqrt{R_1^2 + y_1^2}}\right] \quad (4)$$

The maximum pressure on the wall occurs opposite the midpoint of the line load, that is, when $y_1 = y_o$

Then

$$h_l = 2KP\frac{x^{(2-n)}z}{R_1^4}\left[\frac{R_1^2 y_0}{3(R_1^2 + y_0^2)^{1.5}} + \frac{2y_0}{3\sqrt{R_1^2 + y_0^2}}\right] \quad (5)$$

If the line load is very long, $y_o = y_1 = \infty$ and equation (3) may be written

$$h_l = KP\frac{x^{(2-n)}z}{R_1^4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\theta \, d\theta \quad (6)$$

and

$$h_l = 1.33KP\frac{x^{(2-n)}z}{R_1^4} \quad (7)$$

Pressures defined by this equation check very closely with the measured pressures on the experimental wall when loaded by a parallel line load placed 2 feet from the wall, indicating the validity of the principal of superposition and the premise that a line load acts the same as a series of closely spaced point loads

Likewise an area load may be treated as a series of point loads or as a series of parallel line loads and many problems of interest to the designer may be investigated on this basis Thus, for an area $2y_o$ by $(x_1 - x_o)$ as shown in Figure 9, the normal unit pressure on the wall at any point on the vertical element opposite the center of the area will be

$$h_a' = Kpz\int_{-y_0}^{y_0}\int_{x_0}^{x_1}\frac{x^{2-n}}{R^5}\,dx\,dy \quad (8)$$

in which

h_a = normal unit pressure due to an area load applied at the surface

p = surface load per unit of area

Equation (8) has not been integrated in the x-direction However, this expression may be utilized to obtain qualitative ideas of distribution of pressure caused by various kinds of area loads For example, for a uniformly distributed surcharged load of indefinite extent, the load may be considered as a series of infinitely

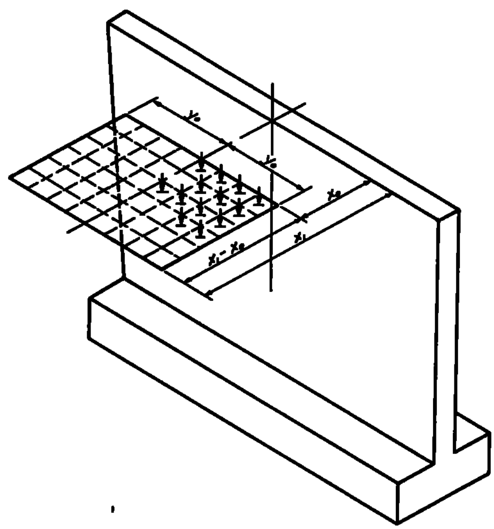


Figure 9 Diagrammatic Sketch of an Area Load Adjacent to a Retaining Wall

long parallel strip loads and equation (8) integrated in the y-direction to yield

$$h_a = 1.33\,KPz\int_0^{\infty}\frac{x^{2-n}}{R_1^4}\,dx \quad (9)$$

This integral is non-existent at the lower limit, and the equation does not indicate the pressure situation near the top of the wall as influenced by the portion of the load immediately adjacent to the wall But by giving dx the finite value of 1 ft, equation (9) can be approximately evaluated arithmetically by summing the pressure caused by a series of strip loads 1 ft. wide and parallel to the wall. The horizontal pressure distribution indicated by this process is considerably at variance with the not uncommon assumption of uniform pressure on a wall due to a surcharged load, as shown in Figure 10. The effect of various distances which the surcharge extends back of the wall is also indicated in this figure.

Another type of problem of widespread interest which may be investigated in this manner is that of a railroad track on the surface of a backfill behind a retaining wall. In this case the load may be considered to be a series of uniformly loaded strips parallel to the wall. As an example, an assumed problem has been worked out with the results shown in Figure 12 for a wall 10 ft. high with the center line of the track 8 ft. back of it. The load was assumed to be equivalent to 1600 lb. per lin. ft. on each of four parallel strips, 2 ft. wide, corresponding roughly to Cooper's E-50 loading with no allowance for impact. The horizontal pressure on the wall for each strip load and the summation of pressure due to all the strips are shown in the figure. Many other special situations

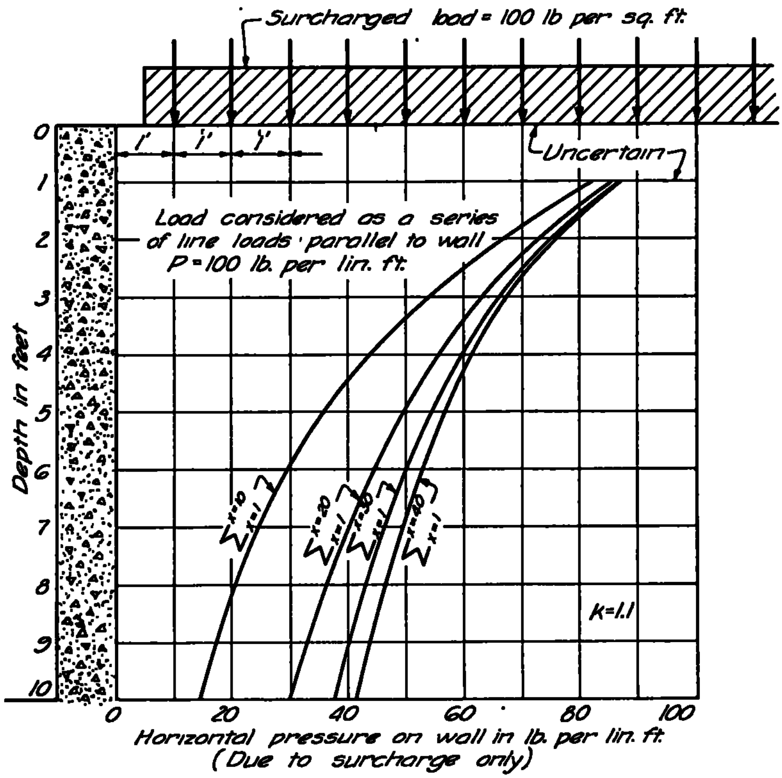


Figure 10. Calculated Pressures on a Retaining Wall Caused by Uniformly Distributed Area Loads Extending Various Distances Back of the Wall

If the surcharged load consists of additional backfill material rising from the wall on a slope as shown in Figure 11, the pressure on the wall may be determined in the same manner as for a uniformly distributed load, except that the strip loads would vary in intensity per unit of length, increasing at greater distances from the wall.

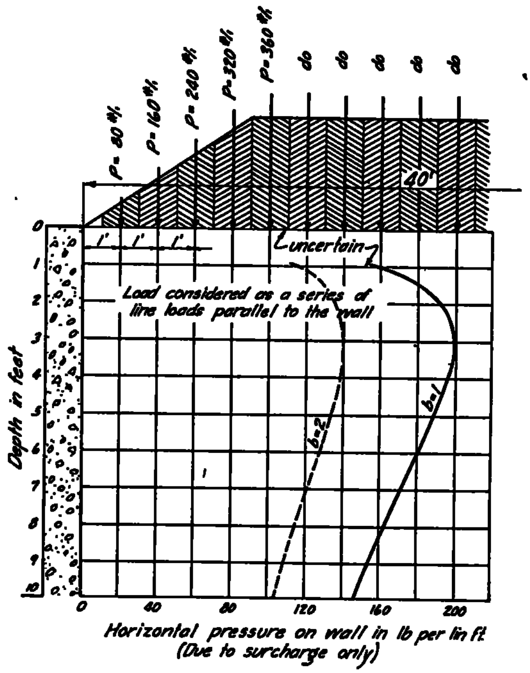


Figure 11. Calculated Pressures on a Retaining Wall Caused by the Portion of a Sloping Surcharged Backfill Which Lies Above the Top of the Wall

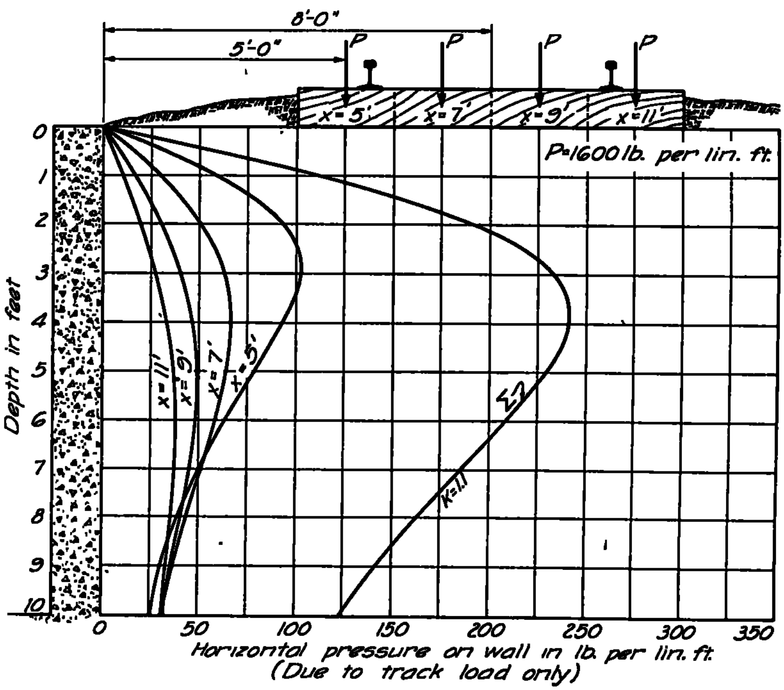


Figure 12. Calculated Pressures on a Retaining Wall Caused by Railroad Track Adjacent to a Retaining Wall

involving surcharged loads may be investigated by this process

It is important to note that the pattern of distribution of horizontal pressures due to surcharged loads is independent of the height of the wall Thus, in the example shown in Figure 12, the center of gravity of the horizontal pressure due to the surcharged load is approximately at the midpoint of the wall, whereas if the wall had been 20 ft high instead of 10 ft, the center of gravity would have been well above the midpoint, because the pressure diminishes rapidly at levels below 10 ft for a track placed 8 ft from the wall

In the research upon which the foregoing method of determining pressures due to surcharged loads is based, remarkable qualitative resemblance was found between the horizontal pressure distribution caused by concentrated surface loads and strip loads and the pressure distribution indicated by the classical Boussinesq equations for stress distribution in an elastic solid This resemblance suggests that a new approach to the problem of retaining wall pressures due to backfill materials might be made upon this same basis In such an hypothesis, the weight of each small incremental volume of backfill material might be considered to be a concentrated load which would transmit an increment of horizontal pressure to a retaining wall through the mass of fill material lying below the increment The sum of the horizontal pressures due to all such increments of volume above any point on a retaining wall would be the pressure at that point

The general form of mathematical expression of this idea would be

$$h = Kw \int_0^z \int_{-\infty}^{+\infty} \int_0^{\infty} \frac{\xi x^{(2-n)}}{(x^2 + y^2 + \xi^2)^{5/2}} \, dx \, dy \, d\xi \qquad (10)$$

in which

h = horizontal pressure on a retaining wall at any depth z below the surface, due to the backfill material

w = unit weight of backfill material

ξ = vertical distance from any incremental volume of backfill down to the depth z

For restricted volumes of backfilled material other appropriate limits than those shown in equation (10) should be used A consideration of this proposition indicates that the pressure on the wall may be affected by the distance which the backfill extends back of the wall to a much greater extent than is indicated by the orthodox wedge theories This hypothesis may provide the basis for an interesting field of experimentation