

STABILITY OF BRIDGE PIERS

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The problem to be considered is that of the stability of bridge piers as dependent on:

1. The supporting earth below the foundation
2. The earth surrounding the shaft of the pier where the subsurface explorations show that the cohesive soils underly the pier to a considerable depth.

It is our purpose to consider first the analysis of the stability of bridge piers founded directly on the soil as a spread foundation as a background for consideration of the factors which enter into the analysis of the stability of bridge piers on pile foundations.

GENERAL

The purpose of a bridge pier is to transmit the heavy concentration of the bridge reaction consisting of vertical and horizontal forces through the pier into the underlying soil. The importance of the bridge foundation problem is indicated by the fact that, while a majority of bridge piers may represent satisfactory construction, a sufficient number have been constructed, where detrimental settlement and lateral movement have occurred to warrant serious consideration. The value of experience gained in the construction of bridge foundations would be immeasurably increased, if systematic, accurate settlement and lateral movement records were kept on all important bridge piers. Such investigations would help toward a better understanding of the phenomena related to the settlement and lateral movement of bridge piers and the information would be very important for design practice because it would enable the engineer to use his judgment and experience in a

more rational way. Such records would be of special value in solving the major problem facing the foundation engineer today, that is in bringing experimental and theoretical soil mechanics to the point where direct application can be made with certainty in bridge foundation design.

The bridge engineer designs a bridge for the particular conditions at the site such as, type and purpose of the bridge, type of loading, the span, the height for clearance, and the conditions of exposure to winds, etc. He furnishes the foundation engineer with the bridge reactions due to dead, live, wind, traction, and impact loads and with the dimensions of the bridge seat.

The foundation engineer takes these design factors, adding to them the particular conditions of exposure of the pier itself to wind, ice, and current pressures and the lateral earth pressures to be found at the site in order to obtain the resultant vertical and lateral forces and the moments acting on the pier. It is usual to consider only a certain fraction of the live load as acting continuously on the foundation (for example 20 per cent). He then selects the type of bridge foundation best suited to meet the particular subsurface conditions at the site.

The design involves an analysis of the stability of the bridge pier under the given forces and moments as affected by the subsurface conditions. This analysis may consist of only the simplest calculations of stability based on the "middle third theory". The pier is considered stable provided that the resultant on the base falls within the middle third and the maximum soil pressure does not exceed some presumptive unit value, which is considered to be safe for the given soil.

Where unusual or doubtful subsurface conditions are encountered such a simplified analysis is entirely inadequate. The situation in certain cases may be such that a rational basis for design is almost impossible.

preliminary subsurface exploration and of the laboratory investigations are:

1. To disclose the nature of the problem
2. To provide the engineer with information so that he can select

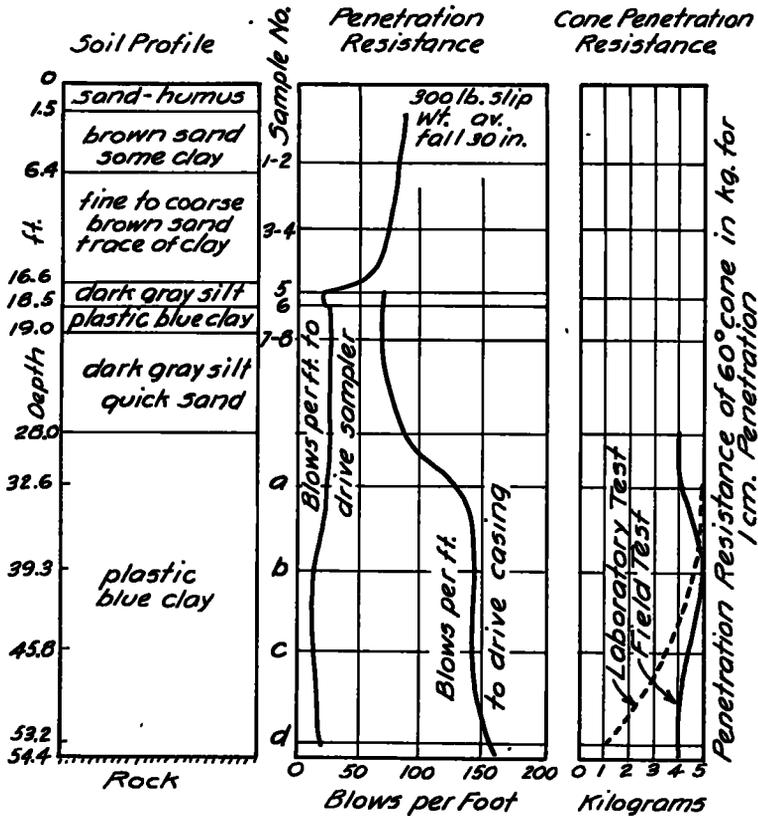


Figure 1. The Soil Profile Showing the Materials Penetrated and the Variation of Consistency as Indicated by the Number of Blows per Foot to Drive the Sampler and the Casing and by the Cone Penetration Test Made in the Field When the Sample Was Taken.

SUBSURFACE SOIL CONDITIONS

The trend in engineering is to make preliminary soil explorations for every important undertaking. Safe and satisfactory construction requires accurate and complete knowledge of subsurface conditions and of the forces and physical factors with which the engineer has to deal, to such depths as may in any way affect the structure. The purpose of the

the most suitable and economical type of foundation.

3. To provide the engineer with design factors as a rational basis for design.
4. To enable the construction engineer to select the most practical and economical method for construction.

Soil investigations may range from the

simplest types for light structures and conditions known to be good, to extensive explorations and laboratory investigations for important structures and serious or doubtful soil conditions. Mohr (1)¹ has pointed out that the extent of exploration necessary in any given case, cannot always be known in advance, but should be determined as the work pro-

gresses and from the conditions actually disclosed by the borings. The simplest soil explorations should provide the engineer with a true record of the soil in the form of a soil profile, which shows the depth, thicknesses, and classification of all the materials encountered. The importance of some definite indications of the compactness of granular soils and of

the consistency of cohesive soils cannot be over-emphasized. A positive resistance basis to penetration either of the sampling spoon or a penetration device should be adopted and used to give quantitative information as indicated in the soil profile of Figure 1.

For cases where a spread foundation is selected and where a satisfactory bear-

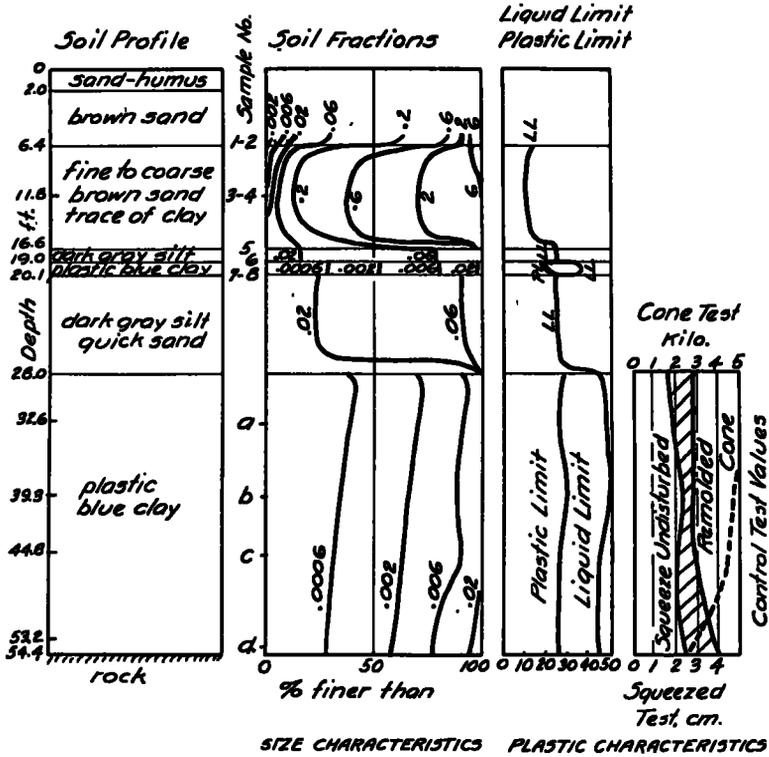


Figure 2. The Physical Characteristics of the Deposit

ing layer is encountered fairly near the surface, the soil explorations may be supplemented by loading tests at the level of the foundation in order to obtain some indication of the allowable bearing pressure. However, this information is of limited value because the soil mass is stressed to only a shallow depth compared with that which occurs under the actual foundation loads.

¹ Numbers in parentheses refer to list of references at end.

Where pile foundations are required,

as is generally assumed for all heavy pier loads and doubtful soil conditions, the soil explorations should be supplemented by carefully conducted pile loading tests. In many cases the number and spacing of the piles have been arbitrarily determined on the basis of an assumed allowable pile load. The piles are then driven in the field until they develop this sup-

layers. The laboratory soil investigations should disclose the nature of the problem—what is physically possible of accomplishment under the given circumstances—and should furnish practical information for design. The record of the physical tests are indicated in the soil profiles of Figures 2 and 3, so that the materials may be positively identified

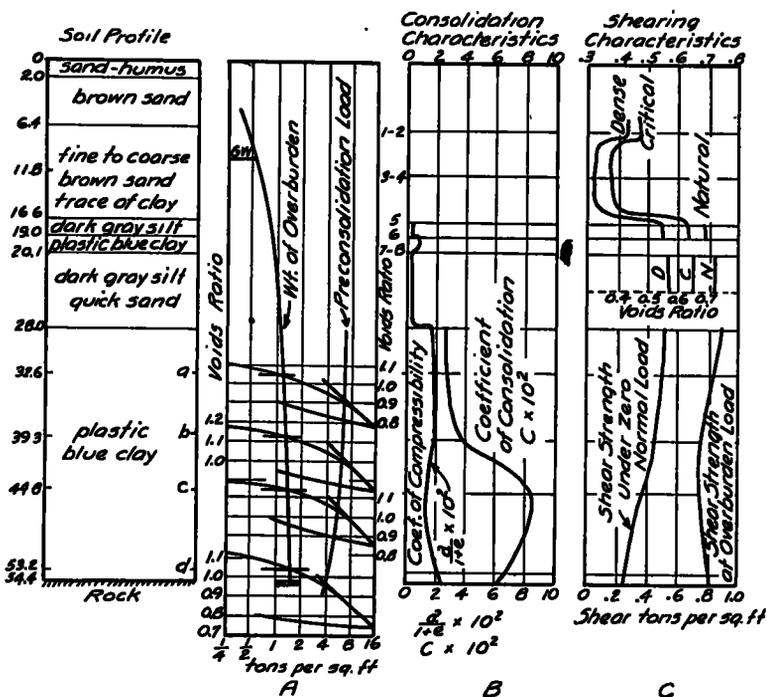


Figure 3. Physical Properties of the Soils in the Deposit. A. Pressure-voids ratio curves, preconsolidation load, weight of overburden. B. Load increment at overburden load, 1-2 tons per sq. ft. C. Shear strength of clays, critical density of the cohesionless soils.

porting capacity as determined by some pile driving formula. Both the number and length of piles required may be badly misjudged under this circumstance. Also the group action of a pile foundation may result in a considerably lower total bearing capacity than that indicated by the assumed value for a single pile.

For all important structures soil conditions should be completely explored with at least one boring to rock to indicate the presence of peat or soft clay

and so that where changes in the character of the soil occur, the nature of the change is definitely indicated. Graphical representation (2) is preferred because it enables one to visualize the subsurface situation better and to study it in detail.

The most important factors are:

1. The variation of the initial stress conditions in the soil, that is, pre-consolidation load, as defined by Casagrande (3). This

pre-consolidation load must be known in order to make a reasonably accurate estimate of settlement due to further consolidation.

2. The variation of compressibility of the different soil layers in the deposit. Tapering or lenticular layers of compressible material will not only cause serious settlement but may cause tilting of the structure by unequal settlement.
3. The variation of the shearing characteristics of the soil as defined by the natural shear strength under the pre-consolidation load and by the apparent angle of internal friction. These characteristics have particular reference to the stability of the pier.
4. The determination of the rate of settlement as defined by the coefficient of consolidation.

A careful study of the variations of these factors with depth and of the critical points of stress is required in order to analyze the problem.

PHYSICAL FACTS FROM SOIL MECHANICS

Settlement-Area Relation

Many of the following physical relations are well established but are brought together here as a background for the analysis. Other relations are obscure and little is known about them. The foundation soil is assumed in this case to be a cohesive soil of clayey character as shown in Figures 1, 2 and 3. Experience shows that the contact pressure of the rigid foundation, of a bridge pier on a stiff clay may be expected to have the distribution shown in Figure 4, (4, p. 66) (5).

Cummings (6, p. 1079, 1028) has shown that the effect of different assumed types of contact pressure distribution become negligible at a depth of twice the footing width or diameter as shown in Figure 5. But the importance of the distribution of contact pressure on settlement has

been pointed out by Terzaghi (7, p. 80) who showed that 80 per cent of the settle-

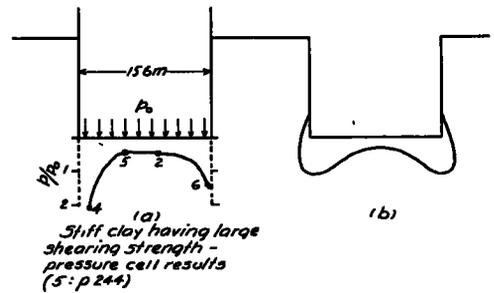


Figure 4. Distribution of Contact Pressure Under a Rigid Foundation on Stiff Clay

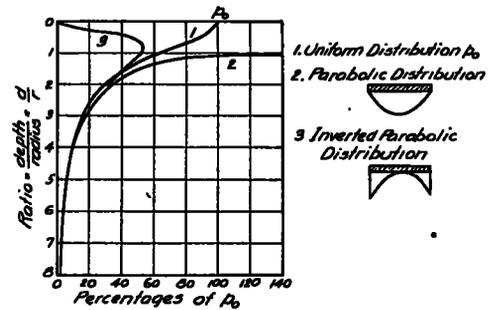


Figure 5. Vertical Normal Stresses as Percentages of p_0 (Ref. 6)

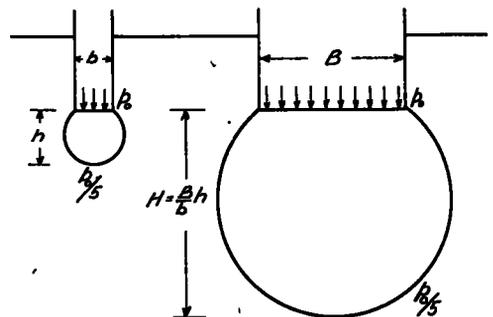


Figure 6. Effect of Size of Loaded Area on the Soil Contributing 80 Per Cent of the Settlement (Ref. 7, p. 80).

ment takes place within a depth of about 1.5 times the breadth of the footing, which corresponds to a pressure contour

of $P/5$ as shown in Figure 6, excepting of course settlement due to consolidation of deeper soil layers. The effect of the size of the loaded areas on settlement has been investigated by Goldbeck (8) Kögler (9) Terzaghi (7) and others, and verified by pressure cells and experience (5). It may be represented by the curves of Figure 7.

The settlement-area relations (11, p. 87) for cohesive soils follows very closely the elastic law expressed by the following

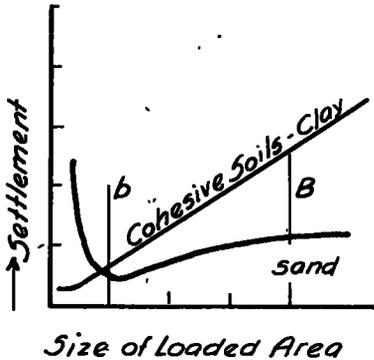


Figure 7. Settlement-Area Relations of Loaded Foundations (Ref. 9, 7). Clay—Settlement increases directly with the width of the foundation. Sand—settlement becomes essentially independent of the width of foundation.

equation (10, p. 339) for the settlement of a rigid circular plate.

$$S = \frac{\pi (1 - \mu^2)}{2 E} p \times b \quad (1)$$

- when p is the unit pressure
- b is the diameter or width of the plate $-\sqrt{A}$, A = Area.
- μ is Poissons ratio
- E is the modulus of elasticity of the soil

The effect of a change of shape of a flexible loaded area from square to rectangular is given theoretically as follows:

$$S_n = \frac{mP(1 - \mu^2)}{E \sqrt{A}} \quad (11, p. 338)$$

TABLE 1
VALUES OF m

Circle	Square	Ratio of Sides of Rectangle					
		1.5	2	3	5	10	100
0.96	0.95	0.94	0.92	0.88	0.82	0.71	0.37

Tilting Effects

The bridge pier problem is further complicated by the fact that the loads are often eccentric and lateral forces produce rotating moments and consequent tilting of the pier. An approximate picture of the distribution of vertical pressure beneath the pier foundation as a result of

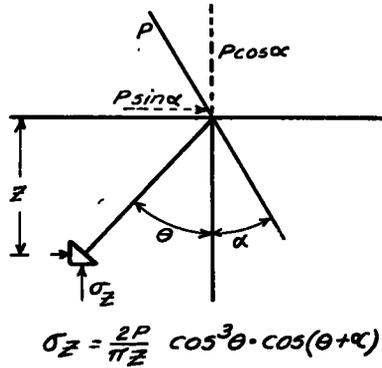


Figure 8. Two Dimensional Elastic Problem of Stress Due to Inclined Loads (Ref. 11, p. 85).

tilting can be obtained from the two-dimensional case of the theory of elasticity (11, p. 82-85) by a combination of line loads perpendicular and parallel to the boundary of the semi-infinite solid as illustrated in Figure 8 and given by Equation (2)

Radial pressure

$$\sigma_r = \frac{-2p}{\pi r} \cos(\alpha + \theta) \quad (2)$$

Vertical pressure

$$\sigma_z = \frac{-2p}{\pi z} \cos^3 \theta \cos(\alpha + \theta)$$

Assuming the contact pressure to be of the form given in Figure 9a, instead of the conventional trapezoidal distribution, the distribution of pressure beneath the pier is obtained by the method of superposition as illustrated in Figure 10.

In view of the fact that there is a concentration of pressure nearer one edge due to eccentricity and rotating moments and because 80 per cent of the settlement occurs within a zone about 1.5 times the

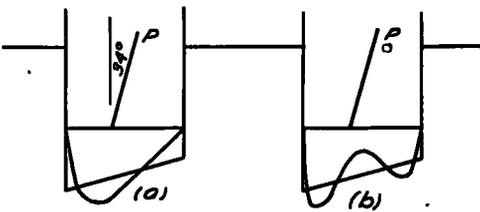


Figure 9. Possible Contact Pressure under Rigid Foundation Due to Eccentric Inclined Loads.

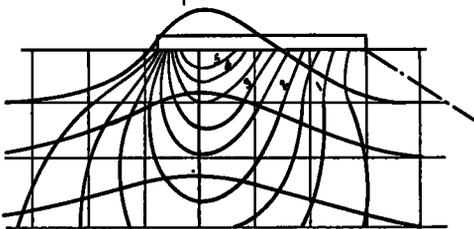


Figure 10. Distribution of Pressure for Eccentric Inclined Loading (Fig. 9(b))

width of the foundation, it may be expected that tilting would be greater for the same reason that the settlement of a larger area is greater than a smaller as indicated in Figures 6 and 7 and by Equation 1. The effect of eccentric loading and of rotating moment on settlement and the amount of tilting, to my knowledge, is not even approximately known. Such relations can only be learned by observation of eccentrically loaded piers.

How much the earth surrounding the shaft of the pier adds to its stability under eccentric loads and rotating moments is

problematical. Lateral loading tests on piles (12, p. 247-252) have shown that quite definite lateral resistance can be counted on, if the unit lateral forces are kept low. In the case of the pier, the shaft is rigid and presents a face of considerable width to the soil. It is reasonable to assume that the pressure developed by lateral movement and tilting is directly proportional to the movement; assuming the modulus of lateral resistance is either uniform or increases with depth. However, an analysis should show first whether this pressure due to tilting is excessive or is reasonably low near the surface of the ground. If it is high, lateral yielding of the soil under the load may decrease this stabilizing pressure so that in plastic soil it should not be counted on for stability.

Settlement Relations

Terzaghi (7, p. 83) has stated that the settlement of a structure founded on a clay deposit may be divided into two component parts S_L and S_V respectively as illustrated in Figure 11. If the material is temporarily incompressible, as is the case for impermeable clays, which consolidate and compress very slowly under load, the increment S_V is temporarily zero and the increment S_L due to lateral yielding without volume change represents the total settlement that occurs immediately under load. The relation between S_L and S_V is defined by Poisson's ratio μ , which is temporarily equal to $\frac{1}{2}$. As consolidation proceeds, it assumes the real value and the vertical settlement increases to $S = (S_L + S_V)$.

Casagrande (13, p. 718) points out that the total settlement may be estimated from the results of the unconfined compression test for S_L and from the consolidation test for S_V . Mason (14, p. 169) has made an important contribution in his correlation of surface loading tests with unconfined compression tests for cohesive soils and as a result has defined

a method for estimating that component of the settlement S_L due to lateral yield from the unconfined compression test. For an isotropic material with Poisson's ratio temporarily equal to $\frac{1}{2}$, the strain as defined by $\frac{S_L}{b}$ is 59 per cent of the

case. The settlement of the pier can be then estimated from the relation:

$$S_L = \frac{(J)}{E_u} pD \quad (3)$$

where J is the strain ratio and E_u is the modulus of elasticity obtained from an unconfined compression test. The strain ratio either has to be assumed or determined from tri-axial loading tests.

The effect of the depth of the foundation below the surface has not been de-

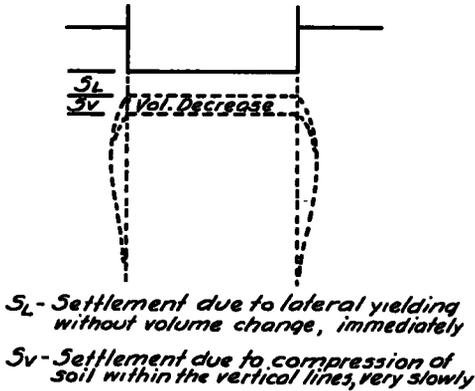


Figure 11. Mechanics of Settlement in Clay (Ref. 7, p. 83)

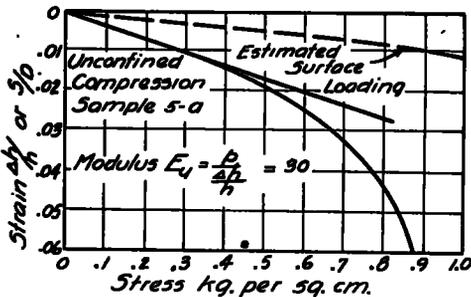


Figure 12. Relation between Unconfined Compression and Surface Loading (Ref. 14, p. 170)

strain obtained in an unconfined compression test as defined by $\frac{\Delta h}{h}$. For natural clay soils which are anisotropic—that is, are stratified and have silt parts—the value of this quantity called the strain ratio- J is as low as 26 per cent as shown in Figure 12, and the settlement consequently is considerably less in this

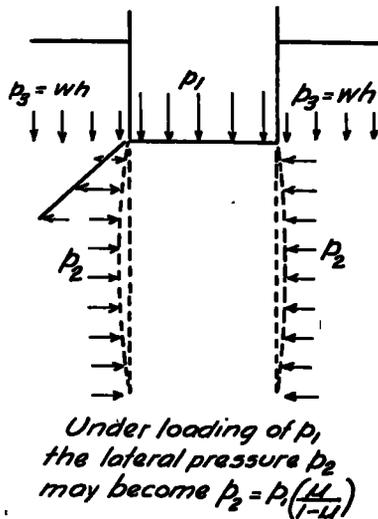


Figure 13. Effect of Depth of Foundation in Decreasing Settlement

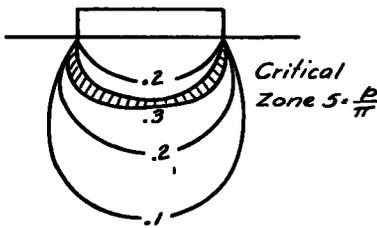
terminated, but the determination is possible from tri-axial loading tests. The conditions in the soil may be represented by Figure 13.

From analogy with the tri-axial compression test, the effect of the surcharge above the level of the foundation, if no appreciable swelling or expansion has taken place in the excavation to cause excessive disturbance, is to reduce settlement because it in effect reduces the value of p in Equation 3 to

$$[p - Cwh] \quad (4)$$

where the value of "C" depends upon the previous history of the deposit and may have the value of unity, where μ is equal to zero, as one limiting value.

The elastic theory, verified by photo-elastic studies, shows that there is a concentration of shear stresses at the edges of a rigid load area (6, p. 149) (15, p. 815-820) (15, p. 227) as shown in Figure 14. Jurgenson (18, p. 215) Hencky, Prandtl, and others have shown that the plastic state begins at the critical point under the edge when the vertical pressure p becomes equal to $\pi \cdot c$ where c is the cohesion or natural shear



Shear Stress in Critical Zone $S = \frac{p}{\pi}$ Plastic State - $p = \pi C$ Rupture - $p > (\pi + 2)C$

Figure 14. Critical Zone of Maximum Shear (Ref. 16, p. 227)

strength. The condition for failure occurs when p exceeds $(\pi + 2)c$ at the edge. This situation is made more serious because of the distribution of contact pressure as illustrated in Figures 4, 9, and 10 where the intensity of the contact pressure near the edge of the footing may be as much as twice the average (5). The effect of lateral confinement due to the surcharge above the level of the foundation increases the stability of the soil mass so that, although the plastic state occurs at the critical points, it is confined and rupture does not take place. An estimate of the effect may be obtained by increasing the pres-

sure from Equation 4, for example, by 100 per cent.

Finally, the settlement due to the consolidation of the underlying soil is computed, having determined the distribution of vertical pressure (17, p. 338-341) and applying the usual method of settlement computation (17, p. 791-797) (18).

PRACTICAL APPLICATIONS

It is assumed that a bridge pier is to be constructed and that Figures 1, 2, and 3 represent the soil conditions at the site. It is further assumed that the pier foundation is to be constructed directly on the stiff clay at a depth of 28 ft. The foundation conditions are analyzed to determine the stability of the pier under the assumed loading indicated in Figure 15. The pressure-voids ratio curves and shear strengths show that this is a highly compressed, stiff clay. The clay deposit probably represents a glacial deposit, which had been consolidated under a temporary re-advance of the ice-sheet in this region.

Foundation Reaction

Settlement of pier,

$$S_1 = \frac{J}{E_u} \times p \times D \quad \text{(Equation 3)}$$

Modulus of elasticity, $E_u = 30$. Figure 12, Sample a.

Coefficient $J = 0.26$, assumed for the clay deposit, which was a varved clay with strong silt partings.

Equivalent width, $D = \sqrt{10 \times 30} = 17.3$ ft.

Settlement,

$$S_L = \left(\frac{0.26}{30} \times 17.3 \right) p = 0.151p \text{ feet.}$$

Foundation pressure, $p = 6.6 S_L$ tons per sq. ft. per ft.

The foundation reaction k_f in $p = k_f S_L$ thus is modified in accordance with

the settlement-area relations for clay soils as indicated in Figure 7. The dimensions of K_f are tons per sq. ft. per ft. of settlement. Under the external loading pressures and tilting moments the

In view of Figure 4 and Figures 9 and 10, however, the tilting will probably be larger than computed, in which case some sort of correction factor should be applied. It seems reasonable to assume in

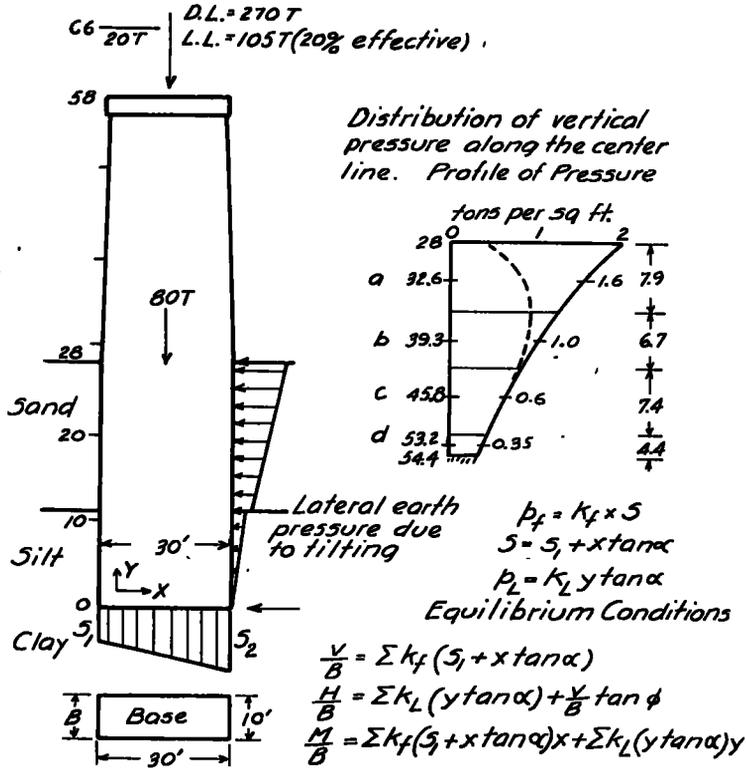


Figure 15. Bridge Pier Design Factors

settlement of the rigid base at any point will be:

$$S_2 = (S_1 + x \tan \alpha)$$

$$S_2 = (S_1 + 30 \tan \alpha) \quad (5)$$

as shown in Figure 15. The conventional trapezoidal distribution of pressure will be given by, (if $\tan \alpha$ is positive)

$$P_{\min} = k_f S_1$$

$$P_{\max} = k_f (S_1 + 30 \tan \alpha) \quad (6)$$

Such equations in effect state that the pressure is directly proportional to settlement, k_f being modified by the settlement-area relations.

such a case with a rigid foundation, that the foundation reaction is a function of (x) , in effect tending to decrease toward the more heavily loaded toe, where the maximum pressure occurs, for example:

$$k'_f = \left(k_f - \frac{Cx}{b} \right) \quad (7)$$

where the constant may be taken as a fraction such as $\frac{1}{10}$, or an inverted parabolic pressure distribution may be introduced.

Lateral Resistance of the Soil

If it is reasonable to count upon the lateral resistance of the soil above the

level of the foundation, then it will furnish a certain resistance against tilting. But one must first be satisfied that there is no possibility of scour or that little plastic flow will take place under the lateral pressures found from the analysis. Furthermore, the conditions of exposure to winds, currents, ice, etc., should be studied to determine how long lateral forces and eccentric loads may reasonably prevail, or whether they are merely momentary conditions. Relations similar to Equation 5 may be assumed for the lateral displacement and lateral pressures developed by tilting.

Lateral displacement, $L = y \tan \alpha$

Pressure, $p = k_L (y \tan \alpha)$ (8)

This assumes that the lateral pressure at any point is equal to the lateral displacement of the rigid pier times the elastic modulus— k_L , of the soil. For sand it has been assumed by a number of authorities that the elastic modulus increases with depth, being zero at the surface where displacement can readily take place. Obviously it will depend on the relative density of the soil. For cohesive soils and stiff clays, it may be assumed that the elastic modulus is constant for each layer.

Sand $k_L = k_f$ (depth to base— y)
const.

Clay $k_L = \text{constant} \times k_f$ (9)

In this case the resistance to driving of the sampling spoon, in Figure 1, indicates that the sand was fairly compact, while the underlying silt was in a loose state and probably above the critical density, as indicated by Figure 3. Because some difficulty would be involved in using an elastic modulus which did not vary according to some simple function, it will be assumed, on the basis of the penetration resistance in the sand, silt, and clay, that in the sand, $k'_L = 1.5 k_f$ and in the silt, $k'_L = k_f$ provided that the pier is

constructed in, and completely fills a sheet pile cofferdam to the level of the ground surface. If on the other hand, the excavation is larger and must be backfilled, then the value of k_L will depend on the backfill material and on its relative density as placed.

On the basis of these assumptions, the following conditions of equilibrium are set up.

$$\frac{V}{B} = \int k_f (S_1 + x \tan \alpha)$$

$$\frac{H}{B} = \int K_L (y \tan \alpha) + \frac{V}{B} \times (\text{coeff. of friction of base on clay})$$

$$\frac{M}{B} = \int k_f (S_1 + x \tan \alpha)x + k_L (y \tan \alpha)y$$

Then integrating

$$\frac{V}{Bk_f} = \left[S_1 x + \tan \alpha \frac{x^2}{2} \right]_0^{30}$$

$$\frac{M}{Bk_f} = \left[S_1 \frac{x^2}{2} + \tan \alpha \frac{x^3}{3} \right]_0^{30}$$

$$+ \left[\frac{k''_L}{K_f} \tan \alpha \frac{y^3}{3} \right]_0^{11} + \left[\frac{k'_L}{k_f} \tan \alpha \frac{y^3}{3} \right]_{11}^{28}$$

$$\frac{455}{10k_f} = 30 S_1 + 450 \tan \alpha$$

$$\frac{8150}{10k_f} = 450 S_1 + 9000 \tan \alpha$$

$$+ 1 \tan \alpha \frac{1331}{3}$$

$$+ 1.5 \tan \alpha \frac{(22000 - 1331)}{3}$$

$$S_1 + 15 \tan \alpha = 1.52/K_f$$

$$S_1 + 44 \tan \alpha = 1.81/k_f$$

$$29 \tan \alpha = 0.29/k_f$$

$$\tan \alpha = 0.01/k_f$$

$$S_1 = (1.52 - 15 \times 0.01)k_f = 1.37/k_f$$

$$S_2 = (1.37 + 30 \times 0.01)k_f = 1.67/k_f$$

$$P_{\min} = (S_1/k_f) \times k_f = 1.37 \text{ tons per sq. ft. approx.}$$

$$P_{\max} = \left(\frac{S_2}{k_f}\right) \times k_f = 1.67 \text{ tons per sq. ft. approx.}$$

$$S_1 = 1.37 \times \frac{12}{6.6} = 2.5 \text{ inches approx.}$$

$$S_2 = 1.67 \times \frac{12}{6.6} = 3.05 \text{ inches approx.}$$

$$\text{Tilting} - 0.01 \times \frac{12}{6.6} = 0.0182 \text{ inches per foot.}$$

The settlement may be somewhat more than these values because of the swelling and expansion of the unloaded foundation soil prior to the time of the placing of the foundation mat, as indicated by the rather steep resaturation and expansion curves in Figure 3.

From Figures 4a and 9b the maximum toe pressure may be twice the above values or about 3.3 tons per sq. ft., which would be considered to be excessive. The maximum shear stress then equals

$$\frac{2p}{\pi} = \frac{2 \times 1.67}{\pi} = 1.05 \text{ tons per sq. ft.,}$$

which exceeds the shear strength of the foundation material of about 0.8 to 0.9 ton per sq. ft., the value under the weight of the present overburden. For a surface loading this would mean that the plastic state had been reached at the toe where the maximum pressure occurs and that rupture was imminent. But at the depth of the foundation assumed, the condition of failure is given by $p_{\max} = 0.8(\pi + 2) = 4.1$ tons per sq. ft. maximum toe pressure, or a slight reserve of shear strength against rupture because

the critical zone is deeper and surrounded by material having a reserve of shear strength.

The lateral pressure on the silt and sand due to tilting is equal to $p_L = k_{LY} \tan \alpha$

$$\text{Silt, } P_{L_{\max}} = 1 \times 11 \times 0.01 = 0.11 \text{ ton per sq. ft.}$$

$$\text{Sand, } P_{L_{\max}} = 1.5 \times 28 \times 0.01 = 0.44 \text{ ton per sq. ft.}$$

The lateral pressure will be the greatest at the surface due to tilting. This value of lateral pressure does not seem excessive for the fairly compact sand encountered here, particularly, since it is somewhat cohesive due to a small amount of clay. The pier will tilt out of plumb under these assumed conditions by $58 \tan \alpha \times 12/6.6 = 1.05$ in., which is not large.

There are two other possibilities, that should be considered concerning the lateral pressure, (1) the lateral resistance can not be counted upon and $k_L = 0$, or (2) the later pressure is low due to a loose backfill. The resultant pressures, settlements, and tilting are as follows under these conditions:

$$(1) K_L = 0$$

$$p_{\min} = 0.65 \text{ ton per sq. ft.}$$

$$p_{\max} = 2.39 \text{ tons per sq. ft.}$$

$$= (\text{times } 2) = 4.78 \text{ tons per sq. ft.}$$

$$\text{Shear stress} = 4.78/\pi = 1.53 \text{ ton per sq. ft.}$$

$$S_1 = 1.2 \text{ in.}$$

$$S_2 = 4.3 \text{ in.}$$

$$\text{Tilting out of plumb in } 58 \text{ ft. of height} = 5.1 \text{ in. (0.105 in. per ft.)}$$

$$(2) k_L \text{ uniform} = k_f/2$$

$$p_{\min} = 1.19 \text{ ton per sq. ft.}$$

$$p_{\max} = 1.85 \text{ ton per sq. ft.}$$

$$S_1 = 2.16 \text{ in.}$$

$$S_2 = 3.41 \text{ in.}$$

$$\text{Tilting out of plumb} = 2.32 \text{ in. (0.04 in. per ft.)}$$

Before deciding on a revision of the size of the base to reduce foundation

pressure or before deciding to use a pile foundation, the settlement due to the consolidation of the underlying clay deposit should be estimated to determine, if the total settlement is excessive. If the foundation pressures are considered to be excessive and the settlement and tilting too large for the structure to take without inducing excessive secondary stresses, then the base must be made larger, or a pile foundation adopted instead. The effect of tilting, unless conditions are prevailing, will not affect the settlement due to consolidation. The pressure distribution along the center line is given in Figure 15. This distribution curve is divided up into a number of sections as indicated for each sample and the average pressure determined as shown. The change in voids ratio from the pressure due to the weight of the present overburden, to the foundation pressures imposed on the soil by the pier are taken directly from the pressure-voids ratio curves similar to those of Figure 3 to a larger scale.

The total settlement, $\sum \Delta h \times \frac{\Delta e}{1 + e}$

Sample a,

$$7.9 \times \frac{0.047}{1 + 1.052} \times 12 = 2.170$$

Sample b,

$$6.7 \times \frac{0.025}{1 + 1.135} \times 12 = 0.942$$

Sample c,

$$7.4 \times \frac{0.012}{1 + 1.15} \times 12 = 0.500$$

Sample d,

$$4.4 \times \frac{0.011}{1 + 1.041} \times 12 = 0.285$$

$$S = \sum \Delta s = 3.897 \text{ in.}$$

The total settlement that might be expected is therefore

$$S_1 = 2.5 + 3.9 = 6.4 \text{ in.}$$

$$S_2 = 3.0 + 3.9 = 6.9 \text{ in.}$$

Whether this settlement is to be considered excessive or not will depend on the type of structure, its rigidity and continuity or lack of continuity, and the effect of relative differential settlement as between adjacent piers upon the secondary stresses induced in the structure.

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