THE STABILITY OF FOUNDATION PILES AGAINST BUCKLING UNDER AXIAL LOAD

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This problem has received but little attention in the literature of foundation engineering in the United States. The subject is frequently mentioned in connection with long slender piles driven through soft soil to bearing on hardpan or rock. It is sometimes assumed that such piles are in danger of buckling as long columns and that the piles must be strengthened or reinforced to prevent this.

It is the purpose of this paper to indicate a method of analysis by which this problem may be studied. In general, the paper will concern itself only with ordinary foundation piles that are completely submerged in soil and are subjected to axial load only. The problem of a foundation pile subjected to a lateral load at the ground surface has been discussed by the writer elsewhere (A).[1]

The analysis for the stability of a pile subjected to axial load will be based on the method developed by Professor S. Timoshenko for the buckling of a bar on an elastic foundation (B). This stability problem has been the subject of a considerable amount of investigation, both theoretical and experimental, by Forssell (C), Granholm (D) and others at the Technical University of Stockholm in Sweden. Because Timoshenko's book is more readily accessible to American engineers, it will be used as the basis of this paper. The Swedish investigations included also the case of a pile partly in and partly above the ground. Such a condition would exist in a dock or a trestle where the piles acted as free-standing columns over a part of their lengths. It is not the purpose of this paper to discuss this problem; the results obtained by the Swedish engineers indicated that it can be analyzed by an extension of the method they used for the completely submerged pile.

THEORY

The theoretical analysis must necessarily be based on certain assumptions about the end conditions of the pile and about the load distribution in the pile as well as in the surrounding soil. Referring to Figure 1, the assumption will be made that the pile is hinged at both ends. It will also be assumed that the entire load, P, is transmitted through the pile to the pile point. In other words, no part of the load, P, is to be thought of as being carried by friction or shear along the sides of the pile. It will be further assumed that the pile is surrounded by a more or less elastic medium that will offer some resistance to lateral displacements of the pile at points along its length.

This resistance of the surrounding medium is one of the most important factors of the problem and it is necessary to consider it in some detail. In practically all of the technical literature dealing with beams and plates on elastic foundations, use is made of a coefficient which is called the modulus of foundation. This coefficient will be designated by K and its meaning will be explained by reference to Figure 2. The explanation will differ slightly from that given by Timoshenko but the reason for the difference will be explained later. In Figure 2(a), the X-Z plane represents the surface of a large body of elastic material. A small part of this surface (QRST) carries a load, w, and this load causes a deflection, y. It is assumed that the

[1] Capital letters in parentheses refer to list of references at end.

deflection is proportional to the force and that the constant of proportionality is the modulus of foundation, K. The relation between w, K and y is then given by the equation

$$w = Ky \quad (1)$$

The modulus of foundation, K, is defined as the force which is required to cause a unit area of the bearing surface to sink a unit distance (E). It is important to keep this definition in mind so that there

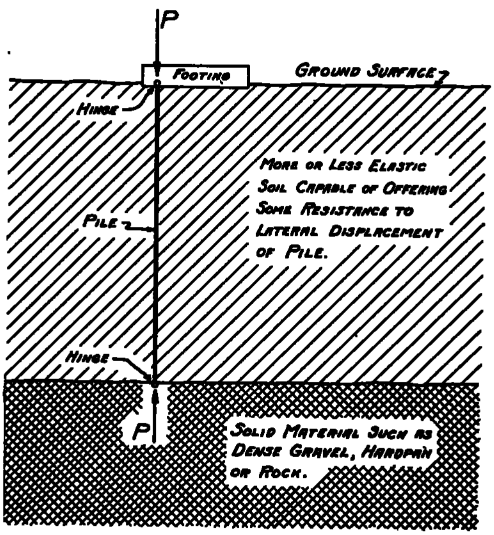


Figure 1

may be no confusion about the dimensions of K. In the figure, pounds and inches have been used for purposes of illustration. Any other units of force and length could be used as long as the units are consistent. The important point is that, by definition, K is a force divided by the cube of a length.

In his discussion of this modulus, Timoshenko expresses w as a force per unit of length so that his K has the dimensions of force per unit of area. In other words, w in Figure 2 (a) would be given in pounds per lineal inch in the X direction. Then, if y were in inches, K would be in pounds per square inch. With this method of expressing K, nothing is said about the third dimension which is in the Z direction. In effect, Timoshenko's method of defining w, y and K represents a two dimensional system. In Figure 2(b), a beam of length, L, and of breadth, b, has been placed on the surface of the large body of elastic

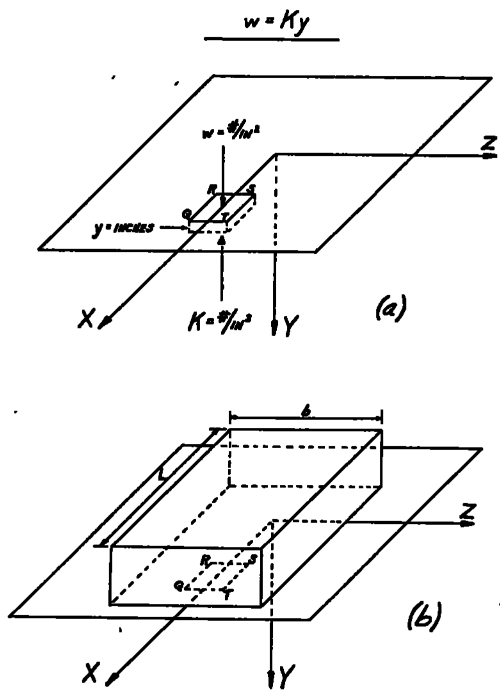


Figure 2

material. When the modulus of foundation, K, is used to study the behavior of this beam, it is necessary to consider the dimensions of the beam in the Z direction as well as those in the X direction. If w has been given as a force per unit of length, it must be understood that some unit of width is included in the definition. This unit of width does not necessarily have to be the same as the unit of length. For example, w could be given in pounds per inch of length per foot of width. However, it is necessary to pay attention to the Z direction because actual problems are usually three dimensional. It is the writer's opinion that the easiest way to avoid dimensional difficulties is to define K as a force divided by the cube of a length as is done in Figure 2(a). It should be noted that Timoshenko's definition of K amounts to the same thing since a definite unit of width must be understood to be a part of his definition of w.

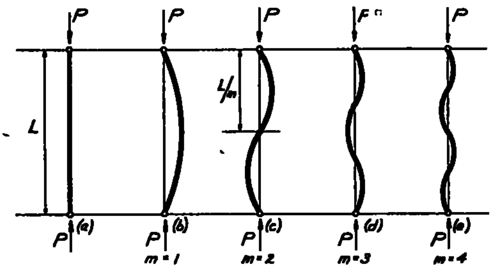
It is easily seen from Figure 2(a) that this definition of K involves an assumption about the nature of the deflection that is produced by the load, w. It has been assumed that deflection occurs only under the area QRST to which the load, w, is applied. Actually, in the semi-infinite elastic isotropic solid, a load, w, applied to the area QRST would cause deflections at other parts of the surface. When this fact is taken into account, the problem becomes involved in serious mathematical difficulties. The analysis leads to integral equations which have been solved only for certain simple conditions on the basis of further assumptions about the nature of the deflections and the elastic properties of the material (F). In this paper, it will be assumed that a modulus of foundation exists as represented by Figure 2 and Equation (1). It will be further assumed that an approximate numerical value for this modulus can be determined by field tests.

Figure 3(a) represents a free-standing column of length, L. The column is hinged at both ends and is loaded with an axial load, P. The critical buckling load for this column is given by

$$P_{CR} = \frac{\pi^2 EI}{L^2} \qquad (2)$$

in which EI is the flexural rigidity of the column. When the column fails it will buckle into a single loop as shown in Figure 3(b). If an elastic restraint were placed at the mid-length of the column, the critical buckling load would be increased and the buckled column would take the form shown in Figure 3(c). With elastic supports at the third points or at the quarter points, the critical buckling load would be further increased and the buckled column would have the forms shown in Figure 3(d) or (e).

In his analysis of this problem, Timoshenko assumes that these elastic sup-



Free-Standing Column (Hinged Ends)

$$(2) \qquad P_{CR} = \frac{\pi^2 EI}{L^2}$$

(P_{CR} = Euler's Buckling Load)

With Surrounding Elastic Medium (Hinged Ends)

(1) $w = Ky$ (K = Modulus of Foundation)

$$(3) \qquad m^2(m+1)^2 = \frac{6KL^4}{\pi^4 EI}$$

(m = Number of Half-Waves)

$$(4) \qquad P_{CR} = \frac{\pi^2 EI}{L^2}\left(m^2 + \frac{6KL^4}{m^2\pi^4 EI}\right)$$

(P_{CR} = Buckling Load)

Figure 3

ports are replaced by a continuous elastic medium which completely surrounds the column. This elastic medium is considered as having a modulus, K, defined by Equation (1). The derivation of the equations is based on a consideration of the strain energy of the system. When the column is deflected, a certain amount of strain energy of bending is stored up in the column. At the same time, the deflections of the surrounding elastic medium cause a certain amount of strain energy to be stored up in the medium. The critical condition for buckling occurs when this internal strain energy is equal to the external work done by the force, P, during the deflection of the column. The solution of the equation obtained by equating the internal and the external work indicates that the column would buckle into a sinusoidal curve. The number of half-waves in the curve is given by the equation:

$$m^2(m + 1)^2 = \frac{bKL^4}{\pi^4 EI} \qquad (3)$$

in which m is the number of half-waves as indicated in Figure 3. It is seen from Equation (3) that the value of m is determined by the elastic properties of the surrounding medium and by the length, the width and the flexural rigidity of the column. It should be noted that the conditions of the problem require m to be a whole number. If Equation (3) leads to fractional values of m, it is necessary to use the next highest whole number.

The critical buckling load for the column completely surrounded by an elastic medium is given by:

$$P_{CR} = \frac{\pi^2 EI}{L^2}\left(m^2 + \frac{bKL^4}{m^2\pi^4 EI}\right) \qquad (4)$$

The factor outside the parenthesis on the right-hand side of Equation (4) is easily recognized as the critical buckling load for the free-standing column with hinged ends. This buckling load is modified by the factor in parenthesis and, since m is always a whole number, it is seen that the effect of the surrounding medium is to increase the buckling load. In the Swedish investigations referred to, the derivation of the buckling load equation was based on the differential equation of the elastic line of the deflected column. This is a fourth order differential equation for which a particular solution was obtained by means of the boundary conditions established by the hinged-end assumption. The resulting equation for the critical buckling load is exactly the same as Equation (4).

NUMERICAL EXAMPLE

Figure 4 is a graphic representation of a set of numerical calculations based on the theory outlined above. The analysis is made for a 12-in. concrete pile of circular cross-section. The pile is 100 ft. long and is hinged at both ends. The Young's modulus of the concrete is taken as 3,000,000 lb. per sq. in. The pile is not reinforced and the moment of inertia of the cross section is 1015 in.^4 The buckling load of this pile as a free-standing column may be calculated from Equation (2) as 20,900 lb.

The curves show the behavior of the pile when it is surrounded by an elastic medium. On the horizontal axis are shown values of w in pounds per square foot when the deflection, y, is 1.0 in. or 0.083 ft. From these values of w and y, the K-curve is calculated from Equation (1). For example, when w is 1,000 lb. per sq. ft. and y is 0.083 ft., K is 12,000 lb. per cu. ft. Since Equation (1) is linear, the K-curve is a straight line.

The m-curve is calculated by means of Equation (3). The area of pile that is bearing horizontally against the soil is taken to be the projected area of the cylindrical pile so that the width, b, is 12 in. With numerical values of b, K, L, E, and I substituted in Equation (3), the right-hand side of the equation reduces to a dimensionless number. The square root of both sides of the equation is then taken and this operation yields a quadratic equation in m. The quadratic equation in m is then solved for m and the positive root is used because the negative root has no physical meaning. Although the m-curve has been plotted as a continuous curve, it must be remembered that m can have only integral values. For example, when w is 500 lb. per sq.

ft. and y is 0.083 ft., the value of K is 6,000 lb. per cu. ft. and m is determined as 3.7 half-waves. This fractional value cannot exist and for this condition m should be taken as 4 half-waves.

The P-curve is determined by Equation (4). This curve shows the great increase in buckling strength that is obtained by surrounding the pile with an elastic medium. For example, consider a soil so soft that a load of 50 lb. per sq. ft. would cause a deflection of 1 in. subjected to a working load in excess of 50 tons. Usually the working load is smaller than this. However, if this 12-in. diameter plain concrete pile 100 ft. long were surrounded by a soil such that a load of 10 lb. per sq. ft. would cause a deflection of 1 in., the pile would be stable against buckling up to a load of about 57 tons. If the surrounding soil were such that a load of 200 lb. per sq. ft. would cause a deflection of 1 in., the critical buckling load would be increased

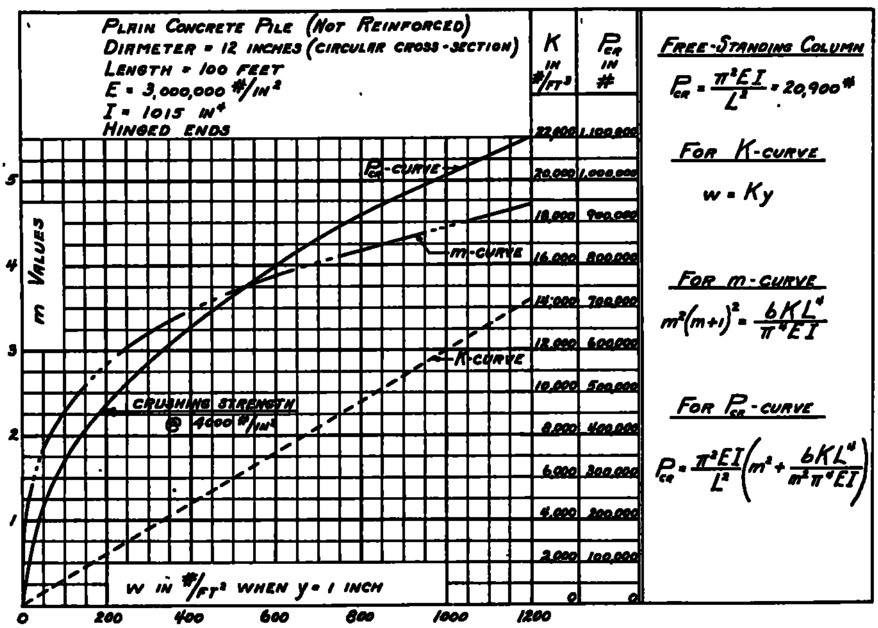


Figure 4

The value of K for such a soil is 600 lb. per cu. ft. For the pile under consideration, the value of m in this soil is 1.88 half-waves. The buckling load is calculated with m taken as 2 and the load is then determined as 236,000 lb. A soil as soft as this would be little better than a swamp and yet, when the pile is surrounded by such a soil, its buckling strength is increased to about 11 times that of the same pile considered as a free-standing column.

Ordinarily, a foundation pile is not to about 450,000 lb. This would be the crushing strength of the 12-in. diameter pile if it were made of 4,000 lb. concrete. From this analysis, it is apparent that the pile would be stable against buckling even though the surrounding soil were exceedingly soft.

In order to check the theoretical analysis, Swedish engineers have made numerous experiments which indicate that the theory is reasonably accurate. In one experiment, (G), a round steel rod $\frac{3}{4}$ in. in diameter was driven vertically through

36 ft. of soft clay to bearing on rock. Because of the manner in which the load platform was fastened to the upper end of the rod, it was assumed that the upper end was fixed against rotation. The lower end was considered to be hinged. For these end conditions, the critical buckling load of the rod as a free-standing column was calculated as 58 lb. The clay with which the rod was surrounded had a natural water content of 45 per cent of the dry weight and its shearing strength was determined as approximately 300 lb. per sq. ft. by the Swedish cone penetration method. The rod was loaded to a total of 6,600 lb. At this load, the measured deflection was 0.29 in. which is but little more than the elastic compression that would occur if the rod were considered as a strut not subject to bending. There was no indication of failure by buckling although the load on the rod was about 113 times the buckling load of the same rod considered as a free-standing column. Many other experiments made with models of various sizes gave similar results and all of the tests demonstrated the validity of the theoretical analysis. Test loads placed on full-sized piles in actual construction work are not usually large enough to be used as a check on the theory. However, Professor Chas. M. Spofford has recently reported (H) a load test on a long slender pile which was driven to rock in Boston Harbor. The pile carried a test load 5 or 6 times as great as its buckling load would have been if it were a free-standing column with hinged ends. There was no indication of failure and stability against buckling was provided by soft blue clay with which the lower $\frac{2}{3}$ of the pile was surrounded. The upper $\frac{1}{3}$ was free-standing.

SUMMARY AND CONCLUSIONS

Before any conclusions are drawn, it seems desirable to review the fundamental assumptions on which the analysis is based. Referring to Figure 1, the basic assumptions were concerned with the load on the pile, the end conditions of the pile and the resistance of the surrounding medium.

As to the load, P, the entire analysis is based on the assumption that all of the load is transmitted to the point of the pile. Ordinarily, this condition does not exist because some of the load is transmitted to the surrounding soil by friction or shear along the sides of the pile. In many cases the pile point does not rest on hard material such as gravel, hardpan or rock. When the total axial load does not reach the pile point, the pile has greater stability than the theoretical analysis indicates. Accordingly, the assumption about P represents the most critical load condition that could possibly exist and the actual conditions will usually be very much more favorable for the stability of the pile.

As to the end conditions of the pile, the assumption was made that both ends were hinged. In some cases, the pile points are driven several feet into hard material and, when this occurs, the lower end of the pile might be considered to be partially fixed against rotation. It could be assumed that the lower end of the pile was elastically clamped. The upper ends of concrete piles and steel piles are usually embedded in concrete for a depth of at least a few inches. The upper ends of wood piles are sometimes embedded in concrete footings and sometimes framed into a wooden superstructure. It would probably not be safe to assume that these fastenings or embedments could fix the upper end of the pile against rotation unless they were actually designed for that purpose. However, the conditions at the upper end of the pile could easily be such that the head of the pile was at least partially restrained against rotation. Any such restraints at the ends of the pile would add to the stability of the pile against buckling. The hinged-end assumption is therefore a more critical condition than would ordinarily occur in actual practice.

Concerning the resistance of the surrounding medium, it was assumed that a modulus of foundation exists and that a numerical value for this modulus could be determined by field tests. The argument for the existence of the modulus is based on the fact that any solid will offer some resistance to volume change and to distortion. A resistance coefficient of some sort must exist and the modulus, K, is simply one form in which this resistance coefficient may be used. The determination of a numerical value for K would probably be made at or near the ground surface. A method for determining the lateral resistance of soils at considerable depths below the ground surface was developed several years ago by Professor F. Kögler (I).

In this connection, it is necessary to keep in mind the fact that the lateral resistance of the soil may vary with the depth. Tests made at the surface of the ground would have to be supplemented by borings or other investigations that would give information about the soil over the full length of the pile. The lateral resistance of the soil might increase or decrease more or less uniformly with the depth or this resistance might vary along the length of the pile in some more complicated manner. In this paper, it has been assumed that K would be constant over the full length of the pile. Therefore, in order to make numerical calculations with Equations (3) and (4), it would be necessary to select some average value of K. It is mathematically possible to consider K as a variable and to define it as some function of the depth. This function would have to be taken into account in the integrations with the result that the mathematical part of the problem might become considerably more complicated, depending on the nature of the function used to express K.

Any analysis of a problem of this kind must necessarily be based on certain simplifying assumptions. However, it is the writer's opinion that the basic assumptions made in this paper represent more critical conditions than any that are apt to be found in ordinary pile-driving operations. It is also the writer's opinion that the foregoing analysis leads to the following conclusions:

(1) In any soil that is capable of supporting an appreciable part of the axial load by friction or shear along the sides of the pile, there is no reason to believe that the pile might buckle as a column or that it should even be considered as a column.

(2) Even in very soft soils overlying rock or hardpan where it is reasonable to assume full point-bearing, the surrounding soil can be exceedingly weak and still be able to provide sufficient lateral stability to prevent buckling under the loads ordinarily used on foundation piles.

(3) In any soil condition where the full point-bearing assumption is justified, a relatively weak soil will provide lateral stability up to the crushing strength of the pile.

In conclusion, the writer wishes to call attention to an important practical matter in connection with this problem. The probability of buckling of foundation piles is often considered by engineers when they are writing specifications or drawing up building codes. Sometimes an arbitrary requirement is set up for the maximum allowable ratio of length to diameter. In other cases, some form of stiffening is required to insure the stability of the pile against buckling. It is the writer's opinion that these requirements are often purely arbitrary and in most cases they are entirely unnecessary.

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