FACTORS AFFECTING THE ACCURACY OF SETTLEMENT RATE FORECASTS

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Great progress has been achieved in the field of settlement forecasts in recent years, as compared to the previous complete inability to make any numerical forecast whatsoever on the matter or even to understand the process of consolidation of clay under applied load. Nevertheless the present known methods do not allow numerically accurate investigation of numerous cases,—I would even be inclined to say of the majority of practical cases. A general review of this field and of its practical limitations may therefore be of interest as well as a discussion concerning the possibility of finding solutions for some special cases not covered by present methods of analysis.

The methods used at present for the numerical forecast of the rate of settlement cover only the settlement component due to the vertical consolidation of a completely water-logged clay layer in direct contact either on one or on both its faces with very permeable sand filter layers.

The engineering profession is indebted to Dr. Terzaghi for the first mathematical analysis of the settlement rate problem under the above special conditions. This analysis, supplemented by laboratory tests, brought out the dependence of the rate of consolidation of a soil layer on its permeability. Various refinements have since been added by several investigators to this first analysis. They concerned variations in the manner of distribution of the total pressure throughout the depth of the layer; that is they covered not only uniform but also triangular and trapezoidal patterns of distribution of the total pressure. An excellent review and new additions to the theory covering the analysis of this special case of perfect filtration at the boundaries of the clay layer has been given in a recent paper by L. A. Palmer and E. S. Barber[1] of the Bureau of Public Roads.

The time required to reach a certain percentage of consolidation increases in proportion to the square of the thickness of the layer and to the first power of its coefficient of compressibility "a"; it decreases with the first power of the coefficient of permeability "k_o"; it further depends on the initial density of the soil, as expressed by its voids ratio.

Figure 1 is intended to help visualise the relationship between the thickness of a clay layer, the time in years required to reach 90 per cent of the final value of consolidation, and the coefficient of permeability of the layer, other conditions being assumed equal and of average value (initial voids ratio $e' = 1.00$; coefficient of compressibility $a = 0.02$; and $N = N_o$, that is, the total pressure is assumed distributed uniformly throughout the depth of the layer.)

It may be seen from Figure 1 that only clays with a reduced coefficient of permeability "k_o" smaller than 1×10^{-1} ft. per yr. have a sufficiently slow rate of consolidation to be of practical importance.

The ideal conditions for the drainage of water from the boundaries of a clay layer, as illustrated by the inset Figure 1a, are not always met in practice. Other conditions may occur and these will now be examined.

RATE OF COMPRESSION OF A CLAY LAYER "SANDWICHED" BETWEEN SOIL LAYERS OF LOW PERMEABILITY

This condition which can very often be met in practice is illustrated by Figure 2.

[1] *Public Roads*, Vol. 18; No. 1, March 1937.

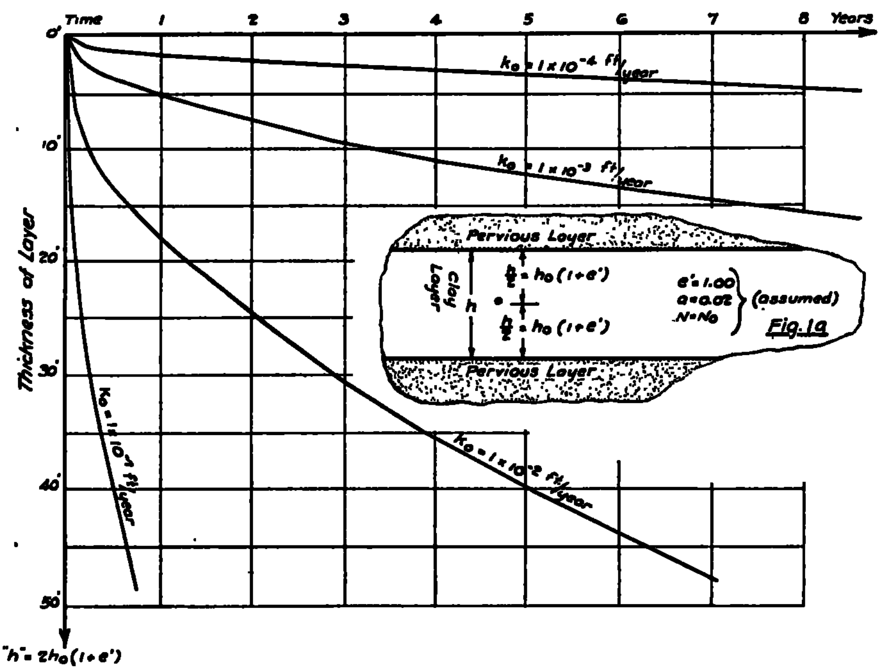


Figure 1. Time Required to Reach 90 per cent Consolidation. Based on permeability only and valid for boundary and other conditions illustrated by 7a

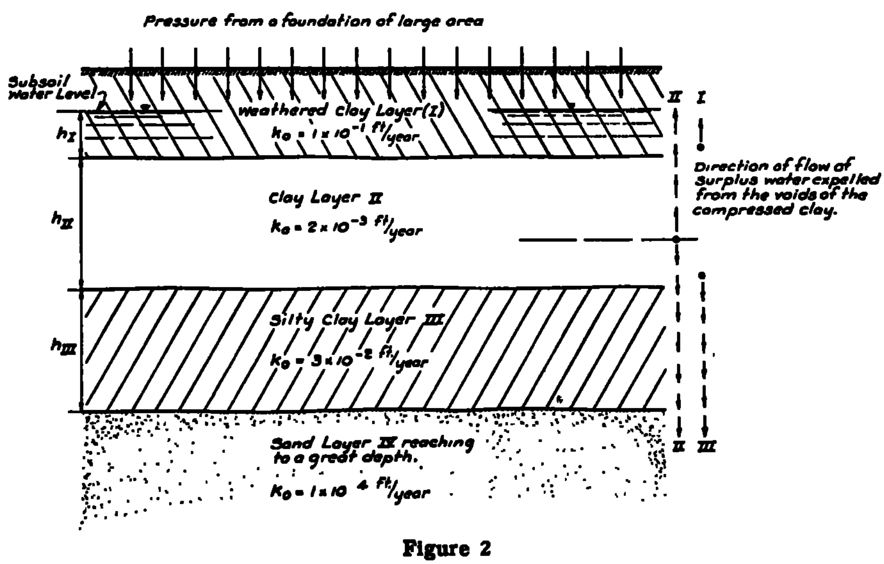


Figure 2

A clay layer (II) is confined between two other clay layers (I) and (III) of greater permeability, which is nevertheless sufficiently low to offer appreciable resistance to the flow of water. All these three layers therefore will consolidate gradually under a newly applied load at a slow but varying rate. The compressibility of the three layers is not the same. The time-settlement curves for each of these layers would have to be determined separately and added together. The direction of the flow of surplus water expelled from the voids of the compressed clay is shown on Figure 2. The surplus water from the clay layer (II) would flow towards both its faces and through the adjoining more permeable clay layers (I) and (III). The surplus water from the latter two would be able to flow only towards one of their faces, away from the layer (II).

Using the methods of computation at our disposal at present, it would be necessary to assume two surfaces of unwatering for layer (II) and one each for layers (I) and (III). These surfaces of unwatering would have to be assumed as providing perfect filtration. For practical purposes this would be approximately correct in respect to the unwatering of layers (I) and (III); but it would not be correct for layer (II).

Let us examine the process of consolidation of this layer, as it would be assumed by the usual methods of computation and as it would be likely actually to occur.

By taking an infinitely small prism and noting that the diminution of the volume of this prism must be equal to the quantity of water that escapes from it, a differential equation has been obtained:

$$c\,\frac{d^2p_w}{dz^2} = \frac{dp_w}{dt} \qquad (1)$$

where

p_w = the unit pressure carried by water alone

z = depth from surface of layer

t = time

c = $\frac{k_o^{II}}{a}$ = coefficient of consolidation.

k_o^{II} = coefficient of permeability of layer (II).

a = coefficient of compressibility of layer (II).

A solution of equation (1) is:

$$p_w = e^{-CK^2t}\,(A\cos Kz + B\sin Kz). \qquad (2)$$

where A, B and K are constants.

The assumption is made in the classical theory of consolidation that the water escapes instantaneously at the boundaries, so that the pressure carried there by the water is always equal to zero. This assumption permits the determination of the constants A, B; and K of equation (2). The above condition of stress distribution throughout the depth of the layer at some intermediate state of its consolidation is illustrated by Figure 3a.

But this ideal condition does not exist in the case of the layer (II) we are considering. The pressure "p_w" carried by the water cannot be equal to zero at the boundaries because of the presence there of layers of low permeability. The distribution of the pressure "p" between the soil skeleton ("$p-p_w$") and the water ("p_w") will therefore follow some pattern similar to that given by Figure 3b.

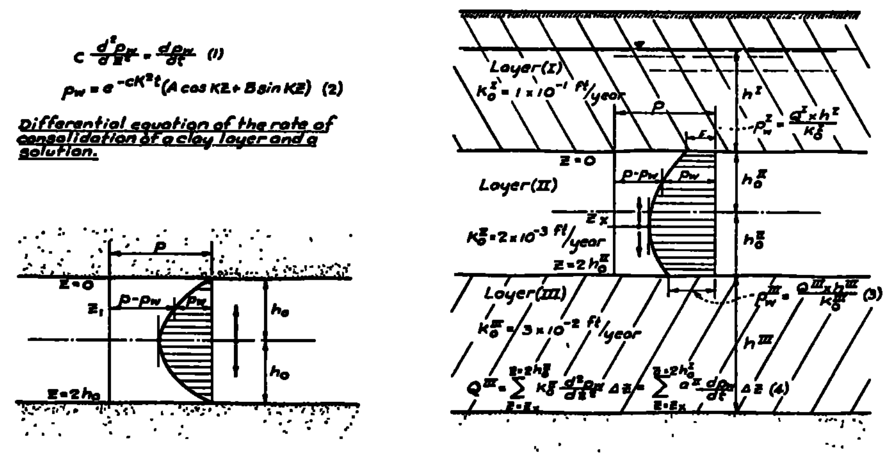
The value of "p_w" at the boundary of the layer "III" is governed by the amount of water "Q_{III}" which flows out in a unit of time from a unit of area:

$$p_w^{III} = \frac{Q_{III} \times h_{III}}{k_0^{III}} \qquad (3)$$

where k_o^{III} is the coefficient of permeability of the boundary layer (III) and h_{III} its thickness. But this amount of water Q^{III} further represents the corresponding decrease in the volume of the layer (II) per unit of time as a result of expulsion of water from its voids. That is, it represents the partial sum of the amounts of water lost in unit time by elementary prisms Δz thick of which the layer is composed:

$$\begin{aligned} Q_{III} &= \sum_{z=z_x}^{z=2h_0^{II}} k_0^{II}\,\frac{d^2p_w}{dz^2}\,\Delta z \\ &= \sum_{z=z_x}^{z=2h_0^{II}} a^{II}\,\frac{dp_w}{dt}\,\Delta z. \end{aligned} \qquad (4)$$

The point “”z_x (see Fig. 3b) where the hydraulic gradient is zero marks the plane below which the water flows towards the layer (III) and above which it flows towards the layer (I). The position of this plane will not coincide with the center plane of the layer but will be displaced towards the greater of the two boundary pressures, —in this case towards “p_w^{III}”—; to an extent depending on their relative values.



a b

Figure 3. (a) Pressure Distribution within a Clay Layer with Ideal Conditions of Drainage at Boundaries. (b) Pressure Distribution within Layer II (See Figure 2) Sandwiched between Layers I and III of Low Permeability.

This consideration and equations (3) and (4) themselves show that the relationship between the factors involved is so complex that the problem of bringing the fundamental equation (2) into a practically usable form does not appear very hopeful of solution in this case. I have not attempted to solve it further; some specialist in applied mathematics may perhaps succeed in doing so.

The preceding discussion as well as a comparison of Figure 3a with Figure 3b shows that in the case represented by Figure 3b the slope of the curve of the pressure “p_w” carried by the water, that is the hydraulic gradient, is much smaller than in the case of Figure 3a which refers to conditions of perfect filtration at the boundaries. It therefore follows that the rate of consolidation of a clay layer confined between soil layers of low permeability should be much slower than will be indicated by the formulas for the rate of consolidation used at present, since these formulas are based on the assumption of perfect filtration at the boundaries.

RATE OF COMPRESSION OF A CLAY LAYER BENEATH A CONCRETE MAT FOUNDATION COVERING A LARGE AREA

A condition similar to the one discussed will exist when a concrete mat foundation of large area rests direotly on a clay layer (Fig. 4) instead of transmitting its loads through an intermediate sand layer providing perfect filtration.

It should be noted here that concrete is far from being perfectly impervious, although this is often assumed to be the case. The permeability of concrete depends on a number of factors, of which the most important ones are the grading of the aggregates and the water-cement ratio of the paste. For normal types of concrete with a water-cement ratio varying from 4.0 to 8.5 gal. per sack the coefficient of permeability of concrete was found by several investigators to vary from 1×10^{-4} ft. per year to 1×10^{-1} ft. per year[2]; that is it may fall within the same range of values as the permeability coefficient of clays likely to cause a rate of settlement sufficiently slow to be of practical importance (see Fig. 1). The reason why concrete is often assumed to be impervious lies in the fact that the coefficient of evaporation is more considerable than the coefficient of permeability of an uncracked concrete. The rate of evaporation is likely to vary approximately from 10 ft. per year to 2×10^{-1} ft. per year, the latter being a value corresponding to an extremely humid surrounding atmosphere. Because of this the surface of normal uncracked concrete will always appear dry,—which is generally interpreted as meaning that it is impervious,—although a steady flow of water may be actually occurring through the concrete towards a surface of evaporation within the mass of the concrete slightly beneath its external surface exposed to air. An interesting analysis of this question is contained in a paper by Dr. Terzaghi published in 1934.[2]

The above considerations tend to show that in the case of a foundation illustrated by Figure 4—that is of a relatively thin concrete mat covering a large area—the surplus water expelled from the voids of the compressed clay layer will flow both downwards towards and into the sand and also upwards towards and through the concrete mat foundation. The conditions of perfect filtration assumed by the consolidation formulas will exist only at the lower boundary of the clay layer where it rests on sand, but this will not be the case at the upper boundary where the clay layer supports the concrete mat. The conditions there will be similar to those already discussed for the case of an adjoining layer of low permeability as shown on Figure 2, illustrated further by Figure 3b. That is, the concrete will offer resistance to the flow of water through it and the pressure "p_w" in the water filling the voids of the clay will, at its boundary, have some positive value greater than zero. Therefore, as shown

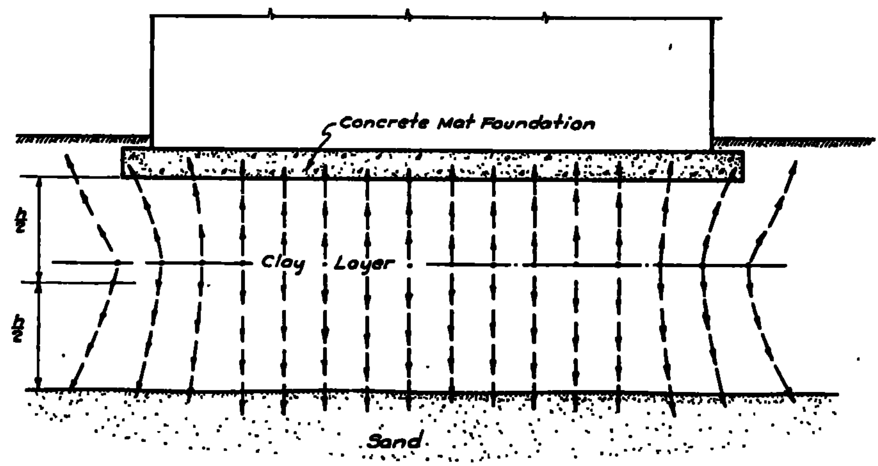


Figure 4. Direction of Flow of Surplus Water Expelled from the Voids of the Compressed Clay beneath a Concrete Mat Foundation of Large Area.

[2] (a) "Zur statischen Berechnung der Gewichtstaumauern" by Dr. K. Terzaghi, in *Die Bautechnik*," 1934, No. 45.

(b) "A simple test for water permeability of concrete" by G. Wiley and D. C. Coulson, *Journal* American Concrete Institute, Sept.-Oct., 1937.

by Figure 4, the separation plane between the upward and the downward direction of flow of the surplus water—that is the plane of zero hydraulic gradient—will not coincide with the center plane of the layer; the length of the path of percolation will be increased and the hydraulic gradient decreased with a resulting slower rate of consolidation.

An exact solution would be just as difficult to obtain here as in the case of an adjoining clay layer of greater permeability already discussed. All that one can say at present is that the rate of settlement should be slower in the case illustrated by Figure 4 than would follow from computations in current use at present; further, that for the same thickness of the concrete foundation mat the above discrepancy should be comparatively small if the depth of the clay layer is great; but that this discrepancy may be quite considerable if the clay layer is comparatively thin.

The value of the ratio of the coefficients of permeability of the clay and of the concrete would also affect the extent of the discrepancy. Concrete less permeable than the clay would increase it and vice-versa. Similar conditions arise during laboratory consolidation tests if porous filter stones of varying thickness and therefore of varying permeability are used for the test. The rate of consolidation may be definitely affected thereby.

RATE OF COMPRESSION OF A CLAY LAYER BENEATH A MASSIVE CONCRETE FOUNDATION COVERING A SMALL AREA

A still more complicated problem is presented by the case of a massive concrete foundation covering a small area, for instance by a bridge pier resting directly on a bed of clay.

Let us first assume that the concrete of the pier is completely impervious and that the bed of clay on which the pier rests either is infinitely deep or that it is underlaid at a finite depth by impervious rock. In this case I am inclined to agree with an opinion I have heard Professor D. P. Krynine express, according to which the surplus water in the voids of the clay would be expelled along the trajectories of minor principal stresses. Figure 5a attempts to illustrate this approximate condition. With the exception of the zone immediately beneath the perimeter of the foundation, at all times the greatest drop in the pressure "p_w" carried by the water—that is the greatest hydraulic gradient within the voids of the clay mass—should coincide

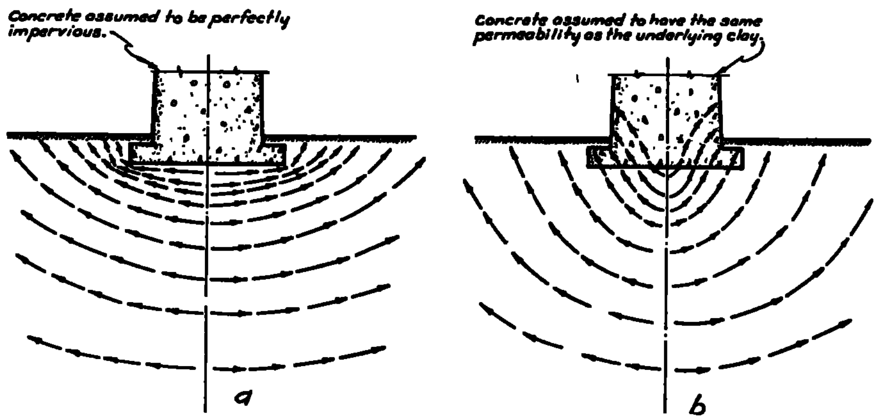


Figure 5. Direction of Flow of Surplus Water Expelled from the Voids of a Deep Clay Layer Compressed beneath a Massive Concrete Pier

with the trajectories of minor principal stresses.

It might conceivably be possible to establish some mathematical expression for the rate of expulsion of water along these trajectories since their approximate shape might be mathematically defined. Needless to say such an expression is likely to be very complicated and cumbersome. I have very little hope that anybody might solve the problem and find a practically usable form for a solution.

Further, even if such a solution were found, it could not be applied in practice because the fundamental assumption on which the flow conditions illustrated by Figure 5a are based—that is of completely impervious concrete in the pier—is incorrect. As already discussed in detail, the permeability of the concrete is likely to be of the same order of dimension as that of the underlying clay. Therefore part of the water would be likely to flow upwards through the concrete. The flow lines would no longer approximately coincide with the trajectories of minor principal stresses in the soil but would follow some pattern similar to the one illustrated by Figure 5b. The exact direction of the flow lines would depend on the relative values of the permeability coefficients of the clay and of the concrete; it would vary with changes in the ratio of these permeability values. As a further handicap, this ratio would be difficult to establish accurately in practice.

The presence of a sand layer at a finite depth beneath the clay would only render things still more indefinite and difficult to express in mathematical form since part of the water would flow downwards towards the sand.

Therefore, there does not appear to be much hope for any exact solution of the case under discussion—that is of the rate of settlement of a massive concrete pier resting on a clay layer. It is however evident that the length of the path of percolation along the flow lines as shown on Figure 5a and b, is greater than would follow from the application of the classical theory to this case—that is if the pier were assumed to provide perfect filtration at its base, so that the water would flow vertically upwards towards it. Accordingly, the hydraulic gradient in the water filling the clay voids would be smaller and the rate of settlement slower for the conditions illustrated by Figure 5 than would follow from the application of the classical methods of computation to this case.

GENERAL REMARKS CONCERNING SETTLEMENT RATE FORECASTS

It should be noted that the rate of the settlement component due to lateral dis-displacement or to plastic flow is not treated by the present methods of analysis. The laws governing the rate of this settlement component have not yet been determined. Full-scale observations appear to show that in some soils this settlement component seems to be considerable (post-office at Bregenz, etc.).

Where the settlement component due to vertical consolidation is concerned, in addition to the special but frequently occurring cases already discussed in this paper, considerable uncertainty exists about the direction of the flow of surplus water in the voids of the stressed clay when piles penetrate into that clay. There is reason to believe that even concrete piles may often act as drainage shafts for the at least partial unwatering of the compressed clay, so that the rate of its consolidation may be substantially accelerated.

Further, little is known concerning the rate of consolidation of a clay layer only partially saturated and containing a large amount of entrapped air in its voids.

Even when a condition fully corresponding to the assumptions of the classical theory of consolidation is actually met in practice—that is when the compressed clay layer is "sandwiched" between two sand layers—an accurate settlement rate forecast still remains difficult. Any engineer who has had the opportunity to carry out himself all the investigations beginning with the extraction of soil samples from bore holes and ending with their testing in the laboratory and tabulation of the results, will know how seldom one meets a really homogeneous clay layer, especially where river deposits are concerned. Variations from 200 to 300 per cent in the values of the coefficients of permeability determined on different samples from the same layer can often be met. The estimation from these isolated and varying results of the average value of the coefficient of permeability of the whole layer is far from being a precise and certain operation. Corresponding discrepancies between the forecast and the observed rate of settlement may therefore be the natural result.

All the preceding discussion serves to show that the question of settlement rate forecasts, like many other problems of applied soil mechanics, seldom lends itself to exact solutions. This fact does not always seem to be realized. For instance, at last year's fall meeting at Boston of the Soil Mechanics and Foundations Division of the American Society of Civil Engineers the writer discussed the so-called "undisturbed soil samplers" and pointed out some inaccuracies in settlement forecasts which had been traced to disturbances caused to the soil specimen in these samplers. A member of the highway department of a large western State then remarked that they had obtained good agreement in their forecasts after work with such samplers and implied that people who did not always get such agreement did not know how to do their tests properly and to interpret them. Other instances may be cited of a similarly dangerous attitude towards all this problem complex.

I said "dangerous," because such an attitude is likely to impress engineers not specialists in this field with an unjustified feeling of security when relying for their designs on numerical data based on present-day methods and tests; also because this attitude creates an erroneous impression that all is well with our present methods of research.

The writer is a firm believer in the great practical value of thorough soil investigations, based on principles of modern soil mechanics, and in the possibility of further advancement of our engineering knowledge through rationally organised research. But he is also convinced that comparatively too much emphasis has been placed lately on the purely theoretical and laboratory phases of research and that the organisation of systematic field observations has been sadly neglected.

In this connection tribute should be paid to the foresight of Dr. Terzaghi,—appropriately referred to recently in the "Engineering News Record" as the "father of soil mechanics." In his book Erdbaumechanik published 13 years ago and giving many outstanding novel contributions to the development of theoretical and laboratory aspects of soil investigations, Dr. Terzaghi nevertheless strongly stressed[3] the fact that soil mechanics could never hope to become an exact science. In its state of perfection it would have to consist of a systematically classified collection of descriptions of separate cases, coupled with correlated records of field observations and laboratory tests. Only a large number of such complete sets of records could give a clear picture of the probable behaviour of structures to be erected under similar conditions in the future. Dr. Terzaghi therefore emphasised the necessity of proper organisation for the collection of such field records and pointed out that further research in soil mechanics was largely a problem of such organisation.

The situation has not changed since

[3] *Erdbaumechanik*, 1925, pp. 5 and 386.

then and Dr Terzaghi has repeatedly expressed such views in his publications Considerable progress has been made both in theoretical studies and in laboratory investigations, but the outstanding efforts of the Foundation Committee of the American Society of Civil Engineers under the chairmanship of Mr Lazarus White for the purpose of collecting settlement records from full-sized structures have been sorely handicapped by the fact that such observations are very seldom made There is little hope of much further advance in the practical application of our knowledge concerning various problems related to settlement of structures unless settlement observations are to be carried out everywhere as a matter of routine on all important new structures

It will be quite a long while before conditions may be created when such observations will become possible on all new private structures But the organisation of such systematic observations should be much easier where government agencies are concerned For instance, the German State Highways have been carrying out for some years routine settlement observations on their new bridges