STRESSES IN CONCRETE RUNWAYS OF AIRPORTS

By H. M. Westergaard

Dean of the Graduate School of Engineering and Gordon McKay Professor of Civil Engineering, Harvard University

SYNOPSIS

The problem of stresses in concrete runways of airports is essentially the same as the problem of stresses in concrete pavements of highways, except that both the wheel loads that must be contemplated and the areas of contact between tire and concrete are greater for runways than for highways. These differences, especially the greater areas of contact, call for a re-examination of the formulas that have been derived for stresses in concrete pavements due to wheel loads. The study is limited to the case of loads at an appreciable distance from edges or joints.

The report of results is a report of "no surprises." The larger area of contact between the concrete pavements of the runway and the tire of each wheel of the landing gear, as compared with the corresponding areas for tires of trucks on highways, call for only very small revisions of the formulas presented by the writer previously, in the Proceedings of the Highway Research Board, 1926, and in *Public Roads,* April 1926 and December 1933. In fact, the needed revisions of the formulas are so small compared with the well-realized uncertainties, including those of the subgrade, that in most cases the corrections may be ignored. Yet the formulas for the corrections are simple and not difficult to use. In the interest of recording the evidence the fairly simple derivations of the formulas are shown at the end of the paper.

The problem of stresses in concrete runways of airports is essentially the same as the problem of stresses in concrete pavements of highways, except that the wheel loads that must be contemplated and the areas of contact between tire and concrete are greater for runways than for highways. Mr. Frank T. Sheets, President of the Portland Cement Association, has pointed out that these differences, especially the greater areas of contact, call for a re-examination of the formulas that have been derived for stresses in concrete pavements due to wheel loads. The study presented here was undertaken at the request of Mr. Sheets.

Analyses of stresses in concrete pavements due to wheel loads were presented by the writer in two articles in *Public Roads* in April 1926 and December 1933 respectively.[1] Extensive tests to examine the validity of these analyses were conducted by L. W. Teller and Earl C. Sutherland of the Division of Tests, Public Roads Administration, and were reported by them in a series of articles under the common title "The Structural Design of Concrete Pavements," *Public Roads,* October, November and December 1935, September and October 1936.

FORMULAS BASED ON THE ASSUMPTION OF A CONSTANT MODULUS OF SUBGRADE REACTION

As in the previous analyses it is expedient to begin by investigating the stresses that are created when the reaction of the subgrade per unit of area at each point is the product of a constant, the modulus of subgrade reaction k, times the deflection z of the concrete slab. The reservations that must accompany the assumption of a constant k were dis-

[1] "Stresses in Concrete Pavements Computed by Theoretical Analysis," *Public Roads,* Vol. 7, No. 2, April 1926, pp. 25-35 (also in *Proceedings,* Highway Research Board, Vol. 5, part I, pp. 90-112); "Analytical Tools for Judging Results of Structural Tests of Concrete Pavements," *Public Roads,* Vol. 14, No. 10, December 1933, pp. 185-188.

cussed at length in the two articles in *Public Roads*, April 1926 and December 1933, in which the previous analyses were presented.

Further assumptions are stated in the following list of notation:

E = modulus of elasticity of the concrete, assumed constant.

μ = Poisson's ratio of the concrete, assumed constant.

k = modulus of subgrade reaction, or reaction per unit of area per unit of deflection, measurable in lb.-in.$^{-3}$, assumed constant.

h = thickness of the concrete slab, assumed constant.

l = radius of relative stiffness, defined by equation (1).

P = total pressure exerted by one wheel, assumed to be applied at a considerable distance from any edge of the slab or joint in the slab.

a = radius of a circle over the area of which P is assumed to be distributed uniformly.

b = radius depending on a and h, defined by equations (2) and (3).

σ = tensile stress at the bottom of the slab directly under the center of the wheel pressure.

σ_1 = value computed for σ by the formulas which were derived in the previous analysis under the assumption of only small values of the radius a.

σ_2 = supplementary stress which must be added to σ_1 to give the proper stress σ; σ_2 will be significant only for larger values of the radius a.

z_0 = deflection of the slab at the center of the load.

z_1 = value of z_0 when a is very small.

The following formulas are quoted from the article in *Public Roads*, December 1933:

$$l^4 = \frac{Eh^3}{12(1-\mu^2)k} \qquad (1)$$

$$b = \sqrt{1.6a^2+h^2} - 0.675h \quad \text{when } a < 1.724h \qquad (2)$$

$$b = a \text{ when } a > 1.724h \qquad (3)$$

$$\sigma_1 = 0.275(1+\mu)\frac{P}{h^2}\log_{10}\left(\frac{Eh^3}{kb^4}\right) \qquad (4)$$

$$z_1 = \frac{P}{8kl^2} \qquad (5)$$

Equations (1), (2), (3) and (5), and the equivalent of equation (4) for $E=3{,}000{,}000$ lb. in.$^{-2}$ and $\mu=0.15$, were also stated in the article in *Public Roads*, April 1926.

The present study has given the following results, satisfactorily applicable when a does not exceed l:

(a) The supplementary stress to be added to σ_1 is approximately

$$\sigma_2 = \frac{3(1+\mu)P}{64h^2}\left(\frac{a}{l}\right)^2 \qquad (6)$$

The result in equation (6) was also derived independently and by a different process of analysis by W. D. Dickinson, Jr., who made a study of the problem at the request of the writer.

(b) Since $1-\mu^2$ varies only slowly with μ, it is justifiable to substitute $\mu=0.15$ in equation (1) for the purpose of computing stresses. This substitution permits the following restatement of equation (6):

$$\sigma_2 = \frac{0.16(1+\mu)Pa^2}{h^3}\sqrt{\frac{k}{Eh}} \qquad (7)$$

(c) The resultant stress

$$\sigma = \sigma_1 + \sigma_2 \qquad (8)$$

is the greatest tensile stress produced by the load P. The radius a would have to be considerably greater than l to cause the maximum tensile stress to occur at any other point than the center of the circle.

(d) The restriction

$$a < l \qquad (9)$$

is found to be approximately equivalent to the condition

$$\sigma_2 < 0.16\sigma_1 \qquad (10)$$

Condition (10) may be used instead of (9) in deciding whether the formulas are within the range of applicability.

(e) The deflection under the center of the load is approximately

$$z_0 = \frac{P}{8kl^2}\left[1 + \left(0.3665 \log_{10}\left(\frac{a}{l}\right) - 0.2174\right)\left(\frac{a}{l}\right)^2\right] \qquad (11)$$

The derivation of equations (4), (6) and (11) is described in the last section of this paper.

NUMERICAL EXAMPLE

To illustrate the use of the formulas the possibility of using a 6-in. slab is investigated. Assume

$$E = 4{,}000{,}000 \text{ lb. in.}^{-2},\quad \mu = 0.15,\quad k = 100 \text{ lb. in.}^{-3} \qquad (12)$$

$$h = 6 \text{ in.},\quad P = 30{,}000 \text{ lb.},\quad a = 13.8 \text{ in.} \qquad (13)$$

The value of a corresponds to a pressure of 50 lb. in.$^{-2}$ over the loaded area. Equations (3), (4), (7), and (8) give

$$\sigma_1 = 626 \text{ lb. in.}^{-2},\quad \sigma_2 = 10 \text{ lb. in.}^{-2},\quad \sigma = 636 \text{ lb. in.}^{-2} \qquad (14)$$

Since σ_2 is less than 16 per cent of σ_1, the formulas are within the range of applicability, which is also verified by computing l from equation (1) and noting that a is less than l. Equations (1), (5), and (11) give

$$l^2 = 858 \text{ in.}^2,\qquad l = 29.3 \text{ in.} \qquad (15)$$

$$z_1 = 0.0437 \text{ in.},\quad z_0 = 0.0404 \text{ in.} \qquad (16)$$

$$\frac{z_0}{z_1} = 0.925 \qquad (17)$$

FORMULA BASED ON COEFFICIENT OF SUBGRADE STIFFNESS

In the article in *Public Roads,* December 1933, it was proposed to introduce a "coefficient of subgrade stiffness," K, defined as

$$K = kl \qquad (18)$$

K is measurable in pounds per square inch and may be less dependent on the properties of the slab than k is likely to be. The following formula is practically a restatement of equation (4) and is quoted from the article in *Public Roads,* December 1933:

$$\sigma_1 = \frac{1.1(1+\mu)P}{h^2}\left[\log_{10}\left(\frac{h}{b}\right) + \frac{1}{3}\log_{10}\left(\frac{E}{K}\right) - 0.089\right] \qquad (19)$$

With the values in the preceding numerical example one finds

$$K = 2940 \text{ lb. in.}^{-2} \qquad (20)$$

and σ_1, σ_2 and σ unchanged.

SUPPLEMENTARY COMPUTATION CONTEMPLATING A REDISTRIBUTION OF THE SUBGRADE REACTIONS

The preceding computations are based on the assumption that the reactions of the subgrade are distributed according to a definite value of the modulus k or the coefficient K. Actually the reactions must be expected to have a different distribution. It is desirable, therefore, to consider the effects of a plausible redistribution of the reactions of the subgrade. The last two sections in the article in *Public Roads,* December 1933, deal with this subject. The following formula for a supplementary stress, to be superposed on σ_1 (or σ) is taken from that article:

$$\sigma_3 = -\frac{15(1+\mu)CP}{h^2}\left(\frac{l}{L}\right)^2 \qquad (21)$$

in which L is a distance and C (denoted Z in the article referred to) is the ratio of reduction of z_1 due to the redistribution of the reactions.

An extreme case is of particular interest; namely, that in which the subgrade behaves as a deep elastic solid with constant modulus of elasticity in compression, E_s, and a constant Poisson's ratio μ_s. The following relations were found to represent this case when a is small.

$$L = 5l, \quad C = 0.3906,$$
$$K = 0.1242 \frac{E_s}{1-\mu_s^2} \tag{22}$$

The present study contemplates larger values of a. It appears to be a plausible estimate that C then should be reduced to the value

$$C = 0.3906 \frac{z_0}{z_1} \tag{23}$$

With the values in the preceding numerical example equations (23) and (17) give

$$C = (0.3906)(0.925) = 0.3613 \tag{24}$$

Then equation (21) gives

$$\sigma_3 = -\frac{15(1+0.15)(0.3613)(30{,}000 \text{ lb.})}{6^2 \text{ in.}^2}\left(\tfrac{1}{5}\right)^2 = -208 \text{ lb. in.}^{-2} \tag{25}$$

so that the resultant stress would be

$$\sigma' = \sigma_1 + \sigma_2 + \sigma_3 = (626 + 10 - 208) \text{ lb. in.}^{-2}$$
$$= 428 \text{ lb. in.}^{-2} \tag{26}$$

The actual resultant stress may be expected to lie between the extremes $\sigma = 636$ lb. in.$^{-2}$ and $\sigma' = 428$ lb. in.$^{-2}$.

LIMITATION OF THE ANALYSIS

It should be kept in mind that the computations presented here deal only with the case in which the load is at an appreciable distance from any edge of the slab or joint. Loads at a joint call for further investigation.

DERIVATION OF THE BASIC FORMULAS

It remains to show the derivation of the basic formulas, (4) and (6) for the stresses and (11) for the deflections.

Let

r = horizontal radial distance from the center of the load, and
z = deflection at any point.

Since z will be a function of r only, Laplace's operator assumes the form

$$\Delta = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} \tag{27}$$

Any unloaded part of the slab must satisfy the equation [2]

$$\frac{Eh^3}{12(1-\mu^2)}\Delta^2 z + kz = 0 \tag{28}$$

By introducing l according to equation (1), equation (28) assumes the convenient form

$$l^4\Delta^2 z + z = 0 \tag{29}$$

Now consider the function

$$Z = z + il^2\Delta z \tag{30}$$

in which $i = \sqrt{-1}$. It is observed that

$$l^2\Delta Z + iZ = i(l^4\Delta^2 z + z) \tag{31}$$

It follows that equation (29) is satisfied if Z is chosen as a function of r satisfying the equation

$$l^2\Delta Z + iZ = 0 \tag{32}$$

and z is taken as the real part of Z. According to equation (30) we may write

$$z = \text{Re}Z, \quad \Delta z = \frac{1}{l^2}\text{Im}Z \tag{33}$$

the symbols Re and Im standing for "the real part of" and "the imaginary part of," respectively.

Equation (32) is satisfied by any Bessel function of order zero with argument $\frac{r\sqrt{i}}{l}$. The Bessel function of interest here is "Hankel's Bessel function" $H_0^{(1)}$. Information concerning this function is given in convenient form in the book of tables by Jahnke and Emde.[3]

[2] See, for example, *Public Roads*, March 1930, p. 4; or, A. Nadai, "Die elastischen Platten" (Julius Springer, Berlin), 1925, especially p. 186.

[3] E. Jahnke and F. Emde, "Funktionentafeln mit Formeln und Kurven" (B. G. Teubner, Leipzig), 1909, especially pp. 96 and 139.

Since Hankel's Bessel function $H_0^{(2)}$ will not be used here, the top index (1) will be omitted in the equations that follow. The function[4]

$$Z=\frac{1}{4kl^2}H_0\left(\frac{r\sqrt{i}}{l}\right) \quad (34)$$

not only satisfies equation (32), but converges toward zero at infinity, and can be shown to have the proper singularity at the origin to account for a concentrated load $P=1$ at that point. For,[5]

$$\frac{dz}{dr}=\frac{d}{dr}(\text{Re}Z)=0 \text{ at } r=0 \quad (35)$$

and for small values of r

$$l^2\Delta z=\text{Im}Z=\frac{1}{4kl^2}\frac{2}{\pi}\log_e\frac{\gamma r}{2l} \quad (36)$$

in which γ is a constant. On a circular section defined by a small constant value of r the vertical shear per unit of length may be computed, by referring to equation (1), as[6]

$$v=-kl^4\frac{\partial\Delta z}{\partial r}=-\frac{1}{2\pi r} \quad (37)$$

making the total shear -1 as it should be.

A load 1 distributed uniformly over the circumference of a circle defined by a constant value of r produces the same values of the deflection z and the curvature Δz at the origin as a load 1 at the origin produces at the distance r from the origin. Consequently, the values z_0 of z and $(\Delta z)_0$ of Δz produced at the origin by a load P distributed uniformly over the area of the circle $r=a$ may be determined by forming the integral

$$W=\int_0^a Z\cdot\frac{P}{\pi a^2}\cdot 2\pi r dr \quad (38)$$

and computing

$$z_0=\text{Re}W \text{ and } (\Delta z)_0=l^{-2}\,\text{Im}W \quad (39)$$

The curvature $(\Delta z)_0$ at the origin defines the bending moment m_0 at the origin per unit of length of section by the formula.

$$m=-\frac{1+\mu}{2}kl^4(\Delta z)_0 \quad (40)$$

This bending moment defines the tensile stress at the bottom at $r=0$:

$$\sigma=\frac{6m_0}{h^2} \quad (41)$$

Consequently

$$\sigma=-\frac{3(1+\mu)kl^2}{h^2}\,\text{Im}W \quad (42)$$

The next step is to express W. One finds by equations (34) and (38)

$$W=\frac{P}{2kl^2a^2}\int_0^a r dr\, H_0\left(\frac{r\sqrt{i}}{l}\right) \quad (43)$$

or

$$W=\frac{P}{2ika^2}\int_0^{\frac{a\sqrt{i}}{l}}\frac{r\sqrt{i}}{l}d\left(\frac{r\sqrt{i}}{l}\right)H_0\left(\frac{r\sqrt{i}}{l}\right) \quad (44)$$

The integral in equation (44) can be expressed in terms of Hankel's Bessel function H_1 of order one, as follows:[7]

$$W=\frac{P}{2ika^2}\left[\frac{a\sqrt{i}}{l}H_1\left(\frac{a\sqrt{i}}{l}\right)+\frac{2i}{\pi}\right] \quad (45)$$

which may be restated as

$$W=\frac{P}{2kl^2}\left[\frac{1-i}{\sqrt{2}}\frac{l}{a}(\text{Re}+i\text{Im})\,H_1\left(\frac{a\sqrt{i}}{l}\right)+\frac{2}{\pi}\left(\frac{l}{a}\right)^2\right] \quad (46)$$

[4] Compare the footnote on page 34 in the article in *Public Roads*, April 1926, which contains references to earlier investigations. Further information concerning the technique of analysis of problems of this type is found in the book by F. Schleicher, "Kreisplatten auf elastischer Unterlage" (Julius Springer, Berlin), 1926, 148 pp., which contains extensive analyses of problems of circular slabs on elastic support.

[5] Jahnke and Emde, *loc. cit.*, pp. 97 and 141.

[6] See for example, *Public Roads*, March 1930, p. 4, where in equations (16), by a misprint, the symbol ∂ is omitted at two places. The first equation should be

$$V_x=-N\frac{\partial\Delta z}{\partial x}$$

[7] Jahnke and Emde, *loc. cit.*, pp. 166 and 98.

By referring to equations (39), (42) and (46) one finds the deflection at the center of the loaded area

$$z_0 = \frac{P}{2kl^2}\left[\frac{l}{a\sqrt{2}}\,(\mathrm{Re}+\mathrm{Im})\,H_1\left(\frac{a\sqrt{i}}{l}\right) + \frac{2}{\pi}\left(\frac{l}{a}\right)^2\right] \quad (47)$$

and the tensile stress at the bottom of the slab under the center of the load

$$\sigma = \frac{3(1+\mu)P}{2h^2}\,\frac{l}{a\sqrt{2}}\,(\mathrm{Re}-\mathrm{Im})\,H_1\left(\frac{a\sqrt{i}}{l}\right) \quad (48)$$

Equations (47) and (48) apply regardless of the size of the loaded circular area, except that when the radius a is small, the value a should be replaced by b defined in equation (2), for reasons explained in the article in *Public Roads*, April 1926.

The function $H_1\left(\frac{a\sqrt{i}}{l}\right)$ can be expressed[8] as the sum of a convergent series of uneven powers of $\frac{a}{l}$ plus the factor $\log_e \frac{\gamma a}{2l}$ times another convergent series of uneven powers of $\frac{a}{l}$; the number γ being such that

$$\log_e\left(\frac{\gamma}{2}\right) = -0.1159 \quad (49)$$

Consequently z_0 and σ in equations (47) and (48) can be expressed similarly by means of series of even powers of $\frac{a}{l}$. These series converge so well that for $a<l$ very good approximations are found by including only the terms of power zero and two. Thereby the following formulas are found:

$$\sigma = \frac{3(1+\mu)P}{2\pi h^2}\left[\log_e \frac{2l}{\gamma a} + \frac{1}{2} + \frac{\pi}{32}\left(\frac{a}{l}\right)^2\right] \quad (50)$$

$$z_0 = \frac{P}{8kl^2}\left[1 + \frac{1}{2\pi}\left(\log_e \frac{\gamma a}{2l} - \frac{5}{4}\right)\left(\frac{a}{l}\right)^2\right] \quad (51)$$

By use of equation (49) equations (50) and (51) are converted readily into equations (4), (6), (8) and (11).

That the bending moments have their maximum value at the center of the loaded circle when a does not exceed l, is ascertained by an examination of the moment diagrams shown in the article in *Public Roads*, April 1926. By interpreting these moment diagrams as influence diagrams it is observed that when a does not exceed l, a small movement of the loaded circular area will always reduce the moments at the original center.

DISCUSSION ON STRESSES IN CONCRETE RUNWAYS

Dr. D. L. Holl, *Iowa State College:* If we were to examine some of the elements in the formula presented, our questions would be with reference to a matter of certain judgments that enter into the expressions involved. In granting that all of the assumptions employed in the original solution are valid, we come to the place where a disturbing infinite stress value arises due to a point load. Here the author introduces another radius known as the equivalent radius of loading b. Am I misinformed that this radius b was derived for slabs having no distributed support but is utilized here with a reaction load underneath the slab?

Dr. H. M. Westergaard, *Harvard University:* Yes, that is quite true. The reaction can be interpreted as a distributed load and the behavior of the slab under a distributed load is reasonably well in ac-

[8] Jahnke and Emde, *loc. cit.*, pp. 97 and 141; numerical table on p. 140.

cordance with the ordinary theory of reflexive slabs so therefore whatever equivalent radius you find for a slab separated at the edge should also be reasonably well applicable to a slab that has distributed support; but that is not so much the point in the present paper. Here we are dealing with large areas of contact; in other words, a load is farther from being a bound load than in the case of a highway and there is no necessity for making a correction.

Dr. Holl: In the introduction of supplementary stresses, it appears to me that the paper would be considerably improved if we could have some rational basis for the judgment factors that enter such as the reduction constant "C," the factor 15 and the radius "L," which are to be employed by the designer in his flexural stress calculations; for example I would not know whether I should use $\left(\frac{l}{L}\right)$ to be $\frac{1}{2}$, $\frac{1}{5}$ or $\frac{1}{10}$, in which case the square of such a small fraction could disturb the equation considerably, or why should we choose that factor $\left(\frac{l}{L}\right)$ and then determine the range within which the reduction constant "C" is employed? I think it would greatly enlarge the interest and usefulness of the paper if we knew these bases.

Dr. Westergaard: I refer you to *Public Roads* December 1933 in which I think a rational reason is given for certain values. The reason is given in that earlier paper. The present paper would have been too long if I had put it in again.

Mr. R. D. Bradbury, *Wire Reinforcement Institute:* It so happens that the factor which Dr. Holl questions was not involved in Dr. Westergaard's paper, since the areas of contact applied by the landing wheels of airplanes are so large that the radius of contact area may be used instead of the so called equivalent radius "b" as is used in analyzing stresses in concrete roads.

Professor M. G. Spangler, *Iowa State College:* The formula which Dr. Westergaard propounded a number of years ago for a corner load situation, indicated that the locus of maximum stress produced by a corner load was some distance back from the corner. Later on, experiments conducted at Ames, in which stresses in the top surface of the pavement slab were measured, verified that conclusion. Also, if my memory serves me correctly, the experiments conducted by the Public Roads Administration did likewise. But it has been noticed that a lot of the pavement corner breaks which develop under traffic do not occur at an appreciable distance back from the corner, but seem to begin fairly close to the corner and the failure progresses toward the interior of the slab until a considerable area adjacent to the corner is affected. Some engineers have called this type of failure, "Map Cracking" although this term is not limited to the cracking in the corner region to which I have reference, and which is illustrated in Figure 1. I have

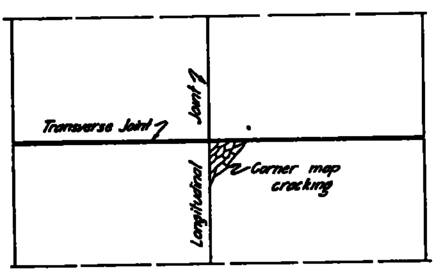

Figure 1. Corner Map Cracking of Concrete Pavements

been very much interested in this phenomenon, wondering why many of these corner failures begin so close to the corner rather than at some appreciable distance back from the corner as indicated by both theory and experiment.

I do not pretend to know the answer to this problem, but would like at this time simply to suggest that perhaps we have not been taking into account all the forces to which a pavement is subjected when a wheel load passes over a corner of the slab, in the development of a stress theory and in the conduct of our experiments. In addition to the vertical gravity loads, both static and impact, which are taken into account in corner load stress calculation, there are certain horizontal or surface shear loads which may exist at the area of contact between a tire and the pavement which are not ordinarily taken into account. These tangential forces may be due to the tractive effort or resistance between the tire and the pavement, and, probably to a much less extent, to stresses accompanying the flexure of a tire as it grips the pavement.

The tangential forces due to tractive effort between a driving wheel and a pavement may be quite high. As a maximum situation, consider a wheel load of 8,000 lb. operating on a dry, rough, concrete surface. Under these conditions, the coefficient of friction between tire and pavement may be as high as 0.7 or 0.8, and if the vehicle is being accelerated rapidly enough, the tangential force may be roughly 5,600 to 6,400 lb. This represents a maximum possible situation and not necessarily a probable case. The direction of this force, when the vehicle is accelerating, will be opposite to the direction of travel. From this possible maximum, the tangential force may range downward through zero, when the vehicle is rolling along without acceleration or deceleration, to a maximum force of equal magnitude in the opposite direction when the vehicle is decelerating or braking at its maximum rate.

Now these tangential forces will produce either tensile or compressive stresses in the top surface of the slab, depending upon their direction, which will combine with the tensile bending stress produced by the vertical load. It has been shown experimentally that the bending stresses due to vertical loads change very slowly along the corner bisector and they may be nearly as great right near the load as at the locus of maximum stress at some distance away from the load, as shown in Figure 2. Therefore, in the immediate vicinity of the load, or even within the contact area, the stresses due to the tangential force may combine with the bending stresses due to the vertical load and reach a maximum combined stress which is greater than maximum stress due to vertical loads alone. These combined stresses may be responsible for the occurrence of "map cracking" in a corner region.

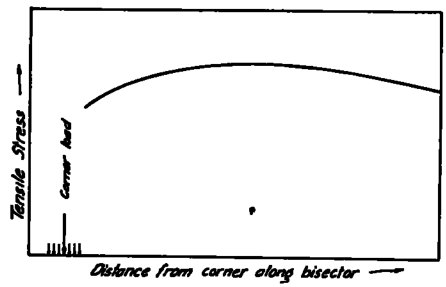

Figure 2. Variation in Bending Stress Caused by Vertical Corner Load

Mr. Bradbury: I have often noticed the condition to which Mr. Spangler refers. It seems to me that these breaks, which occur only a few inches from the corner, might be more a case of aggravated spalling than a true flexural corner break. If they were structural breaks, they would be expected to occur 2 to 3 feet from the corner.

Mr. E. C. Sutherland, *Public Roads Administration:* The Public Roads Administration has made certain studies of the stress conditions occurring in the vicinity of interior corners of concrete pavements. This study included both the stresses caused by loads acting near the corners and those caused by restrained temperature warping. It was found that the longitudinal and transverse joint connections between the four interior corners cause that part of the pavement in the vicinity of the interior corners to act in a manner similar to that of the interior of a pavement slab. That is, a load placed near an interior corner frequently causes a critical stress condition directly under the load while the stresses occurring along the bisector of the corner angle, which are normally high on free corners, may be low at interior corners. This is shown by a specific test in which loads were applied at both the free and interior corners of a pavement slab and the stresses measured directly under the loads and along the bisector of the corner angles. At the free corners the stress directly under the load was so small that it was not measurable while the maximum stress on the bisector of the corner angle, some distance from the load, was 321 pounds per square inch. At the interior corners the average stress directly under the load was 272 pounds per square inch, while the maximum stress on the bisector of the corner angle, some distance from the load, was 84 pounds per square inch.

More complete data, on this subject, were presented in the reports of the Arlington test pavement.[1] In this report the following statement is made concerning the stress conditions at interior corners of concrete pavement slabs.

"It is interesting to note that at the inside corners, where the load stresses along the bisector of the corner angle are very low, the stresses directly under the load become relatively high. This is due to the action of the joints causing the slab at point D to behave more in the manner of the interior of the slab. One joint acting effectively will cause the stresses at this point to be distributed as at a free edge, while with both joints effective a stress distribution more like that which exists in the case of an interior loading is created. Thus the position and magnitude of the critical stress at a slab corner depend upon the action of the joint or joints at that corner. Joints that are very effective in controlling the stress along the bisector of the corner angle may cause a critical stress condition under a load acting near a corner.

"It has already been shown that from the standpoint of reducing warping stresses, free action of the corners at point 'D' is desirable. Such construction would likewise reduce the load stress just discussed and increase slightly the load stress along the bisector of the corner angle of the slab. Therefore, as far as both warping and load stresses are concerned, the joints should be so designed that resisting moments that prevent free flexure are not developed in the joint."

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[1] The Structural Design of Concrete Pavements by L. W. Teller and Earl C. Sutherland, Part 4, *Public Roads*, Vol. 17, Nos. 7 and 8, September and October 1936, Pages 185-6.