THE DISTRIBUTION OF LATERAL EARTH PRESSURES

By M. G. Spangler

Research Associate Professor, Engineering Experiment Station, Iowa State College

SYNOPSIS

Experience in the construction of retaining walls often indicates that the quasi-hydrostatic pressure distribution predicted by the Rankine theory does not always obtain. This theory indicates that the center of the resultant pressure against a retaining wall would be applied at the top of the lower third of the wall. Various experiments have been made that show this center of pressure to be nearer the top of the wall in some cases. It is the author's belief that some of the discrepancies between the Rankine theory and actual experience may be clarified by the hypothesis advanced in this paper, which considers only the horizontal component of the earth pressure.

The resultant of the lateral pressure against a retaining wall may be considered to consist of two components, one due to the pressure of the backfill below the level of the top of the wall, and the other due to the loads superimposed above this level. The first of these components acts at approximately the lower ⅓ point of the wall, the pressure being distributed in accordance with Rankine's analysis for level backfill. The second component may act at any point above the base of the wall, depending upon the point of application of the superimposed load. The pressure distribution will be dependent on the type of the load. The resultant of these two loads may act at almost any point on the wall, but usually in the middle third. In general, the larger the superimposed load and the closer it is to the wall, the higher the resultant pressure will be on the wall.

The character of the lateral forces exerted by earth against any structure which restrains the earth from lateral movement has engaged the interest and attention of scientists and engineers for more than two centuries and the literature is replete with hypotheses, theories, and accounts of experiments and observations of the performance of retaining walls, abutments, and trench sheeting. In spite of this sustained and widespread interest, a basic discrepancy between the classical earth pressure theories of Coulomb and Rankine and the observed performance of some actual structures continues to exist without adequate or widely accepted explanation, while at the same time, many structures do behave as though acted upon by pressures distributed substantially in accordance with these classical theories. Until these contradictory facts are satisfactorily explained, new ideas will continue to be offered for discussion and comparison with experience, and it seems likely that the hope expressed by Mr. George Paaswell (1),[1] who, in his discussion of Mr. Jacob Feld's (2) retaining wall experiments, said " . . . . , makes one hope that it marks the end of the search for the proper method of determining the thrust on a retaining wall," may not be realized for some time to come.

In presenting the hypothesis of this paper it is not the intent of the author to recommend that the classic earth pressure theories be discarded forthwith and a new untried theory substituted therefor. Rather he hopes to supplement these theories in such a manner that the principal differences between them and actuality may be reconciled. Until this or some other plausible supplementary hypothesis has been substantiated by both experiment and the observation of actual performance, certainly the classical theories should not be discarded. They have served engineers too long and too well to be lightly overthrown. However, their age and past usefulness do not

[1] Numbers in parenthesis refer to list of references at end.

exempt them from critical examination and improvement, if that is possible.

The principal discrepancy between Rankine's theory and observed performance seems to be in the distribution of lateral earth pressure which is reflected in the position of the line of action of the resultant pressure against the lateral supporting structure. Mr. J. C. Meem (3), writing as a result of extensive experience in subway and retaining wall construction, concluded that the resultant of the lateral earth pressure to which the sheeting of an open trench or a retaining wall was subjected, acted at an elevation much nearer the top of the trench or wall than is indicated by theory, and, therefore, that the pressure near the top of the structure was much greater than is indicated by the quasi-hydrostatic pressure distribution of the classical theories. On the other hand, Mr. E. P. Goodrich (4) in discussing Mr. Meem's paper, stated: "In a few instances, in the speaker's experience, he has encountered earth pressures which were evidently greater near the ground surface than at lower levels. On the other hand, the opposite condition has been observed in a considerably greater number of cases, and numerous experiments of a very careful nature have shown the close agreement between fact and theory in this regard." The varying experience of Messrs. Meem and Goodrich seems to be indicative of the state of our knowledge of the distribution of lateral earth pressures at the present time.

The experiments conducted by Mr. Jacob Feld at the University of Cincinnati in 1921 and 1922 revealed a similar contradictory situation. Feld's measurements of the position of the resultant of the horizontal pressure on vertical walls loaded by backfills having horizontal surfaces showed the resultant to be at or very near the theoretical height indicated by Rankine's theory and formula. However, in one significant test (test No. 5), wherein additional fill material in the form of a sloping surcharge was placed above the level of the top of the wall, the position of the resultant pressure was raised to a point considerably higher than the lower one third point. In this experiment the backfill was constructed to a height of 5 ft. on the experimental wall with the top surface of the fill sloping downward and backward at an angle of 33 deg. 40 min. Additional material was then added to the backfill and the slope of the surface was varied by about three-degree increments up through the horizontal and on upward until a slope of 34 deg. above the horizontal was attained. The height of fill at the wall remained constant throughout this process. The position of the resultant lateral pressure on the wall was observed to be at the lower one third point for all backfill surface slopes below and including the horizontal. As the upward slope increased, the position of the resultant gradually rose, reaching a height above the base of the wall equal to 0.43 H when the slope was 34 deg. above the horizontal. The data in Table 1 are taken directly from Table 8 on page 1475 of Feld's article. These experiments were very carefully performed, using a "damp" to "humid," nearly cohesionless sand for backfill material.

In 1925, Mr. J. V. McNary (5) published the results of some measurements of lateral pressures on the back of two bridge abutments with horizontal backfills, in which the pressures were measured by means of Goldbeck pressure cells. These measurements revealed a distribution of lateral pressures substantially in accord with that indicated by the Rankine theory.

Mr. Terzaghi (6) has attributed the apparent contradictions between theory and practice to variations in the amount and character of the horizontal yield or movement of a retaining wall or other type of lateral support. His theory of indeterminacy is somewhat difficult to comprehend and apply. It is recognized that every wall will yield to some extent, but it is impossible to predict with any reasonable degree of certainty what the magnitude of lateral yield of a retaining wall will be, within the degree of precision which Terzaghi indicates is necessary. So much depends upon the character of the soil upon which the base of a wall rests.

TABLE 1

DATA SHOWING OBSERVED POSITION OF LATERAL PRESSURE RESULTANT IN TEST NO. 5 OF FELD'S CINCINNATI EXPERIMENTS

Slope of backfill surface	Position of resultant. Ratio of height above base to total height of wall
− 33° 40′	0.33
− 31°	0.33
− 28° 5′	0.33
− 25°	0.33
− 21° 50′	0.33
− 18° 30′	0.33
− 14° 55′	0.33
− 11° 20′	0.33
− 7° 40′	0.33
− 3° 50′	0.34
0°	0.34
+ 3° 50′	0.35
+ 7° 40′	0.35
+ 11° 20′	0.40
+ 14° 55′	0.40
+ 18° 30′	0.40
+ 21° 50′	0.42
+ 25°	0.40
+ 28° 5′	0.44
+ 31°	0.44
34°	0.43

If the author understands correctly the procedure followed in the experiments conducted by Terzaghi (7) and Darwin (8), which demonstrated the effect of the lateral yield of a wall upon the lateral pressure, the experimental walls were held rigid while the backfills were placed behind them. Then with the backfills completed, the walls were permitted to yield by loosening certain restraining devices. The order of occurrence of the yield and the time relationships between pressure and yield as permitted to develop in the experiments, therefore, were not analogous to the situation which prevails in actual retaining wall construction, wherein a substantial proportion of the total yield develops gradually as the backfill is placed. It is at least possible that the effect of yield on pressure on the wall may not have been the same in the experimental trials as in most actual situations. The pressure on a wall which yields gradually as a fill is constructed would probably be greater than the pressure on a wall which is permitted to yield after the fill is completed.

The foregoing discussion of a few selected references in the literature is given for the purpose of illustrating the situation in regard to the relationship between Rankine's theory and actual practice, and the results of experimental research, as it exists today. In brief this situation may be summarized by saying that there is considerable evidence to the effect that the Rankine distribution is realized in many actual construction and experimental cases. At the same time, there is evidence that the Rankine distribution is not realized in other cases, and when it is not, the line of action of the resultant pressure is always above the lower third point and the unit pressure near the top of the restraining structure is greater than indicated by theory.

The author (9) has, during recent years, conducted some experiments at the Iowa Engineering Experiment Station in which the distribution of lateral pressure upon a retaining wall was measured, when loads of various types were applied to the horizontal surface of the gravel backfill behind the wall.

These experiments brought into focus a new concept of the lateral pressure exerted upon a retaining wall by superimposed or surcharged loads applied near the wall, and it is possible that as this concept is developed and grows more clear, some of the discrepancies between the Rankine theory and practical experience, may be clarified. The experiments indicated that the pressure on a retaining wall due to a concentrated load applied at the surface of a level backfill is distributed substantially in accordance with the empirical formula

$$h_c = \frac{KPx^2z}{x^nR^5} \qquad \ldots\ldots\ldots\ldots 1$$

in which

h_c = normal unit pressure on the wall at any point.
P = applied concentrated load.
x = distance from load to back face of wall.
y = lateral distance from any point on the wall to the normal vertical plane containing the load.
z = vertical distance from any point on the wall to the horizontal plane containing the load.
$R = \sqrt{x^2+by^2+z^2}$
K, n, and b are empirical constants. K is a dimensional constant which may be written KC^n, where C is the number of units of length in 1 ft. The other constants n and b are abstract.

The results of these experiments are, in a general way, confirmed by the fact that the pressure distribution indicated by formula 1 is very similar to that indicated by the Boussinesq theory of distribution of a point load through an elastic solid, and by the fact that some laboratory experiments reported by Emil Gerber (10), show lateral pressure distribution similar in both character and magnitude to that measured for one of the load positions in the Ames experiments.

The values of the empirical constants, K and n, observed in these experiments were 1.1 (foot units) and ½ respectively. The evidence concerning the value of b, which has to do with the rate at which pressure due to a concentrated surface load dissipates along the wall in a longitudinal direction, was somewhat limited in the experiments. Such evidence as is available indicates a value of b not less than 1.0 nor more than 2.0. None of these empirical values is conclusive, since only one type of backfill material was used in the study. Much more experimental work and study of retaining wall performance will be necessary before design values of the constants can be recommended. However, the principles upon which the hypothesis of this paper is based have been adequately demonstrated.

If it be assumed that a uniformly distributed parallel strip load acts the same as a series of equal, closely spaced, concentrated loads, formula 1 may be integrated in the y direction,

$$h_1 = KP\int_{-y_1}^{y_0} \frac{x^{(2-n)}z}{R^5}\,dy \qquad \ldots\ldots\ldots 2$$

When y_1 and $y_0 = \infty$, equation 2 becomes

$$h_1 = \frac{1.33KP}{\sqrt{b}}\,\frac{x^{(2-n)}z}{R_1^4} \qquad \ldots\ldots 3$$

in which

$$R_1 = \sqrt{x^2+z^2}$$

The measured pressure on the experimental wall, when a uniformly distributed strip load about eight inches wide and ten feet long was applied on the backfill surface and parallel to the wall, was found to be equal in both magnitude and distribution to the pressure calculated by formula 3, which indicates the validity of the assumption that a strip load acts the same as a series of concentrated loads.

With this experience as a background, we may speculate as to the pressure on a wall due to an area load, such as a building foundation, a railroad track, or simply additional or surcharged fill material, by assuming that an area load acts the same as a series of parallel strip loads. Mathematically, this idea may be expressed by the integral of equation 3 in the x-direction

$$h_a = \frac{1.33KPz}{\sqrt{b}} \int \frac{x^{(2-n)}}{R_1^4} dx \ldots\ldots 4$$

This expression has not been integrated, but it may be evaluated by making a summation of finite load strips about one foot wide in accordance with the formula

$$h_a = \frac{1.33KP}{\sqrt{b}} \sum \frac{x^{(2-n)}z}{R_1^4} \ldots\ldots 5$$

A few experiments, very limited in scope, were conducted in 1938 in which the distributions of the horizontal pressures on a retaining wall were measured when area loads were applied to the horizontal surface of the gravel backfill behind the wall. The pressures were measured by means of stainless steel friction tapes in a manner similar to that reported in Bulletin No. 140 of the Iowa Engineering Experiment Station.

The loads were applied by constructing bins on the backfill surface at the level of the top of the wall and filling them with gravel, which was weighed as it was placed. Then the total weight of bins and gravel was assumed to be uniformly distributed over the area of the bins. These bins were 12 ft. long parallel to the wall and extended back from the wall as much as 20 ft.

The measured horizontal pressures on the wall for various area loads are shown in Figures 1 to 6 and are compared with the pressures calculated by equation 5. Although very limited in extent, these measurements reveal a distribution of horizontal pressure which is similar in character to that indicated by equation 5, and it seems likely that the assumption that an area load acts the same as a series of parallel strip loads will be substantiated when more research data become available.

In all of the experiments and the empirical equations resulting therefrom, and the Boussinesq theory as well, it is evident that the horizontal pressure dis-

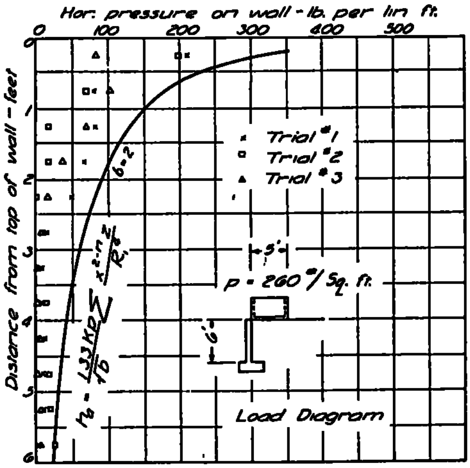

Figure 1

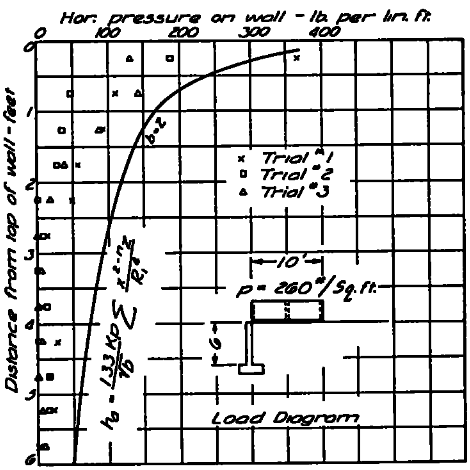

Figure 2

tribution on a wall due to a surface load is not dependent upon the total height of the wall, but that it is dependent to a very great extent upon the position and magnitude of the super-imposed load.

On the basis of the results of these experiments and their extension into the field of area loads, and a study of the evidence provided by the accounts of previous researches and observations of retaining wall performance as they appear in the literature, the following hypothesis which has to do only with the horizontal component of the earth pressures is proposed. The presence or absence of a vertical component of the earth pressure, whether due to friction between the earth and the wall or to some other cause is not under consideration.

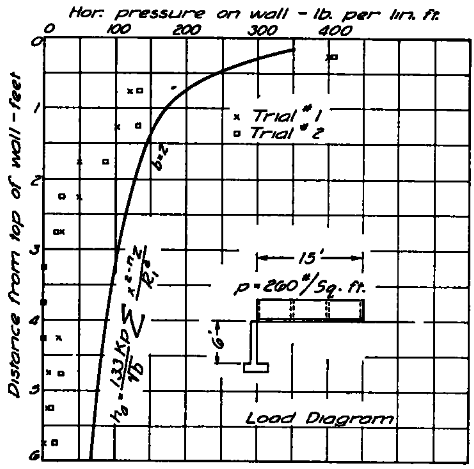

Figure 3

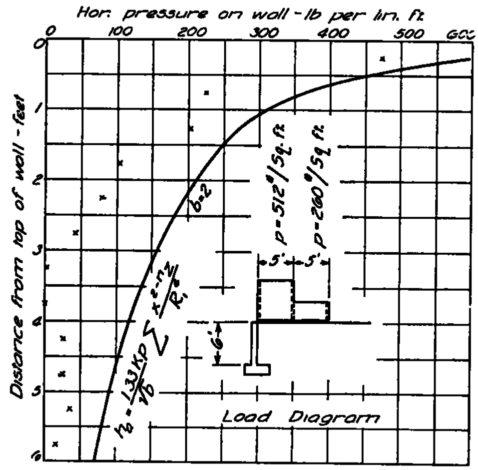

Figure 5

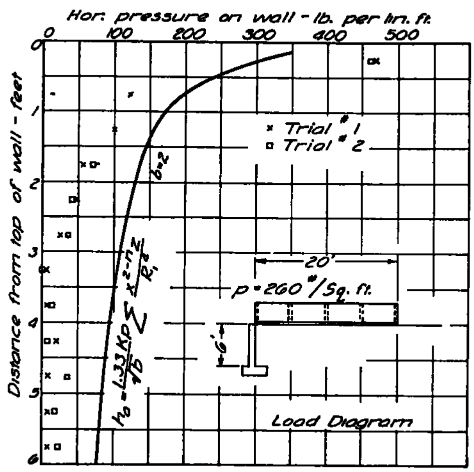

Figure 4

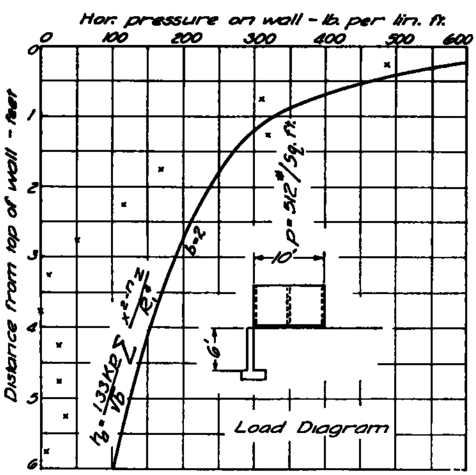

Figure 6

The resultant of the lateral pressure on a retaining wall may be considered to be composed of two components; one due to the pressure of the backfill below the level of the top of the wall and the other due to loads superimposed above the level of the top of the wall and adjacent to it, including surcharge fill loads. The first of these components appears to act approximately at the top of the lower $\frac{1}{3}$ height of the wall, and the pressure is distributed in a quasi-hydrostatic manner in accordance with Rankine's analysis for a level backfill. The second component may act at any distance above the base of the wall depending upon the position, both horizontally and vertically, of the super-imposed load relative to the wall, and the distribution of pressure will be as indicated by formula 1, 3 or 5 depending upon the type of the load. The resultant of these two components may act at almost any surface of the backfill is shown for several positions of the foundation and for two magnitudes of foundation pressure. In each case the lateral pressure of the earth backfill is assumed to be distributed in accordance with Rankine's theory, with the resultant pressure acting at the top of the lower ⅓ height of the wall. Then the lateral pressures due to

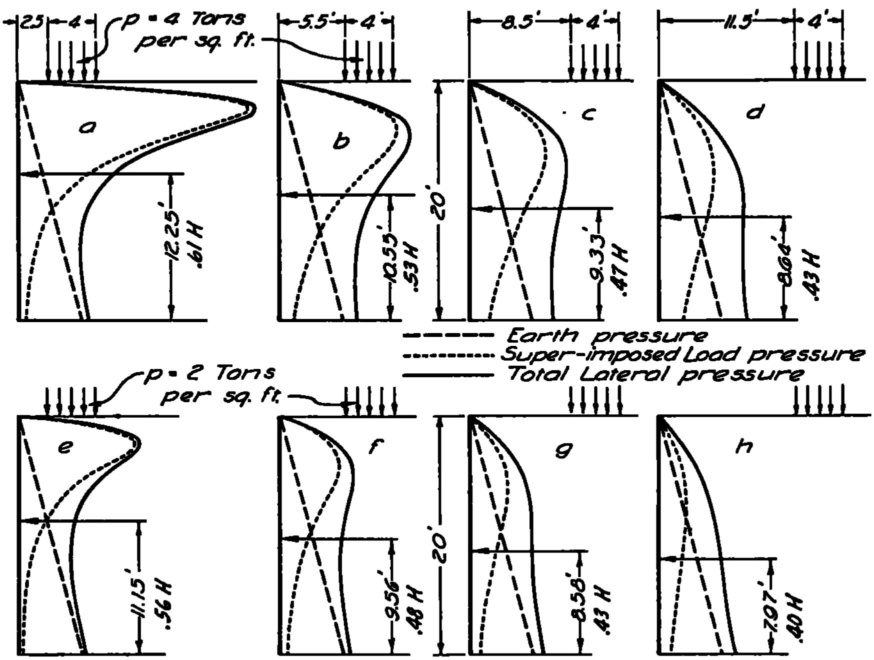

Figure 7

elevation, depending upon their relative magnitude and position, but will usually act somewhere within the middle ⅓ height of the wall. In general, the greater the magnitude of the super-imposed load and the closer it is to the wall, the higher will be the position of the total resultant pressure and vice versa.

As an illustrative example of the application of this hypothesis, several purely imaginary cases are shown in Figure 7. The distribution of the total lateral pressure on a vertical wall 20 ft. high, when a super-imposed load such as a building foundation is applied at the the super-imposed foundation load is added to the earth pressure, giving the total pressure distribution curves as shown. These total pressure curves vary widely in shape within the range of conditions assumed, and illustrate the important effect of the magnitude and position of super-imposed loads upon the distribution of lateral pressure and upon the position of the line of action of the resultant of the lateral pressure.

It is not possible to make a direct quantitative comparison between this hypothesis and the results of Mr. Meem's experience as reported in reference (3) but it seems quite probable that the contradiction between his observations and Rankine's theory may have been due to the lateral pressure effect of superimposed loads, since most of his experience was in connection with subway construction in city streets with buildings occupying the adjacent property.

A comparison between the hypothesis of this paper and Mr. Feld's (2) experiment No. 5 is shown in Figure 8. When

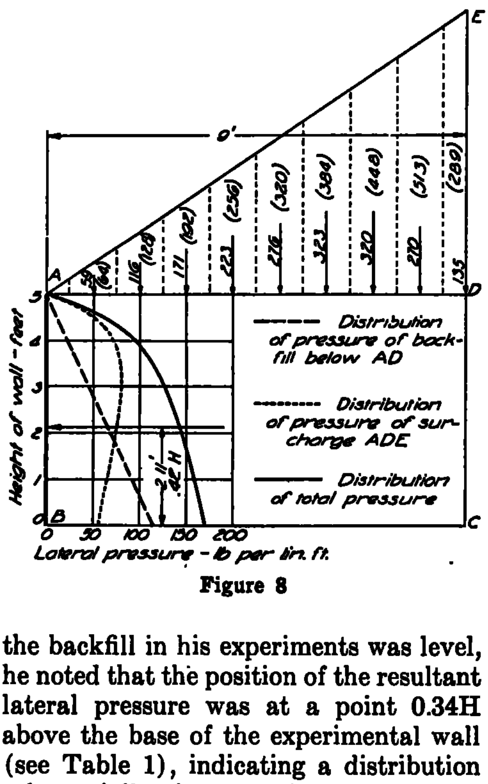

Figure 8

the backfill in his experiments was level, he noted that the position of the resultant lateral pressure was at a point 0.34H above the base of the experimental wall (see Table 1), indicating a distribution substantially in accordance with the Rankine theory, as shown by the dashed line in the figure. Then when the backfill was built upward on a slope of 34 deg. with the horizontal, the position of the resultant was observed to rise to a point 0.43H above the base.

If we consider the surcharged backfill material above the level AD as a superimposed load and calculate the lateral pressure on the wall due to this load by means of formula 5, the pressure distribution indicated by the dotted line in the figure is obtained. Then adding the pressures due to the material below the level AD to the pressures due to the surcharge, the distribution of the total pressure is obtained and is shown by the solid line. The resultant of this total pressure on the wall acts at a point 0.42H above the base, which is nearly coincident with the position observed by Feld.

In making these calculations for distribution of pressure due to the surcharged fill load of Feld's test No. 5, the weight of the fill above the top of the wall (shown in parenthesis in the figure) has been reduced by the probable frictional support at the sides and back of the bin in accordance with the principles of Janssen's analysis of pressures in grain bins and Marston's theory of loads on underground conduits. In all probability there were other friction losses along the sides of the bin below the level AD and back of the friction reducing section of the sidewalls provided in Feld's apparatus which cannot readily be calculated, and these losses may account for the fact that the total calculated lateral pressure on the wall due to the surcharge is about 345 lb. per lin. ft., whereas Feld measured only 214 lb. per lin. ft. However, it is not believed that the general shape of the total pressure distribution curve or the position of the resultant pressure would be greatly affected by these friction losses.

In presenting this hypothesis to the profession, the author hopes that it will stimulate discussion both from the theoretical and the practical standpoint. Particularly it would be interesting to hear from engineers who have observed pressure distribution which is at variance with classical theory. In such cases, have there been appreciable super-imposed or surcharged loads in the vicinity of the retaining walls or other types of restraining structures? Where possible, a quantitative comparison of actual pressure distribution with that calculated by this hypothesis would be very interesting and valuable. Also, this hypothesis suggests a wide open field for both experimental and analytical research, which may lead to a more satisfactory reconciliation between theory and practice than exists at present.

REFERENCES

1. Discussion by George Paaswell ("Lateral Earth Pressure" by Jacob Feld). *Transactions*, Am. Soc. C.E., Vol. LXXXVI, p. 1514, 1923.
2. Feld, Jacob. "Lateral Earth Pressure": "The Accurate Experimental Determination of the Lateral Earth Pressure, Together with a Résumé of Previous Experiments," *Transactions* Am. Soc. C.E., Vol. LXXXVI, p. 1448, 1923.
3. Meem, J. C. "The Bracing of Trenches and Tunnels," *Transactions*, Am. Soc. C.E., Vol. LX, June, 1908, p. 1.
4. Ibid., p. 59.
5. McNary, J. V. "Earth Pressures Against Abutment Wells Measured with Soil Pressure Cells." *Public Roads*, Vol. 6, No. 5, July, 1925.
6. Terzaghi, Karl. "A Fundamental Fallacy in Earth Pressure Computations." Bul. No. 182, Graduate School of Engineering, Harvard University, 1936.
7. Terzaghi, Karl. "Large Retaining-Wall Tests." Engr. News-Record. 112: 136-140, 259-262, 316-318, 403-406, 503-508, 632-636; Discussion 112: 747; 113: 55, 119, 535. 1934.
8. Darwin, G. H. "On the Horizontal Thrust of a Mass of Sand." *Minutes of Proceedings*, Inst. of C.E., Vol. LXXI; 350-378, 1883.
9. Spangler, M. G. "Horizontal Pressure on Retaining Walls Due to Concentrated Surface Loads." Bul. 140, Iowa Engineering Experiment Station, Iowa State College, Ames, Iowa. 1938. Also, Discussion by M. G. Spangler ("Lateral Earth and Concrete Pressures," by Lazarus White and George Paaswell). *Proceedings*, A.S.C.E., Vol. 65, No. 1, p. 130, Jan., 1939.
10. Gerber, Emil. "Untersuchungen uber die Druckverteilung im ortlich belasteten Sand." Diss. a.-g. gebr. Leeman, Zurich. 1929.

DISCUSSION ON DISTRIBUTION OF LATERAL EARTH PRESSURES

PROF. KARL TERZAGHI: In connection with the effect of surcharges on lateral pressure, I suggest that Mr. Spangler should include in his paper a comparison between his theory and the test results obtained by E. Gerber (Untersuchungen über die Druckverteilung im örtlich belasteten Sand. Zürich, 1929), because these results were obtained by means of remarkably sensitive measuring devices, and the scattering of the recorded values was far less important than in Mr. Spangler's test.

Mr. Spangler refers repeatedly to the contradictions between the classical earth pressure theories and reality. Contradictions of that type are invariably due to lack of clarity concerning the limits of validity of the theories involved. Since these limits have been known for several years, we can no longer speak of contradictions. A brief account of these limits is contained in my paper, "General Wedge Theory of Earth Pressure," *Proceedings* Am. Soc. C. E., October 1939. Figure 9 of this paper also contains the results of the measurement of the lateral pressure on the timbering of a cut and in response to Mr. Spangler's inquiry, it may be stated that there was no surcharge on either side of the cut. Otherwise this very essential detail would have been mentioned.

PROF. D. P. KRYNINE, *Yale University:* I wish to emphasize the following important features of this interesting paper.

First. Professor Spangler has concluded that a loaded area behind a retaining wall acts as a series of strip loads. This means that the principle of superposition holds in the case of sands and gravels, at least as far as horizontal pressures are concerned. This conclusion is very important since sands and gravels are supposed not to obey Hooke's law and from the point of view of classical theories the principle of superposition cannot be applied to them.

Second. Experimental curves obtained by Professor Spangler are very similar to Boussinesq's pressure distribution. In all cases Spangler's ordinates are greater than those furnished by the theory of elasticity. This circumstance has been already discussed in engineering circles; but no definite conclusion has been reached. It should be very clearly borne in mind that Professor Spangler's empirical formula is valid only under given circumstances and cannot be generalized.

Third. Professor Spangler's basic hypothesis is quite correct but only in the case when the pressure on the retaining wall as caused by the backfill is hydrostatic. In other words, the retaining wall in question must be perfectly immovable or move only in a certain prescribed way. Professor Spangler's hypothesis does not hold in the case of flexible walls as he himself states.

Fourth. Professor Spangler's curves have maximum ordinates at the top of the wall. The classical theories require, however, that the horizontal pressure at the top of a backfill loaded at the surface with external loads be zero. There is no contradiction there however: Professor Spangler's experimental curves, if prolonged, would probably tend to the outside point at the top of the loading bins. This is an additional proof of the statement that in the case of an earth mass loaded also with earth, stresses must be computed considering the earth mass and the load (for instance, an embankment) as a whole.

Mr. Spangler: We might go back a little in connection with Dr. Terzaghi's discussion and point out that comparison was made several years ago between our retaining wall experiments at Ames and the laboratory work of Emil Gerber at Zurich to which he refers. That comparison was shown at the meeting last year and has been published in the proceedings of the 18th annual meeting on page 58 of part II, and elsewhere.

I am very much interested in Professor Krynine's explanation of these pressure curves which bend so sharply to the right near the top of a retaining wall when the backfill is loaded with an area load contiguous to the structure and I am sure his suggestions will help to clarify some of our thinking on this subject.