LIMIT DESIGN OF FOUNDATIONS AND EMBANKMENTS

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SYNOPSIS

Procedures based on limit design have been in use for some years in soil mechanics. The main contribution of this paper which outlines the basic features of designing on the limit principle, is a proposed method for combining failure criteria and a factor of safety. It is presented with the belief that before using any method of analyzing earth structures, it should be emphasized that accuracies of results can be no higher than the accuracy with which the strength properties are known and that any factor of safety which is used must cover numerous items of uncertainty.

An important factor in the setting up of any method of design is the choice of what the criterion of failure shall be. Two criteria have often been proposed and have been applied in designs of structures, using from the commonest building materials,—steel, concrete and wood, to the more natural materials,—rock and soil. The choice between these two criteria may be worded in this way: shall failure be associated with the condition wherein the most dangerously stressed point just reaches its limit of strength, or shall it be considered that failure has not occurred until the point which first attains the plastic state has undergone further strain, under constant or decreased strength, until all reserve of strength of surrounding points has been expended? The first criterion involves only the limiting value of the stress, the second brings in relationships between stresses and strains. When a design uses this second failure criterion it is spoken of as a "Limit Design."

Much recent discussion of the relative merits of the two criteria has taken place. A stimulating paper by J. A. Van den Broek [1] has recently presented a strong argument for Limit Design in steel structures and has been the cause of considerable discussion.

In the field of soil mechanics it has repeatedly been emphasized that stress-strain relationships are of outstanding importance as soil properties. Much research has been devoted to determinations of shearing strength of soils and in spite of controversial points of view on numerous details, it is generally admitted that real progress has been made in the general knowledge of the stresses which soils can withstand. However, in addition, steadily increasing emphasis has been placed on the importance of the strains associated with these stresses. It must be recognized that a clear understanding of strength is impossible without thorough knowledge of the associated volume changes and shearing strains.

To express the shearing strength of soils, the empirical law of Coulomb is commonly used. It is written

$$s = c + p \tan \phi$$

wherein s is the shearing strength, c the cohesion, p the normal intergranular stress on the incipient shearing surface and ϕ the angle of internal friction of the soil. The soil properties c and ϕ must not be spoken of as constants as they vary widely for the different conditions under which shear may occur. However, for any given homogeneous embankment under a given set of conditions, there probably are values for c and ϕ which will represent the situation with reasonable accuracy. Typical curves of shearing stress against shearing strain are shown in Figure 1. Loosely deposited sands and soft clays have stress-strain

[1] *Proceedings* A.S.C.E., February 1939.

curves of the shape of curve I. Dense sands and many clays which in their natural state show appreciable structural strength, reach a maximum shearing strength at some certain shearing strain, but undergo a decrease in strength at larger strain, as shown by curve II. Unless otherwise stated it will be assumed that the curve under consideration is of the shape of curve I.

As an example of the type of analysis wherein all stresses are kept below the strength of the material, the case of the earth dam of Figure 2, founded on a deep deposit of highly cohesive clay will be used. This analysis was first presented by L. Jürgenson.[2] To simplify the analysis it is assumed that the second term on the right hand side of Coulomb's expression is negligible or that the shearing strength is constant throughout the clay at 1000 lb. per sq. ft.; that the embankment of approximately triangular shape produces a triangular distribution of load on the surface of the clay stratum and that the stresses produced by this loading in the clay stratum are equal to those which would occur if the clay were a truly elastic mass. Computations based on the theory of elasticity give the stresses shown in Figure 2. It is seen that the maximum shearing stress of 920 lb. per sq. ft. is slightly lower than the shearing strength of 1000 lb. per sq. ft.

In order to express the degree of safety, some form of "Factor of Safety" must be introduced. In determinate trusses, the use of a safety factor leads to little confusion. However, the greater the degree to which a structure is indeterminate, the harder it is to obtain a clear understanding of the degree of safety and the more obscure becomes the meaning of "Factor of Safety." If there are one-hundred possible ways in which a structure as a whole could fail, then at least one hundred different factors of safety could be described. Yet the use of a "Factor of Safety" is an engineering necessity. Therefore, whenever the term is used, if any possibility of confusion exists it is

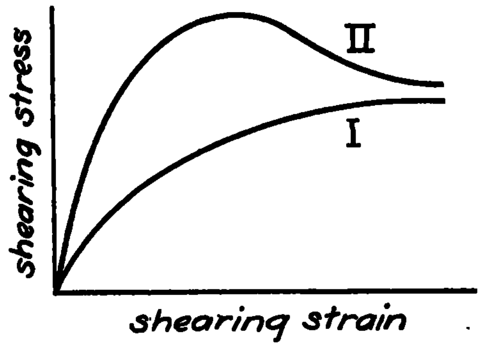

Figure 1

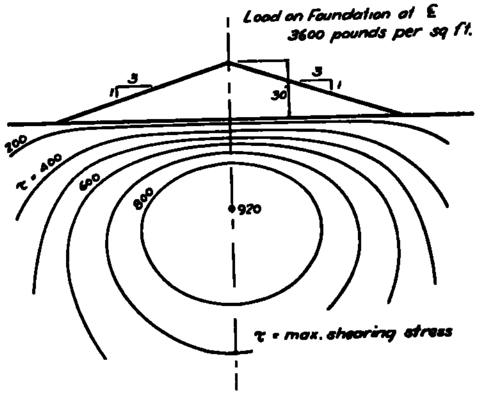

Figure 2

evident that a careful description must be given.

In any soil stability study, it is a statically indeterminate problem that is dealt with, and the degree to which it is indeterminate is itself indeterminate. Therefore, any factor of safety, before being used, must be fully described. The scheme in common use for cases such as that above is to use the ratio between shearing strength and maximum shearing stress; in this case $1000 \div 920 = 1.1$.

[2] "The Application of Theories of Elasticity and Plasticity to Foundation Problems" (Fig. 9) by L. Jürgenson, *Journal*, Boston Society of Civil Engineers, July 1934, Vol. XXI, No. 3.

The value of 1.1 is sometimes spoken of as the *point* factor of safety. It could also be called the factor of safety against the beginning of a plastic condition.

It should be realized that in any stability analysis on soils there are many variations from the idealized conditions which are assumed and which the factor of safety must absorb. Therefore, two significant figures for the above factor are sufficient in view of the low degree of probable accuracy.

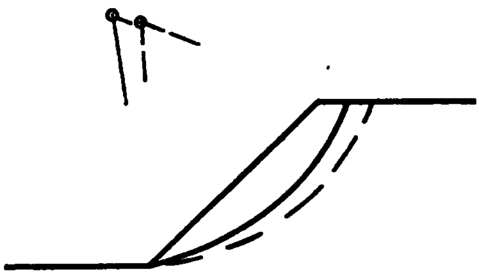

Figure 3

If the dam of the above example were gradually built to steeper slopes on the same base width, the stress would increase in proportion to the load. After a point factor of safety of unity is attained a plastic zone would develop and further load would cause this zone to expand until, at the limit load, the plastic zone would reach ground surface at some point. For soils represented by curves of form I of Figure 1, the limit load would be considerably larger than the load causing the most highly stressed point to just reach its limit of strength. For a soil with a stress strain relationship similar to that of curve II the limit load would exceed the load causing point failure somewhat. However, this excess might be very small if the stress-strain curve shows a large stress drop after reaching the maximum value.

A limitation of the above analysis is that it involves the assumption of elastic material and can be used only for sections where the elastic theory or photoelastic studies are capable of supplying the stress pattern. In soil mechanics studies it has been used only for foundation analyses although its use has been suggested for embankments.[3] Van den Broek claims that the point type of analysis is not as rational as the limit design method. The writer accepts this contention but believes that in soil stability studies the outstanding disadvantage of the point scheme as illustrated above is that results by it can only be as accurate as the degree to which soil behaves like a truly elastic mass.

Limit design in soils wherein the complete shearing strength is developed over an entire failure surface or throughout a failure zone, can best be illustrated by an analysis of a simple slope.

Field investigations of many slope failures in Sweden have demonstrated that if a slope such as that shown in Figure 3 were to fail, rupture would occur along a surface which can be approximated with reasonable accuracy by a circular arc. The "Circular Arc Method," which originated from this evidence, was developed by W. Fellenius.[4] A general solution based on this method for a simple slope of homogeneous soil such as shown in Figure 3 has been developed by the writer.[5] The procedure in brief is to locate in some way that circular arc which is most liable to rupture, assume all shearing strength is being utilized at every point along the arc and analyze the conditions which must exist if the mass is just at the point of failure. The analysis consists of expressing the equilibrium of moments about the center of the circle. The solution is of the form

[3] See Discussion on Soil Mechanics Symposium by T. T. Knappen *Transactions* A.S.C.E., Vol. 103, p. 1451.

[4] "Erdstatisch Berechnungen mit Reibung und Kohäsion," W. Fellenius, Berlin 1927.

[5] "Stability of Earth Slopes," D. W. Taylor, *Journal*, Boston Society of Civil Engineers, July 1937, Vol. XXIV, No. 3.

$c = \gamma HK$, wherein c is the cohesion required to just maintain equilibrium, γ the unit weight of the soil, H the vertical height of the slope and K a term which depends only on the slope angle and the angle of internal friction. The expression $\frac{c}{\gamma H} = K$ is a dimensionless constant to which the writer has given the designation of "Stability Number." For any given inclination of slope, in a homogeneous material which has an angle of internal friction ϕ, the stability number may be obtained from Figure 4.

In Figure 3 the slope is 45 deg. and the friction angle will be assumed to be 20 deg. From Figure 4 the stability number is 0.062 and any combination of values of c, γ and H which give $\frac{c}{\gamma H} = .062$ will describe a condition which is just at the point of failure. The example which will be used is $c = 500$ lb. per sq. ft., $\gamma = 120$ lb. per cu. ft. and $H = 67$ ft. This example of limit design illustrates the commonest type of stability analysis of earth structures and has been in use since the earliest days of the branch of engineering we now call soil mechanics.

Before the above analysis may be used for an actual engineering problem some rational method of expressing a factor of safety must be determined upon. At first glance it might appear that there would be a factor of safety of 2 if, instead of 120, the unit weight were to be 60 lb. per sq. ft., a reasonable value if the slope were the bank of a canal and completely submerged below free water surface. Similarly if the height were but 33.5 ft. instead of 67 ft. or again if the cohesion were 1000 instead of 500 lb. per sq. ft., the actual stability number would be twice the limit value. It is seen that such changes are equivalent to decreasing the loading by half, but reference to Coulomb's law shows that any decrease in loading also causes a decrease in the strength of the soil. Thus the gain in safety is not so large as would first appear. For case of a crib carrying a 1000-lb. load, an extreme example of this point, the resting on a rock slope which is so steep that the crib is just at the point of slipping, may be considered. The factor of safety against slip is just unity. Reducing the load will not improve this

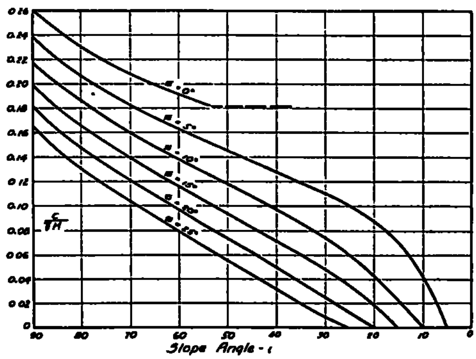

Figure 4. Chart for Stability No. $\frac{c}{\gamma H}$

factor as slip will still be incipient so long as the coefficient of friction is a constant. However, in the simple slope under consideration, if by any one of the three ways mentioned the actual stability number were to be doubled, the factor of safety that would result is sometimes spoken of as 2. It is important that a better terminology be introduced for use here, such as "Factor of Safety with respect to Cohesion" or "Factor of Safety with respect to Height."

For cases such as the above the writer advocates a different approach using a "Factor of Safety with respect to Strength." If the value of this factor is $1\frac{1}{3}$, then the strength must be $1\frac{1}{3}$ times as great as is required to just avoid failure, or if all strength were to be divided by $1\frac{1}{3}$ a condition of failure, or limit condition, would result. Any actual case wherein the factor of safety with respect to strength is $1\frac{1}{3}$, is statically indeterminate. However, if in the actual case the soil were to be replaced by a hypothetical soil with a strength equal to the actual strength divided by $1\frac{1}{3}$, a determinate case would result. The ideas are the same as those pointed out by Van den Broek in an example on rivet design.

Experience has shown that for practical and economic reasons, factors of safety with respect to strength in embankment analyses must frequently be limited to values of the order of 1.1 to 1.5. Again referring to Figure 3 and assuming a 45-deg. slope, a friction angle of 20 deg., a cohesion of 500 lb. per sq. ft. and a unit weight of 120 lb. per cu. ft. and introducing a factor of safety of $1\frac{1}{3}$ by using a working value of cohesion of $\frac{500}{1.33} = 375$ lb. per sq. ft. and a working value of friction angle which has a tangent equal to $\frac{\tan 20}{1.33}$, or 15.2 deg., the allowable height may be determined as follows: The stability number for a 15.2-deg. friction angle is 0.082, and

$$0.082 = \frac{c}{\gamma H} = \frac{375}{120H} \text{ whence } H = 38 \text{ ft.}$$

Actually the most dangerous circle for a 15.2 degree friction angle is the dotted arc of Figure 3 while for 20 deg. it is the full line arc, but this need not be considered in the present discussion. Since the height which would just cause incipient failure was 67 ft., the factor of safety with respect to height for a 38 ft. embankment is 67/38 or $1\frac{3}{4}$; as has already been pointed out $1\frac{3}{4}$ may also be called a factor of safety with respect to cohesion. At the same time the factor of safety with respect to strength is $1\frac{1}{3}$. Failure of this embankment would occur if any one of the following changes should occur; height multiplied by $1\frac{3}{4}$; unit weight multiplied by $1\frac{3}{4}$; cohesion divided by $1\frac{3}{4}$; shearing strength divided by $1\frac{1}{3}$.

If a soil has a stress-strain curve similar to curve II of Figure 1, since the shearing strength is developed progressively, the maximum shear at the instant of complete failure will occur at the point on the failure arc where the shearing-strain is smallest. The average stress at this limiting condition falls somewhere between the maximum value of stress and the ultimate value, its location between these two limits being statically indeterminate. In fact a determination of the stresses would require complete knowledge of stress-strain relationships of the soil and of the strains developed all along the failure surface. Although the discussion given above mentions stress and strength frequently it does not do so at the expense of consideration of strain. Strains and their effect are completely covered and a true limit condition is under consideration when full shearing strength is assumed over the entire arc. In soils, strains are of such an involved nature that it would be difficult to talk in terms of them.

An outstanding example of limit design which is in common use by structural designers is that of retaining wall pressures. Dr. Karl von Terzaghi[6] proved by his experiments on the large model retaining wall at the Massachusetts Institute of Technology that the lateral pressure is very dependent on the amount of yield of the wall, or, what is the same, the amount of strain in the backfill. Sufficient yield away from the fill leads to a condition of complete summoning of shearing strength in the backfill and to a minimum pressure known as the active pressure. This is shown diagramatically by Figure 5. Similarly, sufficient wall displacement toward the fill leads to "passive" pressure. From this it may be seen that the pressure acting on a wall will be greater than the active pressure unless the wall is just barely strong enough to withstand active

[6] *Engineering News-Record,* Spring 1934, Vol. 112.

pressure. Thus any wall which has a factor of safety, resists greater than active pressure, yet designs are based either on active pressure theories or on rule of thumb methods which spring at least partly from active pressure conceptions. The saving feature is that if the pressure at any time happens to overtax the wall, the wall yields just enough to relieve the pressure to a value which no longer overtaxes it.

Many types of factor of safety have been proposed for use in retaining wall design. The writer can suggest no more rational approach than to divide the limit strength of each unit,—soil, wall material, etc.—by whatever factor of safety with respect to strength is reasonable for that unit. Then proceed to the setting up of a design wherein as many as possible of these working values of strengths are just attained and others are on the side of safety. Thus the design value for pressure on the wall would not be active pressure but would be the value the active pressure would assume in a hypothetical soil wherein the shearing strength equals the quotient of the strength of the true soil divided by a factor of safety with respect to strength.

A method of limit design for sheet pile bulkheads was suggested by Dr. Glennon Gilboy a few years ago in his graduate soil mechanics course at the Massachusetts Institute of Technology. By this scheme the piling is first figured as though driven only to the toe-hold required to just give sufficient passive pressure to prevent failure at the toe. For this case the wall pressures are determinate and the piling and tie-rods can be designed by conventional methods. Subsequently the piling is actually made longer by an amount sufficient to give a reasonable factor of safety against failure at the toe. Study of the interrelation between wall-yield and wall-pressure indicates that this added depth at the toe relieves the stresses in the tie-rods and piling and thus gives an indeterminate amount of added safety with respect to these items. The writer would like to suggest that here again the use of the hypothetical soil, with a strength which is expressed by the quotient of true strength and a relatively small safety factor with respect to strength, could be introduced in place of the first computations of the above

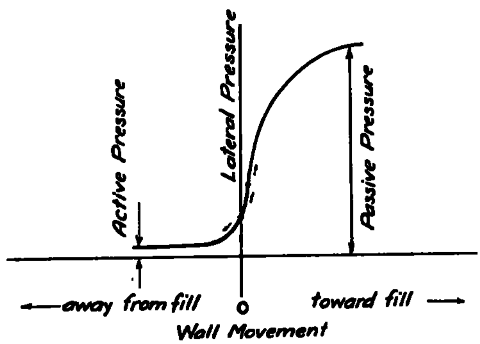

Figure 5

scheme. The resulting wall would have the desired factor of safety in every respect, including that against toe failure.

In conclusion, it must be admitted that rule of thumb methods are needed for simple routine designs. However, in important structures where a good visualization of the interrelated behavior of the numerous elements is important, the limit design for earth structures at least is the rational approach. In any analysis which involves criteria of failure some form of factor of safety must be used and the form which is chosen should be clearly defined. The writer has found that where structural elements have strengths which depend on the loading, the most satisfactory approach is to describe working values of strength as equal to the actual strength divided by a factor of safety with respect to strength; then to proceed with a limit design wherein these working values are treated as though they were the only strength in existence.