SHEARING STRESSES AT THE BASE OF AN EMBANKMENT

By D. P. Krynine

Research Associate in Soil Mechanics, Yale University

SYNOPSIS

In most of the existing methods of designing foundations for embankments it is assumed that the embankment exerts only vertical pressures on its base. Particularly, this is done in applying the theory of elasticity to the design of embankments. As a result of such an assumption, shearing stresses at the base of the embankment should vanish, but in reality they exist. The purpose of this paper is to furnish a quick method of estimating these shearing stresses, and it is believed, that the solution thus obtained is satisfactory for all practical purposes.

A highway embankment or an earth dam tends to slide sidewise. The chief reason for it is lateral pressure which exists in every earth mass. In German literature there are some rather complicated methods of computing the shearing stresses at the base of an embankment.[1] So far as the writer is aware, this problem has not yet been theoretically approached by engineers in English speaking countries. In this paper a simple solution is proposed. It is approximate and its accuracy may be questioned from the point of view of exact elastic theories. It furnishes, however, a fairly clear view of the situation and allows an estimate of the maximum shear at the base of an embankment.

TRIANGULAR CROSS SECTION

To simplify the reasoning consider first an embankment of triangular cross section (Fig. 1(a)). Let its height be H ft., and designate by α the angle made by the slope with the horizontal. Suppose that the slope in question is characterized by the ratio: one height for n bases. Then:

$$\tan \alpha = 1:n \qquad (1)$$

Designate furthermore with γ the unit weight of the material so that 1 cu. ft. weighs γ lb. Imagine that the embankment is cut into two halves along its center line and consider conditions of equilibrium of one half only, for instance, of the left (Fig. 1(b)). Because of the symmetry, the left half pushes the right one horizontally, and vice versa; and there cannot be any vertical component at the center line. Hence the right part can be removed and replaced by the horizontal thrust, F_o, which it exerts on the left half. This thrust can be computed using the Coulomb formula for the horizontal backfill. This is accurate enough for all practical purposes; and it should be noticed that the Rankine formula furnishes the same result in this case:

$$F_o = \frac{\gamma \cdot H^2}{2} \cdot \tan^2\left(45^\circ - \frac{\phi}{2}\right) \qquad (2)$$

As known, the symbol ϕ in Formula (2) means the average angle of friction of the given earth material. Place for the sake of brevity:

$$\tan^2\left(45^\circ - \frac{\phi}{2}\right) = K_a \qquad (3)$$

so that Formula (2) will be:

$$F_o = \frac{\gamma \cdot H^2}{2} \cdot K_a \qquad (4)$$

This thrust is resisted by the adherence of the embankment to its base; so that the average shearing stress, τ, at the base will be obtained by dividing the value of the thrust, F_o, by the half width of the embankment which equals $H \cdot n$. Hence, the value of the average shearing stress,

[1] For instance, Leo Rendulic, Der Erddruck im Strassenbau und Brückenbau. Berlin, 1938.

τ, at the base of a triangular embankment is:

$$\tau_{av} = \frac{F_o}{H \cdot n} = \frac{K_a}{2n} \cdot \gamma \cdot H \ldots\ldots (5)$$

Each vertical section of the embankment such as AB (Fig. 1(b)) may be visualized as a retaining wall *dx* thick. Thrust, F, acting on this retaining wall from the center line toward the toe is greater than the opposite thrust F-dF. Suppose that the thrust F (or F-dF) makes an angle ϕ, that of internal friction of the earth material, with the normal to the retaining wall, AB. The horizontal force, $dF \cdot \cos \phi$, which pushes the retaining wall, AB, toward the toe of the embankment, is balanced by the shearing force $\tau \cdot dx$ at the base of the embankment:

$$dF \cdot \cos \phi = \tau \cdot dx \ldots\ldots\ldots (6)$$

from which:

$$\tau = \cos \phi \cdot \frac{dF}{dx} \ldots\ldots\ldots (7)$$

It is known that the thrust, F, is proportional to the square of the height of the retaining wall, AB, and the latter equals $\frac{x}{n}$ (Fig. 1(b)). Were the slope, MN, infinitely long, thrust, F, at all vertical sections such as AB, would be proportional to x^2 or in other words, according to Equation (7) the shearing stress, τ, at the base of the embankment would be proportional to x, because the derivative $\frac{dF}{dx}$ is proportional to x. In reality, slope MN is finite, and starting at a certain point, P, the shearing stress, τ, decreases towards the center line of the triangular embankment where it becomes zero due to symmetry. To locate point P, it is necessary to know the angle QPN, N being the top of the embankment. The slope of the line PN may be determined analytically or graphically by using one of the numerous methods to locate the Coulomb wedge behind a retaining wall with sloping backfill. The simplest solution would be to trace through N a line making an angle $45° - \frac{\phi}{2}$ with the vertical as in the case of a horizontal backfill. Then area QPN would correspond to the possible maximum wedge behind hypothetical walls imagined within the

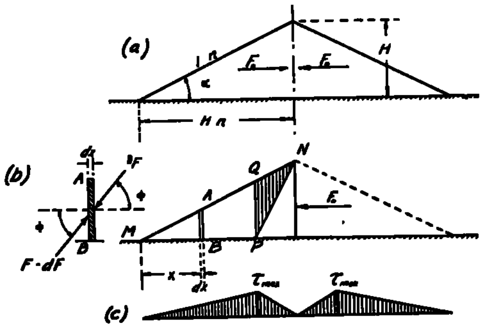

Figure 1. Triangular Cross-Section

embankment and it may be assumed that the difference in thrusts, F and F-dF, is also maximum at the section QP. The decrease of the shearing stress, τ, from point O toward the center line of the embankment does not follow a straight line law any longer. Nevertheless, for the sake of simplicity, the curve expressing the value of the shearing stress, τ, between point P, where this stress reaches a maximum, and the center line of the embankment, will be expressed by a straight line. Thus the diagram of the shearing stress, τ, for each half of the embankment will be a triangle (Fig. 1(c)). It is obvious that according to this visualization the maximum value of the shearing stress, τ_{max}, equals twice the average value, τ_{av}:

$$\tau_{max} = 2\tau_{av} = \frac{Ka}{n} \cdot \gamma \cdot H \ldots\ldots (8)$$

Recapitulating: In the case of a triangular embankment the shearing stress is at a maximum at a point of the base which may be approximately determined by tracing through the top point of the embankment a line making an angle of $45° - \frac{\phi}{2}$ with the vertical. The maximum shearing stress, τ_{max}, may be expressed in terms of $\gamma \cdot H$, where γ is the unit weight of the earth material expressed, for instance in lb. per cu. ft.; and H the height of the embankment (in ft.). It equals $\frac{K_a}{n} \cdot \gamma \cdot H$ where K_a is determined by Formula (3), and n is the coefficient of

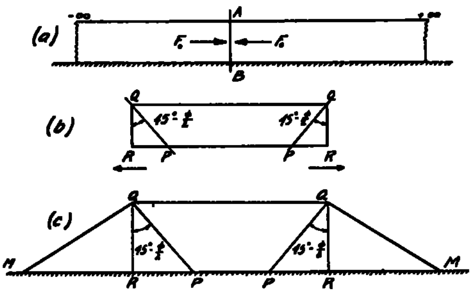

Figure 2. Trapezoidal Cross-Section

the slope. Thus for $\phi = 30°$ and for a slope one height to two bases (n=2);

$K_a = 0.33$ and $\tau_{max} = \frac{1}{2} \times 0.33 \cdot \gamma \cdot H$
$= 0.165\gamma \cdot H$

TRAPEZOIDAL CROSS-SECTION

Consider first an infinitely long fill of uniform height (Fig. 2(a)). At any vertical section, AB, there are two opposite and mutually equal horizontal thrusts, F_o. There is no shear at all at the base of such an embankment.

If, however, the embankment is finite though very wide, there is shear close to its edges. By tracing a line, QP, through the outside upper point, Q, of the embankment, a point, P, may be found where the shear stress, τ, starts to increase from zero to a certain value at the edge, R (Fig. 2(b)). The maximum value of the shearing stress in this case is at the edge. If a slope is added (Fig. 2(c)), there would be a quite small shearing stress at point, R, because the pressures from both sides of the section, QR, are almost equal. Hence in this case distance, PR, takes up but a very small part of the total pressure pushing the embankment toward its toes; and it seems advisable to neglect its effect at all. Thus the problem is reduced to the study of stability of the triangular prism, MQR, i.e. to the preceding problem.

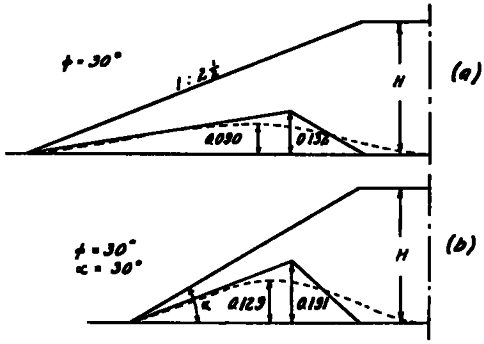

Figure 3. Shearing Stresses at the Base, in terms of 8 · H. (Broken Lines are Rendulic's Data).

The situation differs very little if the crown is narrow. To be on the safe side, it is better to neglect the influence of the central part and consider the stability of the slope only, as in the case of a wide crown.

COMPARISON OF DATA

In Fig. 3(a) and (b) diagrams of shearing stresses as given by Rendulic and as computed by the proposed method are shown. The value of the maximum shearing stress, τ_{max}, as computed by Formula (8) is about 50 percent in excess of that given by Rendulic. One must take into consideration on one hand the uncertainty in the actual distribution of those shears, and the difficulty in determining in the laboratory the shearing resistance at the boundary of the fill and the earth mass, and on the other hand the simplicity of the method proposed. Under the circumstances, it seems advisable to give preference to the simpler method of computing stresses which at the same time furnishes a certain margin of safety.

SHEAR STRESSES IN PRACTICE

The existence of shearing stresses at the base of an earth dam has been proved by Dr. Terzaghi by observing movement of a pipe line under a dam in Africa as reported by him in one of his public conferences.

Shearing stresses at the base of an embankment may be dangerously increased by the overrolling during construction. The overrolled material tends to expand, and this causes the lateral sliding as happened in the case of the Tappan Dam.[2] This type of failure probably unknown to most engineers, merits attention.

The bond between a fresh fill and the existing earth mass increases as the time goes. Hence the maximum danger of sliding apparently corresponds to the construction period; therefore it appears proper not to consider the influence of the live load (traffic) on the shear at the base as done in this paper.

DISCUSSION ON SHEARING STRESSES AT THE BASE OF AN EMBANKMENT

MR. L. A. PALMER, *Public Roads Administration:* According to equation (2) of this paper, the lateral earth pressure is the vertical one multiplied by $\tan^2(45° - \phi/2)$. This could be approximately true if the material comprising the fill is cohesionless, if the supporting soil is unyielding and if the movement of fill material in the horizontal direction is of a magnitude characteristic of a condition of failure. Actually, the fill material is seldom cohesionless, the supporting earth yields in the vertical direction under the fill load and the lateral movement of fill material at mid-section may vary from zero to almost any value.

It may be expected that settlement near the center line of the undersoil is of considerable magnitude and that as a consequence the material at the longitudinal mid-section of the fill will tend to drop vertically downward. Thus there is a condition comparable to the yielding of a trap door at the base of bin of sand and arching at the mid-section of the fill results. Due to this arching action, the thrust F_0 (Figures 1a and 2a) is increased considerably. It is likely that under these conditions the lateral pressure at mid-section may exceed the vertical pressure. Marston[3] has shown that due to the large lateral thrust in the fill material above culverts the load on the culvert is in some instances materially decreased and that in other instances it is considerably increased, the increase or decrease of load on the culvert depending on the relative settlements of the material in the vertical section including the culvert and in the material adjacent to this section.

In equations (3), (4), (5) and (8) K_a is the active earth pressure ratio. If the supporting soil is unyielding the earth pressure ratio is K_a for the condition of failure but for small earth movements in the fill the earth pressure ratio would be intermediate in value between K_a and the coefficient of earth pressure at rest, assuming no displacement of the supporting earth.

Due to arching action when there is vertical displacement of the supporting

[2] Knappen, T. T., "Calculation of the Stability of Earth Dams." Second Congress of Large Dams, Washington, D. C.. 1936, Vol. 4.

[3] See for example Bulletin No. 31, Engineering Experiment Station, Iowa State College, A. Marston and A. O. Anderson.

soil, the vertical pressure at the base of the longitudinal mid-section of the fill is decreased and the vertical pressure at the ground level and to right and left of the longitudinal mid-section is materially increased. This condition would be temporary however. The pressure intensity at the surface of supporting earth would tend to fluctuate as these conditions alternate in their occurrence.