DESIGN GRAPHS FOR HIGHWAY SUSPENSION BRIDGES OF MODERATE SPAN

By Conde B. McCullough

Assistant Chief Engineer, Oregon State Highway Department

SYNOPSIS

The exact or rigorous determination of stresses in suspension spans is somewhat tedious and involved For this reason it has been general practice to make certain approximations, particularly in the design of structures of comparatively short span. Such approximations, in certain cases, involve errors of considerable magnitude, and errors which affect different portions of the structure in different degree Because of this the Oregon Highway Department in 1937 initiated certain researches looking toward the derivation of exact and rational design methods for highway suspension bridges and the development of design graphs which would shorten the time and lessen the labor involved This work was undertaken by Mr G S Paxson, Bridge Engineer, and Mr. Dexter R Smith, Structural Research Engineer for the Oregon Department, under the general direction of the author, and is completely described in Oregon Technical Bulletin No 13, entitled "Rational Design Methods for Short-span Suspension Bridges for Modern Highway Loadings," and Technical Bulletin No 14, entitled "The Derivation of Design Constants for Suspension Bridge Analysis (Fourier-series Method)" from which much of the material presented hereinafter has been extracted

The present paper summarizes the mathematical theory underlying the exact or rigorous method of stress analysis, utilizing the Fourier-series expansion of the deflection term proposed and developed by Timoshenko and Priester A comparison of the results obtained by this method with those derived by the approximate method indicates a degree of divergence amounting, in certain cases, to over 250 per cent, as shown by Table I of this article, which table is based on a comparison of 24 separate designs. The degree of error increases with the span length and decreases as the rigidity of the stiffening frame is increased. The large degree of maximum error, together with the variation in percentage of error for different portions of the same structure warrants the conclusion that the approximate or "elastic-theory" method is of questionable value in any case

A recommended design procedure based upon the results of the Oregon studies is indicated and certain design graphs are presented which may be used with reasonable accuracy for two-lane roadway structures designed for standard highway loadings

DESIGN PROBLEMS AND OBJECTIVES

The suspension bridge because of the comparative flexibility of its long elastic cable system presents certain design problems uniquely inherent in the type. Such an elastic system becomes appreciably distorted under load, thus rendering it necessary, for exactitude in analysis, to compute and consider the displacement of all load points. It is true that such elastic displacements constitute an essential ingredient in the analysis of all statically indeterminate structures, but in most structural types these distortions are of a comparatively low order of magnitude, and, while it is necessary for an exact determination of internal stress trajectories to consider both the equilibrium and the compatibility of all strain components, it is universal practice to assume that such strains do not appreciably affect the position in space of the external load system in reference to its supports.

When this assumption is applied to the *suspension* type, however, errors of considerable magnitude are involved, although such errors decrease as the stiffness of the ensemble is increased. As a consequence, two distinct methods of analysis have developed through the years. The first is the exact or rigorous method whose underlying theory will be briefly summarized in the paragraphs which follow This method takes into consideration the effect of elastic load-point displacements on resulting moment lever arms. The second method is the approximate or so-called elastic-theory method. It disregards the effect of the distorted geometry of the frame, and treats the suspension span in exactly the same manner as any other statically indeterminate frame type.

The relative precision of these methods may be roughly visualized from an

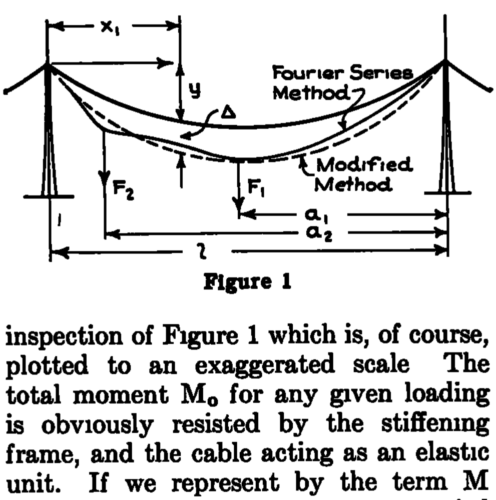

Figure 1

inspection of Figure 1 which is, of course, plotted to an exaggerated scale The total moment M_0 for any given loading is obviously resisted by the stiffening frame, and the cable acting as an elastic unit. If we represent by the term M that portion of the total moment carried by the stiffening frame, the rigorous or exact method writes

$$M = M_O - H_L(y + \Delta) - H_D\Delta \quad (1)$$

The approximate theory disregards the effect of the deflection Δ, and writes.

$$M = M_O - H_Ly \quad (2)$$

In addition to these two methods there is a modification of the rigorous method which merely disregards the secondary distortions assuming the cable always to remain in parabolic form. This method is nearly as accurate as the rigorous treatment, but, since the work involved in its application is nearly as great, it will not be considered in the comparisons which follow.

Because of the development of these radically diverse methods of attack, and because of the rather extended calculations involved in the application of the rigorous method, the Oregon Highway Department in 1937 initiated a research project looking toward the development of rational design methods for highway suspension bridges and the derivation of certain design graphs which would lessen the labor involved. These researches, undertaken by Mr Glenn S. Paxson, Bridge Engineer for the Oregon Highway Department, and Mr. Dexter R. Smith, Structural Research Engineer, under the general direction of the author, are completely described in Oregon Highway Department Technical Bulletin No. 13, entitled "Rational Design Methods for Short-span Suspension Bridges for Modern Highway Loadings," and Oregon Technical Bulletin No. 14, entitled "The Derivation of Design Constants for Suspension Bridge Analysis (Fourier Series Method)." Much of the material presented hereinafter has been extracted from these published reports. Among the purposes and objectives stated were the following.

1 To summarize and outline the mathematical theory underlying the exact or rigorous method of stress analysis
2 To compare the results obtained by this method with those based upon the hypothesis of an unstrained load-point geometry (the so-called elastic theory).
3 To develop conclusions and recommendations for design practice together with certain design graphs for the purpose of expediting the work involved

These graphs were developed with particular reference to two-lane roadway spans designed for modern highway loadings, but with certain modifications their use may be extended to other loadings and roadway widths

The discussion which follows is of necessity somewhat abbreviated, no attempt being made to include the various mathematical derivations in complete detail nor to consider many of the phases included in the reports For a further discussion of these, the reader is referred to Technical Bulletins Nos 13 and 14

THEORY

The fundamental theory underlying the rigorous method of analysis is based upon the energy balance existent in any structural frame at rest and under load. When the equilibrium of such a frame is disturbed by temperature change or by the addition, alteration or removal of live loading, it is immediately set in motion, the point of application of each external load is forced to execute a small displacement, and each of the internal members is required slightly to change shape in order to conform to its newly distorted position Each internal stress is therefore displaced through a short distance represented by its corresponding strain, thus absorbing energy, and the motion (deflection) will continue until such energy absorption is sufficient to balance the kinetic energy generated by the motion of the loads. For any elastic frame under load and at rest, the work (W_E) done by the deflection of the external loads may therefore be equated to the internal energy (W_I) absorbed by and stored within the frame Moreover, this equality will hold not only for the entire structure but for any portion of it, considered as a "free body in elastic equilibrium," provided the action of the balance of the frame upon the segment under consideration is represented by the corresponding external force or forces

Figure 2 is a layout of a single-span suspension bridge under the action of a uniform live load extending from point a($=k_1l$) to point b($=k_2l$), and Figure 3 is the same structure separated into two components [1]

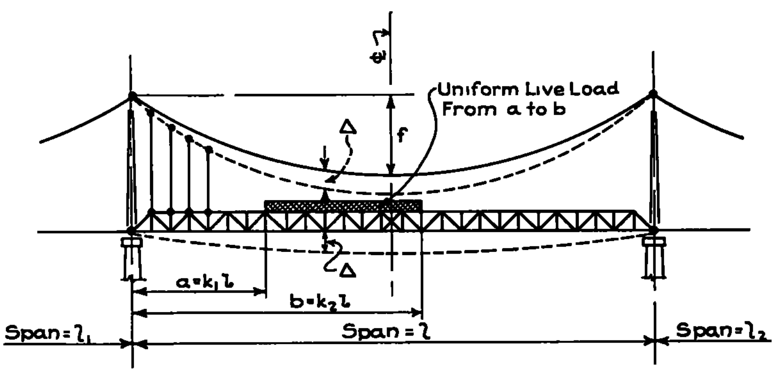

Figure 2

Let H_D = horizontal component of cable stress due to dead load,

H_L = horizontal component of cable stress due to live load,

Δ = deflection of cable at any point,

A = area of cable,

E_c = elastic modulus of cable,

w = dead load per unit length of span,

q = that portion of the live loading (p) transmitted

[1] In the interest of simplicity the backstay or side spans are not considered. These will be considered later

from the stiffening frame to the cable,

$\beta = H_L/H_D$.

The following expressions, representing respectively internal and external energy may be easily derived from elementary mechanics:

$$W_I = H_D^2(1 + \beta/2)\frac{\beta}{AE_C}\int_0^l \frac{ds^3}{dx^2} \quad (3)$$

$$W_E = \int_0^l (w + q/2)\Delta\, dx \quad (4)$$

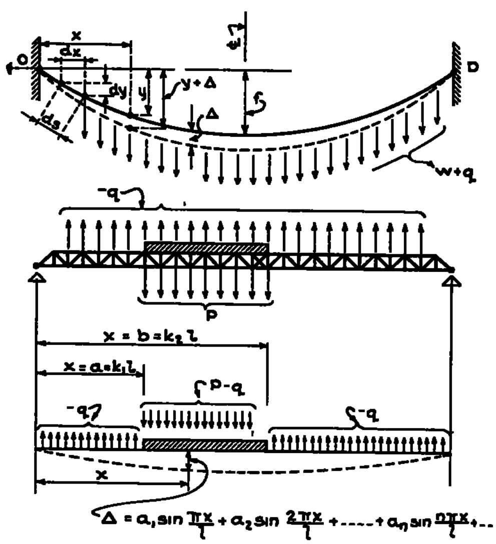

Figure 3

Moreover, the term q representing the distributed (but nonuniform) live load transmitted to the cable may be represented by the term

$$q = \beta w - H_D(1 + \beta)\frac{d^2\Delta}{dx^2} \quad (5)$$

For elastic equilibrium, therefore, we may write:

$$\frac{H_D^2}{AE_C}(1 + \beta/2)\beta\int_0^l \frac{ds^3}{dx^2} = \int_0^l \left[w + w\beta/2 - \frac{H_D}{2}(1 + \beta)\frac{d^2\Delta}{dx^2}\right]\Delta\, dx \quad (6)$$

Equation 6 is not in shape to handle because of the fact that it involves the deflection term Δ which is not only unknown but also variant from point to point along the span.

Dr. Timoshenko and Dr. Priester have developed a very ingenious method of handling this equation by utilizing a sine-series expansion of the unknown deflection term, Δ, that is to say, writing:

$$\Delta = a_1 \sin\frac{\pi x}{l} + a_2 \sin\frac{2\pi x}{l} + a_3 \sin\frac{3\pi x}{l} + \cdots + a_n \sin\frac{n\pi x}{l} + . \quad (7)$$

In order to evaluate the unknown coefficients a_1, a_2, a_3, etc., they utilized the following device

Considering the stiffening truss as a free body in elastic equilibrium, the internal work generated by its deflection may be represented by the term.

$$W_1 = \frac{1}{2}\int\frac{M^2 dx}{EI} = \frac{EI}{2}\int_0^l\left[\frac{d^2\Delta}{dx^2}\right]^2 dx \quad (8)$$

which may be transformed by means of the above Fourier expansion into the form

$$W_I = \frac{EI\pi^4}{4l^3}\sum_{n=1}^{n=\infty} n^4 a_n^2 \quad . \quad (9)$$

Differentiating this expression with respect to any particular Fourier coefficient a_n yields:

$$\frac{dW_I}{da_n} = \frac{EI\pi^4 n^4 a_n}{2l^3} \quad (10)$$

It can also be shown that

$$\frac{dW_E}{da_n} = p\int_a^b \sin\frac{n\pi x}{l}dx - q\int_0^l \sin\frac{n\pi x}{l}dx \quad (11)$$

The right-hand terms of equations 10 and 11 may now be equated and solved for the Fourier coefficient a_n, whence is derived the following.

For "n" odd:

$$a_n = \frac{2l^4[p(\cos n\pi k_1 - \cos n\pi k_2) - 2\beta w]}{EI\pi^5 n^5 + H_D(1+\beta)n^3\pi^3 l^2} \quad (12)$$

For "n" even:

$$a_n = \frac{2l^4[p(\cos n\pi k_1 - \cos n\pi k_2)]}{EI\pi^5 n^5 + H_D(1+\beta)n^3\pi^3 l^2} \quad (13)$$

We may now go back to equation 6 and substitute for the term Δ, its equivalent in terms of the sine series, whence we derive

$$\frac{H_D^2\beta}{AE_C}(1+\beta/2)\int_0^l \frac{ds^3}{dx^2} = \frac{2wl}{\pi}[1+\beta/2]\left[a_1 + \frac{a_3}{3} + \frac{a_5}{5} + \cdots\right] + H_D(1+\beta)\frac{\pi^2}{4l} \cdot [a_1^2 + 2^2 a_2^2 + 3^2 a_3^2 + \cdots] \quad (14)$$

This is the final equation from which the term β for any single span may be developed. If temperature changes as well as gravity loadings are considered, this expression becomes:

$$\frac{H_D^2}{AE_C}\beta(1+\beta/2)\int_0^l \frac{ds^3}{dx^2} + ctH_D[1+\beta/2]\int_0^l \frac{ds^2}{dx} = \frac{2wl}{\pi}[1+\beta/2]\left[a_1 + \frac{a_3}{3} + \frac{a_5}{5} + \cdots\right] + H_D(1+\beta)\frac{\pi^2}{4l} \cdot [a_1^2 + 2^2 a_2^2 + 3^2 a_3^2 + \cdots] \quad (15)$$

Considering the interaction of the side spans, the above expression is expanded into the following form:

$$\frac{H_D^2}{AE_C}\beta(1+\beta/2)\left[\int_0^l \frac{ds^3}{dx^2} + 2\int_0^{l_1} \frac{ds^3}{dx^2}\right] + H_D(1+\beta/2)ct\left[\int_0^l \frac{ds^2}{dx} + 2\int_0^{l_1} \frac{ds^2}{dx}\right]$$

$$= \frac{2wl}{\pi}[1+\beta/2]\left[a_1 + \frac{a_3}{3} + \frac{a_5}{5} + \cdots\right] + H_D(1+\beta)\frac{\pi^2}{4l}[a_1^2 + 2^2 a_2^2 + 3^2 a_3^2 + \cdots] + \frac{4w_1 l_1}{\pi}[1+\beta/2]\left[b_1 + \frac{b_3}{3} + \frac{b_5}{5} + \cdots\right] + H_D(1+\beta)\frac{2\pi^2}{4l_1} \cdot [b_1^2 + 2^2 b_2^2 + 3^2 b_3^2 + \cdots] \quad (16)$$

The integrals $\int_0^{l_1}$, the terms w_1 and l_1 and the coefficients b_1, b_2, etc. refer to the side spans, these latter coefficients being calculated in exactly the same manner as for the main span.

The fundamental equations suffice for the determination of the term β representing the ratio of live to dead load cable stress. Obviously, with this ratio determined, the stress distribution throughout the entire structure is determinate from statics. Since the term β occurs in both sides of equation 16, it must be determined by a process of trial and error However, this process is not particularly complicated or tedious.

The foregoing derivation, except for the neglect of hanger distortions is rigorous and exact, and its application is considerably less cumbersome than the solution of the linear differential equation involved in the Melan method.

COMPARISON OF EXACT AND APPROXIMATE METHODS

In order to effect this comparison, 24 separate designs were analyzed by the exact or rigorous method and also by the approximate method, or so-called elastic theory. Table 1 indicates for four typical designs in each length group the maximum degree of error introduced by the use of the elastic theory method. It will be observed that the degree of error increases with the span length, and decreases as the rigidity of the stiffening frame (as evidenced by the value I_m) is increased, which results are logical and to be expected.

The investigation also developed the fact that even for the same design the percentages of error for different portions of the structure were widely variant so that it is not feasible to apply any fixed coefficient of correction. This fact, coupled with the large degree of maximum error,

TABLE 1

MAXIMUM PERCENTAGE OF ERROR IN ELASTIC THEORY[1]

Design No	Main Span Length	Mom of Inertia of Stiffening Frame (I_m)	Maximum Percentage of Error—Elastic Theory		
			Positive Moment	Positive Shear	Cable Stress
	ft	biquadratic in	%	%	%
5	450	72,000	40 56	32 57	0 74
6	450	144,000	23 33	20 63	0 70
7	450	216,000	17 51	15 00	0 62
8	450	288,000	14 50	12 08	0 56
17	900	108,000	198 51	108 76	0 73
18	900	216,000	106 61	73 91	0 67
19	900	324,000	73 01	58 22	0 63
20	900	432,000	53 84	48 59	0 54
21	1,800	720,000	252 86	112 99	0 55
22	1,800	1,440,000	140 62	91 31	0 53
23	1,800	2,160,000	100 24	75 52	0.49
24	1,800	2,880,000	79 08	61 34	0 46

[1] The above data are for the main span. The percentages of error for the side spans are of a similar order of magnitude.

appears sufficient to support the conclusion that the approximate method is of practically no value whatsoever even for preliminary work.

RECOMMENDED DESIGN PROCEDURE

Based upon the results of these investigations, the recommendation was made that the exact or rigorous method of analysis be used in all cases since, as before stated, the elastic theory method involves errors of material and variant magnitude. The following procedure for analysis and design was recommended.

1 Sketch a general layout of the structure to fit the particular needs of the site, and calculate the dead load cable stresses in the usual manner.

2. Using an average value of β (from tables given in Bulletins 13 and 14), compute the approximate maximum cable stress, and select an approximate value for the cable area A.

3. Using different values of the moment of inertia I, develop a series of grade change and moment graphs such as illustrated in Figure 4, and with the known maximum grade change as previously determined from traffic necessities, or as given in the design specifications for the structure in question, enter the diagram and determine the necessary values of I_m and I_s.

4. With these values, test both main and side spans for maximum fiber stress induced by positive moment.

5. If one or both of these stress values exceed the safe allowable limit, make such adjustments as are necessary to bring them into conformance. The economy and feasibility of modified frame depths and also of the employment of alloy steels should be investigated in this connection. It should be remembered that since the stiffening frames are structurally redundant, more liberal unit working stresses may be employed than are ordinarily permissible. It should also be remembered that the moment values in the stiffening frame are functions of the moment of inertia I of the frame in question, and when these I values are changed during the process of balancing the design, moment values must be adjusted accordingly.

When the design has been finally selected and balanced, as described heretofore, it will, of course, be necessary to test it for negative moment and also for shear. However, positive moment stress is generally the controlling factor.

DESIGN GRAPHS

Throughout the research which forms the basis of this report, there was evidence of a relationship between maximum stress and strain and certain design constants representing relative loading, stiffness and other related data. This led to an increasing degree of hope regarding the possibility of developing certain design graphs at least of sufficient accuracy for preliminary purposes, and for estimates. Obviously the only method of approach was that of repeated trial, plotting the

pression· Grade change $= \pi/l \cdot (a_1 + 2a_2 + 3a_3 + \text{etc.} \ldots)$

The Fourier coefficients a_1, a_2, a_3, etc , involve the terms.

- p representing the specified unit live load,
- l representing the span length,
- k representing the proportionate loaded length,
- E representing the elastic modulus of the frame,
- I representing the moment of inertia of the stiffening frame,

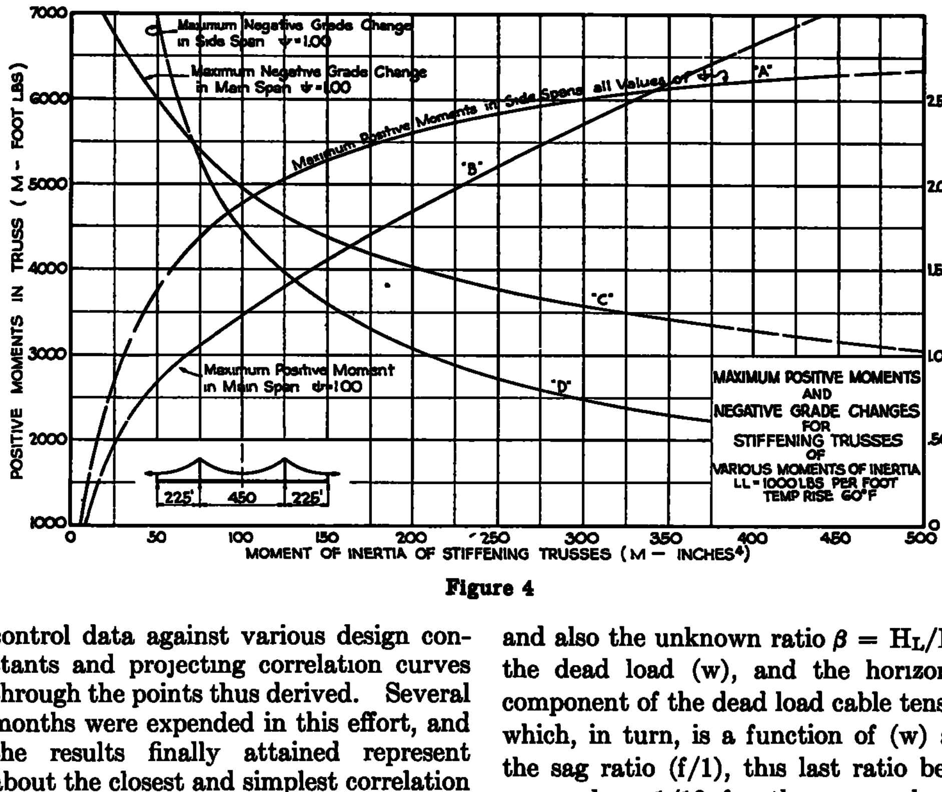

Figure 4

control data against various design constants and projecting correlation curves through the points thus derived. Several months were expended in this effort, and the results finally attained represent about the closest and simplest correlation possible in view of the many factors involved. The following description of the development of these design graphs is taken almost verbatim from Bulletin No. 14.

Grade-change Values

The maximum grade change occurs at the towers, and is represented by the ex-

and also the unknown ratio $\beta = H_L/H_D$, the dead load (w), and the horizontal component of the dead load cable tension which, in turn, is a function of (w) and the sag ratio (f/1), this last ratio being assumed as 1/10 for the curves herein developed.

After repeated trial, a design constant hereinafter termed the load-stiffness factor was developed. This constant is represented by the formula.

$$\text{L.S.F.} = \left[\frac{EI}{wl^3}\right]^{1/2} + \left[\frac{EI}{pl^3}\right]^{1/2} \cdot \left[\frac{w}{p}\right]$$

Against this constant, the corresponding values of maximum grade change were plotted, assuming a side-span length (l_1) equal to 50 per cent of the main-span length (l). The degree of correlation is quite close, as will be observed from Figure 5, wherein the correlation graph is seen to lie quite close to the individual data points.

Similar graphs for maximum grade changes in the side spans are indicated in

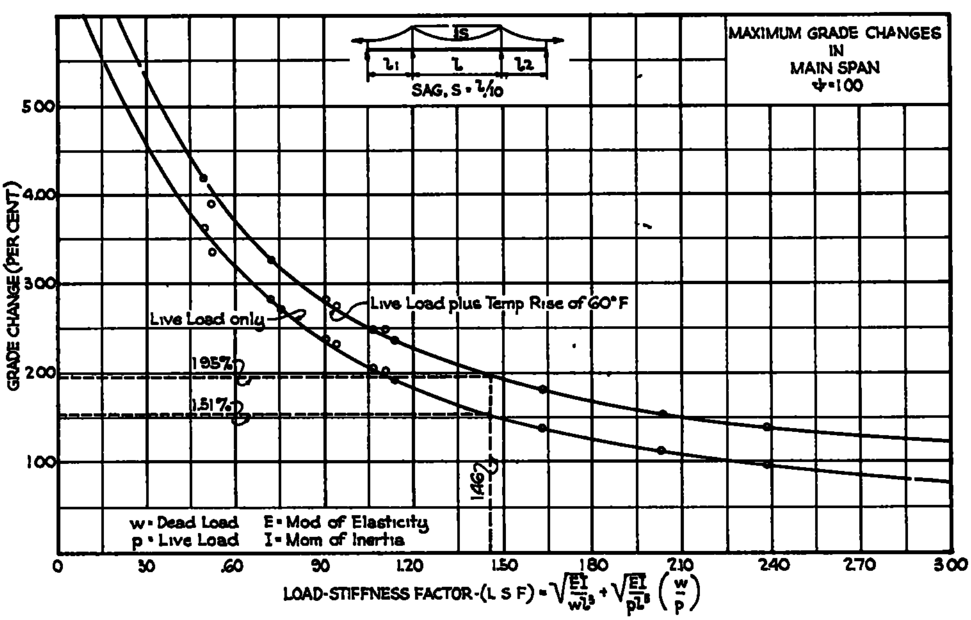

Figure 5

Figure 6. In this latter case, the design constant employed is represented by the formula.

$$\mathrm{L_1S_1F.} = {}^*\left[\left(\frac{EI_s}{8w_1 l_1^3}\right)^{1/2} + \left(\frac{EI_s}{8p_1 l_1^3}\right)^{1/2}\right]\left[\frac{w_1 + p_1}{p_1}\right]$$

In each of the above figures, two graphs are indicated, one for live load only, the other for live load in combination with a 60°F. rise in temperature.

* The subscripts are to distinguish the values of I, w, l, etc , for the side spans.

Values of k for Maximum Positive Moment in the Main Span

The graphs given in Figures 7 and 8 are correlation curves for two design constants. As abscissae are plotted, the values of the so-called stiffness factor EI/l^3 (pounds per linear inch) while as ordinate is plotted, the design constant

$$Z = \frac{k^2EI}{l^3}.$$

It is only necessary, therefore, to compute, for the design at hand, the value of EI/l^3, enter the diagram with this abscissa and determine for the particular section x/l under investigation, the value of Z. The loaded length k is then determined from the formula:

$$k = \left(\frac{Zl^3}{EI}\right)^{1/2} \text{ as indicated in the figures.}$$

Figure 7 is for stiffness factors ranging from zero to 10 lb. per lin. in., while Figure 8 carries the correlation graphs up to a value of 55 lb. per lin in.

Maximum Positive Moments in the Main Span

With the values of k determined from Figures 7 and 8, the next four graph sheets (Figures 9 to 12) may be employed to calculate maximum positive moment values for each tenth point along the main-span frame. As abscissae are plotted the values $EI/l^3 \cdot k$; as ordinates are plotted, the term

$$Y = \left(\frac{MEI}{k^2 p^2 l^5}\right)^{1/3}$$

Figures 9 and 10 are for live load plus temperature, while Figures 11 and 12 are for live load only.

Maximum Moments in Side Spans

Since the criterion for the maximum positive side-span moment is full live load over the entire span, no graphs are necessary to the determination of k. The design constant finally selected for the correlation graphs is represented by the expression:

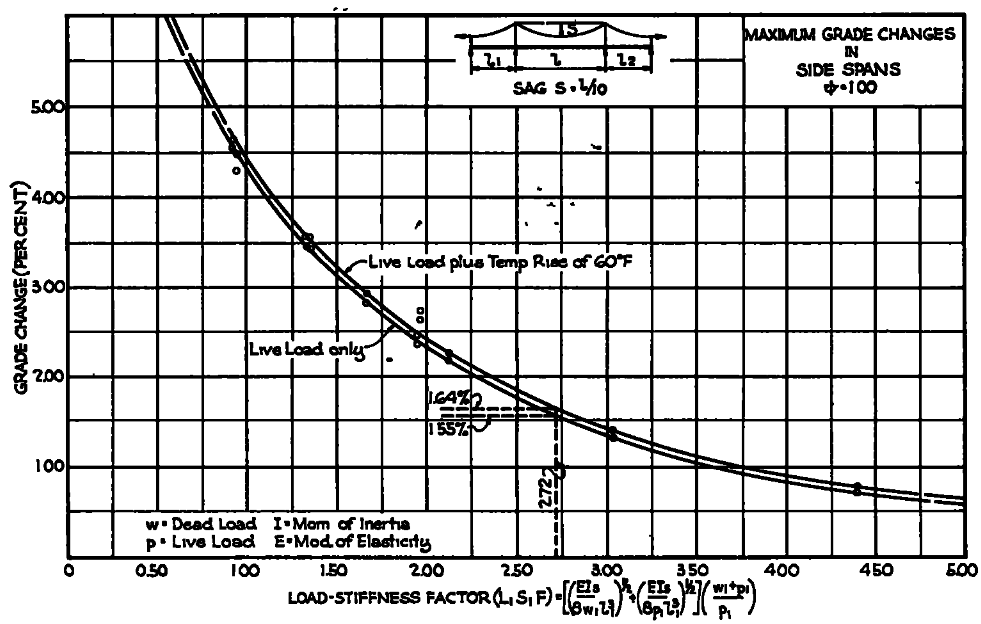

Figure 6

It is only necessary, therefore, to (1) determine k from Figures 7 or 8; (2) calculate $\frac{EI \cdot k}{l^3}$ for the design at hand; (3) enter the diagram with the above value and determine Y for the section x/l under investigation, and (4) determine M from the formula:

$$M = \frac{Y^3 k^2 p^2 l^5}{EI}$$

$$M_1L_1S_1F. = \left(\frac{EI_s}{8w_1 l_1^3}\right)^{1/2}$$

and has been termed the side-span moment load-stiffness factor.

Against this design constant as abscissa, a moment coefficient $h_L = \left(\frac{M_L}{p l_1^2}\right)^{1/3}$ has been plotted as ordinate in Figure 13, correlation graphs being developed for each tenth-point section x/l along the span. It is only necessary, therefore, to

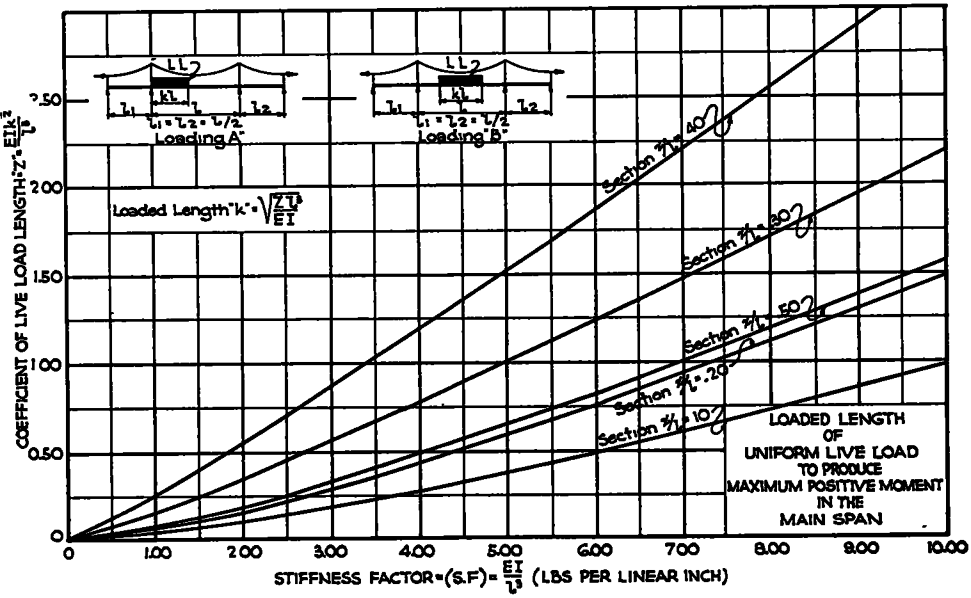

Figure 7

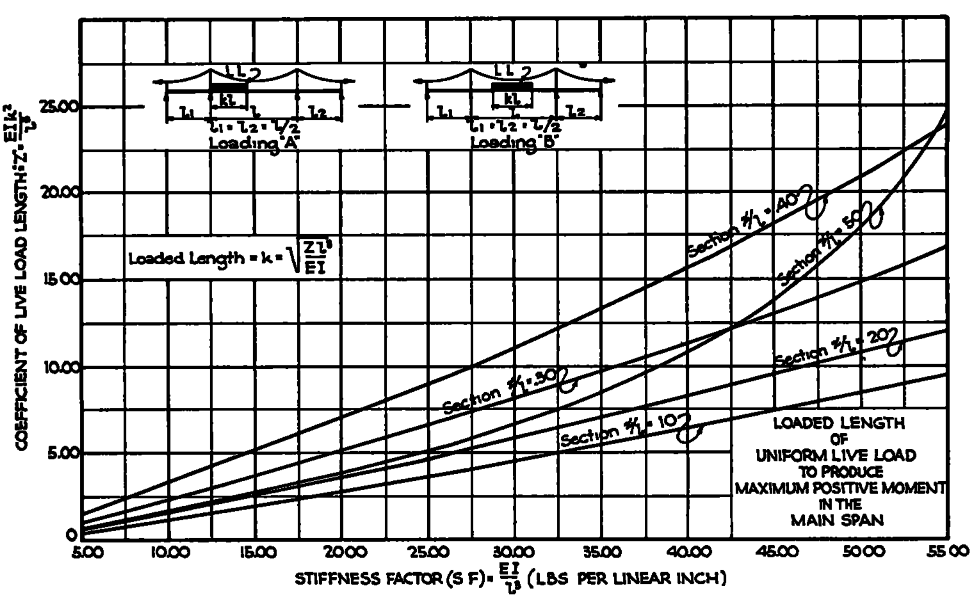

Figure 8

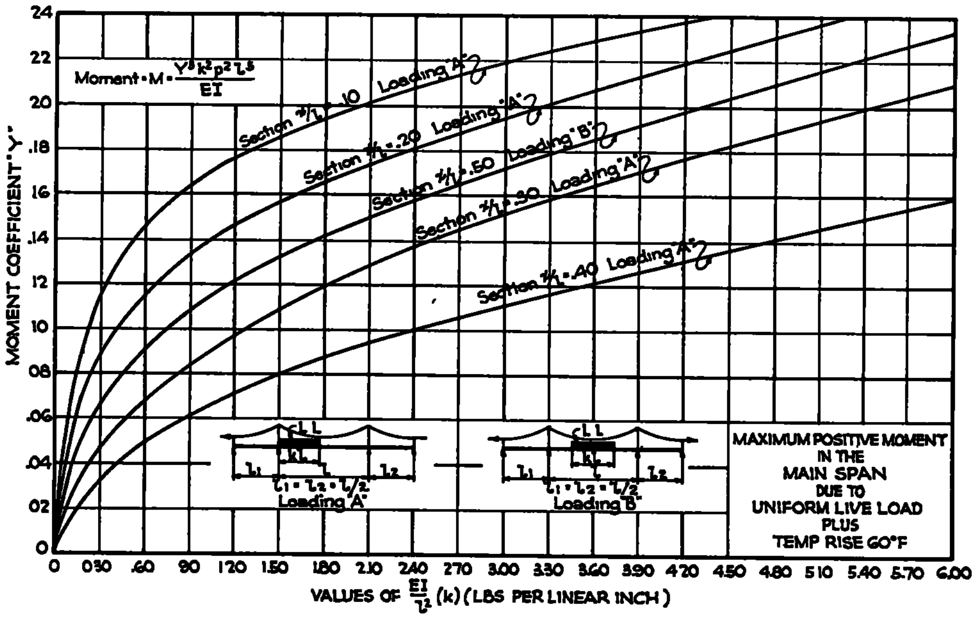

Figure 9

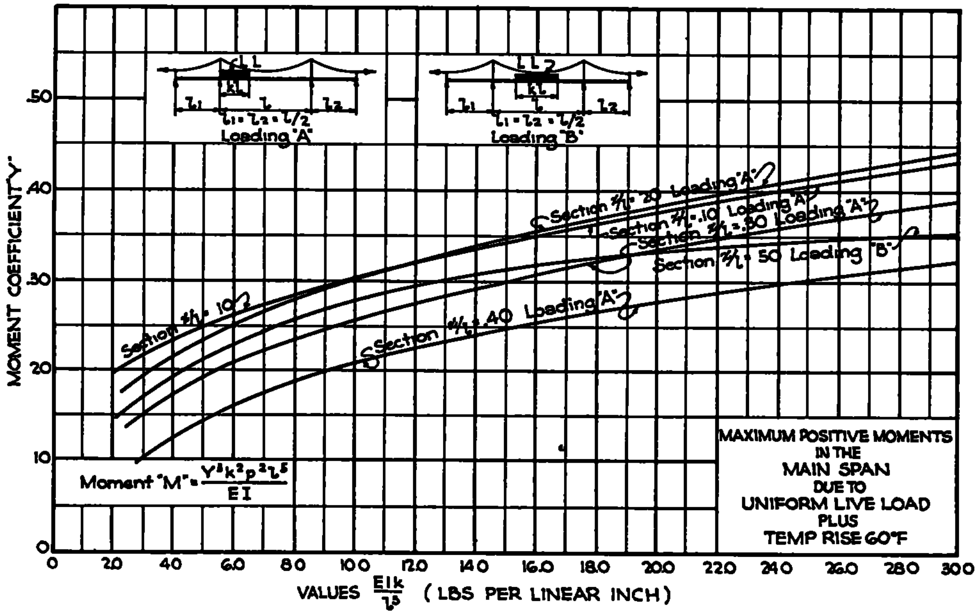

Figure 10

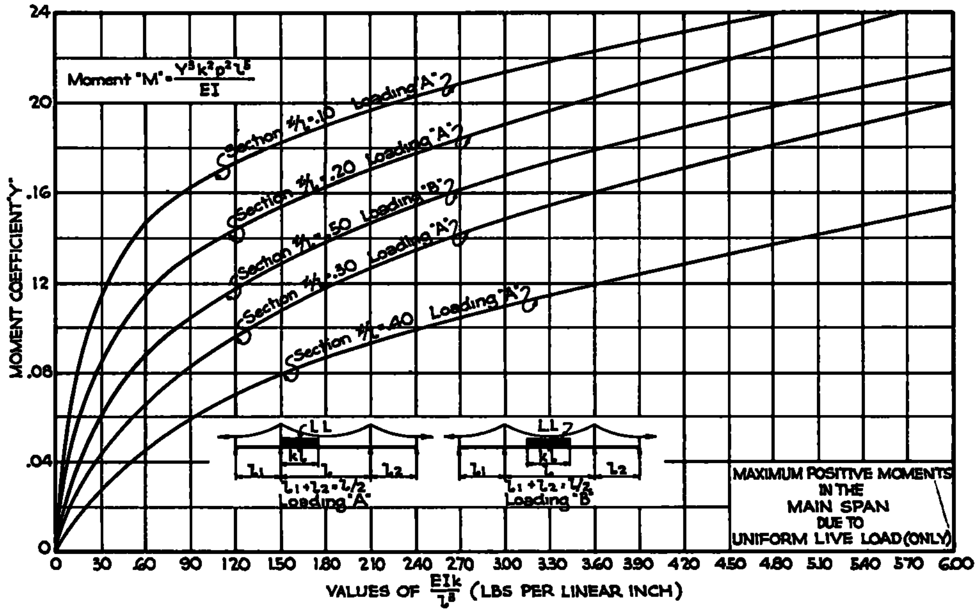

Figure 11

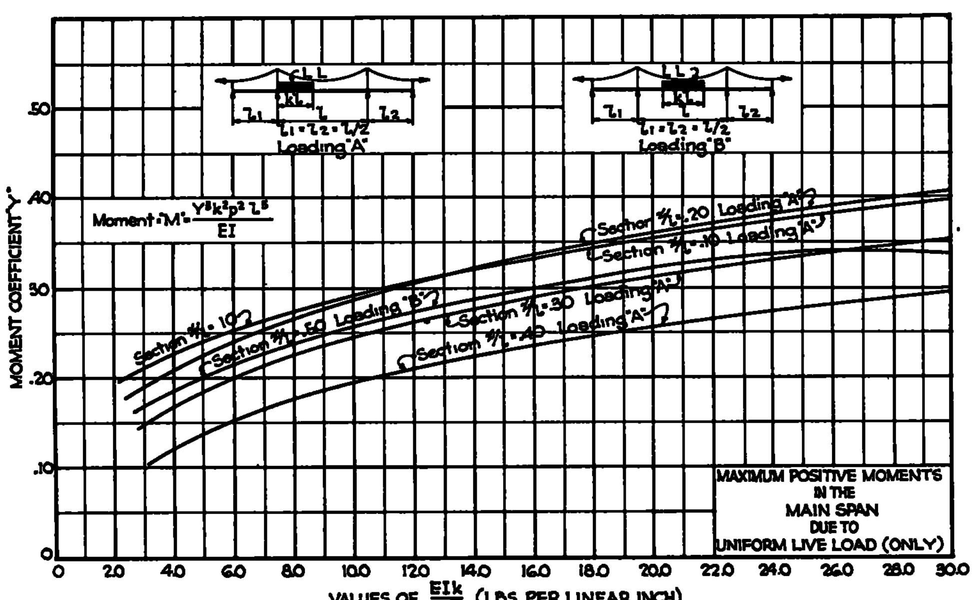

Figure 12

compute from the design data the term $M_1L_1S_1F. = \left(\frac{EI_s}{8w_1l_1^3}\right)^{1/2}$ enter the diagram with this value, and for any section x/l, determine h_L. The side-span moment at the section considered is clearly given by the expression.

$$M_L = h_L^3 p l_1^2$$

The term M_L given in Figure 13 represents the side-span moment due to live load only. Similar correlation graphs for the side-span moment, M_{LT}, representing the combined effect of live load and a 60°F. temperature rise, are given in Figure 14.

β Values

The correlation graph for β values is indicated in Figure 15. As abscissae are plotted, the values of the stiffness factor: S.F. = EI/l^3 in lb. per lin. in. As ordinates are plotted, the corresponding values of the coefficient (O) where

$$O = \frac{EIw\beta}{l^3p}$$

It is only necessary, therefore, to enter the diagram with the proper value of S.F.(= EI/l^3) and determine the corresponding value of O. The value of β is given by the expression

$$\beta = \frac{Ol^3p}{EIw}$$

Figure 16 is similar to Figure 15, except that the values of S.F.(= EI/l^3) have been carried up to a maximum of 60 pounds per linear inch It will be observed that the β correlation graphs are straight lines

Figures 15 and 16 give the values of β for full live load on the span in combination with a temperature drop of 60°F. In Figures 17 and 18 are plotted β values for a uniform live load placed in such position as to produce maximum positive moment in the main span. In these latter curves, the values of the stiffness factor S.F.(= EI/l^3), as abscissae are plotted against the coefficient $b = \frac{\beta EIw}{l^3p}$ one correlation graph being plotted for each tenth-point section x/l along the span It is only necessary, therefore, to enter the diagram with the proper stiffness factor (S.F.), determine the corresponding value of b for the section x/l under consideration, and calculate β from the formula:

$$\beta = \frac{bl^3p}{EIw}$$

Limitations to Employment of Graphs

The correlation graphs and design data described have been developed from the particular designs investigated, and are therefore somewhat limited in their application. Specifically, these limitations are as follows

(a) The graphs are smooth curves projected through a group of individual data points, as will be seen from an inspection of the figures. Their use, therefore, leads to results which are approximate only, and any design developed therefrom must be checked by a detailed analysis. Their principal utility is to enable the designer rapidly to develop a preliminary design sufficiently exact for estimating purposes.

(b) The graphs are limited to structures having the same relative geometric properties as those considered herein, to wit, a main-span sag ratio of one-tenth, and side-span length ratios of 50 per cent.

(c) The graphs are based upon an assumed value of $\psi(= I_m/I_s)$ equal to unity. The results, therefore, must be adjusted to take into account the variance in side-span interaction if other values of the ratio ψ are found to be necessary in the final design. It is generally found, however, that the effect of varying ψ values is slight.

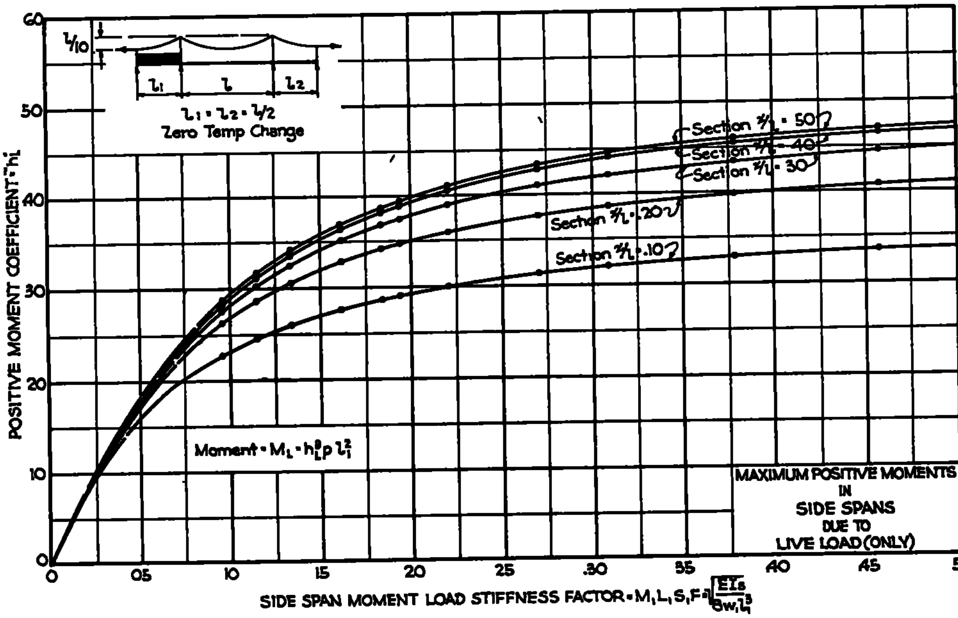

Figure 13

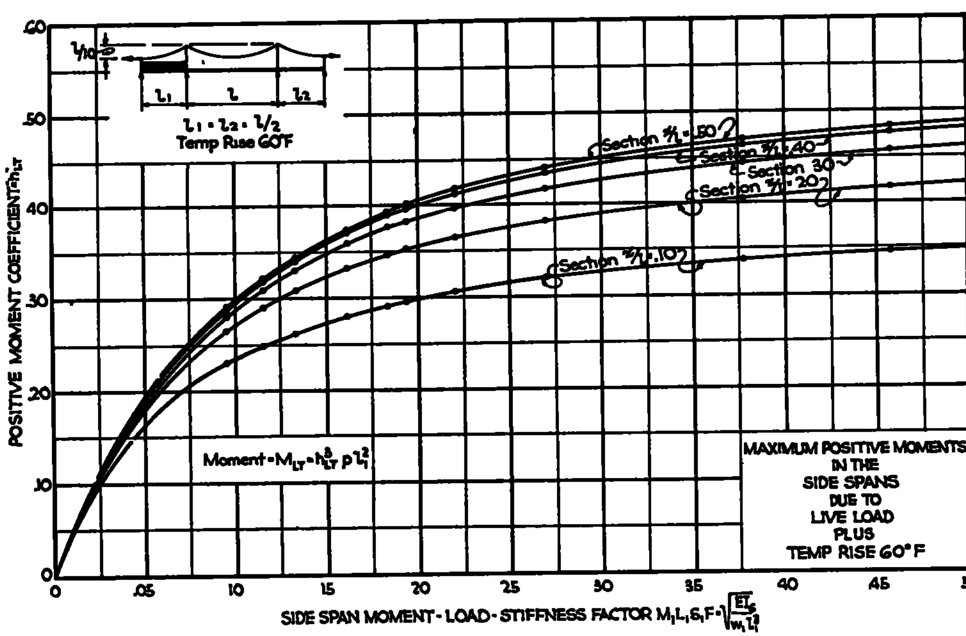

Figure 14

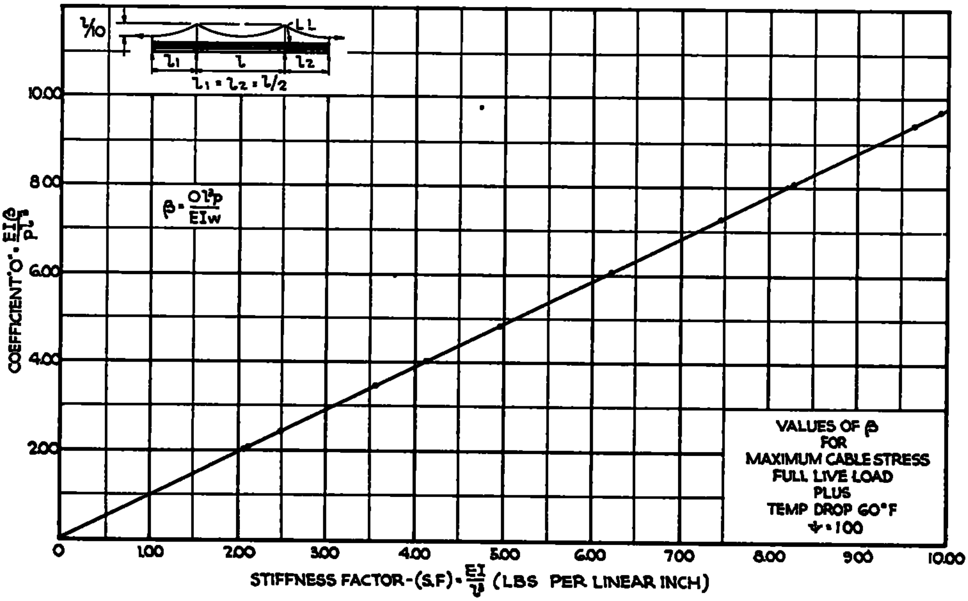

Figure 15

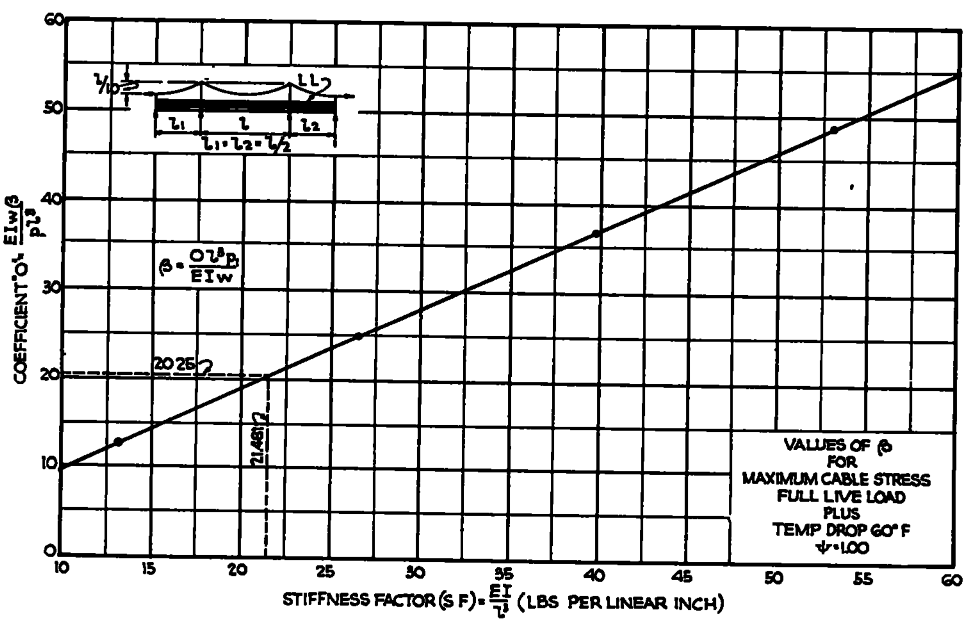

Figure 16

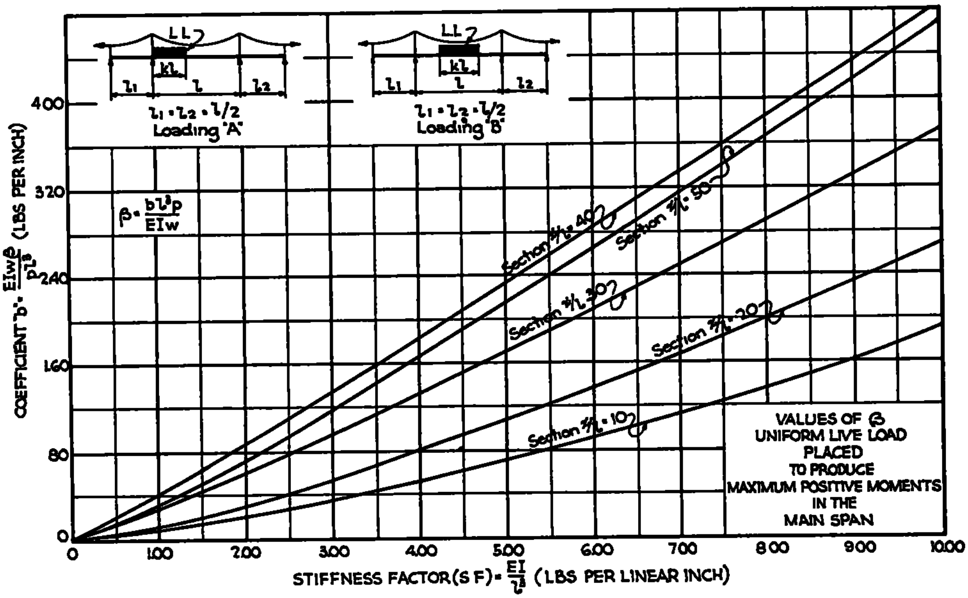

Figure 17

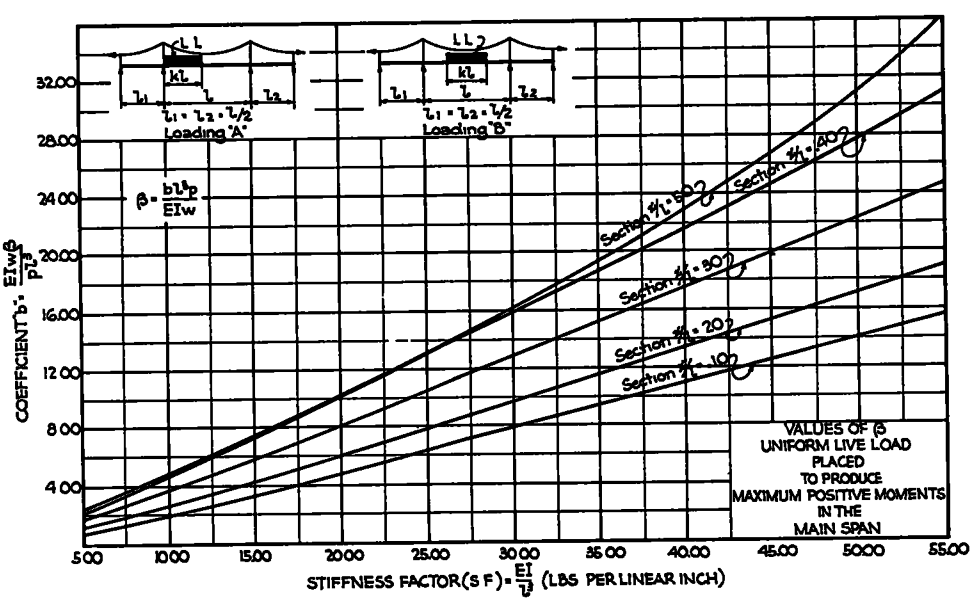

Figure 18

Illustrative Problem

In order to illustrate the use of the graphs, let us consider the design of a specific structure having the following properties:

Main span	500 ft
Side spans	250 ft
Sag ratio	1/10
E	29,000,000 lb per sq in.
Dead Load	2,200 lb per lin. ft of frame
Live Load	1,000 lb per lin ft of frame
Temperature variation 60°F.	

As before stated, the design procedure involves four initial steps, as follows:

(a) The development of four graphs or curves, such as are shown in Figure 4, to wit:

Main span grade change vs. I_m
Main span positive moment vs I_m
Side span grade change vs. I_s
Side span positive moment vs. I_s

(b) The selection of I_m and I_s for balanced grade-change values.

(c) The testing of the above values to determine their adequacy, as regards positive moment capacity, in other words, to see if either of the frames selected is overstressed for moment.

(d) To make such adjustments as are necessary to effect the greatest degree of balance consistent with safe unit stresses and maximum economy of materials.

Let us first consider the development of the four basic data graphs:

(a) *Main-span grade-change graph*—The grade changes in the main span plotted as functions of the so-called load-stiffness factor are indicated in Figure 5.

Assuming an arbitrary value of I_m equal to 160,000 biquadratic inches

$$\left(\frac{EI_m}{wl^3}\right)^{1/2} = (.11717)^{1/2} = .3423$$

$$\left(\frac{EI_m}{pl^3}\right)^{1/2} = (.25778)^{1/2} = .5077$$

$$w/p = 2.2$$

Load-stiffness factor = .3423 + 2.2 (.5077) = 1.46, and the corresponding grade-change value from Figure 5 is 1.51 per cent for live load only, and 1.91 per cent for live load plus 60°F. temperature rise, as indicated by the dotted lines plotted thereon

The foregoing determination establishes one point on the grade change-I_m graph (Figure 19). Assuming varying values of I_m and repeating the above operation, other points are established and the curve constructed as shown in Figure 19.

(b) *Main-span moment graph*—The next step is the development of the main span positive moment vs. I_m graph. In order to determine the points on this curve, it is of course necessary first to determine the critical values of k for maxima.

The curves of Figures 7 and 8 give the value of k for varying values of the so-called stiffness factor EI/l^3. Using the same trial value of I_m (160,000 biquadratic in.) we find:

$$S.F. = \frac{29{,}000{,}000 \times 160{,}000}{216{,}000{,}000{,}000} = 21.48$$

Utilizing this value of S. F., the data in Table 2 are obtained from Figure 8:

Using the values of k in Table 2 the term EIk/l^3 is computed for each section x/l to be investigated, and from Figure 9 or 10, the corresponding values of Y are determined. From these last values, the positive moment values, $M = \frac{Y^3k^2p^2l^5}{EI}$ are readily derived, the calculations, in this case being given in Table 3.

The maximum-positive moment occurs at section x/l = 0.30, and amounts to 55,672,000 in.-lb. This may not be the absolute maximum since the sections between x/l = 0.20 and x/l = 0 30 on the left and between x/l = 0 30 and x/l = 0.40 on the right have not been investigated. For absolute accuracy, it would be necessary to investigate two or more

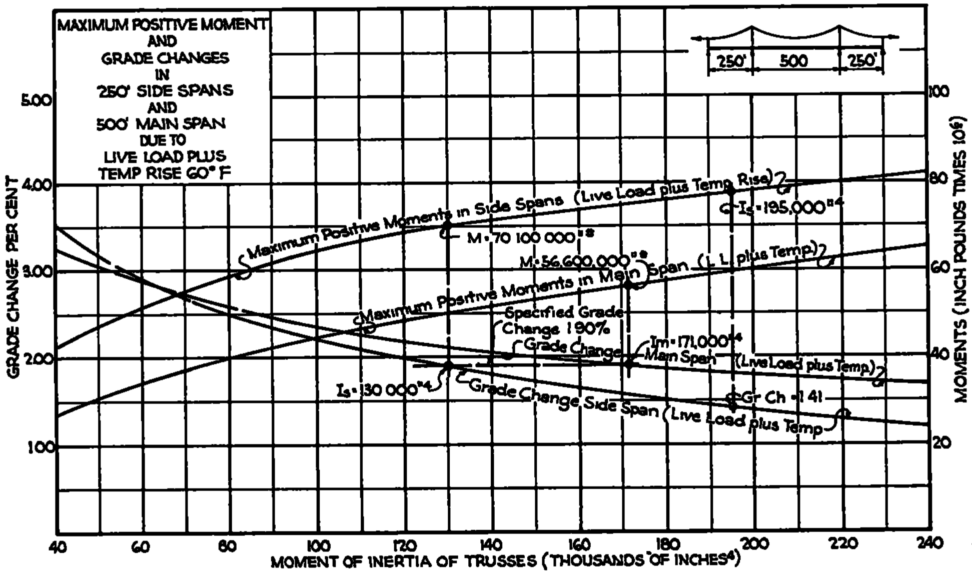

Figure 19

intermediate points and to plot a moment graph from which the absolute maximum could be determined graphically. The above value, however, is sufficiently exact for our present purpose.

This procedure establishes one point in the Positive Moment vs. $-I_m$ graph for the main span. Proceeding in exactly the same manner with other values of I_m, the graph is developed as indicated in Figure 19.

(c) *Side-span grade-change graph*—In Figure 6 are plotted the grade-change values in the side spans as functions of the factor.

$$L_1S_1F = \left[\left(\frac{EI_s}{8w_1 l_1^3}\right)^{1/2} + \left(\frac{EI_s}{8p_1 l_1^3}\right)^{1/2}\right]\left[\frac{w_1 + p_1}{p_1}\right]$$

Assuming

$w_1 = w = 2200$ lb. per lin. ft., and

$p_1 = p = 1000$ lb. per lin. ft., and

taking the first trial value of I_s as 160,000 biquadratic inches, we derive the following value for the load-stiffness factor:

$$\left(\frac{EI_s}{8w_1 l_1^3}\right)^{1/2} = .3423$$

$$\left(\frac{EI_s}{8p_1 l_1^3}\right)^{1/2} = .5077$$

$$\frac{w_1 + p_1}{p_1} = 3.20 \quad \text{and} \quad \text{L.S.F.} = [.3423 + .5077]3.20 = 2.72$$

TABLE 2

Section x/l	Z	l^2Z/EI	k	k^2
0 10	2 92	0 1359	0 369	1362
0 20	3 83	0 1783	0 422	1780
0 30	5 52	0 2570	0 507	2571
0 40	7 51	0 3496	0 591	.3517
0 50	4 14	0 1927	0 439	1927

TABLE 3

Section x/l	$\frac{EIk}{l^3}$	Y	Y^2	k^2Y^2	Moment (in. lb.)
0 10	7 927	282	022426	0030535	35,536,000
0 20	9 087	290	024389	0043639	50,786,000
0 30	10 891	265	018610	0047837	55,672,000
0 40	12.739	230	012167	0042785	49,793,000
0 50	9 430	271	019903	0038357	44,640,000

From Figure 6, we find the corresponding grade-change values to be 1 55 per cent for live load only, and 1 64 per cent for live load plus temperature, as indicated by the dotted lines shown thereon. Proceeding in like manner with other values of I_s, the side-span grade-change graph is developed, as indicated in Figure 19.

(d) *Side-span moment graphs*—In Figure 14 are plotted the values of the side-span positive moment coefficient

$$h_{LT} = \left[\frac{M_{LT}}{p_1 l_1^2}\right]^{1/3}$$

against values of the so-called side-span moment load-stiffness factor

$$\left(\frac{EI_s}{8w_1 l_1^3}\right)^{1/2}$$

Using the value assumed for I_s (160,000 biquadratic inches), the factor evaluates to 0 3423 as noted Using this value, and the section $x/l = 0\ 50$ (which is the point of maximum moment in the side spans) we determine (from Figure 14) the value of h_{LT} to be 0.462.

The maximum positive moment in the side span (for I_s = 160,000 biquadratic inches) is, therefore

$$M_{LT} = h^3 p l_1^2 = (.462)^3 \cdot \left(\frac{1000}{12}\right) \cdot (250 \times 12)^2 = 73{,}958{,}000 \text{ in.-lb.}$$

The foregoing procedure establishes one value of M_{LT}. Proceeding in like manner, but with other values of I_s, the side-span moment graph is developed as indicated in Figure 19.

With the graphs of Figure 19 once developed, the remaining procedure is largely one of adjustment for maximum balance and economy. The detailed calculations for this adjustment are given in Bulletin No. 14, pages 228, et seq