DESIGN OF PIPE CULVERTS FROM STANDPOINT OF SOIL MECHANICS

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SYNOPSIS

The purpose of this paper is to give a review (a) of existing methods of analysis of external forces acting on highway pipe culverts and (b) of some experiments along these lines These methods and experiments are examined using simple statics with addition of some elementary concepts of the mechanics of earth masses, such as limit equilibrium or active and passive pressure. No attempt has been made to propose a new theory.

"Sand Arching" and "Bin Effect." If part AB of the bottom of a box filled with dry sand is removed (Fig. 1) material below curve ACB falls out and the rest may form a convex surface. Since the latter is free of loads, the only possible stress at any point C of curve ACB is the major principal stress σ_1 tangent to that curve. The sand material along curve ACB is thus compressed as in an arch. Therefore this phenomenon is often termed "sand arching" though many engineers object to this term as incorrect. This so called "arching" cannot happen if the earth mass does not fail at least partially and fall down at least through a very small distance. If a tunnel is constructed in fragmental material, a part of the material is removed to make the tunnel and this is tantamount to a partial failure as in Figure 1. Hence in the case of a tunnel there may be sand arching; but it cannot happen over a highway pipe culvert Suppose that a pipe is placed on the earth surface (the so-called "100 per cent projection") and covered with an embankment (Fig. 2a). Unless this pipe breaks down and gives way to the material of the embankment, there may be no failure of the mass and hence no possibility of "sand arching." However, due to the action of friction between sand particles, the pipe may carry a weight less than that of earth which it supports. This is the so-called "bin effect" observed in grain elevators. In the case shown in Figure 2b the weight of the material in the bin, W, is carried by both the bottom AB (reaction R_b) and the friction against the walls (reactions R_f). Admittedly, the walls transmit pressures equal to R_f to the foundation. In what proportion the weight W is subdivided between the bottom and the walls, depends considerably on the properties of the bottom. Apparently, the weaker the bottom, the greater the part of the weight carried by the walls. This statement needs further research proof, however.

Bin Formula Examined. The bin formula is based on the studies by Janssen and Airy (1),[1] and is in some relation with the Forchheimer's study of pressure on the roof of a tunnel (2). Its application to pipes placed in ditches will be considered hereafter.

A thin element of the earth mass (thickness, one unit) occupying the whole horizontal section of one running unit of a ditch (Fig. 3, plan) is under the action of the unit vertical pressure, σ_z, acting downward (Fig. 3, elevation); a unit vertical reaction $\sigma_z + d\sigma_z$, acting upward; upward friction around the perimeter (area 2 sq. units) and the weight of the element $B \cdot \gamma$, where B is the width of the ditch in given units and γ—the unit weight of the backfill. It is assumed in computing the friction, that the unit lateral pressure equals the product of the vertical pressure, σ_z, by a constant, K_a (Rankine active value):

$$K_a = \tan^2\left(45° - \frac{\phi}{2}\right)$$

[1] Figures in parentheses refer to list of references at end.

where ϕ is the angle of internal friction of the back-fill material. Strictly speaking, the Rankine formula corresponds to the state of *limit equilibrium*; and since the walls of the ditch do not move laterally, this state is never reached. The lateral pressure is not proportional to the value of K_a, but to some other value, K, termed "coefficient of pressure at rest" or "natural hydrostatic pressure ratio." The value of K is somewhat greater than

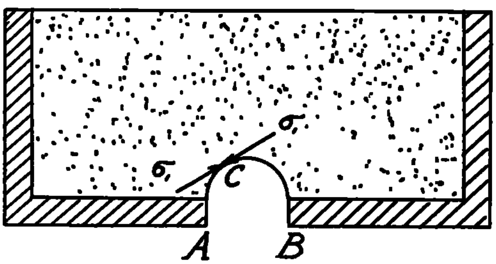

Figure 1. "Sand-Arching"

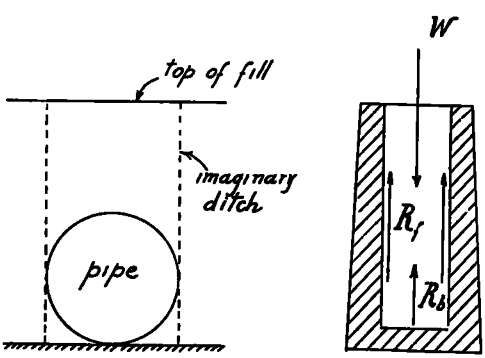

Figure 2. "Bin Effect"

the Rankine active value K_a, perhaps by 15 or 20 per cent. To find the value of the frictional reaction R_f per running unit of the perimeter of the ditch, the lateral pressure $K_a\sigma_z$ is generally multiplied by $\tan \phi_1$ where ϕ_1 is the angle of friction between material within the ditch and the walls of the latter. As already stated, this friction is variable and depends on the properties of the bottom, or in this particular case, on those of the pipe culvert Consequently, it is very questionable whether the full value of the angle of friction ϕ_1 can be developed. Hence, in the value of the frictional reaction

$$R_f = K_a\sigma_z \tan \phi_1. \quad .. \quad 1,$$

factor K_a is too small, and factor $\tan \phi_1$ is too large. This fact may bring the frictional reaction R_f computed using Formula 1, and hence the bottom reaction R_b, more or less back to their true values.

If the pipe culvert is not placed in a ditch but is exposed to the action of the weight of the embankment ("100 per cent projection"), there is no reason to believe that the imaginary ditch above the pipe

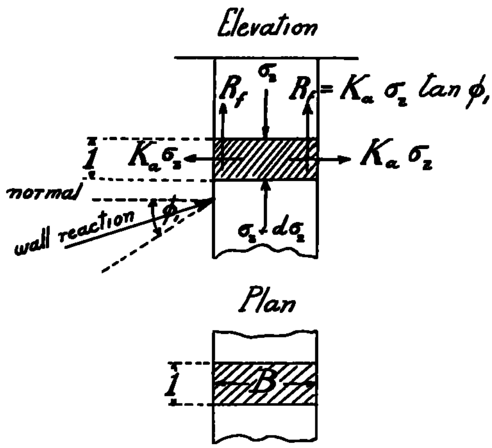

Figure 3. Bin Formula

culvert possesses vertical walls. These may be curved surfaces or oblique planes. Vertical walls are assumed only for convenience. Such an assumption termed "vertical prism assumption" hereafter was introduced by Dean Marston (3) and used by his co-workers (4, 5) Voellmy in Switzerland (6, 7) considers oblique planes which bound a wedge above the pipe culvert. Voellmy's assumption will be termed "wedge assumption" hereafter.

Classification of Pipe Culverts. Pipes may be· (a) rigid and semi-rigid and (b) flexible This classification depends on the capacity of a pipe to change its shape without injurious cracks. Of great im-

portance are also bedding conditions which, however, will not be discussed here in detail.

Structural Failure of a Pipe. It will be assumed that a pipe culvert fails when its waterway area is obstructed by the earth material, inside or near the pipe and when, in addition, the pipe itself is badly damaged or displaced. From this point of view cracks in a concrete pipe do not represent a failure if the pipe keeps more or less its original shape. But heavy longitudinal settlement of a concrete pipe under an embankment placed on a weak foundation is a failure. Especial attention should be called to this type of failure which at the present time are relieved only by "cambering" during the construction.

Furthermore, in the case of flexible pipes both smashing of the pipe by a heavy load and distortion of the shape of a shallow pipe due to eccentricity of the live load are also failures.

Rigidity of the Pipe and Internal Friction Interconnected. If the pipe is flexible (for instance, a corrugated pipe under a highway embankment), earth over it settles more than the adjacent mass (Fig. 4 (a) and (b)) and friction against the direction of this settlement is developed along the faces of the prism or of the wedge, respectively. In other words, friction in this case is directed upward and decreases the weight of the overburden. Should the structure be rigid or semi-rigid (for instance, a reinforced concrete pipe under an embankment) as shown in Figure 4(c) and (d) the settlement over the structure is less than in the adjacent mass, and the latter tends to settle. Friction is directed against this movement i.e., downward and increases the weight of the overburden. In this case friction is negative in the same way as friction along a pile driven into a deposit which is being consolidated. Designations R in Figure 4 correspond to the two reactions,

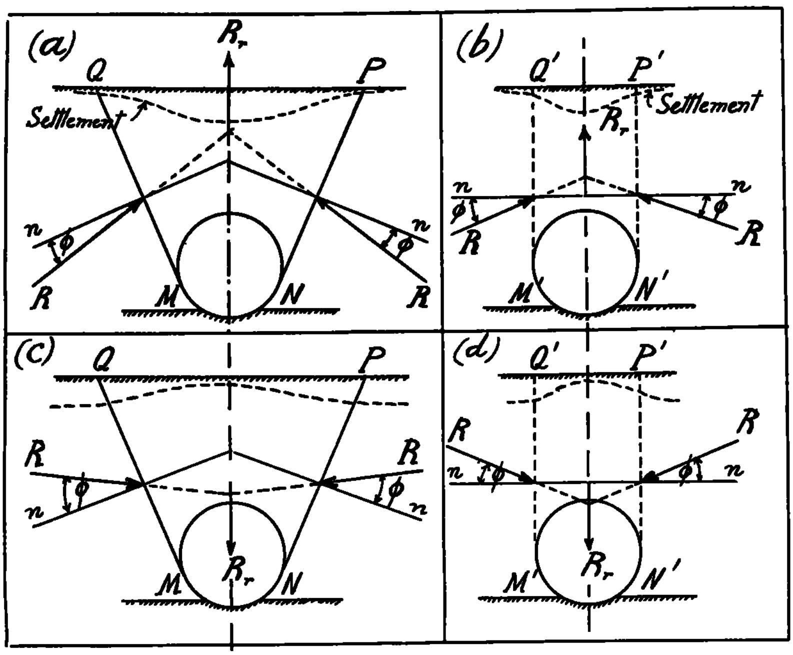

Figure 4. Rigidity of the Pipe and the Carried Load

R_r being their resultant; n–n is the designation of the normal; ϕ is the angle of internal friction of the earth material.

From this point of view slight cracking of a concrete pipe is rather beneficial since in such a case the pipe loses its rigidity and pressure on it automatically decreases.

It may be seen from Figure 4d that a rigid pipe should carry the weight of the vertical prism above it plus some additional load R_r, whereas a flexible pipe carries the weight of that vertical prism minus some other additional load, R_r (Fig. 4b). In the case of the "wedge assumption" (Fig. 4a and c) the pipe should carry the weight of the wedge plus or minus some additional load R_r, according to the steepness of the slopes. Apparently it cannot be found from experiments whether the "vertical prism assumption" or the "wedge assumption" is more correct. Since the "vertical prism assumption" is simpler, more convenient for computations and agrees more or less with experimental data, its use is preferable.

A very strong objection, however, should be made to Figure 4 in which reactions R make an angle ϕ, that of internal friction, with the normals n–n to the faces. This means that either wedge MQPN or prism M′Q′P′N′ are in the state of limit equilibrium; and there is no reason to believe so. It is quite evident that either portion QP or portion Q′P′ may be reasonably loaded without separating the respective bodies MQPN or M′Q′P′N′ from the rest of the mass which would necessarily happen were these bodies in the state of limit equilibrium. Angle ϕ as shown in Figure 4 is less than the angle of internal friction of the material, in the same way as in the bin in Figure 3.

Experiments at Iowa State College. The following is an abstract from Dean Anson Marston's report in the "Highway Magazine" for 1922. First the "projected condition" was duplicated. Ten 40-in. diameter by 2-ft. long sections of pipe made up of wood plank of iron ribs, were set between concrete head walls, 20 ft. high. The load on each section was carried by a beam to a platform scale housed in the head wall. The culvert was covered by a sandy soil, the depth of cover varying from 4 to 20 ft. As a result, each 2-ft. section carried nearly twice the weight of the earth above it. The 20-ft. fill over the test culvert was permitted to settle for a period of 50 days with a resulting increase in pressure of 1,457 lb., the final maximum value being 12,257 lb. The corresponding weight of material above the culvert was 6,240 lb. or slightly over 50 per cent of the vertical pressure.

To imitate the "ditch condition" a compacted fill about 8 ft. high was formed, and a vertical trench was dug in it for the culvert. After placing the culvert in position, the trench was backfilled with loose material and the rest of the fill was placed by the usual dumping process, to a height of 20 ft. The pressure was now 6,200 lb. and settlement for 95 days was accompanied by a steady increase to a maximum value of 7,300 lb. The values given are the average pressures on the middle eight sections.

Other work at Iowa State College has been already mentioned (4, 5). Figure 5 represents the values of radial pressure on corrugated iron pipe culvert measured by pressure ribbons (3).

Experiments at the University of North Carolina. The tests at Chapel Hill were conducted on 20 and 32 in. pipes of various materials (8). The general features of the work were: (a) pipes were placed under the fill in a 100 per cent projection condition; (b) vertical pressure was measured by means of scales as at the Iowa State College; (c) radial pressures were measured by means of Goldbeck cells fastened on the test pipe at 45 deg. intervals around its circumference; (d) the decrease in diameter was measured with a Browne and Sharp inside pipe microme-

ter. Table 1 prepared on the basis of these experiments shows displacements Δ and cell pressure readings at the ends of the horizontal diameter of a pipe The former are expressed in percentage of the diameter *d* of the pipe and the latter in lb. per sq. in. The first line corresponding to each type of pipe, shows values of Δ and the second line shows pressures. For designations, see Figure 6. Each pipe was gradually loaded and unloaded, the height of the sand fill reaching 12 ft. Unit weight of sand used was from 99 to 112 lb. per cu ft Notice that the concrete pipe referred to in the table, cracked under the maximum load.

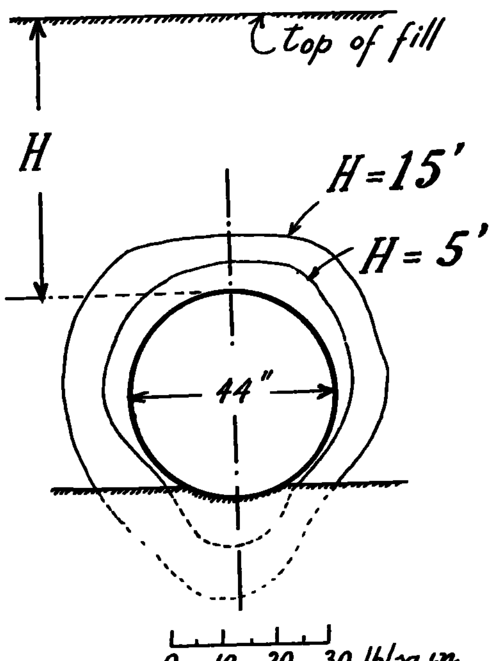

Figure 5. Radial Pressures on Corrugated Iron Pipe Culvert (Bull. 96, Iowa State College)

Experiments of A. R E. A at Farina, Illinois. In these experiments (9) no direct weighing was done. The lower 8 ft. of the experimental fill were made of the top soil of a borrow pit (unit weight 87 lb. per cu. ft), and the rest of clay with an average unit weight 112 lb per cu. ft. Pressure measurements by Goldbeck-Smith pressure cells were made on six corrugated pipe culverts from 24 to 48 in. diameter, under 6.0 to 34 9 ft height of embankments. Similar measurements were made on a 27 in. concrete culvert and a 42 in. cast iron culvert. The average pressures are represented in Table 2. In the latter the second line corresponding to each height shows the ratio of a given pressure to the weight of the fill.

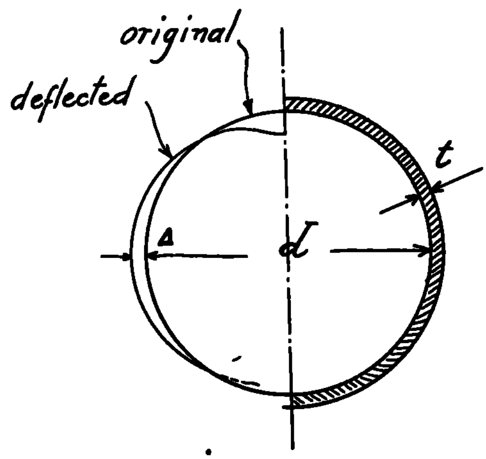

Figure 6. Deflected Pipe

Experiments in Switzerland. These experiments (6) were made by Voellmy in the laboratories of the Zurich Technical University To prove his wedge assumption Voellmy used a box 2 ft. long, 8 in wide and 8 in. deep with a hole in the bottom closed with a somewhat movable plug The angle of friction of the experimental sand being from 33 to 34 deg., the failure lines (or faces of the wedge) formed angles from 4.2 to 19 5 deg. to the vertical. Voellmy makes reference to Dean Marston's theory stating that the assumption of the latter contradicts his findings, though otherwise it is a satisfactory approximation.

Experiments in Russia. These experiments (10) were made in the laboratories of the Moscow Municipal Academy. A pipe model was centrifuged during seven minutes on a centrifuge with a radius of rotation about 20 in. Centrifugal force replaced the forces acting on the pipe and the ditch. Special dynamometers were designed to measure pressures and shears. Some of the conclusions are: (a) Marston's theory has proved to be more satisfactory than theories used in tunnel design; (b) besides normal pressures on the walls of the pipe there are tangential stresses (shears) so that the direction of the full stress at a point of the pipe forms an angle with the normal to the surface of the pipe. This angle is at a maximum at the ends of the horizontal diameter where it approaches the angle of friction. In other words, at the ends of the horizontal diameter of a pipe stresses are close to the state of limit equilibrium.

Conclusions from the Experiments. (a) Experiments confirm that pipes under embankments carry the weight of the vertical prism above it plus some additional weight in the case of rigid pipes and minus some additional weight in the case of flexible pipes.

TABLE 1

DISPLACEMENTS AND PRESSURE-CELL READINGS AT THE ENDS OF A HORIZONTAL DIAMETER OF A PIPE (CHAPEL HILL EXPERIMENTS)

Type of Pipe	Height of Sand Fill in Ft.						
	0	4	8	12	8	4	0
Smooth iron, d = 30 in.; t = 0.109 in	0 00 0 0	0 53 3 8	0.81 5 8	1 10 8.5	1 09 7 1	1 09 5 3	0 95 0 0
Smooth iron, d = 20 in., t = 0.076 in.	0 0 0 0	0 51 4 2	0 91 6 1	1.24 8 3	1 23 7 2	1 22 5 8	0.75 0 0
Corrugated metal, d = 31.5 in ; t = 0.103 in	0 0 0 0	0 20 3 9	0.36 5 7	0.51 8.1	0 51 6 1	0 50 4 2	0 13 0 0
Corrugated metal, d = 21 in , t = 0.076 in.	0.0 0 0	0 16 3 3	0 32 5.2	0 51 6 7	5 3	2.5	0 0
Steel tube, d = 30 in , t = 0 349 in	0 0 0 0	0 13 3 6	0 26 5 1	0 39 8 2	0 41 6 2	0.38 3.3	0.07 0.0
Cast iron, d = 32 in.; t = 1.0 in.	0.0 0 0	0 04 2 0	0.10 2.9	0.16 4.1	0 14 2 3	0.10 1 0	0 02 0.0
Concrete, d = 29 in ; t = 2.5 in.	0 0 0 0	0 01 2.7	0 03 4 4	0.16 5 7	0 10 3 8	0 08 3 4	0 08 0 0

(b) The vertical diameter of a loaded pipe decreases and the horizontal increases Measurements of deflections were made at Chapel Hill only. According to the measurement on pipes from 20 to 32 in. in diameter, the maximum one-way horizontal displacement Δ of a point of a smooth iron pipe is not over 1 24 per cent of its diameter under the weight of a 12 ft. high fill. This value was 0.51 per

cent for corrugated pipes; 0.39 per cent for steel tubes and 0.16 per cent for cast iron and concrete pipes.

(c) In pressing against the earth mass a pipe develops contact (passive) pressures which increase as the horizontal displacements of the pipe increase. In this way the shape of a deflected flexible pipe (pipes of smooth iron, those of corrugated metal and to a certain extent steel tubes) approaches the pressure line corresponding to the gradually increasing lateral pressure. If a concrete pipe cracks, but not badly, it may behave in the same way.

TABLE 2

Pressures on a Pipe in Lb per Sq. in in the Farina, Ill Experiments of A R E.A.

Height of fill above the pipe	Weight of the fill	Vertical Pressure		Horizontal Pressure	
		Rigid culvert	Flexible culvert	Rigid culvert	Flexible culvert
ft.					
35	26.1 1.00	41.0 1.58	14.0 0.54	12.0 0.46	17.5 0.67
30	22.3 1.00	33.9 1.52	11.8 0.53	9.6 0.43	14.6 0.65
25	18.4 1.00	26.4 1.43	9.6 0.52	7.4 0.40	11.7 0.63
20	14.5 1.00	19.5 1.34	7.4 0.51	5.4 0.37	8.8 0.60
15	10.6 1.00	12.6 1.19	5.2 0.49	3.9 0.36	5.7 0.53
10	6.7 1.00	6.7 1.00	3.4 0.50	2.5 0.37	3.4 0.50
5	2.9 1.00	2.9 1.00	1.7 0.58	1.3 0.45	1.7 0.58

(d) The results of Marston's pipe culvert analysis are practically in agreement with experimental data, and at the present stage of soil mechanics research, should be considered as adequate for all practical purposes.

(e) The attention of investigators should be called to possible tangential stresses at the circumference of a loaded pipe. Apparently, pressures on a loaded pipe are not only radial as in Figure 5 but may form an angle with the normal to the surface of the pipe.

(f) Experiments at Iowa State College have shown that highway traffic has no appreciable load effect on pipe culverts when the height of the embankment exceeds 5 ft. In the case of a pipe buried in a ditch this height may be even less.

Additional References. A series of articles on culvert pipe analysis has been published in France by the Hungarian engineer Enyedi Bela (11) and by M. A. Guerrin (12). Information may be also found in the catalogues of pipe manufacturers. (13, 14, 15.)

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