DISCUSSION ON FLEXIBLE SURFACES

Prof. W. S. Housel, *University of Michigan:* As has been previously stated by several investigators, the design of a flexible surface is no different than the design of any other structure and consists of two essential parts. first, determination of the stresses resulting from the loads which are to be carried; and second, correlation of these stresses with the resistances which may be furnished by the materials of which the structure is built.

While this general statement holds, the problem at hand is complicated by two special conditions. First, instead of an articulated structure made up of an assembly of structural members which may be rather easily isolated and placed in equilibrium, a flexible surface involves stresses in an indefinite mass and the equilibrium of elements of mass which must be so selected as to properly represent the dimensional effects that are involved Second, instead of homogeneous materials to which the normal laws of elasticity may be applied the soil and soil mixtures used in the structure are not elastic, and most of all they involve definite discontinuity of stress relationships which prevent the logical application of the laws of elastic behavior in any but qualitative terms. Qualitative similarity has some value but a rational design is impossible unless stress and resistance can be correlated in quantitative terms measurable by available test procedures.

Considering the two major subdivisions of structural design the six papers presented make up an exceptionally well balanced program. Two of the papers, the one by Spangler and Ustrud and the other by Benkelman and Lancaster, deal with stresses imposed on the flexible surface and subgrade or methods of determining those stresses. Two more of the papers, the one by Palmer and Barber and the other by Burggraf, are concerned with available resistance of the materials in the structure and methods of measuring and integrating this resistance. The remaining two papers by Hubbard and Field and by Goldbeck are a direct attack on the problem of design of the complete structure and suggest methods of evaluating the combined resistance of surface and subgrade.

WHEEL LOAD STRESS DISTRIBUTION

In the paper on "Wheel Load Stress Distribution beneath Flexible Type Surfaces," Spangler and Ustrud have reported results of a preliminary series of tests performed in the laboratory on a dry clay subgrade compacted to an approximate dry density of 114 lb. per cu. ft. This subgrade was placed in a concrete bin on a 6-in. layer of coarse gravel and was 2.5 ft. thick. Three types of surfaces were used in the reported experiments consisting of two well graded gravel base courses 3 and 4 in. thick and a well graded sand-clay base course 5 in thick.

The results of the tests indicate what might be anticipated from the procedure used, that the subgrade was relatively rigid and pressure distribution curves are qualitatively similar to those obtained from numerous series of such experiments that have been conducted since the first recorded work by Kick and Steiner in 1879. Spangler and Ustrud indicate that quantitatively the results are not entirely adequate as some experimental difficulties were encountered with the carbon stack measuring devices. The protection of these carbon stacks by rubber packing is open to serious question as a source of differential deformation which would concentrate pressure on the more rigid carbon plates.

One is forced to agree with Spangler that the results presented represent a progress report and throw some light on the experimental difficulties which must be overcome in order to obtain authentic data on the pressures acting at the contact plane of the flexible surface and subgrade. One is also forced to agree with the authors that the application of these data to actual field conditions is not possible

In this connection the writer would like to emphasize several statements by the authors that have been recognized by other investigators but not generally recognized by the profession as a whole In the introductory discussion it is stated, "the relative stiffness of a pavement and its subgrade is one of the reasons why final studies of subgrade stresses should be made on actual pavement structures in the field rather than depending wholly upon laboratory studies where it is impossible to duplicate actual subgrade and base course conditions" "In general it seems probable that the less stiff the subgrade in relation to the base course, the wider will be the subgrade stress distribution and the lower the maximum stress at the subgrade surface directly beneath the load"

The extent to which these statements may be true was clearly demonstrated to the writer in a series of load tests made in the field on gravel bases and oil aggregate surfaces built over both plastic clay and sand subgrades These tests were conducted by the Michigan State Highway Department and have been reported in a preliminary way [1] There seems little necessity for duplicating the discussion referred to, and it is sufficient to say that in most cases in this series of tests the mats were sufficiently rigid in relation to the subgrade to reverse the usual sequence of stress reactions It was found that the load transmitted to the subgrade immediately under the bearing area was a minimum and did not reach maximum values until loaded areas had punched through the base This disturbing behavior has been under study but final conclusions have been delayed largely because of time consuming work on related laboratory resistance tests

However, these tests confirmed the writer's long-maintained position on load tests and bearing capacity of foundations in general, namely, that it was impossible to duplicate field conditions adequately in the laboratory and impractical to find a substitute until we had identified the responsible factors of resistance more completely than is generally the case today

There are several other statements and tentative conclusions made by Spangler and Ustrud that might be discussed with profit After having recognized the possible effect of relative rigidity of pavement and subgrade the authors proceeded to make tests under limited conditions of relative rigidity and draw some conclusions which were general even though tentative

This statement applies to conclusions 1, 2 and 4 given in the paper The writer definitely questions the general validity of the statement "the maximum pressure occurs on a relatively small area directly beneath the contact area" in the light of the authors' previous statement and some experimental evidence cited above. To go further and even suggest a tentative formula for the maximum pressure relating pavement thickness and giving constants for different total loads obtained under the limited test conditions seems like a dangerous procedure It is true that the tentative conclusions are prefaced by a concise statement of their limitations, but the temptation for the uninitiated to use the formula may very easily be too great

With respect to the first conclusion and the applicability of the Boussinesq solution and Griffith's concentration factor there is much to be said and a great deal has been said in reports of previous investigations and reviews on pressure distribution This question may be discussed in

[1] Superior figures refer to list of references on page 324

conjunction with some of the introductory remarks in the authors' discussion. After referring to a number of previous attempts to formulate a design procedure for flexible surfaces they state, "these suggestions..... need not be discussed here It is sufficient to say that all of these suggested formulae are based on speculative concepts of flexible pavement performance and that none of them has an adequate experimental background. Very little experimental work has been done in the field. A few pertinent data were published by Goldbeck in 1923, Older in 1924, Spangler in 1930 and 1940, and Goldbeck in 1937 and 1940"

This rather sweeping dismissal of previous attempts seems to be wholly unjustified particularly when it is considered that the present investigation itself is limited to the restricted phase of pressure distribution which probably has the most complete experimental and theoretical background of any problem in soil mechanics. Undoubtedly there is a definite need for additional experimental data under the specialized conditions of a pavement surface supported by a plastic subgrade, but it appears that this need will only be met by the introduction of a considerable range of variation in the relative rigidity of the component parts and not by the repetition of experiments which have been quite adequately covered by previous investigations.

Excellent reviews of both experimental work and theoretical developments of pressure distribution have been made available by Kogler and Scheidig[2] and by Cummings.[3] A long list of names which are associated with pressure distribution studies could be added to those given by the authors but suffice it to say that the repeated verification of results qualitatively and quantitatively establish the adequacy of the experimental background for pressure distribution under idealized conditions. When special conditions are selected as they usually have been, it appears that the various elastic equations including the original Boussinesq formula and all extensions of that solution reproduce the results of observation with quite remarkable accuracy.

The problem of interpreting this knowledge and extending it to more practical conditions has been discussed at length, and a number of important conclusions have been generally recognized. Cummings states the two most important conclusions as, "(1) the manner in which the pressure is distributed over the contact area must be considered and (2) the equations of the theory of elasticity must be modified before they can be applied to soil." It is generally recognized that the elastic equations become inadequate at points close to the load application which is particularly significant because it is in this disturbed zone that the soil is overstressed and the limit of load carrying capacity is established.

It is also generally admitted that the basic assumptions of the theory of elasticity are all violated in the critical region of stress Under these important practical conditions the soil is not elastic and definite discontinuity of stress-strain relationships have been recognized since the early work of Strochschneider. Cummings notes two general methods of further progress in calculating stress distribution in soils under these conditions. The first is to retain the general framework of elastic theory introducing additional constants which alter the elastic equations. The second is to eliminate the elastic theory as the method of attack and proceed on a "rational" basis which recognizes the failure of the basic assumptions.

In a discussion of this subject the writer supported the latter method and expressed the opinion that the value of elastic theory had been exhausted.[4] The imperfections in elastic theory are too deep-seated when applied to a plastic medium subjected to discontinuities of stress and strain to permit further extension The writer still holds this opinion and believes the criticism valid even when applied to alteration of elastic equations by stress concentration factors such as proposed by Griffith and Froehlich

The writer's viewpoint was presented in some detail in a discussion of the Cummings paper and the essential elements of a proposed method of pressure distribution have been incorporated in a method of design of flexible surfaces [5] There should be little need to duplicate this development here as the objective of this discussion is simply to establish the fact that pressure distribution representing the first phase of the problem of designing a pavement structure does have a well developed experimental background A careful study of this background reveals data which indicate the variations which may be anticipated under the special conditions of flexible surfaces and more flexible subgrades While more experimental data are unquestionably needed, available information points the way to future investigation which will contribute the greatest progress

In this connection the authors of the paper under discussion appear to be in agreement with a general concensus that the most productive research will be that made in the field under natural conditions which are difficult if not impossible to duplicate in the laboratory. It is not fair to assume without some investigation that no experimental evidence of this character is available Reference to one such investigation has been made in the present discussion but a complete report of these tests has not been published. There may be other such experimental projects completed or in progress that are not yet available to the profession, and while they serve no immediate purpose the progress in this direction should be noted.

On the other hand, there is a considerable amount of valuable experimental data on load tests made for the purpose of evaluating the bearing capacity of foundations which have been published and which are a source of information capable of interpretation in terms applicable to flexible surfaces During the past twelve years the writer has conducted approximately 20 complete series of load tests under a wide variety of soil conditions and used these data in the design of surface structures Most of these tests have been reported in publications in this country and abroad and it is believed that they do supply experimental information applicable to the design of flexible surfaces [5, 6, 7, 8, 9, 10, 11] One such series of tests was performed under conditions which simulate quite closely the elements of a flexible surface A complete analysis of this series of tests is presented in the paper on flexible surfaces previously mentioned and appears to verify the suggested formula for thickness within reasonable limits [5]

The analyses of load test data and the application to substructure design represents a direct experimental approach that differs materially from that suggested by the authors of most of the papers on the present program Instead of attempting to measure subgrade pressures directly and evaluate pressure distribution by conventional equations or approximations of these equations, the stress reactions of the soil mass are evaluated from the variation in bearing capacity of different sizes of bearing area This variation has been expressed by a linear equation for bearing capacity in terms of stress reactions, perimeter shear and developed pressure, which are the integrated result of the variable resistances mobilized in the disturbed zone of the soil mass These stress reactions have been correlated with pressure distribution as formulated in the investigations previously cited and limiting values have also been correlated with soil resistance measurable by available test procedures.

The methods developed have been the source of considerable controversy but the fact remains that structures of some magnitude have been designed on the basis of these methods, and the settlement of the structures has been predicted from the test results within close limits without a single notable exception. While there are some complicating features in the case of flexible surfaces there has been no evidence produced up to this time to indicate that the same method of approach will not prove equally valuable in pavement design.

While being a proponent of one method of approach to the design of flexible surfaces, the writer fully recognizes the value of other methods of attacking the problem. When pressure distribution at the contact plane of the pavement surface and subgrade is measured under conditions properly reproducing field behavior, the results should prove most enlightening. Unfortunately the experimental difficulties appear to be even greater than in the interpretation of the load tests.

SOIL PRESSURE CELLS

The careful and complete study of "The Design and Use of Soil Pressure Cells" by Benkelman and Lancaster is a case in point. The writer did not have a copy of this paper available for study but the results presented at the meeting were extremely interesting and illustrated the possibility for considerable variation which may be present in the cells themselves or in the installation of the cells. Certainly the data presented demonstrate the necessity for eliminating or controlling variations in the pressure cells before attempting to obtain authentic data on actual pressures under a flexible surface.

FIELD TESTS

The second phase of the design of flexible surfaces is that having to do with resistance developed by the subgrade soil in conjunction with the pavement surface. Mr. Burggraf presented a paper on a field test to evaluate the shearing resistance of subgrade soils and the pavement surface both separately and in combination. Copies of this paper were not available for general distribution but the presentation at the meeting and familiarity with the development of the test through the past several years furnishes some basis for comment. In the first place, the test is designed to measure physical properties under field conditions not reproducible in the laboratory. In this respect the procedure conforms to the concensus that future progress lies in taking the laboratory into the field.

Mr. Burggraf has covered a great deal of territory in the past several years conducting tests where road surfaces have failed and accumulating the most vital information on limiting design conditions which are usually quickly buried by the necessity of immediate correction. He has presented a number of interesting and valuable correlations between road surface failures and test data using the special device described in the paper. Among other things these data bring out the importance of relative rigidity of the surface and subgrade which is admittedly a primary consideration.

Up to the present time correlation of test data has necessarily been empirical but nevertheless has shown great promise. In attempting to evaluate Burggraf's results from the viewpoint of a rational design the writer is somewhat confused as to whether loading of the surface and subgrade soil in a horizontal direction is sufficiently representative of the stress conditions under vertical load. It is desirable, of course, to measure resistance in quantitative terms which may be used directly in a formula for thickness. It may be that continued investigation will show that results from such a test may be applicable, and if this proves to be the case, the device and procedure will be an extremely practical and valuable working tool

SOIL DISPLACEMENT

The paper by Palmer and Barber on "Soil Displacement under a Loaded Circular Area" is an attempt to integrate soil resistance in terms of the mathematical theory of elasticity by correlating stress and deformation in the disturbed zone. The writer remarked at the time of the meeting that this mathematical development led to the same basic relationships which had been accepted as a self-evident fact from the variation in bearing capacity on loaded areas of different size and shape The mathematical treatment does supply a missing link between theory and experiment and the fact that it leads to the same qualitative results should be reassuring from both viewpoints

Conflict arises, however, when the formulae based on elastic theory are extended into the realm of quantitative measurement In this phase it is the writer's opinion, as previously stated, that elastic theory breaks down It is the purpose of this discussion to point out the similarity of general relationships and demonstrate the failure of elastic theory to provide for observed behavior when discontinuities of stress and strain are encountered at boundaries of the loaded elements

The authors' mathematical development leads to an equation for settlement which is a special case of the general equation for settlement of a loaded area developed from load test analysis[6]

REDUCTION OF GENERAL EQUATION TO SPECIAL CASE

$$\Delta = \frac{Wh}{I(b^2 + brh)} = \frac{K_1 p}{1 + K_2 \frac{P}{A}}$$

Δ = settlement
W = total load
I = modulus of incompressibility
b = width or diameter
r = slope for any angle of distribution
h = any depth
K_1 = coefficient of settlement
p = applied pressure
K_2 = stress-reaction coefficient
$\frac{P}{A}$ = perimeter-area ratio

$$K_1 = \frac{h}{I},\ K_2 = \frac{rh}{4},\ \frac{K_2}{K_1} = \frac{rI}{4}$$

When h is taken infinitely large and for a circle of radius a the general equation reduces to

$P = 2\pi a$
$A = \pi a^2$

$$\Delta = \frac{p}{I \frac{r}{2} \frac{P}{A}} = \frac{2}{r} \frac{pa}{I} \qquad (1)$$

$$S_L = F \frac{pa}{C},$$

Palmer and Barber Equation (2)

Eqs 1 and 2 are identical, with one exception to be elaborated on later, inasmuch as Δ and S_L both denote settlement or deformation and I and C are the same by definition and take the place of modulus of elasticity in compression

The one exception involves different methods of evaluating those quantities which depend upon the physical properties of the soil Even here the same general relationships are involved though couched in different terms and measured by different tests Palmer and Barber, after using an infinite depth, h, which produces the special case of an infinitesimal bearing area, evaluate a settlement factor, F, in terms of Poisson's ratio and the principal stress difference which varies with the depth, z, but is subject to a maximum value equal to twice the shearing resistance of the soil The solution is further limited to dealing with vertical pressures computed only for the vertical axis and does not provide for variation in pressure distribution over the entire bearing area. The modulus of deformation, C, is evaluated from stabilometer tests by the use of a secant modulus which is a stress-strain ratio averaged from tests at different lateral pressures.

In the writer's general equation from load tests there are three factors, r, h, and I, which depend upon the material and which must be evaluated although they are not subject to direct measurement under practical conditions by any method other than an actual load application which measures their combined and integrated effect.

In this solution there are introduced two soil resistance coefficients K_1, coefficient of settlement, and K_2, stress-reaction coefficient where Palmer and Barber introduced a settlement factor, F. The relationship between K_1 and K_2 and the stress reactions, m and n, measured by load tests has been given,[6] $\left(K_1 = \frac{\Delta}{n}, K_2 = \frac{m}{n}\right)$. K_1 is also related to the modulus of incompressibility being proportional to $\frac{1}{I}$ and thus includes the authors' C. K_2 is a factor which provides for treating pressure over the loaded area as a whole by including the boundary stress concentration in the average bearing capacity by adding it to the developed pressure arising from the principal pressures developed as the maximum stress difference and including static head. Developed pressure which is triaxial compression is independent of the size of the area and is the stress reaction supposedly duplicated by the stabilometer test.

Palmer and Barber evaluate these principal pressures in terms of assumed values of Poisson's ratio to obtain the effect of lateral pressures, combined with the depth function, $\frac{z}{a}$, to obtain decreased pressures with the depth. The writer has used a similar method of pressure distribution with a linear approximation for the decrease in pressure ordinates with the depth. This has been referred to as an "assumed cone of pressure" but it should be pointed out that the cone of pressure is not an assumption but a name given to a stress variation that is deduced directly from all of the mathematical and experimental investigations of pressure distribution that are now on record. Any assumptions involved have to do with values assigned to the angle of distribution. In the development of the general equation for settlement from load tests no specific value has been given to this angle, and it remains as one of the factors to be measured as an integrated effect.

Poisson's ratio has not been used directly in the writer's analysis of load tests and the impression sometimes exists that it has not been considered. This is incorrect as the pressure developed under the bearing area is a direct function of the supporting lateral pressure, a relation which is implied in the general equation. When developed pressure is segregated from the boundary stresses and evaluated by direct load tests, the manner in which lateral pressure comes into play reflects the actual stresses that are developed in the disturbed zone and is more accurately portrayed than when it is evaluated in terms of assumed values of Poisson's ratio. There has been a great deal of intelligent guessing as to the proper value of Poisson's ratio for soils but so far as the writer is aware there have been no actual measurements for this ratio in terms of lateral deformation which would be applicable to the complex conditions of pressure concentration beneath a loaded area.

The relation of Poisson's ratio to stress reactions evaluated from the analysis of load tests has been developed in another publication.[8] In the generalized treatment of soil resistance the following equation relating the modulus of incompressibility, I, and modulus of rigidity, R, indicates the replacement of Poisson's ratio by a ratio of measured stress reactions.

$$I = 2R\left(1 + \frac{nA}{mP}\right)$$

From elastic theory the relation between the modulus of elasticity, E, and the modulus of rigidity, E_s, is given by the equation

$E = 2E_s\ (1 + \lambda)$

λ = Poisson's Ratio

The relation between modulus of incompressibility and modulus of rigidity developed from soil mechanics is similar to that given for elastic solids, except that the ratio of total force carried by developed pressure to total force carried by perimeter shear replaces Poisson's ratio in the elastic equation. Poisson's ratio is defined as the ratio of lateral deformation to deformation in direction of the applied load. The total load carried by developed pressure represents the amount of lateral resistance developed and the ratio of this force to the load carried by perimeter shear is the ratio of lateral resistance to resistance in direction of the applied load However, it is expressed in terms of force rather than deformation, and includes factors which express the boundary conditions. If it is accepted that stress is proportional to strain, the ratio of resistance is strictly comparable to Poisson's ratio providing further that the boundary conditions are not to be ignored

The separation of boundary stresses from developed pressure exposes the manner in which lateral pressure is actually developed and may be illustrated as shown by the typical stress reaction curves in Figure 1. In this chart settlement, Δ, is plotted horizontally and the stress reactions, m and n, vertically as shown by the typical m and n curves. The major portion of the applied load may be carried by perimeter shear at small settlements and pressure is developed gradually after some settlement due to lateral yielding and compression of the soil. The writer has always maintained that the sequence with which these stress reactions are developed in the ground cannot be satisfactorily reproduced in the laboratory under artificial conditions. The manner in which soils develop pressure with respect to relative deformation varies through a wide range for different types of soil. It is hard to conceive how this variation could ever be reproduced by assumed values of Poisson's ratio.

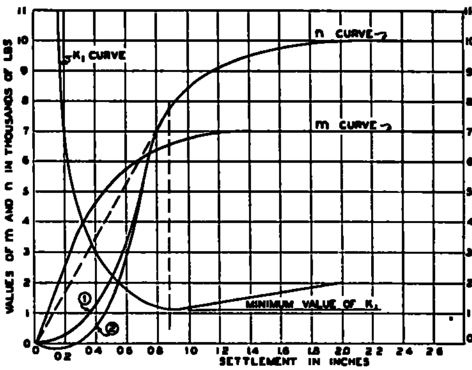

Figure 1

As an illustration of this very important point one need only refer to the sequence with which the stress reactions, perimeter shear and developed pressure, are mobilized as shown in Figure 1 In several of the series of load tests the developed pressure curves swing down into negative values for low settlements as shown by Curve 2 There is a perfectly logical explanation for this in compressible soils which develop a large elastic depression. In the lower range of load, if the relative rigidity of the bearing plate or a pavement surface is great with respect to the subgrade, there is a tendency to develop resistance at the boundary of the loaded area and bridge the relatively large elastic depression in the supporting mass As shown in Figure 2 bearing is developed first around the boundaries and the central portion is pulled down by vertical components of load emanating from the perimeter Thus tension or negative developed pressure is reflected in abnormally high boundary stresses and compensating negative pressure ordinates

This behavior, which is quite marked under the proper combination of relative rigidity of the loaded surface and supporting soil, illustrates how completely assumed values of Poisson's ratio would fail to reproduce the actual lateral components developed in the soil mass. Poisson's ratio $\left(\frac{nA}{mP}\right)$ would in this case have to be negative. It also illustrates the necessity for considering the pressures

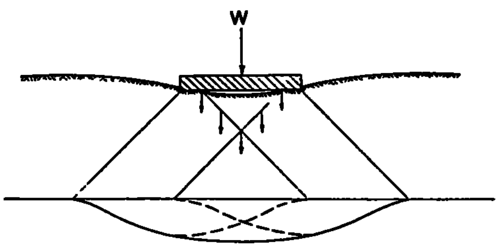

Figure 2

over the whole bearing area rather than just the theoretical pressures on the central vertical axis which in this case would also be negative.

The series of tests which have previously been referred to as simulating the action of a flexible surface provide another excellent illustration [5, 7] As shown by the developed pressure curve in this series of tests, there are certain stages of loading in which there is no increase and even a slight recession in the developed pressure and equivalent lateral pressure. Such variation would require that Poisson's ratio be zero or negative in this range. Some might assume that results which are so far out of agreement with the usual theoretical conception of Poisson's ratio must be seriously in error. It may be pointed out, however, that in this particular instance the footings for a large grade separation were designed on the basis of these unusual results for a predicted settlement of approximately 0 15 in. and that the measured settlements on this structure after a period of some years are remarkably close to the prediction.

Referring again to Figure 1 there is another interesting similarity between treatment used by Palmer and Barber and the criteria developed by the writer. The authors have used a secant modulus based upon a load deformation diagram from the triaxial compression test. The settlement coefficient, K_1, used as a criterion in load tests makes use of the developed pressure curve but is identical, however, with the secant modulus referred to above. The minimum value of K_1 which establishes the ultimate bearing capacity corresponding to the maximum stress difference on the loaded element is the cotangent of the angle between the horizontal axis and the secant line drawn from the origin to the designated point on the developed pressure curve.

There is one other statement in the paper under discussion which further illustrates the failure of elastic theory adequately to describe the behavior observed in the load tests. The authors state that the shape of the loaded area has little effect on settlement and cite as authority for this statement deflections computed by Timoshenko by application of the equations of elasticity to different shapes of bearing areas. There can be little question of the correctness of the mathematics which may be unusually rigorous. But one has only to review the results of actual tests on different shapes of bearing area to find a wide variation due to shape which is, however, quite adequately described by dimensional effects of the areas. In the final analysis the problem clearly reduces itself to a case of definite conflict between fact and an application of theory which latter, in the writer's opinion, has been extended beyond the limitations of the assumptions on which it is based

DESIGN METHODS

Goldbeck follows the more or less conventional method of approach, and while he recognizes the relative rigidity of the pavement surface as a factor to be considered, this recognition is not incorporated in the design procedure in any way that would provide for such phenomena as have been observed in load tests in the field The pressure distribution curves obtained in his tests conform to those obtained by other investigators and he makes use of an average pressure distribution at a 45-deg line as in comparable design formulae by Harger and Bonney, and Gray [12]

In passing it may be noted that the use of an average pressure distribution has been quite generally rejected as inadequate in any problem involving a direct correlation between the pressure transmitted and soil bearing value, particularly under conditions which involve concentration of pressure in the vicinity of the loaded area. Even when provision for a varying angle of distribution is made as in Hawthorn's design formula, such average distribution is incompatible with the conditions of static equilibrium outside of the elastic range.[8]

Goldbeck introduces a ratio, k, of the maximum subgrade pressure measured by pressure cells to the average pressure from load applications on synthetic subgrades built up in the laboratory. The direct experimental determination of k might establish validity of method if three conditions were satisfied First, the type of pressure distribution should be a conventional type with maximum pressure on the vertical axis Second, as stated by the author, the correct value for k should be known for each particular combination of surface and subgrade. Third, and most important, the bearing capacity of the subgrade must be measured in comparable terms.

It would appear to the writer that the third condition touches on the weakest point in Goldbeck's proposed method. The use of a single size of bearing area to measure bearing capacity provides no basis for measuring a property which varies as a function of the size The use of 100 sq in, which is approximately an average size of the tire contact area, partially compensates for the inadequacy of a single test but allows no opportunity to adjust the design for other sizes. The selection of a change in the curvature of the load-settlement diagram and the short time interval used in the tests also makes the bearing value a very uncertain determination.

The method proposed by Hubbard and Field embodies the direct experimental approach in which the writer has long been interested as an investigator. As stated by the authors, "the nature and distribution of stresses within the pavement and within the soil, as well as in the mechanism of stress distribution of the soil . . . may then be eliminated from consideration insofar as the present problem is concerned "

In discussing this statement it is logical to consider the results of the authors' tests and their present conclusions in the nature of a progress report which is complete as far as it goes but has not yet reached its ultimate objective. The writer regards the results of these carefully controlled tests as a convincing demonstration of the validity of load-area relationships and an independent verification of an experimental technique which has been the main subject of the preceding discussion They also demonstrate that these relations can be extended to the special conditions of a pavement surface on a relatively plastic subgrade

However, the proposed method assumes that the ultimate objective is achieved by conducting load tests on substantially the same type and size of pavement structure that will be built in the field. It has always been the writer's hope that the stresses imposed on the surface and subgrade could be isolated and correlated with resistance of the materials involved so that relatively simple resistance tests could be incorporated in the design procedure. For example, the bearing capacity of the subgrade might be evaluated in terms of shearing resistance measured by a test such as proposed by Burggraf, by a penetration method, or by a simple transverse shear test such as proposed by the writer.[13, 14] The ability of the surface to distribute the concentrated load over the subgrade might also be evaluated by a transverse shear test on samples prepared in the laboratory to the same density produced by ordinary compaction procedure in the field and may reproduce the resistance to punching shear observed by Hubbard and Field.

It is along this line that the laboratory research of the Michigan State Highway Department and the University of Michigan has been directed during the past several years. The results of these studies appear promising and it is hoped that the results may be reported within the next year. In the present discussion it may be pointed out that if this type of design procedure is the ultimate objective it is necessary that the nature of stresses within the pavement and within the soil be known for the various conditions of relative rigidity of subgrade and surface that will be encountered in the field. The limitations of reproducing actual pavement surfaces in the laboratory have been recognized by all investigators including Hubbard and Field, and it seems that the effort to develop more practicable test procedures should be continued.

There is another phase of the investigation by Hubbard and Field which should be discussed and that is the selection of the deflection of 0.5 in. as representative of the allowable load to be carried by the pavement. The fact that the same critical settlement was observed regardless of the size of the bearing plates is in substantial agreement with load test observations which indicate that the settlement coefficient, K_1, is constant for all sizes of bearing area. However, Hubbard and Field did not demonstrate that the minimum value of K_1 which establishes the ultimate capacity of the subgrade or yield value of the soil coincided with the settlement of 0.5 in. Due to the short time interval between load increments, the question may be raised as to whether or not the lateral yielding of the subgrade soil was completely evaluated. It might well be that a longer time interval would have produced failure in punching shear at a lower intensity of load.

The variation in K_1 and the determination of its minimum value identifies the yield value of the soil. The determination of linear equations for the entire range of settlement and analysis of the corresponding variations in perimeter shear, m, and developed pressure, n, is the only positive basis for evaluating the critical load. It may very well be that a settlement of 0.5 in. under the test conditions used is beyond the ultimate capacity of the subgrade. While actual wheel loads may be applied for only short periods of time the accumulation of infinitesimal but permanent displacements caused by instantaneous subgrade stress in excess of the yield value may eventually lead to failure of the surface. It is to be hoped that Hubbard and Field may extend their analysis of the present tests to determine the criterion mentioned and verify the selection of the specific settlement used as a basis for design.

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14 "The Shearing Resistance of Soils—Its Measurement and Practical Significance," *Proceedings*, American Society for Testing Materials, Vol 39, Pt. II, 1939

MR L A PALMER, *Public Roads Administration, Discussion on Interrelationship of Load, Loaded Area, Stress and Deformation of Soil*· Assuming a linear stress-deformation relationship, the following device serves to illustrate the effect of area and load on deflection

With reference to Figure 1, consider the same unit load, p, on the same soil and on two different circular areas having radii, a, case 1 and Na, case 2, N being greater than unity The points A_1 and A_2 on the axes of symmetry and at depths a in case 1

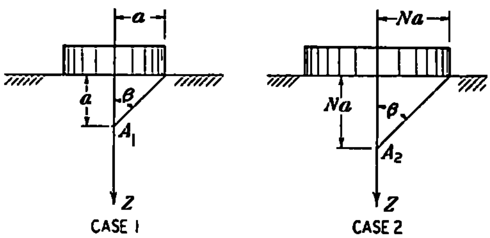

Figure 1

and Na in case 2 are homologous points. The vertical stress, p_z, is the same at A_1 as it is at A_2. In each case,

$$p_z = p\,(1 - \cos^3\beta) \qquad (1)$$

and since β in the figure is 45° in each case,

$$p_z = p\left[1 - \left(\frac{1}{\sqrt{2}}\right)^3\right] = 0.65p.$$

An element of earth at A_1 is stressed to the same extent as the homologous element at A_2 But there are N times as many elements between the load and A_2 as there are between the load and A_1 Hence, the deflection along the axis in case 2 is N times the corresponding deflection in case 1

If the same total load is on each circular area, Figure 2, cases 1 and 2, then the unit load, p_1, of case 1 exceeds the unit load, p_2, of case 2. In case 1, $p_1 = \dfrac{P}{\pi a^2}$ and in case 2, $p_2 = \dfrac{P}{\pi N^2 a^2}$. Then

$$p_1 = N^2 p_2 \qquad (2)$$

The vertical pressures at A_1 and A_2 are expressed then by the relations,

$$\text{At } A_1,\ p_z = p_1(1 - \cos^3\beta) = N^2 p_2(1 - \cos^3\beta).$$

$$\text{At } A_2,\ p_z = p_2(1 - \cos^3\beta)$$

The pressure at A_1 is N^2 times that at the homologous point, A_2. But there are still N times as many elements between the load and A_2 as there are between the load and A_1. Apparently then the deflection on the axis in case 1 is $\frac{N^2}{N} = N$ times the corresponding deflection in case 2.

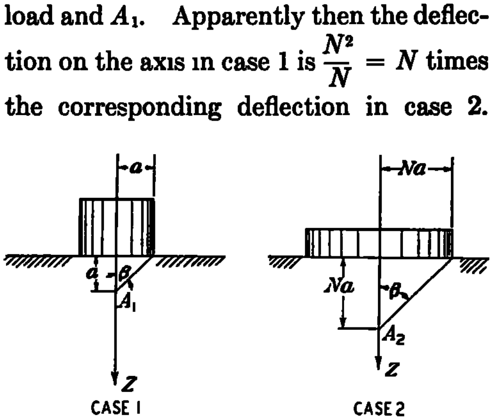

Figure 2

Summarizing and remembering that a linear stress-deformation relationship is assumed,

1. For the same unit load, the deflection increases in direct proportion to the diameter of the loaded circular area.

2. For the same total load, the deflection is inversely proportional to the diameter of the loaded circular area.

Suppose, for example, that $N = 2$. Denote the deflections by S_L (1) in case 1 and S_L (2) in case 2. Then for the same unit load,

$$\frac{S_{L(1)}}{S_{L(2)}} = \frac{1}{N} = \frac{1}{2}$$

or

$$S_{L(2)} = 2\,S_{L(1)}$$

whereas, for the same total load,

$$\frac{S_{L(1)}}{S_{L(2)}} = \frac{N}{1} = 2$$

or

$$S_{L(2)} = \tfrac{1}{2}\,S_{L(1)}.$$

It is not necessary that the load distribution over the circular areas be uniform for these relationships to hold. The only requirement is that the load distribution be symmetrical and of the same shape in both cases.

In 1929, J. H. Griffith[1] derived stress equations involving a parameter, n, which may be adjusted to fit the behavior of materials other than the elastic isotropic solids. For the latter materials, $n = 3$, and Griffith's expressions for stresses reduce in this case to those of Boussinesq. For a parabolic load distribution over a circular area of radius a, the vertical pressure, p_z, at a depth z on the axis is

For $n = 3$,

$$p_z = 2p_0\left[1 - 2\frac{z^2}{a^2} + \frac{2z^2}{a^2\left(1 + \frac{z^2}{a^2}\right)^{1/2}}\right] \quad (3)$$

For $n = 4$,

$$p_z = \frac{2p_0}{1 + \frac{z^2}{a^2}}\ . \quad (4)$$

For $n = 5$,

$$p_z = 2p_0 + \frac{4p_0 z^5}{3a^5\left(1 + \frac{z^2}{a^2}\right)^{3/2}} - \frac{4p_0 z^2}{3a^2} \quad (5)$$

For $n = 6$,

$$p_z = p_0\left[2 + \frac{z^6/a^6}{(1 + z^2/a^2)^2} - \frac{z^2}{a^2}\right] \ldots (6)$$

where p_0 is the average pressure over the loaded surface, that is, the average contact pressure.

It is interesting to compare the values for vertical pressures reported by M. G. Spangler with those computed from the

[1] J H Griffith, "Pressures Under Substructures," *Engineering and Contracting*, March 1929, pp 113-119

above equations. The values computed by equation (5) are in fair agreement with the reported observed values. Values computed by equations (3), (4) and (6) deviate considerably from the observed values.

Aside from this theoretical consideration, M. A. Biot[2] showed that the presence of solid rock below a soil layer loaded at the surface tended to concentrate the stresses in the neighborhood of the axis of symmetry. This deduction has no

TABLE 1

OBSERVED AND COMPUTED VALUES OF p_z ON CENTERLINE UNDER A WHEEL LOAD OF 3000 LB

Area of contact	Equivalent radius, a	Depth to pressure cell	Observed p_z	Computed values of p_z: Parabolic loading, $n = 5$	Computed values of p_z: Values computed by equation (1) and multiplied by 1.7
sq in	*in*	*in*	*lb per sq in*	*lb per sq in*	*lb per sq in*
55.3	4.20	3	67	80	74
60.4	4.38	3	67	75	70
69.8	4.71	3	65	68	62
60.4	4.38	4	59	61	58
60.4	4.38	5	50	50	48
60.4	4.38	6	37	41	40
60.4	4.38	8	25	28	28
60.4	4.38	10	19	20	19

relation whatsoever to Griffith's concentration factor, n, applicable to the case of soil of infinite depth. Biot assumed two possibilities, (a) perfect friction between overlying soil and rock and (b) no friction between these two materials. For his case (b), the vertical pressure at the rock surface and on the axis of loading is shown to be 1.7 times its value at the same spot if the rock were not there. This comparison is, however, for a point load and not for a load distributed over a circular area.

If the values of p_z as computed by equation (1) for a uniform load are each multiplied by 1.7, the results compare favorably also with Spangler's observed values. These results are shown in Table 1.

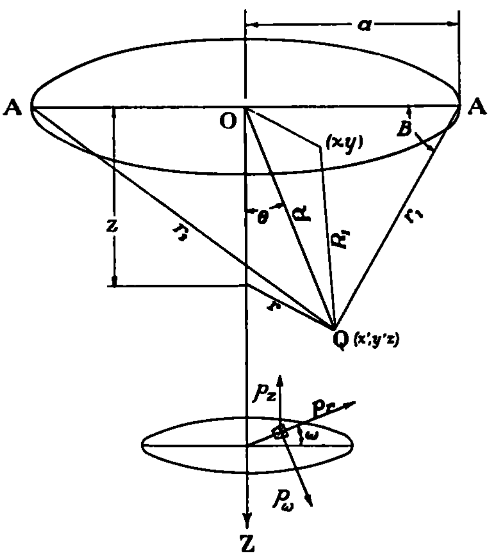

Figure 3

The fact that it is possible to reconcile theoretical and observed pressures by multiplication of theoretical values by a constant factor is of particular interest. This same principle will most likely hold when theoretical and actual deflections under wheel loads are compared.

For a uniform load on a circular area special theory is required for computing the vertical pressures at points removed from the axis of symmetry. With reference to Figure 3, the value of p_z at the point Q is computed as follows:

$$p_z = -\frac{p}{2\pi}\left(z\frac{\partial w}{\partial z} - w\right) \qquad (7)$$

[2] M. A. Biot, "Effect of Certain Discontinuities on the Pressure Distribution in a Loaded Soil," *Physics*, December 1935.

where

w = solid angle subtended at Q by the loaded circular area

z = the depth of Q below the ground surface.

p = unit load on the circular area.

To evaluate w and $\frac{\partial w}{\partial z}$ at any point Q, the following relations are used:

$$w = 2\pi - 2\pi\left[\frac{R}{a}P_1(\cos\theta) - \frac{1}{2}\left(\frac{R}{a}\right)^3 P_3(\cos\theta) + \frac{3}{8}\left(\frac{R}{a}\right)^5 P_5(\cos\theta) - \frac{15}{48}\left(\frac{R}{a}\right)^7 P_7(\cos\theta)\right] \ldots \quad (8)$$

and

$$\frac{\partial w}{\partial z} = -\frac{2\pi}{a}\left[P_0(\cos\theta) - \frac{3}{2}\left(\frac{R}{a}\right)^2 P_2(\cos\theta) + \frac{15}{8}\left(\frac{R}{a}\right)^4 P_4(\cos\theta) - \frac{105}{48}\left(\frac{R}{a}\right)^6 P_6(\cos\theta)\right] \quad (9)$$

For the case, $R < a$, where $R = OQ$, Figure 3

For the case, $R > a$, the expressions are:

$$w = 2\pi - 2\pi\left[1 - \frac{1}{2}\left(\frac{a}{R}\right)^2 P_1(\cos\theta) + \frac{3}{8}\left(\frac{a}{R}\right)^4 P_3(\cos\theta) - \frac{15}{48}\left(\frac{a}{R}\right)^6 P_5(\cos\theta) + \frac{105}{384}\left(\frac{a}{R}\right)^8 P_7(\cos\theta)\right] \ldots \quad (10)$$

and

$$\frac{\partial w}{\partial z} = -\frac{2\pi}{a}\left[\left(\frac{a}{R}\right)^3 P_2(\cos\theta) - \frac{3}{2}\left(\frac{a}{R}\right)^5 P_4(\cos\theta) + \frac{15}{8}\left(\frac{a}{R}\right)^7 P_6(\cos\theta)\right] \ldots \quad (11)$$

Taking only the first significant terms in the infinite series, the Legendrian Coefficients are evaluated from the following relations:

$P_0(\cos\theta) = 1$

$P_1(\cos\theta) = \cos\theta$

$P_2(\cos\theta) = \frac{1}{4}(3\cos 2\theta + 1)$

$P_3(\cos\theta) = \frac{1}{8}(5\cos 3\theta + 3\cos\theta)$

$P_4(\cos\theta) = \frac{1}{64}(35\cos 4\theta + 20\cos 2\theta + 9)$

$P_5(\cos\theta) = \frac{1}{128}(63\cos 5\theta + 35\cos 3\theta + 30\cos\theta)$

$P_6(\cos\theta) = \frac{1}{512}(231\cos 6\theta + 126\cos 4\theta + 105\cos 2\theta + 50)$

$P_7(\cos\theta) = \frac{1}{1024}(429\cos 7\theta + 231\cos 5\theta + 189\cos 3\theta + 175\cos\theta)$.

With w and $\frac{\partial w}{\partial z}$ thus evaluated and the depth z at the point known, p_z is determined from equation (7).

TABLE 2

VERTICAL PRESSURES UNDER A UNIFORMLY CIRCULAR AREA (SEE FIG 3)

R in terms of a	θ Degrees	$\frac{p_z}{p}$	R in terms of a	θ Degrees	$\frac{p_z}{p}$
$R = 2a/3$	0	0 79	$R = \sqrt{2}a$	0	0 48
"	30	0 84	"	30	0 40
"	45	0 86	"	45	0 34
"	60	0 91	"	60	0 21
"	80	0 99	"	80	0 04
"	90	1 00	"	90	zero
$R = a$	0	0 65	$R = 3a$	0	0 15
"	30	0 63	"	30	0 11
"	45	0 61	"	45	0 06
"	60	0 58	"	60	0 03
"	80	0 53	"	80	0 002
"	90	0 50	"	90	zero
$R = 2a$	0	0 28	$R = 4a$	0	0 09
"	30	0 22	"	30	0 06
"	45	0 16	"	45	0 03
"	75	0 02	"	60	0 02
"	90	zero	"	80	0 001
			"	90	zero

Table 2 contains some values of p_z in terms of p for various points under a uniformly loaded circular area. These values have been published more completely by S D Carothers [3]

C. A. Hogentogler, Jr., *University of Maryland*: By including a stiffness factor with formula 6, of Palmer's and Barber's excellent approach, an expression for pavement thickness is obtained as follows:

$$t = \left[\sqrt{\frac{(3P)^2}{(2\pi CS)} - a^2}\right]\left[\sqrt[3]{\frac{C}{C_p}}\right] \ldots (1)$$

in which,

P = wheel load

C_p = modulus of deformation of pavement

C = modulus of deformation of soil

S = assumed settlement of pavement

a = radius of the assumed area of tire contact

For assumed conditions of load, settlement and pavement and subgrade moduli of deformation values of pavement thickness may be obtained as shown in Table 1

For an assumed pavement thickness of 6 inches, subbase thickness may be computed from the following formula.

$$t_s = (t - 6)\sqrt[3]{\frac{C_p}{C_s}} \ldots \quad .(2)$$

in which,

t_s = thickness of sub-base

C_s = modulus of deformation of subbase

Resulting values are shown in Table 2.

Before values thus computed can be used with confidence in design, additional information is needed as follows:

1. Comprehensive data on values of the modulus of deformation for the range of soils met in airport construction
2. The range of permissible pavement settlements
3. Accurate information on the effect of stiffness of pavements.
4. Data on the validity of theory.

[3] S D Carothers, "Test Loads on Foundations as Affected by Scale of Tested Area," *Proceedings*, International Mathematical Congress, Toronto 1924, pp 527–549

TABLE 1

Thickness of Pavement with C_p = 4,000,000

S	Wheel Loads, lb			Wheel Loads, lb		
	8,000 a-5 6 in	15,000 a-7 75 in	30,000 a-11 0 in	8,000 a-5 6 in	15,000 a-7 75 in	30,000 a-11 0 in.
	Subgrade C = 100,000 lb per sq ft			Subgrade C = 500,000 lb per sq ft		
0 1	14	26	51	4	8	17
0 2	7	13	26	—	—	7
0 3	4	8	17	—	—	—
0 4	3	6	13	—	—	—
0 5	2	5	10	—	—	—
	Subgrade C = 200,000 lb per sq ft			Subgrade C = 600,000 lb per sq ft		
0 1	8	16	32	3	6	15
0 2	4	8	16	—	—	5
0 3	—	4	10	—	—	—
0 4	—	3	7	—	—	—
0 5	—	—	5	—	—	—
	Subgrade C = 300,000 lb per sq ft			Subgrade C = 700,000 lb per sq ft		
0 1	6	12	24	—	6	13
0 2	—	3	11	—	—	3
0 3	—	—	7	—	—	—
0 4	—	—	4	—	—	—
0 5	—	—	—	—	—	—
	Subgrade C = 400,000 lb per sq ft			Subgrade C = 900,000 lb per sq ft		
0 1	5	10	20	—	—	10
0 2	—	4	9	—	—	—
0 3	—	—	5	—	—	—
0 4	—	—	—	—	—	—
0 5	—	—	—	—	—	—

TABLE 2

THICKNESS OF SUB-BASE FOR PAVEMENT 6 IN. THICK

$C_p = 4{,}000{,}000 \quad C_s = 2{,}000{,}000$

S	Wheel Loads, lb			Wheel Loads, lb.		
	8,000	15,000	30,000	8,000	15,000	30,000
	Subgrade C = 100,000 lb per sq ft			Subgrade C = 500,000 lb per sq ft		
0.1	10	25	54	—	3	14
0.2	1	8	25	—	—	2
0.3	—	3	14	—	—	—
0.4	—	—	9	—	—	—
0.5	—	—	5	—	—	—
	Subgrade C = 200,000 lb. per sq ft			Subgrade C = 600,000 lb. per sq. ft		
0.1	3	13	33	—	1	11
0.2	—	2	12	—	—	—
0.3	—	—	5	—	—	—
0.4	—	—	1	—	—	—
0.5	—	—	—	—	—	—
	Subgrade C = 300,000 lb per sq ft			Subgrade C = 700,000 lb per sq ft		
0.1	—	8	23	—	—	9
0.2	—	—	7	—	—	—
0.3	—	—	1	—	—	—
0.4	—	—	—	—	—	—
0.5	—	—	—	—	—	—
	Subgrade C = 400,000 lb per sq ft			Subgrade C = 900,000 lb per sq ft		
0.1	—	5	18	—	—	5
0.2	—	—	4	—	—	—
0.3	—	—	—	—	—	—
0.4	—	—	—	—	—	—
0.5	—	—	—	—	—	—

MR. E. S. BARBER, *Author's closure, Comments on Discussion by C. A. Hogentogler, Jr.:* To estimate the displacement under the center of a loaded circular area on a semirigid pavement which rests on a subgrade, the pavement thickness may be considered as equivalent to a thickness of subgrade of the same stiffness. Thus

$$t_p = t_s \sqrt[3]{\frac{C_s}{C_p}\frac{1-\mu_p^2}{1-\mu_s^2}}$$

wherein

t_p and t_s = thickness of pavement and subgrade equivalent

C_p and C_s = stress-strain moduli of the pavement and subgrade corresponding to magnitude and duration of imposed load

μ_p and μ_s = Poisson's ratio for pavement and subgrade.

Taking Poisson's ratio as ½, for simplicity and because it has a minor effect in this problem, the above formula may be combined with equation 6 to obtain a formula for pavement thickness. Thus

$$t_p = \frac{a}{\sqrt[3]{C_p/C_s}}\sqrt{(p/q)^2 - 1} \ldots \quad (1)$$

TABLE 1

p/q	$\sqrt{(p/q)^2 - 1}$	$\sqrt{p/q} - 1$
1	0	0
2	1.73	0.41
3	2.83	0.73
4	3.87	1.00
5	4.90	1.24
10	9.95	2.16
20	19.97	3.47

wherein

$q = \dfrac{C_s S}{1.5a}$ = allowable bearing pressure on subgrade

S = allowable displacement of subgrade

a = radius of circular loaded area

p = average pressure on loaded area.

This formula is quite similar to one derived from more direct assumptions used by G. E. Hawthorn, W. S. Housel, B. E. Gray, and others. That is

$$t_p = \frac{a}{\tan\theta}\left(\sqrt{p/q} - 1\right) \ldots \quad (2)$$

wherein θ = angle of pressure distribution in pavement which is a measure of the stiffness of the pavement. The functions of p/q have a fairly constant ratio as indicated by Table 1.

The chief difference between the approaches to formulas 1 and 2, is in the method of determining the soil factors If the value of $\sqrt[3]{\frac{C_p}{C_s}}$, formula 1, is equal to approximately four times tan θ, formula 2, resulting values of t_p would be comparable This would be true for a $C_p = 6,400,000$ lb per sq. ft, a $C_s = 100,000$ lb per sq ft and an angle of pressure distribution, $\theta = 45°$ If the value of $\sqrt[3]{\frac{C_p}{C_s}}$ becomes less than about four times tan θ, the thickness of pavement according to formula 1 becomes proportionately greater than that indicated by formula 2

MR L A PALMER, *Author's Closure* Professor Housel, in claiming agreement between the formula

$$\Delta = \frac{2}{r}\frac{pa}{I} \qquad (1)$$

and the theoretical expression

$$S_L = F\frac{ps}{C} \qquad (2)$$

apparently overlooks the fact that the authors use C as a "modulus of deformation", a term that implies that strict adherence to the theory of elasticity is not the authors' procedure

Professor Housel has published very useful test data and the entire profession is indebted to him for this valuable contribution In considering merely the principles of mechanics from the theoretical standpoint, the authors did not feel that there was occasion for reference to these or to other load test data

As pointed out by Professor Housel in his bulletin (Ref. 6 of the discussion) the expression,

$$\Delta = \frac{Wh}{I(b^2 + brh)} \cdots\cdots \qquad (3)$$

from which equation (1) is derived, was presented by Professor C C. Williams as a discussion[1] of a paper by Terzaghi[2] appearing in the Proceedings of the American Society of Civil Engineers, in 1927

Since laboratory tests may be made with all soil conditions known and controlled, the authors feel that much benefit may be derived by adapting equation (2) to various practical field conditions For example, Table 2 of the paper shows an interesting relationship between soil density and moisture and settlements which cannot be disclosed by loading tests per se Loading tests supplemented by laboratory data could provide this information but loading tests alone cannot.

There is no doubt a similarity in form between equations (2) and (1). However, r is not in any sense identical to F, which is a function of depth, and a, as well as Poisson's ratio Clearly, r is independent of h and a, according to Professor Williams' derivation. It would be extremely difficult to establish the true relationship between the moduli, I and C

In paper E-3, "Tangential Stresses Under a Spread Foundation", volume 1, page 63, Proceedings of the International Conference on Soil Mechanics and Foundation Engineering, D P Krynine has shown that a straight line pressure distribution is possible only for a point load. Hence, for all practical purposes, r is not a constant but varies with depth If one is inclined to reject the principles of mechanics, he can ignore complicated interrelationships and set up various empirical expressions that account for any given set of data obtained under more or less limited conditions The objection to this procedure is that there is then likely to be as many different empirical expressions as there are varied conditions A rational expression, derivable from sound principles of mechanics has more

[1] Williams, "The Science of Foundations", Discussion, *Proceedings* A S C E, February, 1928

[2] Terzaghi, "The Science of Foundations—Its Present and Future", *Proceedings* A. S. C E, November, 1927

general utility than an empirical one. However, the rational expression usually must be modified and adapted to apply to actual conditions that most often are far from the ideal ones that are assumed. The whole point is that it is always well to begin with the principles of mechanics but not to stop there. These principles suggest methods of experimentation that otherwise would not be considered.

It is apparent from both equations (2) and (1) that the settlement increases as the radius a increases. The reason for this is shown in the author's discussion of the paper by Spangler and Ustrud. This relationship is experimentally true when the unit load is well below the supporting power of the soil and it is a relationship that has been observed by various engineers.

PREVOST HUBBARD AND F. C. FIELD, *Authors' closure:* In reviewing Professor Housel's discussion of the paper by Hubbard and Field it appears that the authors may not have adequately explained their conception of the utility of the method which they have followed. This method is not suggested as a means of evaluating the inherent load bearing capacity of any given soil nor of its inherent ability to carry loads transmitted through overlying pavement structures of different thicknesses. It is quite possible that the dimensional effects of the soil box or container may influence test results so as not to represent the load bearing capacity of the same soil as determined by field tests. This is immaterial, however, provided laboratory tests show normal relationship to field tests insofar as characteristic soil behavior is concerned.

Thus, for a given bearing area the laboratory test might rate a given soil with a 30 psi value, although the same soil in the field might by test develop only a 20 psi value for the same bearing area. It is apparent that necessary pavement thickness for that particular soil, as determined by laboratory test, would be inadequate for field use. However, it seems reasonable to assume that necessary pavement thickness indicated by laboratory tests for a soil with a laboratory rating of 20 psi would be applicable to the soil with the same psi value as determined in the field. The method therefore is suggested only for the accumulation of laboratory data which may be applied to soil ratings determined in the field. Such rating can be made by load settlement tests in the field or quite possibly by other tests, such as suggested by Professor Housel, provided the matter of critical deflection is taken into account.

With this understanding it seems unnecessary to take into account, in the accumulation of laboratory data by means of this test, any demonstration as suggested by Professor Housel that "the minimum value of K_1 which establishes the ultimate capacity of the subgrade or yield value of the soil coincides with a settlement of 0.5 in. Whether or not ultimate settlement has occurred in the laboratory test it seems unnecessary to attempt to determine accurately. However, in the laboratory procedure relatively small load increments are slowly applied with a three-minute load maintenance period for each, and tests so far conducted have indicated little or no further settlement for each increment.

If the method proves reliable for the accumulation of sufficient data to develop diagrams of required pavement thickness for soils of different ratings the method itself will have served its purpose and may thereafter be abandoned. Of course, it will be highly desirable eventually to correlate the laboratory method with a sufficient number of full-scale field tests to demonstrate its degree of accuracy.