REPORT OF COMMITTEE ON STRESS DISTRIBUTION IN SOILS

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STUDY OF STRESS DISTRIBUTION IN SOILS FROM 1920 TO 1940

SYNOPSIS

A brief review of the literature covering vertical pressure in homogeneous masses, shearing stresses in homogeneous masses, principal stresses in elastic continua, stresses in an elastic layer resting on rigid or slippery boundary, stresses in an aeolotropic mass, use of photoelasticity, pressure on tunnels and culverts and lateral pressure

The facts established so far are cited briefly with mention of certain strong indications

The report closes with mention of current and needed research

Stresses in earth masses are estimated by figuratively replacing an actual earth mass with an idealized one. The latter is mostly conceived as an homogeneous elastically isotropic body ("elastic continuum") to which the formulas of the theory of elasticity are applicable. An alternative is visualizing the earth mass as made of fragmental material possessing constant angle of friction and constant cohesion throughout. These two kinds of idealized bodies are assumed to be deformable, sometimes deformations in the earth mass are neglected, and it is visualized as a rigid body.

REVIEW OF WORK DONE

Vertical Pressure in Homogeneous Masses

Knowledge of vertical pressure is important in the following three cases: (a) in considering consolidation of a clay layer; (b) in designing a subgrade, (c) in determining pressures on tunnels and culverts

The "Boussinesq formula" proposed about 1885 (1)[1] for determining vertical pressure and other stresses at a point, became familiar to engineers after 1925 when Dr. Karl Terzaghi published his well known book "Erdbaumechanik" (1a). J. H. Michell solved the same problem in two dimensions (1b). To reconcile the Boussinesq formula with the experimental data, Professor John H. Griffith in 1929 (2) and Dr -Ing O. K. Froehlich or Holland in 1932 (3) published equations involving a parameter ("concentration factor") which may be adjusted to suit materials other than the elastic isotropic solids Further analysis by A E. Cummings and others followed (4).

In 1933 the Special Committee on Earths and Foundations, A S.C.E. (5) published the Boussinesq formula in a simplified form, both for a concentrated load and for uniformly distributed load This was followed by a great number of papers and articles discussing stress distribution in an earth mass from uniformly and non-uniformly loaded areas at its surface. These publications have been reviewed by D. P. Krynine, (6) and by N. M. Newmark (6a) both members of this Committee. Besides analytical procedures two graphs were proposed, one by Professor Burmister (7) and the other by Professor Newmark, member of this Project Committee (8).

Simplified methods such as two-to-one and 60-deg. method are sometimes used (21, page 102).

Shearing Stresses in Homogeneous Masses

Knowledge of shearing stresses is important in designing both foundations of structures and slopes in cuts and embank-

[1] Numbers in parentheses refer to list of reference at end.

ments. Shearing stresses may be also responsible for instability of pavements.

The general rule that the maximum shearing stress in a plane problem equals half the maximum stress difference has been known from mechanics. S. Timoshenko determined shearing stresses in a semi-infinite elastic continuum loaded by a uniformly loaded circular disk placed at its surface (9). This problem was studied in some detail by L. A. Palmer, member of this Committee (10). The study of shearing stresses in an earth mass was given special attention at the 18th and 19th meetings of the Highway Research Board (papers by D L. Holl and D. P. Krynine, member of this committee, see ref. 11, 12, and 13). Shearing stresses under a triangular load such as a schematized embankment have been studied by L. Jurgenson (14).

There are numerous papers concerning shearing stresses along the failure line (or failure surface) during a slide. This stress is generally assumed to be uniformly distributed along this line or surface which is an approximation. An outstanding paper on the study of these stresses has been written by Professor D. W. Taylor (15). An extensive bibliography along these lines is given in Taylor's paper; some references are given hereafter under items (16), (17), and (18).

Principal Stresses in Elastic Continua

Formulas for principal stresses in plane problems as functions of the angle of visibility are often attributed to S. D. Carothers (19). Their early development may be found also at other places (for instance, in ref. 20).

At the present time the formulas for principal stresses, together with the use of the Mohr's circle are discussed in textbooks (21, 22). Formulas for principal and other stresses for different cases of surface load distribution have been arranged by Hamilton Gray (18).

Stresses Under an Elastic Layer Resting on Rigid or Slippery Boundary

This problem was solved by S. D Carothers (19) who used hyperbolic functions in his solutions. The stresses at the rigid boundary as computed according to Carothers do not agree with values obtained by E. Melan in Germany (23). The cause of disagreement lies in the difference of assumptions of both authors. Melan was followed by some authors in Germany (24, 25) and tried to clarify his position in a letter to the Editor of the Journal of the Boston Society of Civil Engineers in 1935.

The same problem may be easily solved by reducing the thickness of the elastic layer and computing stresses, using elastic formulas for the reduced depth. A. E. Cummings has shown that the thickness of the layer thus reduced equals 0.75 of the original thickness. Cummings states in his paper (26) that the idea of a depth reduction was first advanced by K. Terzaghi (in 1932).

At the 1938 Meeting of the Highway Research Board, G. Pickett gave a comprehensive theoretical paper on soil masses with rigid boundaries (38).

Stresses in an Aeolotropic Mass

The simplest case of an aeolotropic mass having different moduli of elasticity in vertical and horizontal direction has been discussed by Michell (26a) and later by Wolf (27). Biot studied the influence of non-homogeneity in a mass consisting of an elastic layer underlain by a base of another material (as in the preceding section of this report) and also in a clay mass with thin sand inclusions (28).

Use of Photoelasticity

Experimental determination of stresses in transparent models of structures has been widely discussed (20, 22). F. B. Farquharson and R. G. Hennes, member of this Committee, ingeniously applied the photoelastic method to the determination of stresses around tunnels (29). F. Plummer, also member of this Committee, in a text on soil mechanics and foundations, has published some examples of photoelastic research (30).

Systematization of the Theories Discussed

Considerable work in applying the stress distribution theories to highway engineering has been done by the Public Roads Administration in the past five years. This material has been published in numerous papers in "Public Roads." (31). A considerable part of this work was done by L. A. Palmer, member of this Committee. The Bureau of Reclamation issued a technical memorandum specially dedicated to analytical soil mechanics (32). Work done in the Iowa State College by D. L. Holl is to be noticed (33). Considerable work on gathering together and extending elastic studies as applied to highways was done by Warner Tufts, member of this Committee, when working for the Federal Coordinator of Transportation (37).

An interesting and comprehensive historical review of earlier studies of lateral earth pressure was given by Jacob Feld at the 20th Meeting of the Highway Research Board (36).

Pressure on Tunnels and Culverts

This problem occupies quite a special place in the theory of stress distribution in soils and will not be discussed in this report in detail See references (34) and (35), the former is an analytical study of stresses around a tunnel and the latter is an attempt to put together, in very brief terms, what is known about pressure on pipe culverts, with references to the literature on this subject

Lateral Pressure

M. G Spangler obtained very interesting and important results in his experimental studies of horizontal pressures due to concentrated loads (39). Work done in the past years by Karl Terzaghi, Lazarus White, George Paaswell, Jacob Feld, H. de B. Parsons, Joseph Jáky and others has been reviewed in Chapter X of reference (21). Recently R. B. Peck has made measurements of earth pressures in open cuts of the Chicago Subway. Results have been but partly published (40).

Lateral pressure within an embankment has been studied by J. H. A. Brahtz who introduced the conception of the "compaction factor" (41).

FACTS ESTABLISHED

1. There is no way to compute stresses within an earth mass. They may be estimated only by considering the earth mass as an idealized body.

2. Formulas of the theory of elasticity may be applied to earth masses with certain limitations Particularly, for computing vertical pressure at a certain depth, such as 50 ft. or more, the Boussinesq formula may be used For shallow depths, particularly in the case of the highway subgrade, the Boussinesq formula does not fit experimental results.

3. For computing shearing stresses elastic formulas alone are available. No other way of doing so has been proposed except some graphical or experimental methods based, after all, on the theory of elasticity.

4. For the simplest case of an aeolotropic mass, plane stresses may be computed by replacing this mass by an isotropic one of modified thickness.

5. For the case of an elastic layer on rigid boundary stresses at this boundary may be computed by reducing the thickness of the layer practically to 0.75 of the actual thickness.

6. In using "classical formulas" for lateral pressure, such as Coulomb's or Rankine's, attention should be paid to the fact that these formulas are valid only under the assumptions made in their deri-

vation. Particularly, these formulas cannot be used for flexible walls, for sheet piling and for bracing of cuts and tunnels.

7. In the province of vertical pressure on pipe-culverts, Marston's theory may be considered adequate for all practical purposes.

STRONG INDICATIONS

1. In all cases of estimating stresses, different assumptions should be made in order to obtain two limiting values of the given stress such that the true value will be somewhere between them. The more limited the range of values thus obtained, the better.

2 In all cases, without any exception, mathematical results obtained by the use of formulas, must be checked against sound engineering judgment based on experience which cannot be replaced by theories. Theory and practice should be combined

RESEARCH

Several agencies are working on the determination of pressure on tunnels These data are being gathered, digested, and completed

Analytical study of the influence of the impact and vibrations on the value of stresses in an earth mass is being made.

Information on the design and construction of bridge abutments as influenced by stresses in the adjacent earth mass is being gathered, digested, and completed.

SUGGESTIONS FOR FURTHER RESEARCH

The literature on stresses in earth masses is too voluminous and contains repetitions and complicated formulas. It is desirable to prepare, sometime in the future, a brief manual along these lines, written especially for highway engineers. This manual should be such as to completely replace all articles and papers written before the date of its publication.

A structure causes stresses and strains in the earth mass on which it is built; and in turn, the earth mass causes stresses and strains in the structure. It is proposed to study this contact problem both in its general aspect and in possible highway applications.[2]

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DISCUSSION ON EARTH STRESSES

Mr. L. A. Palmer, *Bureau of Yards and Docks, U. S. Navy Department:* Professor Krynine has presented a very interesting and comprehensive report which indicates that considerable progress has been made and that the Committee has contributed materially to this progress.

The extent to which theory is applicable to foundation problems is mostly a matter of opinion, based on individual experience, which, considering the profession as a whole, may sometimes be quite limited. Furthermore, authentic data pertaining to foundation settlements are still meager.

The limitations of theory in computing earth stresses caused by foundation loads can never be known until accurate and reliable means of measuring these stresses are available and are utilized on a large scale. This can be the only reliable answer to the controversial question. Without such evidence, doubt concerning the reliability of earth stress computations is in itself largely a matter of speculation.

In a discussion of Professor Spangler's paper on wheel loads on flexible pavements presented at the 1940 Highway Research Board Meeting, the writer showed that computed stresses, when multiplied by a constant quantity, agreed closely with observed values [1] If it should happen that this is true generally or in very many cases, then the outlook is quite hopeful.

The degree of reliability of experience alone without reference to principles of mechanics as applied to earth problems may be good or bad, depending on the experience. Experience limited to only a few types of foundations may be exceedingly unreliable when new problems are encountered. Experience without knowledge of soil characteristics as they have actually existed is almost useless. Deep sampling is always necessary and an adequate soil profile study is needed if much is to be obtained from experience alone. Too often, the soil characteristics are studied superficially down to the depth of excavation and the existence of deep seated strata which markedly affect the performance of the structure is not even known. Without such information, the accumulation of reliable experience is hardly possible.

The most hopeful sign today is an ever increasing tendency on the part of design engineers to obtain more and more information concerning the supporting earth down to a depth equal to or greater than the least dimension of the structure. Fortunately, soil mechanics is getting into the hands of men who design structures. The specialist in this field can do the most good and broaden his own knowledge by working closely with design engineers, a procedure that results in a desirable blending of theory and experience.

There is a suggestion concerning the future work of this Committee Specialists in the field of soil mechanics sometimes give too little thought to the structure in the same way that structural engineers may underestimate the importance of the supporting earth. Various expressions for stresses have been derived for different types of load distribution, such as uniform, triangular, trapezoidal, parabolic loading, etc. It is suggested that these types of loading may quite often be nonexistent. It is further suggested that future work include some study of actual contact pressure distribution. The footing and the earth are interdependent. It is likely that the distribution of surface load or contact pressure changes with time. Certainly when there are differential settlements one would expect a redistribution of load, depending on the degree of rigidity of the footing, floor slab, etc. The

[1] L A Palmer, Discussion on "Interrelationship of Load, Loaded Area, Stress and Deformation of Soil," *Proceedings,* Highway Research Board, Vol. 20, p 325.

assumption of perfect rigidity or complete flexibility leads to considerations that tend to have only academic interest.

In this connection there are two possible means of extending our knowledge, (a) photoelastic studies and (b) the application of principles of equilibrium involving the interaction of earth and structure.

To illustrate the latter, consider a concrete slab of uniform thickness, having a modulus of elasticity E, and a moment of inertia, I and supported by earth having a modulus of elasticity, E_s. Assume that the slab is very long in relation to its width. Let the z direction denote depth, the y direction that of the longitudinal axis or center line of the slab and the x direction be horizontal distance from the center line Let $a=\frac{1}{2}$ the width of the slab Let p_s denote the pressure between earth and slab at any horizontal distance x from the centerline. Then clearly at equilibrium p_s is a function of x.

In Technical Memorandum No. 596 of the Bureau of Reclamation, J. H. A. Brahtz has shown that for a unit load at a point x_1, the surface deflection at another point x is given by the expression,

$$\delta v=\frac{1}{\pi E_s}\left[A+\log(x-x_1)^2+\frac{2x}{x_1}\right]$$

one of the constants having been eliminated from the condition that the loading is symmetrical. Then for the entire load, p, the deflection at any point x on the ground surface is obtained from the expression,

$$v=\frac{1}{\pi E_s}\int_{-a}^{a}\left[A+\log(x-x_1)^2+\frac{2x}{x_1}\right]f(x_1)dx_1$$

Here $f(x_1)$ denotes the distribution of contact pressure, that is,

$$p_s=f(x_1)$$

Now if z denotes the deflection of the slab at any point x, then

$$EI\frac{d^4z}{dx^4}=f(x_1)=p_s$$

is a condition for equilibrium. Another condition is that $v=z$ or that $v=z+a$ constant, the constant being the value of v at $x=a$ It then remains to determine $f(x_1)$ from boundary conditions peculiar to the problem For example, one may assume that p_s may be expressed by the polynomial,

$$p_s=f(x_1)=B+Cx_1^2+Dx_1^4+Ex_1^6+\ldots$$

the odd terms not appearing in symmetrical loading. To evaluate the constants one condition that may be used is that at the origin, $x=0$,

$$\frac{dz}{dx}=\frac{dv}{dx}=0,\ x=0$$

Still another condition is that the moment in the slab at $x=a$ is zero. That is,

$$EI\frac{d^2z}{dx^2}=EI\frac{d^2v}{dx^2}=0,\ x=a.$$

Another condition is that

$$\int_{-a}^{a}f(x_1)dx_1=p,\ \text{the total load.}$$

Having actually obtained p_s, it is then possible to obtain earth stresses at finite depths under the slab. The assumption that the surface deflection is directly proportional to k, the modulus of subgrade reaction, is obviously incorrect. To see the fallacy of this assumption one needs only to consider that for any uniform load on the ground surface the settlement of the earth, according to this assumption, would be uniform under the loaded area at all points.

Photoelastic studies to determine p_s would be particularly valuable in cases where existing discontinuities render theoretical analyses almost hopelessly complicated. They also serve as a check on analyses of the simpler cases.

STRESSES UNDER PIERS

Mr. E. S Barber, *Public Roads Administration*: To calculate the vertical pressure at a given depth below a footing, the new Boston Building Code (1)[1] assumes that "the load is spread uniformly at an angle of 60 deg. with the horizontal." In computations for a bridge at New Orleans, W. P. Kimball (2) found a variation of less than 5 per cent in settlement using these stresses instead of stresses from Boussinesq's equation. In order to compare these stresses at various depths below a uniform load on a circular area, the average stresses from the theory of elasticity were obtained by graphical

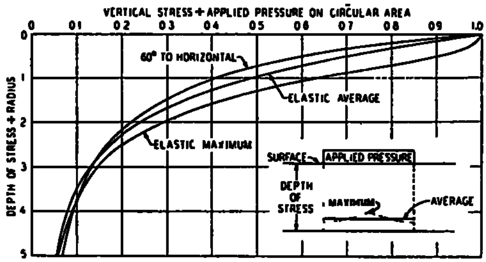

Figure 1. Vertical Stress Below Uniformly Loaded Circular Area

integration from values tabulated by Warner Tufts (3) which were checked by Newmark's chart (4). The stresses for the distribution at 60 deg. to the horizontal were computed from the formula

$$p_z/p=\left(\frac{1}{1+z/a\ \cot 60°}\right)^2 \ldots\ldots 1.$$

in which p_z = vertical stress at depth z below the surface

p = uniform pressure applied on circular area of radius a.

Figure 1 shows these stresses as well as the maximum vertical pressure on the axis below the loaded area from the theory of elasticity. The difference between the average stresses is not large and computations show that it is even less under rigid footings if the reaction of the supporting material is concentrated toward the edge. The method of formula 1 is simple and easily applied to various shapes of footing. The angle of distribution may be used as a parameter to allow for particular conditions.

[1] Figures in parentheses refer to list of references at end.

To get an idea of the stress distribution on the axis below a uniform pressure on a circular area applied below the surface, Mindlin's formulas (5) for the stresses from a force P at a point were integrated from 0 to a after replacing P by $p2\pi r dr$. The following further notation is used:

r = radial horizontal distance from the point of load application at depth c

$R_1=(a^2+(z-c)^2)^{\frac{1}{2}}$

$R_2=(a^2+(z+c)^2)^{\frac{1}{2}}$

μ = Poisson's ratio

E = modulus of elasticity

S = vertical displacement

The radial stress p_r was determined by combining the radial and circumferential stresses from a point load (6) The shearing stress on the horizontal plane and the radial displacement are 0. Others are shown by formulas 2, 3, and 4 in Figure 2.

The pressure immediately under the applied load from formula 2 is less than the applied pressure since part of the load is theoretically carried by tension above as shown on the left in Figure 3. In the practical problem (Figure 3) the load is carried to the surface by a shaft so that as the pier is loaded and displaced downward part of the load is transferred by shear along the sides. The distribution of these stresses is a problem in itself (7, 8) but the foregoing formulas may indicate the conditions below the pier.

The axial vertical stresses from formula 2 for several founding depths and Poisson's ratio of 0.5 are plotted in Figure 4.

$$p_z/p = 1 - \frac{1}{8(1-\mu)}\left[\frac{z(1-\mu)(z-c)}{R_1} - \frac{2(1-\mu)(z-c)}{R_2} + \frac{2(z-c)^3}{R_1^3} + \frac{2(3-4\mu)z(z+c)^2 - 2c(z+c)(5z-c)}{R_2^3} + \frac{12cz(z+c)^3}{R_2^5}\right] \quad (2)$$

$$p_r/p = 0.5 + \mu + \frac{1}{8(1-\mu)}\left[\frac{(c-z)(1+4\mu)}{R_1} + \frac{(z+c)(-7+4\mu+8\mu^2)+6c}{R_2} + \frac{(z-c)^3}{R_1^3} + \frac{3(z-c)(z+c)^2 - 4\mu(z+c)^3 - 6cz(z+c)}{R_2^3} + \frac{6zc(z+c)^3}{R_2^5}\right] \quad (3)$$

$$\delta = \frac{p}{4E}\,\frac{1+\mu}{1-\mu}\left[(3-4\mu)R_1 + (5-12\mu+8\mu^2)R_2 - \frac{(z-c)^3}{R_1} - \frac{(3-4\mu)(z+c)^2 - 2cz}{R_2} - \frac{2cz(z+c)^2}{R_2^3} - 4(1-3\mu+2\mu^2)z + 4(\mu-2\mu^2)c\right] \quad (4)$$

Figure 2

For a depth equal to the radius, the contact pressure is 0.75 of the applied pressure; the other 0.25 is carried by tension above and contributes stress to the axis at greater depths. The pressure for an infinite founding depth is just half the pressure for a surface load. For Poisson's ratio less than 0.5 the stresses are somewhat smaller.

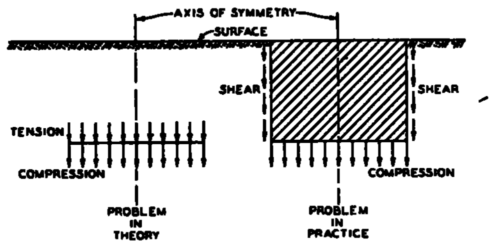

Figure 3. Stress from Loaded Area Below the Surface

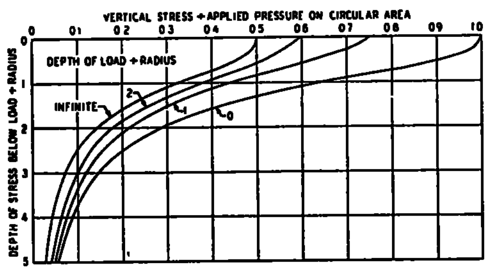

Figure 4. Axial Vertical Stress from Uniform Pressure on Circular Area Below the Surface (Poisson's Ratio=0.5).

The radial stresses are likewise reduced and the cumulative effect is shown by formula 4 for the vertical displacements which are plotted in Figure 5. Poisson's ratio has an appreciable effect at shallow depths only. The figure indicates that in homogeneous materials increasing the founding depth may considerably reduce displacements even though the applied pressure is constant.

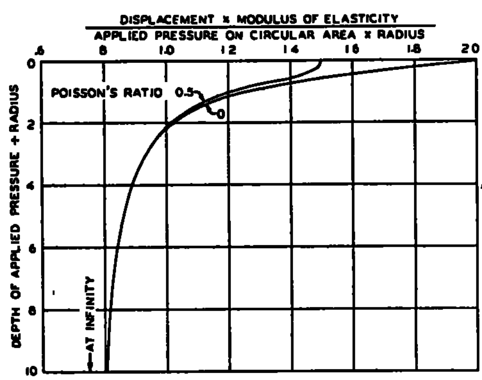

Figure 5. Axial Displacement from Loaded Circular Area Below the Surface

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