# THE STRUCTURAL DESIGN OF FLEXIBLE PAVEMENTS

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## SYNOPSIS

This paper presents a method of design of flexible pavements which follows the orthodox procedures of structural design in the sense that it deals first with the pattern and magnitude of the pressures on the subgrade beneath such a pavement due to wheel loads applied at the pavement surface From these subgrade pressures, the subgrade deformation is derived and this is equated to the safe allowable deflection which the pavement can withstand to arrive at a judgment as to the thickness of pavement required on a given subgrade and to carry a given load

A flexible pavement is defined as a pavement having little or no inherent resistance to deformation under applied load Failure of such a pavement is assumed to be the result of excessive deflection of the subgrade which causes the pavement to develop the widely familiar "alligator cracks" The theory is based on the premise that if these cracks can be prevented the pavement will successfully carry the traffic for a long service life

Measurements have shown that the pattern of subgrade pressure distribution is bell shaped and that the maximum pressure occurs directly under the wheel load These experiments also indicate that the magnitude of this maximum central pressure is dependent on three factors the thickness of pavement, the magnitude of the load, and the stiffness of the subgrade soil An empirical expression for the pressure on the subgrade, which recognizes these factors and which meets the requirements of equilibrium has been devised as follows

$$\sigma_r = \frac{CP}{t} e^{-\frac{\pi C}{t} r^2}$$

in which

$\sigma_r$ = the pressure on the subgrade, p s i
P = the wheel load, lb
t = the pavement thickness, in
r = the radial distance from the center of the load, in
C = the "subgrade stress factor"

$$C = \frac{0070 + 0000068E_c}{\sqrt[3]{\frac{P}{1000}}}$$

$E_c$ = the modulus of compression of the subgrade soil, p s i

If the subgrade soil is assumed to be quasi-elastic in character and proper values of the modulus of compression can be determined, the maximum central deformation of the pavement and subgrade can be determined by the theory of elasticity The value of this maximum central deformation is indicated by the formula

$$\Delta Z_0 = \frac{09\,P\sqrt{\frac{C}{t}}}{E_c}$$

Each type of flexible pavement has a safe allowable deflection which is a function of the pavement thickness If an expression for this safe allowable deflection is substituted for $\Delta Z_0$ in the above equation the design thickness of the pavement may be obtained by solving for t

Little is known at the present time concerning safe deflection values for various kinds of flexible pavements such as those constructed of soil-aggregate, sand-clay, soil-cement, rolled-stone, bituminous-stabilized, etc, and a technique for determining this limiting deflection will need to be developed. The paper includes a discussion of an apparatus by means of which it is believed the facts concerning safe allowable deflection can be determined.

All of the experimental studies of subgrade pressure on which this theory of design is based have utilized a single highway tire as the loading element. Some approximate ideas of the character of subgrade pressure distribution under dual tire highway loads and under large airplane wheel loads are obtained by extrapolating the data for the single tire loading, although the limitations of this extrapolation procedure are fully realized.

A flexible pavement may be defined as one having little or no inherent resistance to deformation under applied load. The principal function of this type of highway or airport runway surfacing is to transmit wheel loads to the underlying soil or subgrade in such a manner that the wheel-load pressure on the subgrade will be distributed sufficiently to prevent the wheels from sinking into or causing objectionable ruts in the highway or airport runway and to provide a smooth, dustless riding surface. In addition to these requirements, if the pavement is to be a relatively permanent structure, it must transmit the wheel-load pressures to the underlying soil in such a manner that the subgrade will not deform a greater amount than the pavement can deform and still retain its structural integrity, since the pavement, being flexible, will conform to the shape of the subgrade.

Observations of the structural failures of flexible pavements disclose the formation of longitudinal cracks in the wearing surface, in the early stages of failure, which are apparently due to excessive deflection of the pavement. At first, these cracks are parallel to the direction of traffic and are in general independent of each other and not interconnected. As the failure progresses, transverse and diagonal cracks develop which connect the longitudinal cracks, forming the typical "map cracking" or "alligator cracking" so familiar to the highway engineer. Under continued traffic and adverse subgrade conditions, these map sections loosen and are thrown out by passing wheels and chuck holes develop, which spread rapidly in area and deepen quickly as the wheels abrade the exposed base course. These chuck holes may spread sufficiently to coalesce with neighboring holes until large areas, involving the whole width of pavement and several hundred feet of length, are completely demolished. There may be other manifestations of advanced stages of failure such as shoving and washboarding, but they are preceded by the same type of alligator cracking. If this preliminary stage of failure can be prevented, it is probable the integrity of the whole structure can be maintained.

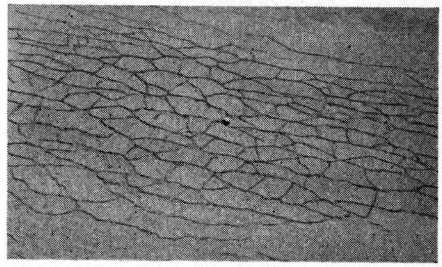

**Figure 1. Early Stages of Failure of a Flexible Pavement Consisting of a 3-In. Gravel Sub-Base, a 3-In. Stabilized Gravel Base Course and a $\frac{3}{4}$-In. Bituminous Wearing Surface. The Cracks Near the Top of the Picture Are Parallel to the Direction of Traffic and Represent a Primary Stage of Failure. Later These Cracks Are Connected by Transverse and Diagonal Cracks Forming the Typical "Alligator" Cracks as Shown in the Center of the Picture.**

Several photographs showing various stages of failure of a stabilized soil-aggregate pavement having a $\frac{3}{4}$-in. bituminous wearing surface are shown in Figures 1 to 3. A diagram of the observed failure of a laboratory experimental 4½-in. sand-clay base course on a very yielding subgrade under a load of 2500 lb. is shown in Figure 4. The load on this pavement was static, causing an approximately circular locus of failure. It is probable that if the load had been moving, longitudinal cracks parallel to

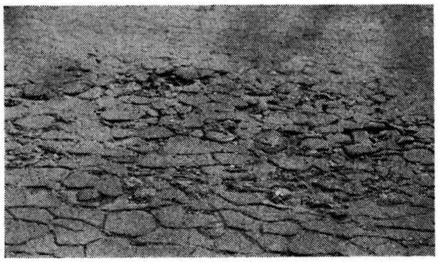

**Figure 2. Continued Deflection Under Traffic Loosens the Map Sections of the Wearing Surface and Deterioration of the Pavement Proceeds Further.**

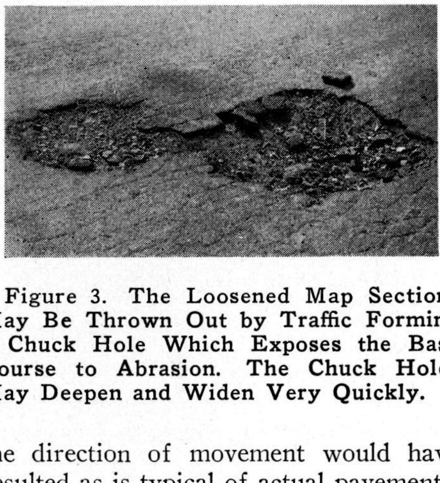

**Figure 3. The Loosened Map Sections May Be Thrown Out by Traffic Forming a Chuck Hole Which Exposes the Base Course to Abrasion. The Chuck Holes May Deepen and Widen Very Quickly.**

the direction of movement would have resulted as is typical of actual pavements. It seems clear from these and other observations of flexible pavement failures that the principal cause of failure is excessive deflection of the subgrade, which causes the pavement to crack. These cracks may permit the entrance of surface moisture which hastens the deterioration of the stabilized pavement. Another observation of importance is the fact that failures of pavements of this type often occur in the spring of the year when moisture in the subgrade is relatively high and therefore the subgrade soil is relatively yielding in character. Also, pavements have been known to remain in excellent condition for a number of years, until, due to some maximum combination of meteorological conditions, the ground water table rose to a height which weakened the subgrade sufficiently to permit excessive deflection of the pavements and bring about their destruction.

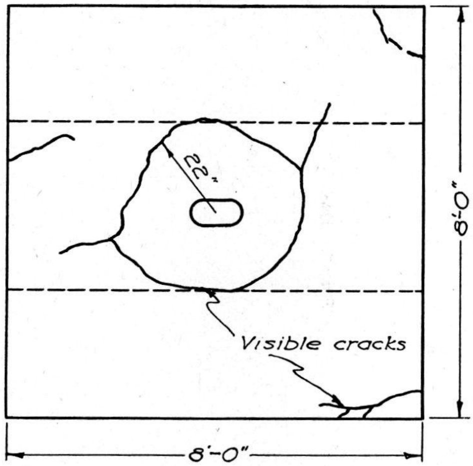


**Figure 4. Failure of 4½-In. Sand-Clay Base Course on Wet Subgrade Under Static Wheel Load. Dotted Lines Indicate Probable Longitudinal Cracks if Wheel Had Been Moving.**

The total flexible pavement structure may consist of four principal elements; a bituminous wearing surface, a stabilized base course, a sub-base and the subgrade or soil stratum on which the other elements rest, as illustrated in Figure 5. The bituminous wearing surface imparts a smooth dustless riding surface and in addition, serves to protect the base course from traffic abrasion and from surface moisture. In highway work, the bituminous wearing surface usually ranges from about ¾ in. to 2 or 3 in. thick. The stabilized base course is the principal structural element of the pavement. It may be constructed of a wide variety of locally available materials and is characterized by very high density and high shearing resistance. It should be thick enough to adequately transmit the wheel load stresses to the underlying soil strata without being subjected to shearing stresses greater than it can withstand. At the same time it should not be too thick, because it must be capable of conforming to the deflected shape of the subgrade under a wheel load without being subjected to excessive bending stresses which may cause cracks and otherwise destroy the structural in-

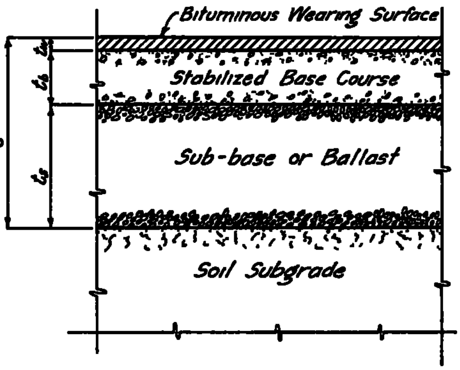


**Figure 5. Elements of a Flexible Pavement Structure**

tegrity of the base course Many successful base courses ranging in thickness from 3 to 6 in. have been constructed for highway use.

A sub-base consisting of coarse grained, dense material, but containing little or no binder material, may be employed in a flexible pavement structure for one or the other or both of two principal reasons First it may be necessary in order to provide ballast or added depth to the pavement structure above the subgrade level which will spread the wheel load pressures on a low strength subgrade more widely. Or it may be used in certain localities and on certain types of subgrades to prevent the occurrence of detrimental frost heave and subsequent frost boils Since many subgrade soils which are susceptible to frost heave are also low in resistance to deformation under load, a sub-base may readily fulfill both of these functions. When the subgrade soil is highly resistant to deformation and is not susceptible to detrimental frost heave, a sub-base is usually not necessary.

The subgrade is the foundation upon which the pavement rests and to which the vehicle wheel loads are transmitted by the pavement. It may be soil in its natural bed or filled material in an embankment, and may or may not be densified or otherwise manipulated or treated to increase its resistance to deformation

This discussion of some of the fundamental aspects of the type of structure under consideration are given as a preliminary to the formulation of a criterion for the structural design of flexible pavements which may be stated as follows. The structural design of a flexible pavement consists of the determination of a suitable thickness of the pavement which will insure sufficiently wide distribution of wheel-load pressures to prevent the subgrade from deforming in a manner which will cause stresses in the stabilized base course in excess of those which it can safely withstand under a large number of repeated loads.

The purpose of this paper is to present a method for the design of flexible pavements in accordance with the above criterion and to suggest procedures, in so far as possible, by which the necessary information concerning the distribution of subgrade pressures, subgrade deformation characteristics, and safe deflections of various types of flexible base courses may be determined Some of these procedures, as will be evident in the following discussion, are fairly well developed and based on reliable experimental and theoretical evidence, while others are less well authenticated and still others are yet in the hypothetical stage Nevertheless, the broad outline of the complete theory has been formulated and will be presented with the expectation that missing links of information and comparisons between the theory and observations of actual performance will be presented from time to time by the author and with the hope that other observers may find it of sufficient interest to be stimulated to add their experiences, interpreted in the light of the theory, to the record.

As is true with practically every theory which serves as an engineering tool, certain assumptions have been made both for the purpose of simplifying the method and to bridge over gaps of uncertainty and lack of information. Mainly, these assumptions are

1 That the subgrade acts as the sole supporting medium for the pavement In other words any resistance to deformation supplied by the pavement itself is neglected. Corollary to this, it is assumed that the pavement deforms with the subgrade and conforms to the contour of the subgrade in the loaded state.

2. That the subgrade soil, after the passage of a limited number of wheel loads, becomes quasi-elastic in character and deformations may be calculated by means of the theory of elasticity if the proper deformation constants are utilized. It is also assumed that the subgrade material is essentially homogeneous in character and that it extends for a considerable depth below the pavement, although suggestions are made for empirically determining the deformation constants in cases of non-homogeneous subgrades.

3 That the horizontal pattern of vertical pressure on the surface of the subgrade is circular in shape, that is, the stress surface is a surface of rotation about the vertical axis through the load.

Admittedly the first of these assumptions is not strictly in accord with fact since even a very flexible pavement has some inherent strength to resist deformation It is a simplifying assumption which contributes a factor of safety of unknown magnitude in the computation of pavement thickness by this method.

The second assumption is likewise not in accord with the widely recognized fact that soils, in general, are not elastic in the sense that they will rebound or recover all deformation when an applied load is removed In this respect they range all the way from the nearly elastic granular types to the highly plastic and inelastic fine-grained silts and clays However, this widespread concept of the inelasticity of soils is based largely upon the performance of the material under a single application and release of load. There is rather convincing evidence, though it is limited in amount, that soils may tend to become nearly elastic under the influence of a fairly small number of repeated applications and release of load, and since the loads on flexible pavements are repeated many times during the life of the pavement, it is this repeated load performance of soils which probably governs in this problem

Older (9),[1] in the famed Bates road tests, found that the subgrade soil in these tests became essentially elastic in character after a relatively few cycles of loading and unloading, as shown in Figure 6 Krynine (15) states that the permanent or plastic increments of deformation of a sand mass decrease and practically disappear after a number of stress repetitions (sometimes over 100), leaving the soil practically elastic under the influence of loads equal to or less than the magnitude of the repeated load Further evidence of the essentially elastic character of subgrade soils may be found in the performance characteristics of successful flexible pavements themselves If a state of quasi-elasticity of the subgrade does not develop under the influence of repeated loading, then the passage of every wheel load would produce a residual def-

[1] Figures in parentheses refer to the list of references at the end of the article

ormation which would be accumulative for subsequently applied wheel loads, and undesirable ruts would develop in heavily traveled lanes of traffic We may visualize this by considering an imaginary case Suppose a subgrade soil exhibits a residual deformation of 0 001 in after the passage of one wheel load One thousand such loads would produce a permanent deformation of 1 in and the pavement on such a subgrade would soon be badly rutted This simply does not happen in many cases, even after the passage of many thousands of wheel loads and the conclusion is inescapable that some and perhaps many subgrade soils become practically elastic under repeated loading

Since the loads on flexible pavements are repeated many times under normal service conditions, this assumption of subgrade elasticity appears to be justified It will be employed in this theoretical treatment of the flexible pavement problem until the sciences of plasticity and rheology are developed sufficiently to provide a better basis for calculating subgrade deformations

The third assumption is readily accepted and is used solely to simplify procedure The area of contact between a highway vehicle tire and a pavement is roughly a rectangle with the ends bulged outward, and the stress pattern on the subgrade reflects the influence of this shape of contact area Actual measurements of pressures on the subgrade indicate that the corners of the rectangular shape are rounded off and the pattern approaches an elliptical shape with the major axis about one and one half times the minor axis By shortening the major axis and lengthening the minor axis, the assumed circular pattern is obtained and no change is effected in the value of the maximum pressure directly under the load Obviously the circular shape is much easier to handle mathematically than the elliptical shape

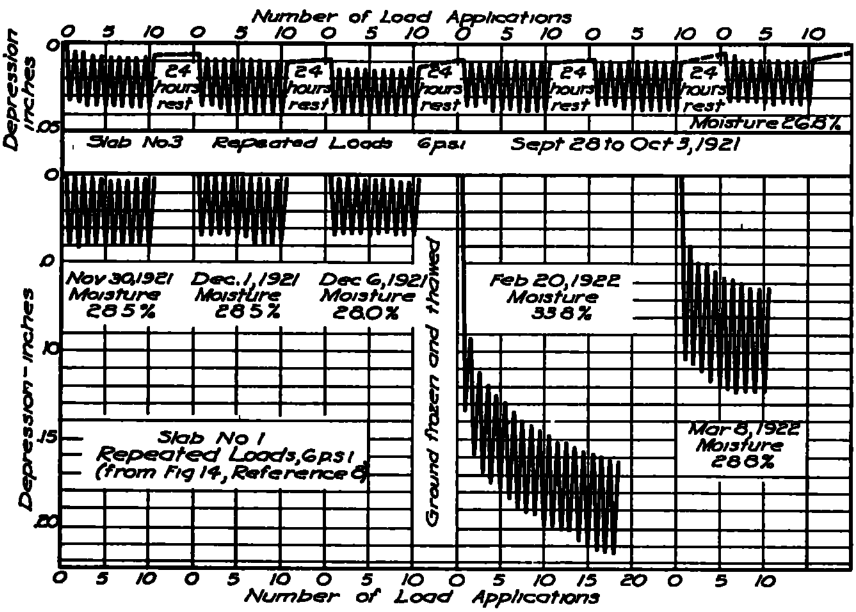


Figure 6

### MAXIMUM SUBGRADE PRESSURE

Extensive experiments by Spangler and Ustrud (12, 13, 14), Kneeland (8), Goldbeck (3, 4) and Bone (1) in which the magnitude and pattern of vertical pressures on subgrades have been measured, have indicated the various factors which control the subgrade stress pattern. These studies show quite definitely that the maximum pressure on the subgrade occurs directly under the center of the load and that the pressures reduce in all directions, forming a bell or helmet shaped surface which becomes tangent to the plane of the subgrade at some distance from the load. They also show that the maximum pressure directly under the load is an inverse function of the pavement thickness, being greater under a thin pavement than under a thick one, for otherwise equivalent conditions

Another factor of great importance which affects the subgrade pressure pattern is the stiffness of the subgrade itself. On a very stiff subgrade the vertical pressures are bunched over a relatively small area, while on a yielding subgrade the larger deformation under the load brings about a redistribution which causes the vertical pressures to spread out over a larger area It was shown by the author at the 1941 meeting of this Board (14) that the maximum pressure under a 4½ in. pavement resting on a very dry and therefore a very stiff subgrade was approximately nine times as great as it was on the same subgrade in a very wet and yielding condition. Also it was shown that in the experiments by Goldbeck (3) which were conducted on subgrade material which had essentially the same textural characteristics as that used in the Ames experiments, the maximum pressure on the subgrade containing an intermediate amount of moisture and therefore being intermediate in stiffness between the very wet and the very dry conditions, was intermediate in amount between the two extreme cases studied at Ames.

Palmer and Barber (10) have suggested that the stiffness of a soil subgrade can be expressed in terms of a modulus of deformation which they define as the ratio of stress to deformation without regard to the nature of the deformation, whether it be elastic or plastic or both and they have indicated a method whereby the value of this modulus can be determined from triaxial compression test data. The author agrees with the reasoning employed by these writers with the added suggestion that, in all probability, the subgrade soil modulus should be determined under repeated load conditions. This suggestion is made in view of the fact that, unless a flexible pavement fails very early in its life, most of the inelastic deformation of the subgrade has taken place at the time of failure and the stress-strain relationship under repeated loads is probably controlling. On the other hand, the indications are that the slope of the stress-strain diagram for the first loading is not so steep as it is for subsequent loadings and therefore the least value of the modulus of compression will be obtained on the first loading. Since a low value of this modulus will yield a thicker pavement than a high value, it may be argued that the initial modulus should be used and any excess thickness indicated thereby be considered as an additional factor of safety. At any rate the nature of the stress-strain characteristics of subgrade soils under repeated loads and their applications to the problem of design of flexible pavements needs to be studied extensively.

The modulus of compression of a subgrade may be determined directly by means of a confined compression test on undisturbed cylinders of the subgrade soil It may be determined indirectly by one or more of several soil tests which have been developed such as Porter's California Bearing Ratio (11) and Boyd's

North Dakota Cone Bearing Test (2). Such indirect determinations will require extensive correlation to properly relate the modulus of compression with the results of these tests Also the modulus may be considered as an empirical physical constant and determined by measuring the deflections of a pavement under an applied wheel load and by the use of formulas 12 and 14 which will be derived later in this paper. This latter method may be particularly applicable in the case of non-homogeneous subgrades from which it would be difficult to obtain representative samples or in cases where the top layer of a subgrade is to be artificially compacted to a greater density than the natural soil. When the empirical value of the modulus is to be determined for a proposed pavement, a test section may be constructed and loaded to obtain the necessary deflection measurements, taking care to insure that the subgrade is in its worst probable state as regards moisture content and other conditions affecting resistance to deformation

A third factor which affects the subgrade pressure pattern is the magnitude of the load itself Since the surface of the subgrade is at a comparatively shallow depth below the pavement surface, it lies within the "zone of disturbance" of Kogler and Scheidig and the principle of superposition is not applicable in the consideration of the distribution of wheel-load pressures through flexible pavements That is to say, the unit pressure on the subgrade is not a linear function of the applied load. In the subgrade pressure measuring experiments conducted at Ames (13), single tire loads ranging from 1000-lb. to 5000-lb were applied and it was found that a relatively greater maximum pressure under the load was caused by the lower loads than by the higher loads. For example on a 4-in thick pavement resting on a very dry and very stiff subgrade, the maximum subgrade pressure directly under the load was about 25 psi under a load of 1000 lb, 43 psi for a 2000-lb. load, and 57 psi for a 3000-lb load. The same phenomenon is apparent in the results obtained by Goldbeck, who measured subgrade pressures under loads ranging from 4000 to 12,000 lb. In his experiments, the maximum pressures due to the 12,000-lb. loads were only about $2\frac{1}{4}$ times greater than the pressures due to 4000-lb. loads

The Ames experiments have led to an empirical formula (13) expressing the value of the maximum subgrade pressure which is

$$\sigma_0 = \frac{CP}{t} \qquad 1$$

in which

$\sigma_0$ = the maximum subgrade pressure (directly under the load, psi)
P = the applied tire load, lb
t = the pavement thickness, in
C = an empirical constant, the "subgrade stress factor"

Tentative value of $C = \dfrac{0\,0000068E_c + 0\,0070}{\sqrt[3]{\dfrac{P}{1000}}}$

$E_c$ = the modulus of compression of the subgrade soil, psi

### THE SUBGRADE STRESS FACTOR

The qualitative character of the empirical constant C, which is called the subgrade stress factor, is revealed by an examination of the subgrade pressures under flexible pavements measured on wet and dry subgrades at Ames and by those measured by Goldbeck on a subgrade having an intermediate moisture content These data are summarized graphically in Figure 7, where values of the subgrade stress factor have been calculated by equation 1 from the measured pressures and plotted against the applied loads They show clearly that the value of C is reduced rapidly as the moisture content of the subgrade soil increases. Also, C decreases as the applied load increases, and in a ratio approximately equal to the reciprocal of the cube root of the load expressed in kips.

While the relationship between the subgrade stress factor and the moisture content of the soil is very marked in these experiments, obviously this property of soil is not a valid criterion upon which to base a general expression for C, since differences in soil texture, mineralogical composition and density will greatly influence the stiffness characteristics of two soils, even though their moisture content may be the same The modulus of compression as defined by Palmer and Barber is a more general property of soil and one which clearly influences the extent to which the pressures are spread out over the surface of the subgrade, and it is therefore a suitable criterion for fixing the value of C in equation 1

An accurate numerical expression of the subgrade stress factor in terms of the modulus of compression of the experimental subgrade soils is difficult to derive because the modulus was not determined in either the Ames experiments or those by Goldbeck. However, what is believed to be a fairly accurate estimate of this property of the Ames subgrades has been obtained by testing cylinders of the soil containing various percentages of moisture in a triaxial compression machine and obtaining the relationship be-

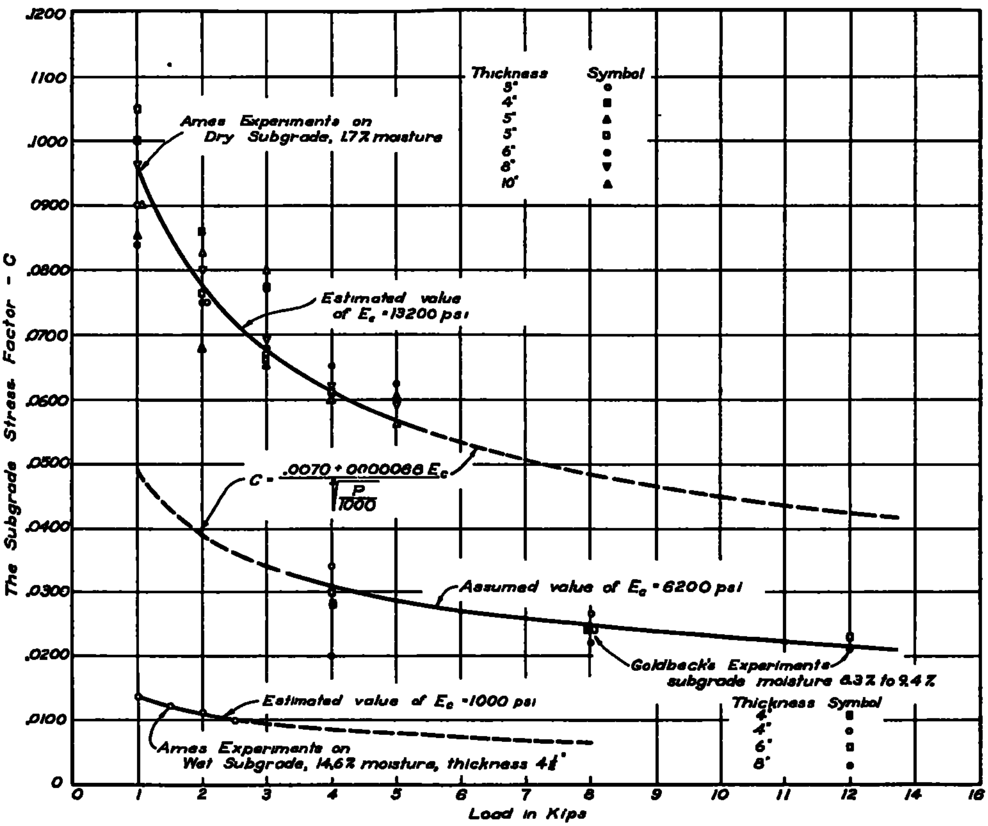


Figure 7

tween moisture content and modulus of compression Four sets of three cylinders each were molded to the density of the test subgrades and each set contained a different moisture content ranging from 5 to 11 percent Each cylinder of a set was tested at different lateral pressures, one at 6 9 psi, another at 13 9 psi and the third 20.8 psi. A typical stress-strain diagram of this series of tests is shown in Figure 8 in which the unit strains are plotted against the principal stress differences. The value of the modulus of compression $E_c$ was taken from the slope of the initial straight-line portion of the curves, and in this case is about 7700 psi

The experimental values of $E_c$ from these tests of four sets of cylinders are plotted against the moisture content in Figure 9 and values of the modulus of the Ames subgrade at the two moisture contents which prevailed in the pressure measuring experiments have been estimated by extrapolating the curve defined by the four test points. The curve has been extrapolated in accordance with the principles governing the change in deformation characteristics of soils with changes in moisture content as defined by Hogentogler, Wintermyer and Willis (6). These writers state "The deformations of either confined or unconfined soil samples under constant load increase with increase of moisture content at a consistent rate until a given moisture content known as the critical moisture is reached. When the moisture content is increased above this value the deformations of the samples increase at a very much greater rate than for similar moisture increases below the critical moisture."

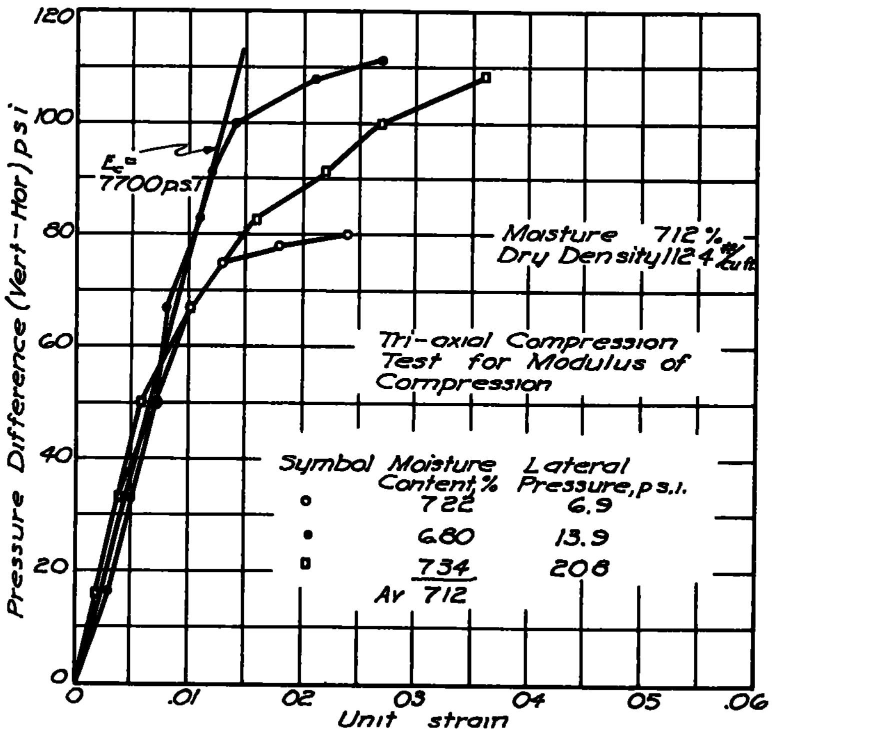


Figure 8

In accordance with the foregoing procedure, the moduli of compression of the dry and wet subgrades used in the Ames experiments have been estimated to be approximately 13,200 psi and 1000 psi, respectively, and using these values the

Further experimentation may dictate modifications of the numerical terms of this equation. It will be especially appropriate to study the moduli of soils under repeated loading for the reasons

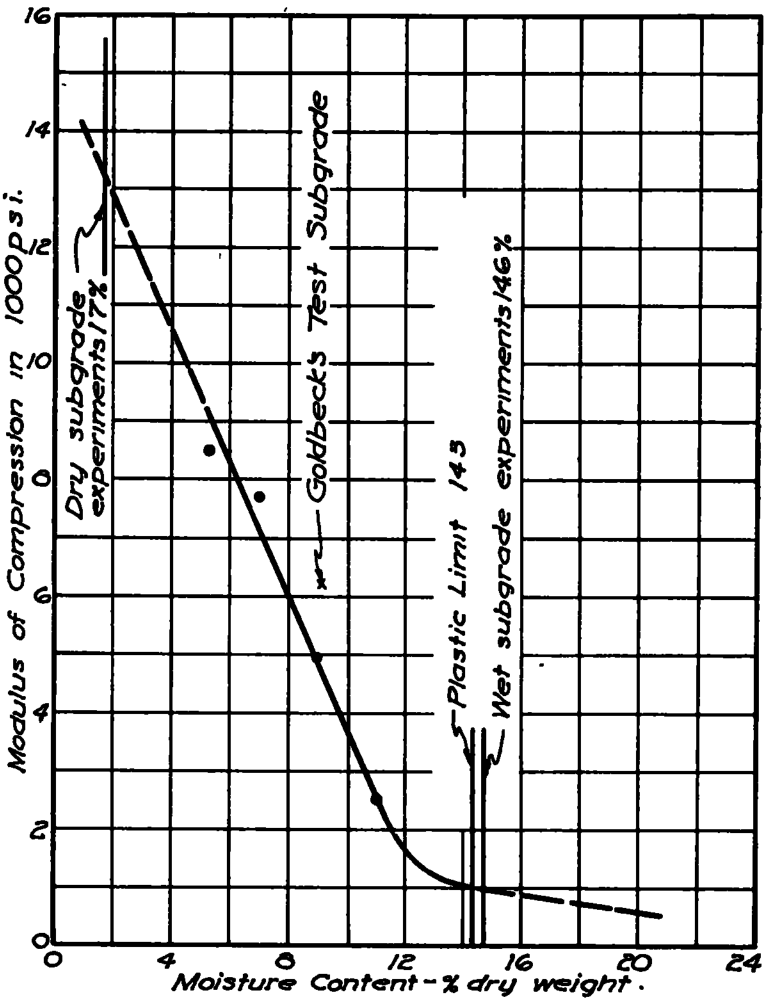


**Figure 9**

empirical expression for the subgrade stress factor for single tire highway loading is tentatively established as

$$C=\frac{0.0000068E_c+0.0070}{\sqrt[3]{\frac{P}{1000}}} \qquad 2$$

previously discussed. Also, it will be noted that this expression for C is dimensionally non-homogeneous It is hoped that future studies will more clearly reveal the true dimensional properties of this constant.

Goldbeck's test subgrade soils, which

were texturally similar to those used in the Ames experiments, and which were described by Goldbeck as being "quite stable," contained about 9 2 percent moisture and according to Figure 9 might be expected to have a modulus of compression of about 4900 psi. However, the modulus calculated from the measured pressures by equations 1 and 2 indicate a value of about 6200 psi, and this value is plotted in Figure 9. It is believed probable that mineralogical differences between these two soils which were from widely separated geographical sources might readily account for this discrepancy

### SUBGRADE PRESSURE DISTRIBUTION

As stated above, there are a number of research studies on record which indicate definitely that the pattern of pressure distribution on a flexible pavement subgrade may be represented by a bell or helmet-shaped surface, and this is in harmony with the facts associated with the general problem of stress distribution in soil masses Therefore, knowing the value of the maximum ordinate to such a pressure surface, as expressed by equation 1, the approximate pressure at any point on the subgrade may be obtained by the use of a probability type of equation having the proper constants to insure equilibrium between the applied load and the integrated subgrade pressures

As a means of defining the vertical pressure at any point on the subgrade in terms of the maximum pressure under the load, Holl (7) has suggested the equation

$$\sigma_r = \sigma_0 e^{-k^2 r^2} \qquad 3.$$

in which

$\sigma_r$ = the subgrade pressure at any point, psi

$\sigma_0$ = subgrade pressure directly under the load, psi

r = the radial distance to any point, in

k = a constant

e = the base of natural logarithms

In order to satisfy the requirements of equilibrium

$$P = \int_0^\infty 2\pi\sigma_0 e^{-k^2r^2} r\,dr \qquad 4.$$

from which

$$P = \frac{\sigma_0 \pi}{k^2} \text{ or } k^2 = \frac{\sigma_0 \pi}{P} \qquad 5$$

substituting the value of $\sigma_0$ as given by equation 1,

$$k^2 = \frac{\pi C}{t} \qquad 6$$

and equation 3 becomes

$$\sigma_r = \frac{CP}{t} e^{-\frac{\pi C}{t} r^2} \qquad 7$$

This is an expression for the vertical pressure on the subgrade due to a single tire load in terms of the load, the pavement thickness, the subgrade stress factor and the lateral distance from the load to any point The computed pressures by equation 7 agree satisfactorily with laboratory measured pressures as is illustrated by several typical examples shown in Figure 10 From this formula for the subgrade pressure, Holl (7) has developed an expression for the subgrade deflection, assuming the subgrade soil to be quasi-elastic as stated in assumption No 2, as follows

From the theory of elasticity the surface deflection at a distance s from a point load is (see Fig 11)

$$\Delta Z_s = \frac{p(1-\mu^2)}{\pi E_c s} \qquad 8.$$

in which

$\Delta Z_s$ — the surface deflection, in

p = a point load, lb.

s = the distance from the load, in

$\mu$ = Poisson's ratio of the subgrade soil

$E_c$ = the modulus of compression of the soil, psi.

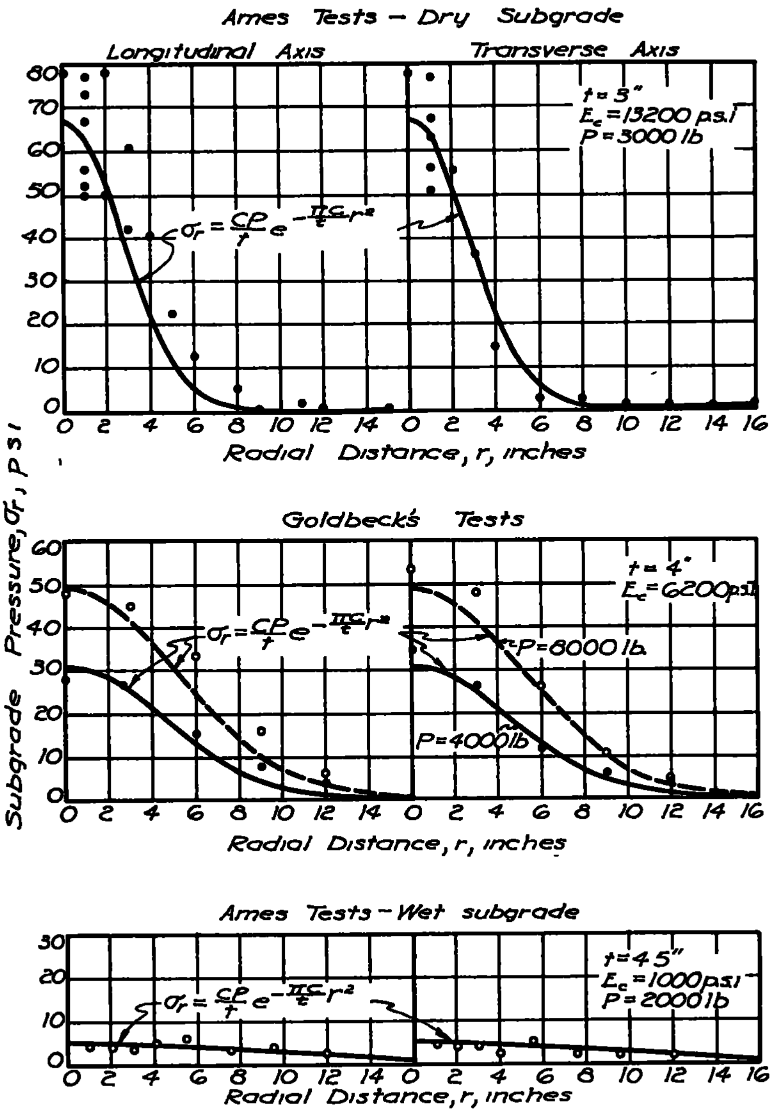


**Figure 10**

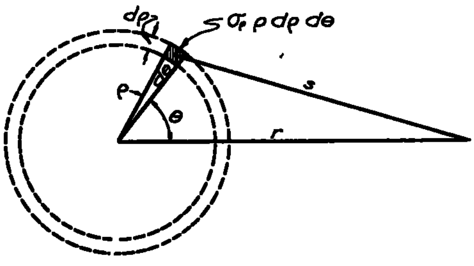


**Figure 11**

Then for an elemental load, the deflection at a radial distance r from the center of loading (the center of the tire load) is

$$d\Delta Z = \frac{\sigma_\rho (1-\mu^2)\rho d\rho d\theta}{\pi E_c s} \qquad 9$$

The deflection for the pressure defined by equation 7 is

$$\Delta Z_r = \int_0^\infty \int_0^{2\pi} \frac{CPe^{-\frac{\pi C}{t}\rho^2}(1-\mu^2)\rho d\theta d\rho}{\pi t E_c \sqrt{r^2+\rho^2-2r\rho\cos\theta}} \qquad 10$$

Integrating

$$\Delta Z_r = \frac{(1-\mu^2)P\sqrt{\frac{C}{t}}}{E_c} e^{-\frac{\pi C}{t}r^2}\left[1+\frac{\pi Cr^2}{2t}+\frac{1\cdot 3\left(\frac{\pi Cr^2}{2t}\right)^2}{(2!)^2}\frac{1\cdot 3\cdot 5\left(\frac{\pi Cr^2}{2t}\right)^3}{(3!)^2}+\quad\right] \quad 11$$

Only the first two terms of the series are significant Therefore equation 11 may be written

$$\Delta Z_r = \frac{(1-\mu^2)P\sqrt{\frac{C}{t}}}{E_c} e^{-\frac{\pi Cr^2}{t}}\left[1+\frac{\pi Cr^2}{2t}\right] \quad 12$$

The maximum central deflection, that is when r=0, is

$$\Delta Z_0 = \frac{(1-\mu^2)P\sqrt{\frac{C}{t}}}{E_c} \quad 13.$$

This expression contains the material constants $\mu$, Poisson's ratio, and $E_c$, the modulus of compression of the subgrade soil In accordance with assumption No 2 and the previous discussion it is recommended that $E_c$ be determined by laboratory triaxial compression tests or by such field tests as the California Bearing Ratio or the North Dakota Cone-Bearing Test, properly correlated to give results in terms of $E_c$. Or as suggested previously, $E_c$ may be considered as an empirical physical constant whose value may be determined by measuring the deflections of a pavement under an actual wheel load and comparing such deflections with equation 12 Also, since the most serious deformation conditions affecting the integrity of a flexible pavement will occur when the subgrade soil is in its most yielding state, it is recommended that the soil be tested in a state or degree of saturation as near as possible to the maximum degree of saturation which it is estimated will prevail during the life of the pavement This moisture content may be greater than that which the soil contains at the time of sampling Also, the repeated load tests reported by Older (9) which are shown in Figure 6, indicate the important effect on the stress-strain relationship of soils immediately after freezing and thawing, and this condition may need to be taken into account in testing the subgrade soil

There is little available information concerning Poisson's ratio for soils. So far as the author is aware the only reported determinations of this property are those by the late Prof. J H. Griffith (5) He measured the ratio of lateral to vertical deformation on a few unconfined compression specimens of yellow and blue clays and of loam at very low stresses ranging up to 1.72 psi These values of Poisson's ratio varied widely and erratically from 0.10 to 0 61 In view of the tenuousness of this property of soils and the difficulties encountered in its measurement, it is recommended that no attempt be made to measure the ratio, but that a reasonable value be assumed for use in calculating pavement thickness

In many analyses of soil problems based upon the theory of elasticity it is assumed that the soil does not undergo any appreciable volume change That is to say, the value of Poisson's ratio is assumed to be 0 5 In this problem if the assumed value of the ratio is greater than the actual value, the calculated deformation of the subgrade will be less than the actual deformation, and the indicated thickness of pavement will be somewhat less than the safe thickness. A value of 0 5 is about the greatest which any material can have, since this represents a condition of no volume change under load. It seems, therefore, that a value less than 0 5 would be a more suitable assumption for this elastic property and a value of approximately 0.3 is recommended. This, it is believed, will contribute a greater factor of safety and yet not an excessive one, to computations of pavement thickness by this method.

In the light of this assumption, equation 13 may be rewritten

$$\Delta Z_0 = \frac{0.9P\sqrt{\frac{C}{t}}}{E_c} \qquad 14$$

sure on the subgrade decreases as the modulus of compression decreases, but that the maximum central deflection increases as the modulus decreases and at a faster rate than the pressure decreases. These facts are shown graphically in Figure 12 where diametral profiles of both stress and deformation distributions are shown for a subgrade under a 4-in. pavement loaded with a single tire load of 8000 lb. and for various values of the compression modulus from 12,000 psi

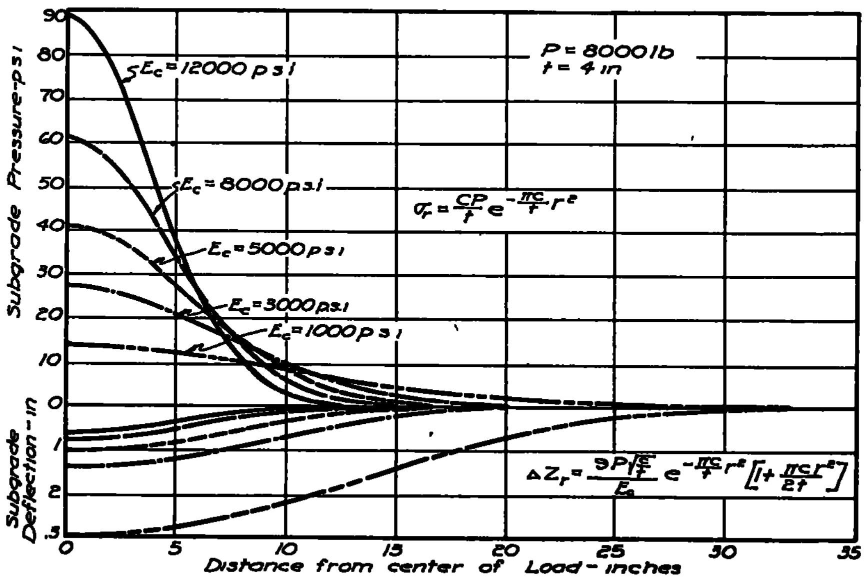


Figure 12

This equation is recommended for calculating the maximum deflection of a subgrade directly under a single tire highway load applied to a flexible pavement in terms of the applied load, the thickness of the pavement, and the modulus of compression of the subgrade soil.

An examination of equations 1, 7, 12 and 14 will reveal the influence which the stiffness of the subgrade exerts upon both the pressures and deformations of the subgrade surface. For given conditions of load and pavement thickness it will be noted that the maximum central pres-

down to 1000 psi. For these conditions, the maximum central pressure varies from 89.4 psi down to 13.8 psi while the maximum central deflection increases from 0.06 in. to nearly 0.30 in.

## SAFE ALLOWABLE DEFLECTION

If the maximum deflection which the base course of a flexible pavement of a given type can successfully withstand under repeated applications of load is known, this safe allowable deflection may be substituted for $\Delta Z_0$ in equation 14 and the required thickness of pavement calculated by solving the resulting expression for t At the present time there is no information available concerning this safe allowable deflection for base courses constructed of various materials such as soil-aggregate, sand-clay, soil-cement, rolled-stone, bituminous-stabilized, etc , and a technique for determining this limiting deflection will need to be developed.

Although the term "safe allowable deflection" is used in this discussion to designate the criterion upon which the determination of pavement thickness is based, actually it is not the deflection of the pavement which causes a pavement to crack, but rather it is the degree of curvature of the deflected pavement and the accompanying stresses and strains as it

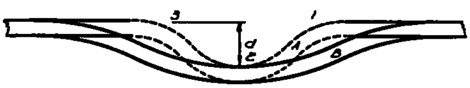

Figure 13

follows the contour of the subgrade under load, which is the critical phenomenon This is illustrated in Figure 13 where it is evident that pavement A will be stressed much greater at points 1, 2 and 3 than will pavement B which has less curvature than A, even though the maximum deflection of the two pavements is the same.

Since the equation for the deflected surface of the subgrade is available (see equation 12) and since in mechanics it is known that the maximum stress (or strain) of a structural unit such as a beam or a slab is a function of the curvature of the stressed unit, it is suggested that the modulus of rupture of the pavement material might be utilized as a criterion for design However, preliminary attempts to derive an expression for maximum stress from the equation for the elastic surface reveal practical difficulties which may or may not be surmountable. In the first place such a process involves the second derivative of the equation for the deflected surface, and this always tends to magnify the errors inherent in an empirical equation. The situation is further complicated by the fact that the subgrade stress factor, C, is dimensionally non-homogeneous. Further attempts will be made to utilize this modulus of rupture criterion, but additional knowledge of the rational relationships involved may be necessary before success is achieved. In the meantime, the concept of a safe allowable deflection seems to offer the greatest promise for practical use

As a general rule in engineering procedure, safe design limits such as the one under discussion can be determined most authoritatively by observations of actual structures in service. In this case, however, it is believed that practical difficulties which would be encountered in an attempt to establish safe allowable deflections by field observations would render the results valueless or nearly so. It is recommended, therefore, that, for the present at least, a laboratory technique be developed for obtaining the necessary information. The essential features of an apparatus by which it is believed the required information may be obtained, but which has not been built, is shown in Figure 14. To use this apparatus a series of specimens of base course of the type under consideration about 3 ft in diameter should be constructed They should be of several thicknesses, covering the range of probable thicknesses of the final base course. They should be built in accordance with the same specifications as those for the actual pavement, both as to materials and manner of placement and compaction and should be allowed to cure in a manner that will produce a final specimen as nearly analogous to the actual base course as possible.

These specimens shall be tested by placing on the ring bearing of the apparatus. They shall also be supported at the center by means of a special steel coil spring which fits snugly against the bottom of the test specimen at no load A micrometer dial graduated to 0.001 in shall be mounted adjacent to the spring by which central deflections of the pavement specimen may be measured. The specimen shall be loaded by means of a vehicle tire placed on top of the specimen and at its center The tire mounting shall

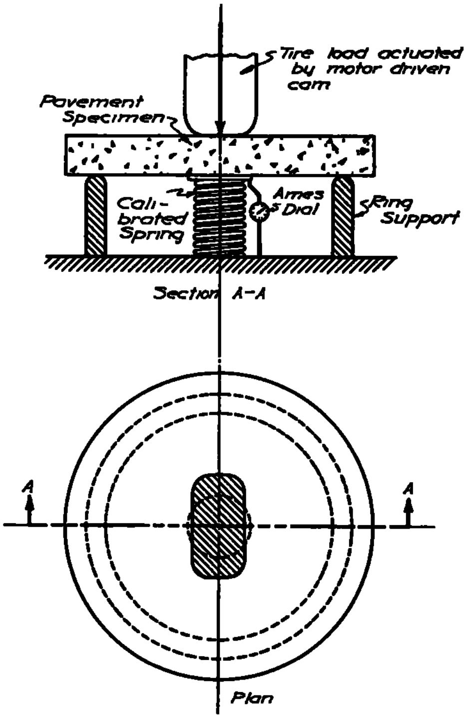


**Figure 14. Apparatus for Determining Safe Allowable Deflection of Flexible Pavements.**

be equipped with a motor driven cam by means of which loads can be applied and released in cycles of about one minute each The throw of the cam and the inflation of the tire will need to be correlated and adjusted so that an applied load of the desired magnitude may be obtained

The steel coil spring which provides the center support for the specimen should be made replaceable and a series of springs provided for the apparatus having a range of stiffnesses such that various amounts of deflection under a given load may be obtained. It is suggested that a series of six springs which will deflect from $\frac{1}{16}$ in to $\frac{3}{8}$ in. by increments of $\frac{1}{16}$ in. under a load of 4000 lb will probably give sufficient range for the purpose at hand, although experience with the apparatus may indicate desirable modifications of this series

A procedure for testing the base course specimens of the flexible pavement will be to place a specimen of a given thickness on the ring with the coil spring of the greatest stiffness (least deflection) in place and apply about 100 or more cycles of load. At the conclusion of this loading, if the specimen is still intact, replace the coil spring with the spring having the next greater stiffness and repeat the loading operations Continue this procedure using a less stiff spring for each loading until the specimen fails. The greatest deflection indicated by the micrometer dial beneath the specimen at which failure does not occur is the "safe allowable deflection" for that particular type of base course of the thickness represented by the specimen Repeat the operation for each thickness of specimen for the range of thicknesses to be studied, and obtain the relationship between safe allowable deflection and base course thickness under the condition of repeated loading.

The safe allowable deflection determined by this method will always be less than the failure deflection of an actual pavement if the radius of the ring support is made approximately equal to the radius of the locus of contraflexure of the pavement, because the laboratory specimen is subjected to tensile stress in the bottom surface only, whereas an actual pavement is subjected to tensile stress in the bottom surface in the immediate vicinity of the load and to tensile stress in the top surface at points beyond the locus of contraflexure. That is to say, the test specimens are the three-dimensional counterpart of a simple beam, whereas an actual pavement is analogous to a continuous beam. For equal tensile stresses, therefore, an actual pavement will deflect more than the laboratory test specimens in the machine described above as illustrated in Figure 15. It is believed that this difference in deflection between the test specimens and an actual pavement of the same material will constitute a suitable factor of safety for the finished pavement if the laboratory failure deflection is used as a limiting design deflection Actual experience with this method of


*Actual Pavement Deflection (Continuous beam type)*


*Safe Allowable Deflection (Simple beam type)*

**Figure 15**

design will, of course, be necessary to verify the adequacy of this factor of safety

In the more orthodox types of structures such as steel beams, concrete slabs, etc., the deflection varies with the reciprocal of the cube of the depth of the beam or slab and it is probable that the safe allowable deflection of a flexible base course will likewise vary approximately with the reciprocal of some power of the base course thickness The equality suggested above will therefore probably be of the form

$$F\left(\frac{1}{t_b^{n}}\right)=\frac{0.9P\sqrt{\frac{C}{t}}}{E_c} \qquad 15$$

in which

$t$ = total pavement thickness (wearing surface + base course + subbase)

$t_b$ = thickness of base course only.

### DUAL TIRED WHEEL LOADS

All of the experimental measurements of subgrade pressures which form the background of this method of structural design of flexible pavements have been made for loads applied through a single highway vehicle tire. Since many trucks transmit their loads to the pavement through wheels equipped with dual tires it will be of interest to examine the subgrade pressures and deformations beneath a dual tire assembly. Although the experiments have indicated definitely that the principle of superposition is not applicable in this problem of stress distribution because of the shallow depth of the subgrade below the surface at which the load is applied, and the marked difference in compression modulus of the subgrade soil and the pavement, it is believed that some valuable ideas of the nature of a first approximation may be gained by adding together the subgrade pressures and deformations due to two single tire loads placed at the dual tire spacing

This procedure has been carried out for an arbitrarily chosen case of a 4 in pavement carrying two 4000-lb. tire loads spaced 10 in. c. to c and supported on a subgrade having a modulus of compression of 5000 psi. The results are shown in Figure 16 Within the limits of the validity of this superposition procedure it is indicated that the dual tire load produces subgrade pressures and deformations which are less than those caused by a single tire load of 8000 lb. and greater than those caused by a single tire load of 4000 lb However, a more important indication is that the dual 4000-lb. load does not appear to be any more injurious to this 4 in flexible pavement than a single 4000-lb. load, since the curvature of the subgrade deflection curve under the dual loading is nowhere any greater than it is under the single tire loading. These findings are applicable only to the case investigated and are not general Also they are only tentative and need to be verified experimentally.

### AIRPLANE WHEEL LOADS

Considerable interest has been stimulated recently in the structural effects of large airplane wheels carrying the very heavy loads of modern aircraft and the importance of this problem and the need for extensive experimentation with loads of heavy aircraft proportions cannot be overemphasized Until reliable experimental evidence is available, some approximate ideas concerning subgrade stresses and deformations under loads of this character can be obtained by extrapolation of existing single highway tire load data. Again calling attention to the limitations of the superposition procedure, we may investigate the subgrade situation under an arbitrarily assumed 30,000-lb plane wheel load which has a pavement contact area of 500 sq. in. by considering the load to be equivalent to ten 3000-lb. highway loads having a contact area of about 50 sq. in., as shown in the sketch in Figure 17. A pavement 6 in thick and a subgrade modulus of 5000 psi have been arbitrarily chosen for this illustrative case. The pressures and deflections of the subgrade along the central transverse element have been calculated by equations 7 and 12 for each of the ten loads and added together to obtain the curves for the total load. This procedure indicates that for this assumed case the maximum central pressure and deflection under the heavy load applied over a large contact area is roughly about 75 percent of the values obtained by considering the plane wheel load as a single load concentrated over a small area at the center. This reduction is qualitatively in the right direction, but is quantitatively not so great as the author expected, and the need for experimental evidence in this field is once more emphasized

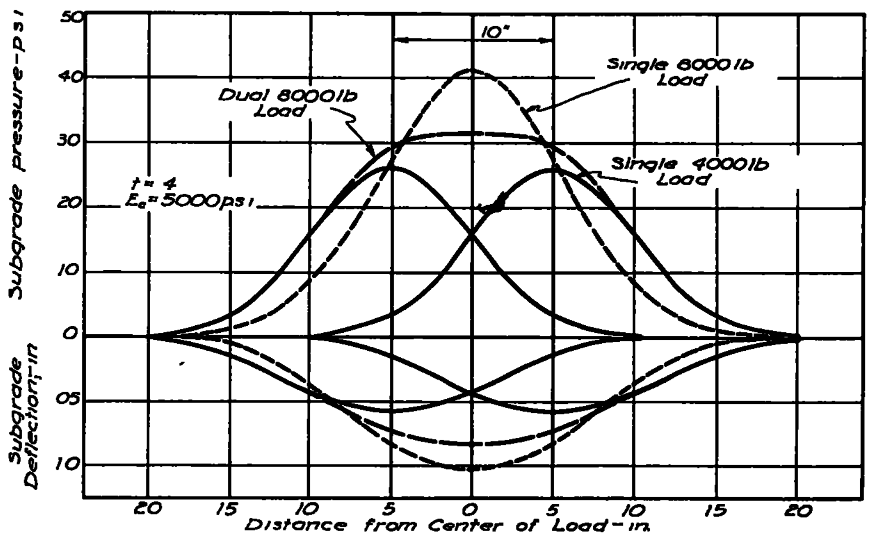


Figure 16

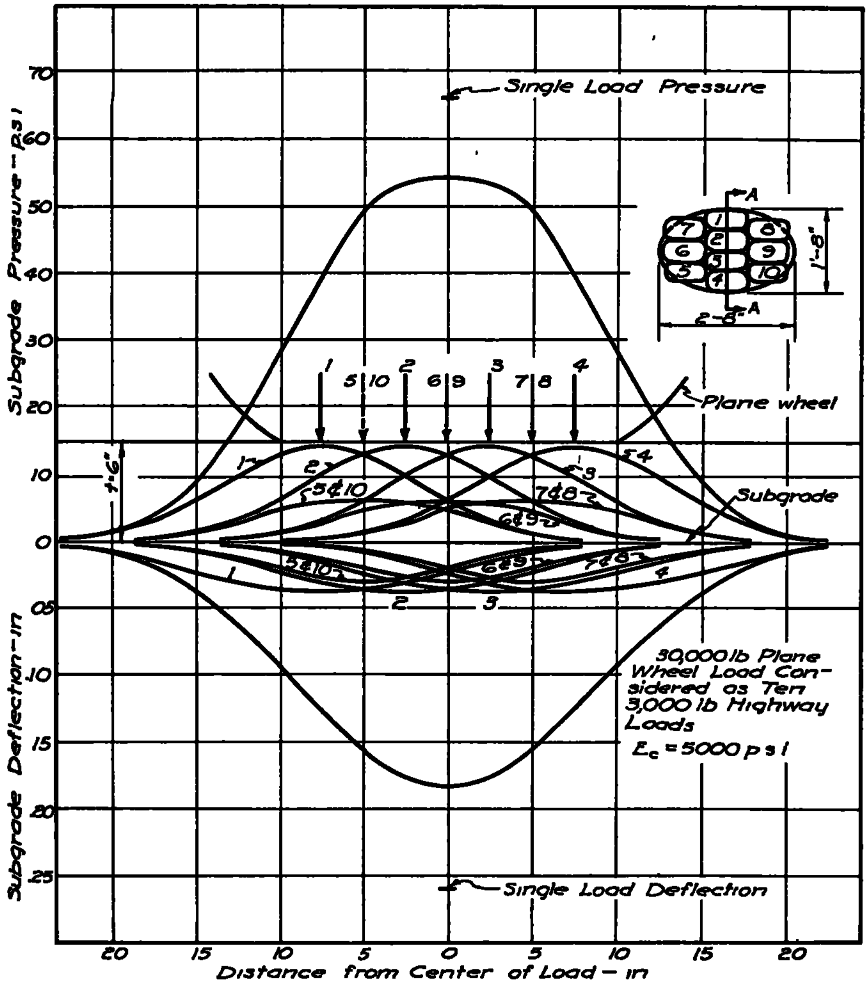


Figure 17


## REFERENCES

1 Bone, A J Personal Communication Jan 1942

2 Boyd, Kieth "An Analysis of Wheel Load Limits as Related to Design," *Proceedings* Highway Research Board, Vol 22 (1942)

3 Goldbeck, A. T. "Studies of Subgrade Pressures under Flexible Road Surfaces," *Proceedings* Highway Research Board, Vol. 19, p 164 (1939)

4 Goldbeck, A. T. "A Method of Design of Non-rigid Pavements for Highways and Airport Runways," *Proceedings* Highway Research Board, Vol 20, p 258-270 (1940)

5 Griffith, J. H "Physical Properties of Earths," Bulletin 101, Iowa Engineering Experiment Station (1931).

6 Hogentogler, C A, A. M Wintermyer, and E A Willis "The Subgrade Soil Constants, Their Significance, and Their Application in Practice," *Public Roads,* Vol 12, p 120, July 1931

7 Holl, D L Personal Communication Jan 1941.

8 Kneeland, Wm. F. "Wheel Load Stresses on Wet and Dry Subgrades under Flexible Type Pavements," Unpublished thesis, Library, Iowa State College (1941).

9 Older, Clifford "Highway Research in Illinois," *Transactions,* Am Soc. C E, Vol 87, p 1180 (1924)

10 Palmer, L A and E S Barber. "Soil Dis-

placement under a Circular Loaded Area," *Proceedings* Highway Research Board, Vol. 20, p. 279-286 (1940).
11. Porter, O. J. "Foundations for Flexible Pavements," *Proceedings* Highway Research Board, Vol 22 (1942)
12. Spangler, M. G. "Preliminary Experiments on the Distribution of Wheel Loads Through Flexible Pavements," *Proceedings* Highway Research Board, Vol 18, Part I, p. 162-171 (1938)
13 Spangler, M G and H O Ustrud "Wheel Load Stress Distribution through Flexible Type Pavements," *Proceedings* Highway Research Board, Vol. 20, p 235-257 (1940).
14 Spangler, M G "Wheel Load Stress Distribution through Flexible Type Pavements," *Proceedings* Highway Research Board, Vol 21, p. 110-117 and 133-136 (1941).
15 Krynine, Dimitri P "Soil Mechanics," p. 130. McGraw-Hill Book Co, Inc New York and London. 1941

## DISCUSSION ON THE STRUCTURAL DESIGN OF FLEXIBLE PAVEMENTS

MR. PREVOST HUBBARD, *The Asphalt Institute*: The speaker wishes to take issue with the first assumption of Professor Spangler "That the subgrade acts as the sole supporting medium for the pavement" and "that any resistance to deformation supplied by the pavement itself is neglected " Although not directly stated, this assumption appears to be implied in the three preceding papers and this discussion is therefore applicable to them also. Professor Spangler admits that this assumption is not strictly in accord with fact, and that it contributes a factor of safety of unknown magnitude.

Tests made in the laboratory of the Asphalt Institute indicate a very material resistance to deformation of hot-mix dense graded asphaltic concrete, as compared with that of mechanically stabilized mineral aggregate. The general method of test which developed this information is the same as described in a paper[1] presented by Hubbard and Field at the 1940 meeting of the Highway Research Board

Figure 1 shows the results of loading tests on 6-in thicknesses of asphaltic concrete and crushed rock resting on various soil supports—tests being made with a circular plate of 8 6-in. diameter, equivalent to the tire contact area of a light truck. At every point of comparison, the deflection is the same for both the asphaltic concrete and the crushed rock. The figures at points indicate the ratio of resistance

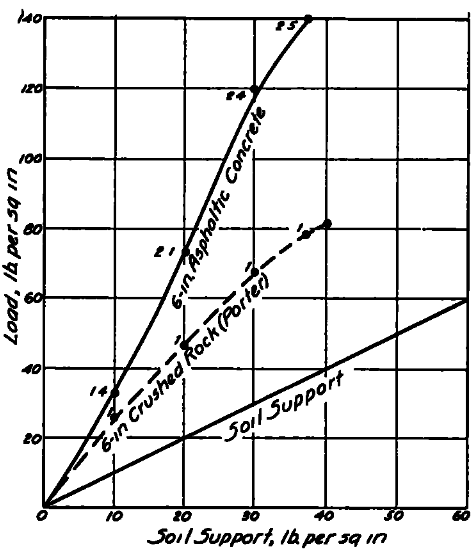


**Figure 1. Relative Increase in Support of 6-In. Thickness of Crushed Rock and Asphaltic Concrete in Relation to Soil Support. Test Head 8.6 In. Diameter.**

offered by the crushed rock to the resistance offered by the asphaltic concrete. It will be seen that, with the exception of the 10-lb. soil support, the asphaltic concrete shows more than twice the increased resistance over the subgrade support than is developed by the crushed rock.

[1] Hubbard and Field, "Required Thickness of Asphalt Pavement in Relation to Subgrade Support," *Proceedings*, Highway Research Board, Vol. 20, p 271 (1940)

In Figure 2, the resistance offered by a 3-in. thickness of asphaltic concrete is plotted in the same way against the resistance offered by 6 in. of crushed rock and 3 in. of crushed rock. It is seen that 3 in. of asphaltic concrete develops somewhat greater resistance than 6 in of crushed rock and that, as compared with 3 in. of crushed rock, from 2 to 5 times as much resistance is offered by the asphaltic concrete.

It is believed that this increased resistance is due to the relatively high tensile strength of asphaltic concrete which develops considerable resistance to bending It would therefore seem only reasonable to make due allowance in the total thickness design where asphaltic concrete is to be used as a surfacing medium. In other words, if a given total thickness includes 10 in. of mechanically stabilized mineral aggregate to afford the necessary support, and a 3-in. asphaltic concrete pavement is decided upon for a portion of the total thickness, it would seem that instead of making the mechanically stabilized aggregate 7 in. plus 3 in of asphaltic concrete, the thickness could be reduced by making the mechanically stabilized aggregate 4 in with 3 in. of asphaltic concrete.

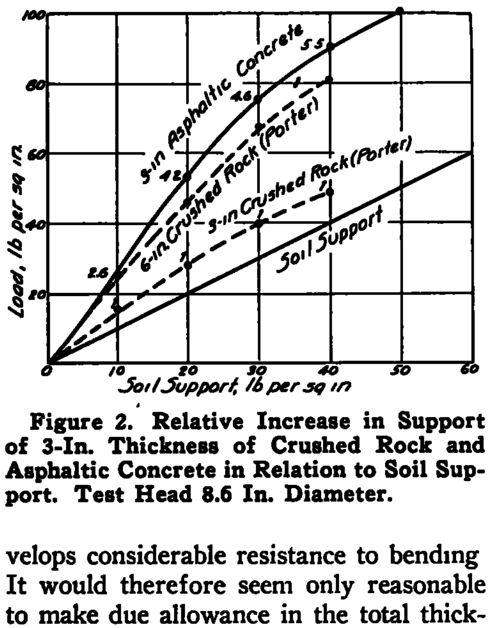


**Figure 2. Relative Increase in Support of 3-In. Thickness of Crushed Rock and Asphaltic Concrete in Relation to Soil Support. Test Head 8.6 In. Diameter.**

Investigation on other than the hot-mix types of asphalt pavements would indicate little or no differential in supporting value as compared with an equal thickness of crushed rock. In such cases, the asphalt pavement could be considered as the equivalent of an equal thickness of crushed rock.

MR. H. G. NEVITT, *White Eagle Division, Socony-Vacuum Oil Company* My comments are primarily in amplification of Mr. Hubbard's remarks concerning Mr. Spangler's assumption that the beam strength of bituminous surfaces is negligible. I might add that this whole matter may be to some extent a question of definition. Mr Spangler groups bituminous surfacings with those of granular aggregates under the description of flexible surfaces. I agree that the latter type come under such a heading and have negligible beam strength, but not the former. I avoided this difficulty in my article, later referred to, by distinguishing between the two. I classified those from granular aggregates without beam strength as flexible surfaces, and those with appreciable beam strength, although still maintaining the ability to adjust themselves to the subgrade over a considerable period and consequently always benefiting from the complete support it could supply, as semi-flexible in type I still think this distinction is warranted and required, although the general practice is to discuss only rigid and flexible pavements without bringing in the intermediate type.

In my early close association with the construction of low cost bituminous roads in the West, I was impressed by the remarkable load carrying capacity of these thin surfaces While we have heard much about the subsequent failure of surfaces after capillary water had accumulated due to the elimination of evaporation, with the implication that the dry subgrade was entirely responsible for the excellent results until it failed due to moisture increase, we never hear of the jobs—and I have seen many of them—which were put down on only moderately good subgrades where the moisture conditions, both before and after oiling, were only fair due to lateral moisture supplies in excess of the evaporation rate. On these jobs the thin bituminous mats also demonstrated surprising carrying capacity, even though failing ultimately under the repetitions of increasingly heavy traffic, and this carrying capacity was far in excess of nearby similar gravel surfaces. A careful structural analysis of this situation led me to the definite conclusion that mere frictional interlock, or granular support, could not account for these capacities, and therefore appreciable beam resistance must be present. Likewise the action over noncohesive sand subgrades, requiring a surcharge effect well in excess of that supplied by the weight of the mat, and hence only due to reverse flexure of the mat, indicated a considerable beam action. A rather elaborate mathematical analysis of the properties of high consistency bitumens definitely indicated that such materials in thin films should display the considerable tensile resistance in bending required for such beam action. I consequently then felt, as I do today, that such an effect exists to an appreciable extent.

As a result of these studies I developed a formula for determining the required thickness of a flexible surfacing which was published in the *Engineering News-Record*[1] some years ago. This formula was not intended as an elaborate or highly scientific approach, in view of our very limited control over the design and construction otherwise, but the allowance of an inch or so of extra thickness to take care of these irregularities should give a conservative design It is interesting to note that its results best correspond to the carrying capacities being demonstrated by the highways in actual use—a point which I felt was in itself a considerable verification of the theory.

In calculations using this formula, such as those published with it, I generally noted that the calculated bituminous mat thickness was ordinarily about one-half the thickness calculated for the corresponding gravel surface, and I used this rough relationship with considerable success I was naturally extremely interested in Mr. Hubbard's demonstration of this same situation, with its further indication of the approximate correctness of the formula

Now for the points I wish to make.

First, high type bituminous mats do have beam strength, even though they are still in the flexible classification.

Second, this beam strength is appreciable. If my formula is accepted as giving some indication, over half the load in many cases is carried by this beam action, with the balance through the granular effect of the aggregate.

Third, the formula indicates that this proportion carried by beam action should be greater with low base course bearing values. Hence in the critical cases (which have been brought out in the discussion of the California bearing value design curves) where very thick total base and surfacing course thicknesses are indicated, the increase in effectiveness of the bituminous surface course becomes even more important, as it tends to offset the need for the extremely thick base courses now being called for. It is possibly one of the reasons for the many early mats on thin bases over such subgrades carrying the increased loads without the failure to be expected from the latest theories. It is true that Mr. Hubbard's results do not quite confirm this phase of the situation, but I think its existence can be qualitatively demonstrated with little difficulty.

I feel it is conclusively indicated that bituminous mats of present day accepted

[1] H. G. Nevitt, "Semi-flexible Road Surfaces as Dynamically Rigid Slabs," *Engineering News-Record,* April 11, 1935.

thicknesses do have beam strength, and of such magnitude that it should be allowed for in the calculations.

MR. V. A. ENDERSBY, *Shell Development Company:* I want to raise a question on Professor Spangler's proposed method of fatigue testing of pavements In normal traffic, on a resilient or partly resilient subgrade, there is an alternating stress, and it is that which Professor Spangler proposes to measure. However, the apparatus, as he shows it, provides a simple beam test; whereas, the road stresses are applied on a continuous beam. The major stresses in a continuous beam will be tensile at the flexure point and tensile at the contra-flexure point; so I would like to ask if such an apparatus should not be constructed with a restraint beyond the contra-flexure point, because we noticed in photographs shown in previous papers, that the cracking was at both points, and that the cracking may be more serious at contra-flexure than at flexure.

MR HUBBARD: As a matter of interest, I might state that quite independently of Prof. Spangler, we have built almost the identical apparatus which he has suggested, except that the spring is at the top instead of the bottom; and we have included the restraining influence mentioned by Mr. Endersby. We have not yet obtained any test results that we can depend upon from this apparatus.

PROF. D. P. KRYNINE, *Yale University*

*Assumptions*

Professor Spangler assumes that the subgrade acts as the sole supporting medium for the pavement. He states that this is a simplifying assumption which contributes a factor of safety of unknown magnitude in the computation of pavement thickness by this method.

The writer believes that this assumption really may increase, but also may decrease the factor of safety As a matter of fact, when the dynamic load, P, is applied at the top of the pavement (see Fig. 1), a certain force F acts on the two neighboring very small equal cubes 1 and 2 and drives them down. Cube 1 belongs to the pavement and because of the difference in unit weights, its mass $m_1$ is larger than the mass $m_2$ of cube 2 which belongs to the subgrade Were cubes 1 and 2 quite free, cube 2 would move down with an acceleration $a_2$ greater than the acceleration, $a_1$, of cube 1. This is because $F = m_1a_1 = m_2a_2$ In such a case, a separation of cube 2 from cube 1 is possible. This separation, however, may last only

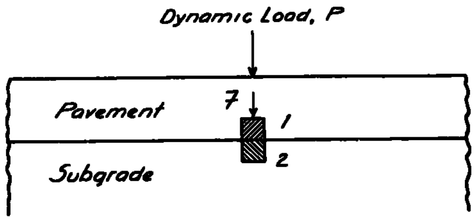


**Figure 1**

an infinitely small fraction of a second since as soon as the wheel leaves its point of application at the top of the pavement an upward movement of the pavement (recovery) will take place. The possibility of the separation increases together with the increase of the difference $m_1 - m_2$. In other words, the heavier and stiffer the pavement, the greater the chance for it being left unsupported.

It should be noticed that our measuring devices are too rough to record that separation, if any If it exists, the action of the pavement decreases the safety factor; if it does not, Professor Spangler's statement about the increase of the safety factor is correct.

As to the second assumption of Professor Spangler, the writer agrees completely that after a passage of a limited number of wheel loads of a certain magnitude, the subgrade soil becomes quasi-elastic. As a matter of fact, the irreversible deformation at each of these passages gradually decreases until it becomes zero. Should afterwards a larger load pass over the pavement, the subgrade, as a rule, will become inelastic again.

*Zones of Disturbance*

A zone of disturbance or simple "disturbed zone" has probably a constant shape under a constant individual load. It becomes quasi-elastic after a certain number of loadings and unloadings using a constant individual load; but the material of this disturbed zone, being non-uniformly compressed, ceases to be isotropic and possesses variable moduli of elasticity throughout. Duplication or triplication of the load makes the zone deeper and inelastic again. It may become quasi-elastic after a certain number of loadings and unloadings using these double or triple loads Certainly, unrestricted use of the principle of superposition in this case cannot be permitted.

The disturbed zone under an actual pavement is possibly deeper at the middle of the roadway where the traffic is intense and may be rather shallow close to the edges. The moduli of elasticity in such a zone may vary much more than in the case of an experimental box loaded at a point.

*Stresses*

In the opinion of the writer there are two different sets of stress in this case: (a) after one (or a few) applications of an individual load when the disturbed zone still is inelastic; and (b) after complete "elasticization" of the disturbed zone applying the maximum load to be used in the experiment These two sets of stresses (and the corresponding moduli of compression) may be quite different.

*Computationns Using the Modulus of Compression and $\mu=0.5$*

Apparently the idea of such computations was first advanced by Terzaghi and Frohlich[1] and subsequently very ably developed by Palmer and Barber in this country. Professor Spangler proposes to use the value of $\mu=0.3$. In using this method it should be assumed that the load acts only once and is not removed; hence the actual material may be considered as elastic independently of what it represents in reality, and consequently elastic formulas for deflections may be used, placing in them the modulus of compression instead of the modulus of elasticity. Such a procedure may be acceptable in the case of a massive structure such as a bridge pier, when the probable value of an eventual settlement is to be computed; but in the case of a pavement it furnishes the value of the virgin deflection only. Actual deflections decrease as the pavement is used and finally become constant.

In an actual case it would be perhaps more practical to pretest the subgrade by rolling it and by loading and unloading it. The modulus of compression should be determined on an undisturbed sample taken from such pretested material which for testing purposes may be prepared in the laboratory.

*Deflections vs Curvature*

It is often assumed that the strength (or safety) of a pavement is characterized by the value of the deflection under a given load. What does count in reality is not exactly the value of the deflection, but the curvature of the deflected pavement as Professor Spangler ably shows in Figure 12. This sound view should be shared by all highway engineers.

*Observations on Actual Pavements*

Professor Spangler leads the reader very ably along a logical chain "stress—stress distribution—deflections" Some links of this chain are rather weak, as Professor Spangler himself states and

[1] Terzaghi-Frohlich "Theorie der Setzungen von Tonschichten," Leipsig und Wein, page 10 1936.

need further research The writer disagrees with Professor Spangler's statement that at the present time the laboratory technique may furnish more valuable information than field observations on pavements In all cases of mass production—and pavement building undoubtedly is a case of mass production—much more evidence is furnished by the production itself than by mathematical analysis or laboratory experiments

PROFESSOR SPANGLER: In no sense do I disagree with the contentions of Mr. Hubbard and Mr. Nevitt concerning the development of beam or slab strength by high type bituminous mats, particularly the dense-graded hot-mixed asphaltic concretes. The only purpose of assumption No. 1 was to delimit the area of the discussion and to fix the conditions under which the developed theory is applicable. It was not my intention to imply that all pavements which do not fall in the rigid classification are of the flexible type, as defined in the paper. I think Mr. Nevitt's suggestion provides a solution for the problem raised by these discussions and that we may need to recognize at least one and perhaps two intermediate types between the rigid type of pavement exemplified by concrete slabs and the flexible type represented by mechanically stabilized gravel pavements with bituminous surface treatment. The quantitative basis of classification and the line of demarcation between, for example, a flexible and a semi-flexible pavement will be more or less arbitrary for some time to come. Nevertheless, there are semi-flexible pavements and the fact needs to be recognized as pointed out by these discussers.

The theory as presented, however, directly applies only to those pavements which have little or no beam strength and if it is applied to a semi-rigid pavement, some arbitrary modification of the derived thickness will need to be applied properly to take into account the semi-rigidity of the pavement. Otherwise the factor of safety introduced by assumption No. 1 will, in all probability, be too great for proper economy in design.

The suggestion offered by Mr. Endersby to the effect that the stress situation in the specimens tested in the apparatus shown in Figure 14 is not analogous to the stress situation in an actual pavement is entirely correct and the matter is discussed quite fully in the paper. The simple beam type of specimen was suggested because it was thought the test procedure would be simplified thereby in comparison to the procedure which would be required if a continuous beam type of test were employed. Simplified tests in which the stresses in the specimens are not exactly similar to those existing in the actual structure are very common in engineering practice. For example sewer pipe are usually tested in a 3-edge bearing test, wherein the loads and reactions are applied in a manner radically different from that in which they are applied in an actual structure. Properly correlated, such tests are very useful and because they are relatively simple, it is probable that a greater number of products are tested than would be the case if more complicated testing procedures were required. In all probability the test for safe allowable deflection as outlined in the paper will undergo considerable revision and modification if it is put into actual use in the future.

The comments offered by Professor Krynine are all very helpful and constructive and are greatly appreciated by the author as are all of the discussions offered in connection with this paper