UNCERTAINTIES IN DESIGN OF CONCRETE PAVEMENTS DUE TO DIFFERENTIAL SETTLEMENTS AND VOLUMETRIC CHANGES

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SYNOPSIS

The paper presents a qualitative and quantitative study of the effect of some variables on the design of concrete runways. Studied is the sensitivity of computed and measured strains, deflections, and subgrade reactions to uncertainties in variables of weather, differential settlements, and properties of materials.

Strains caused by volumetric changes of the material of the pavement and of subgrade may be, and often are, of primary importance. In some regions the design of a runway may be for weather and then modified for loads of air carriers Strains caused by wheel loads, however, are of interest as they may affect progressive demoralization of the runway Clearly the importance of each source of strain depends on the scale, relative magnitude, range of uncertainty, and also on the character of the source. Design requirements of runway and subgrade for purposes of weather may be opposite to those for purposes of load.

Geometry of the deflected pavement is the dominating characteristic of deformations produced by volumetric changes of subgrade or of the material of the pavement. Strains produced by these sources are studied essentially as problems of geometry and not of stress.

The purpose of this discussion is to study the qualitative and quantitative effects of uncertainties in some variables on computed and measured strains, deflections, and subgrade reactions of concrete runways of airports. The study here is restricted to variables of weather, differential settlements, and properties of materials Previously studied were variables of static load, runway, and subgrade.[1]

The effects of weather on structural integrity have been discussed by various architect-engineers and laymen in histories of architecture, in the technical literature, and in novels. Important sources of evidence as to the effects of weather are structures and monuments of previous generations of architect-engineers Much evidence also is available in the literature reporting on weathering of rocks Little has been done to correlate the effects of season, weather, and climate on structures in different regions

In some regions runways may be designed for weather and then modified for loads of air carriers Strains caused by wheel loads, however, may affect progressive demoralization of the runway Weather, although discussed, has been often discounted or neglected as a factor in design. Little is known of designing structures for weather Uncertainties exist in weather, in the resulting strains, and in the interpretation of these strains.

Design requirements of runway and subgrade for purposes of weather may be opposite to those for purposes of load Desirable properties of a pavement for purposes of weather are not known with any degree of certainty. Yet, in some cases, the properties of pavement and subgrade desirable for purposes of weather conflict with those required for load Also where weather is an important factor the use of structural elements at joints may be harmful rather than helpful.

Strains caused by volumetric changes or differential settlements may be of primary importance. Clearly the importance of each source of strain depends on the scale, relative magnitude, range of uncertainty, and also on the character of each source Care must also be observed in properly adding effects from the various sources and interpreting the results.

Hardy Cross has often stated that in weighing evidence as to strength, a distinction must

[1] F M. Baron, "Variables in the Design of Concrete Runways of Airports," *Proceedings*, Highway Research Board, Vol. 22, p 225 (1942).

be made as to whether the dominating characteristic of a source is essentially a stress or a strain. When statics dominates, the problem is essentially one of stress However, when geometry dominates, the problem is essentially one of strain. It may be naive to add numerically a stress produced by one source to a strain produced by another. Strains in general are changed into computed stresses by multiplying them by a ratio of stress to strain. This ratio is commonly called E. The ratio of stress to strain in a fiber of a structure in the field may be quite different from that obtained in a test conducted on a coupon specimen in a laboratory. Included among factors apparently affecting this ratio are the magnitude of stress or strain, the duration of a stress or strain, the rate of application of a load or movement, moisture content, chance cracking of concrete, and also the procedure of pouring and of curing concrete Much uncertainty exists also in the bending of a pavement per unit of moment per unit of length.

Geometry is the dominating characteristic of deformations produced by volumetric changes of subgrade or of the material of the pavement. Deformations produced by these sources are then essentially problems of strain and not of stress.

Important questions exist as to the tensile strength of concrete. Significant are the differences in the tensile strength of a specimen tested in direct tension, in the modulus of rupture of a beam, and in the modulus of rupture of a slab. The concept of a modulus of rupture of a slab is important. Yet, it is difficult to conceive how the modulus of rupture can be defined or measured. Some factors apparently affecting the modulus of rupture of a slab are the sources and distribution of strains, history of the strains, procedure of pouring and of curing concrete, age of concrete, moisture conditions, and proportions of mix.

The tensile strength of concrete has been usually stated in pounds per square inch In structural problems where statics is the dominating characteristic, it seems reasonable to correlate strength in terms of pounds per square inch. In those problems where geometry dominates, such as strains resulting from volumetric changes, limiting strains are involved rather than limiting stresses.

Hatt and Mills defined extensibility as the ability of concrete in tension to withstand deformations without the appearance of cracks or fissures.[2] Hatt and Mills reported that under fairly rapid loading (too rapid to permit plastic flow) plain concrete beams extended about 150 to 190 millionths of an inch per inch before the appearance of cracks visible to the naked eye of the observer (cracks open about 1½ thousandths of an inch).

J. L. Savage reported tests at the Bureau of Reclamation on sealed cylinders of concrete subjected to direct tension applied in increments of 50 p.s.i. at intervals of 28 days until failure occured[3] The maximum strains at time of failure ranged from 70 to 110 millionths of an inch per inch. R. E. Davis has reported tests of specimens loaded in tension at a rate which required 2 to 3 months to produce failure[4] Reported was a range in extensibility of 80 to 160 millionths of an inch per inch Lower values were reported for a rapid loading.

STRAINS RESULTING FROM DIFFERENTIAL SETTLEMENTS

Differential settlements may be an important source of objectionable cracking. The subgrade may be disturbed by a freeze or a thaw forming lenses of ice or soft spots, respectively. Volumetric changes of the subgrade may be produced by changes in moisture conditions resulting from a wetting, a drying, or changes in ground water conditions. The subgrade may further be disturbed by repeated movements of planes over soft spots. Loss of support may occur due to structural damage to drains. Poor practices observed in

[2] W. K. Hatt and R. E. Mills, "Physical and Mechanical Properties of Portland Cements and Concretes," Bulletin No. 34, Purdue University, November, 1928, p. 53.

[3] J. L. Savage, "Special Cements for Mass Concrete," United States Department of the Interior, Bureau of Reclamation, Denver, 1936, especially p. 91. See also, "Concrete Manual," United States Department of the Interior, Bureau of Reclamation, October, 1942, p. 36.

[4] R. E Davis, "Cement and Concrete Investigations for Bonneville Dam," Final Report to Corps of Engineers, U. S. Army, Second Portland District, February, 1938, pp. 115–118. Reference obtained in "Concrete Manual," United States Department of the Interior, Bureau of Reclamation, October, 1942, p. 36.

the preparation of a subgrade may be an important source of differential settlements. Volumetric changes may also be produced by vibrations from planes—particularly in "warm-up" areas.

Cracks may be objectionable for reasons of structural satisfactoriness, maintenance, or operation. A cracked pavement may not look good. Strains produced by wheel loads may accelerate cracking of the pavement. Easy entrance of water may produce leaching of tlements resulting from the various sources. Uncertain are the probable distances between high areas of a deflected pavement, maximum deflections of low areas relative to these high areas, distribution of angle changes or curvatures along the horizontal dimensions of the pavement, distribution of subgrade reactions, variations in the properties of concrete and of subgrade along the pavement, chance cracking, and possible loss of contact at certain areas between pavement and subgrade.

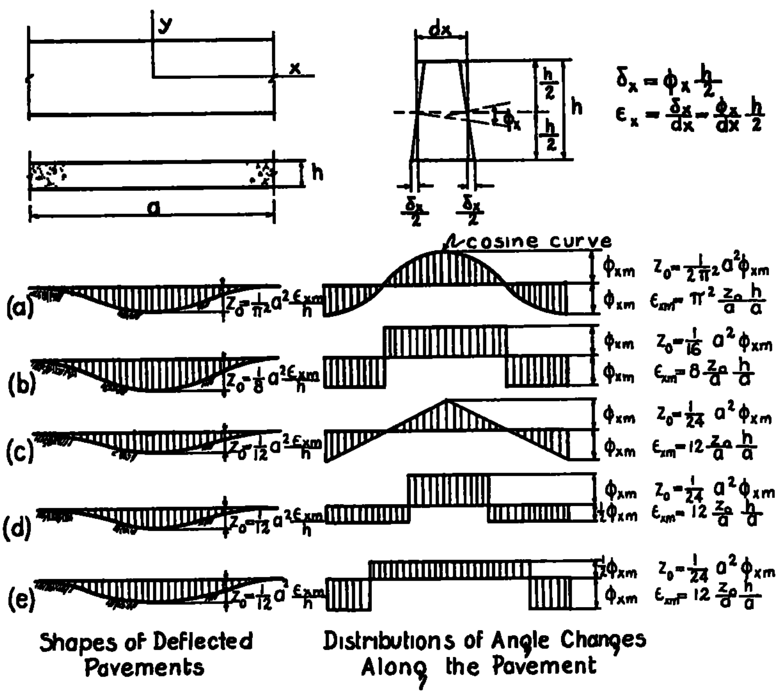

Figure 1. Effects of Distributions of Angle Changes Along the Pavement on Maximum Unit Strains and Deflections

concrete or corrosion of steel. Alternate expansion and contraction may break down the bond between the cement paste and aggregate. The subgrade condition may be disturbed sufficiently by uncontrolled water to be the cause of further cracking. Loose particles on a runway or apron may be a source of increased maintenance to propellers, leading edges of wings and tails, and of danger to personnel. Also, possible ice sheets or wet areas in the vicinity of cracks or of settled areas may endanger landings of planes.

Much uncertainty exists in differential set-

Of importance is the sensitivity of deflections, unit strains, and subgrade reactions to uncertainties in the above variables. The relative sensitivity of results to uncertainties or accidental combinations of variables may give scale to the validity of certain measurements and to problems of design. Clearly the interpretation of results depends upon their use, sensitivity to uncertainties in variables, and on the range and magnitude of these uncertainties.

The sensitivity of strains to uncertainties in the deflected shape of a pavement may be studied as a problem of geometry[5] Consider a differential element of a pavement with curvatures only as in Figure 1 The thickness is h and the horizontal dimensions are dx and dy It is assumed that the shortenings of fibers on one side of a horizontal neutral surface are equal to the lengthening of corresponding fibers on the other side of the neutral surface. By definition of a small angle, the angle changes, ϕ_x and ϕ_y, between corresponding straight lines normal to the neutral surface are then

$$\phi_x = \frac{2\delta_x}{h} = \frac{2\epsilon_x\, dx}{h} \qquad (1)$$

$$\phi_y = \frac{2\delta_y}{h} = \frac{2\epsilon_y\, dy}{h} \qquad (2)$$

where δ_x, δ_y and ϵ_x, ϵ_y are changes in length and unit strains, respectively, of surface fibers of the differential element It may be of some interest that angle changes per unit of length have the same meaning as curvatures

Figure 1 indicates several possible shapes of deflections of a pavement as a result of volumetric changes of subgrade Corresponding diagrams of the distribution of angle changes per unit of length along the pavement are shown in the same figure It is assumed for convenience in the present discussion that the angle changes, ϕ_y, are zero. As a consequence of geometry and definition of a small angle, the deflection z_o at $x = o$ may be written in general as

$$z_o = c_x \cdot a^2 \phi_{xm} \qquad (3)$$

$$z_o = 2c_x \cdot \frac{a^2}{h}\, \epsilon_{xm} \qquad (4)$$

where c_x is a constant depending on the distribution of angle changes along the x axis, ϕ_{xm} is the maximum angle change in a differential length, a is the horizontal dimension as shown in Figure 1, and ϵ_{xm} is the maximum unit strain. The maximum unit strain is then

$$\epsilon_{xm} = \frac{1}{2c_x} \cdot \frac{z_o}{a} \frac{h}{a} \qquad (5)$$

[5] H Cross and N. D. Morgan, "Continuous Frames of Reinforced Concrete," John Wiley and Sons, Inc., 1932 See especially pp 26–46 on the geometry of deflected structures

It is important to note that the maximum unit strain is directly proportional to the ratio $\frac{z_o}{a}$ and the ratio $\frac{h}{a}$. For a constant limiting unit strain and a given distribution of angle changes, the maximum deflection is directly proportional to a^2 and inversely proportional to h

The values of c_x for the assumed distributions of angle changes along the pavement as in Figures 1(a) to 1(e), inclusive, vary between $\frac{1}{24}$ and $\frac{1}{16}$. Figure 2 compares the resulting deflections for a constant value of maximum

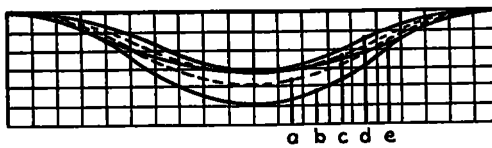

Figure 2. Comparison of Deflected Shapes of Pavements for Distributions of Angle Changes Along the Pavement as in Figure 1.

unit strain It is to be recognized that the value of c_x cannot be predicted: The distribution of angle changes along the pavement are consequent upon the properties of the material along the pavement, chance cracking, depth of tension cracks, distortion of plane sections at cracks, and properties of subgrade. However, for an uncracked pavement with a deflected shape resembling those of Figure 1, the maximum deflection and unit strain may be computed for purposes of scale from Equations 6 and 7, respectively

$$\frac{z_o}{a} = \frac{1}{10} \cdot \frac{a}{h}\, \epsilon_{xm} \qquad (6)$$

$$\epsilon_{xm} = 10 \cdot \frac{z_o}{a} \frac{h}{a} \qquad (7)$$

For example, values of $z_o = \frac{1}{2}$ inch, $h = 6$ inches, and $a = 500$ inches, give computed values of $\frac{z_o}{a} = \frac{1}{1{,}000}$, $\frac{h}{a} = \frac{12}{1{,}000}$, and a maximum unit strain of 120 millionths of an inch per inch. It is doubtful whether the horizontal distance, a, between high areas of deflected pavements can be predicted. Much uncertainty exists also in the value of the maximum unit strain at which a pavement would crack.

For the assumed distributions of angle changes in Figures 3(a) to 3(c), inclusive, the value of c_z varies between $\frac{1}{12}$ to $\frac{1}{8}$. Figure 4 compares the resulting deflections for a constant value of maximum unit strain. The maximum

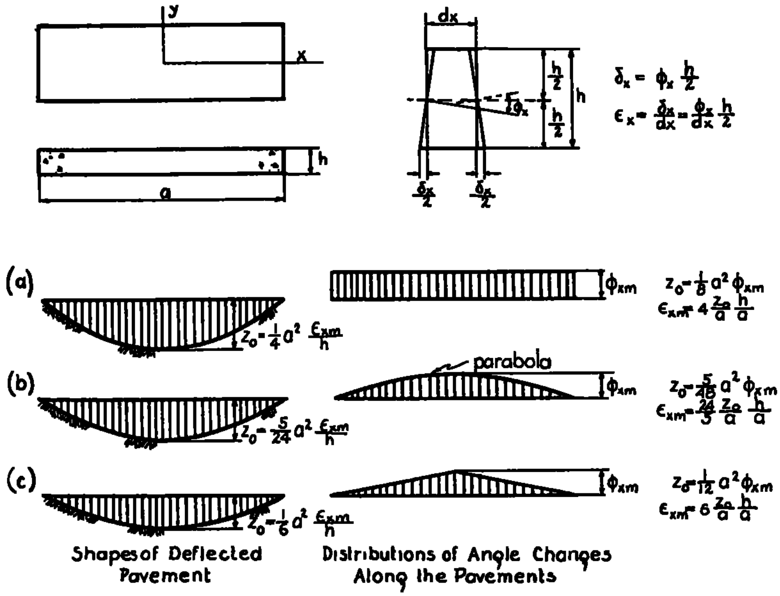

Figure 3. Effects of Distributions of Angle Changes Along the Pavement on Maximum Unit Strains and Deflections

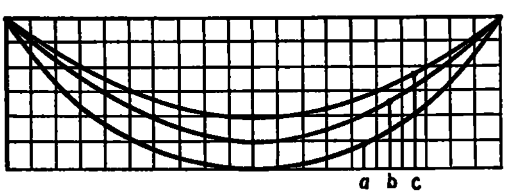

Figure 4. Comparison of Deflected Shapes of Pavements for Distributions of Angle Changes Along the Pavement as in Figure 3.

deflection and unit strain may be approximated for pavements deflected as in Figure 3 by Equations 8 and 9.

$$\frac{z_0}{a} = \frac{1}{5} \cdot \frac{a}{h} \epsilon_{xm} \qquad (8)$$

$$\epsilon_{xm} = 5 \cdot \frac{z_0}{a} \frac{h}{a} \qquad (9)$$

Values of $z_o = \frac{1}{4}$ inch, $h = 6$ inches, and $a = 250$ inches, give computed values of $\frac{z_o}{a} = \frac{1}{1{,}000}$, $\frac{h}{a} = \frac{24}{1{,}000}$, and a maximum unit strain of 120 millionths of an inch per inch. The same may be said of the uncertainties here as above. For example, consider two equal segments of a pavement with no curvature but with a crack between them transverse to the longitudinal axis of the runway and extending through the thickness of the pavement. The maximum deflection, z_o, relative to a chord through the high areas of the pavement is then

$$z_o = \tfrac{1}{4}\phi a \qquad (10)$$

where a is the length of chord and ϕ is the angle formed between the two segments of pavement. Thus, for $a = 250$ inches and $\phi = 4$ thousandths of a radian, the maximum deflection is $\frac{1}{4}$ of an inch.

In general the analysis of a pavement deals with three sets of conditions. Namely, (1) static equilibrium, (2) geometry of the deflected pavement, and (3) properties of the material. The conditions of statics and geometry are interrelated through the defined properties of the material. Thus knowing with certainty the distribution and magnitudes of any two of the above conditions along a pavement, the distribution of the third set of conditions may be obtained. Knowing with certainty only one set of conditions, different possible combinations for the other two sets may be obtained provided that they are consistent. Also knowing the uncertainties in any two sets of conditions along a pavement, the resulting uncertainties in the third set may be studied. Clearly the interpretation of constant. Attempts to deduce the distribution of loads from measurements of deflections or unit strains are of doubtful value.

Figure 5 compares possible distributions of moments and subgrade reactions along a pavement deflected as in Figure 5(a). The corresponding properties of the material along the pavement, namely the relation between mo-

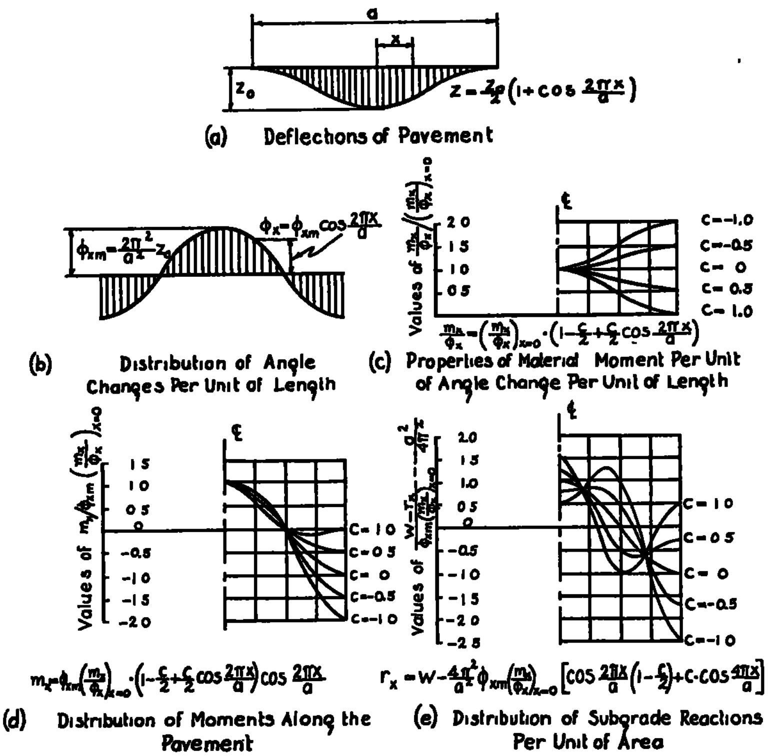

Figure 5. Effects of Variations along the Pavement in Properties of Material on the Distribution of Moments and Subgrade Reactions: Notations as shown: Also, m_x = moment per unit of width, c is a number $\leq$ 1, r_x is the subgrade reaction per unit of area, w = weight of pavement per unit of area.

results depend on the sensitivity of the results to uncertainties in variables.

An inspection of Figures 1(c), 1(d), and 1(e), shows that computations for the distribution of angle changes and unit strains are sensitive to small uncertainties in the deflections along the pavement. Also, computations for the distribution of subgrade reactions are sensitive to small uncertainties in the deflections or unit strains along the pavement: It is assumed for the present that the properties of the material along the pavement are ment and change in angle per unit of length, are as in Figure 5(c). An inspection of Figure 5 shows that the distributions of moments and subgrade reactions are sensitive to uncertainties in the properties of the material along the pavement. Uncertainties in the properties of concrete along the pavement and of chance cracking make attempts to correlate differences in measured and computed subgrade reactions seem of doubtful value.

Figures 5 and 6 show the same results. However, the results in Figure 6 are expressed in numbers rather than symbols. Numerical values have been assumed in Figure 6 for the physical and geometrical variables of a particular deflected pavement. Also, the moment needed to produce a unit change in angle per unit of length was assumed to be equal to

$$\frac{E_x h^3}{12(1 - \mu^2)} \qquad (11)$$

where E_x defines the distribution of the modulus of elasticity along the pavement and μ is Poisson's ratio of the material.

subgrade may be obtained. Little is known of the physical properties of a subgrade when volumetric changes are occurring. Suppose, however, that

$$r_x = k_o\, f(x) \cdot z_1 \qquad (12)$$

where r_x defines the distribution of subgrade reactions along the deflected pavement, $k_o\, f(x)$ defines the distribution of the subgrade modulus along the pavement, and z_1 defines the deflections of the unloaded subgrade measured with respect to the loaded subgrade.

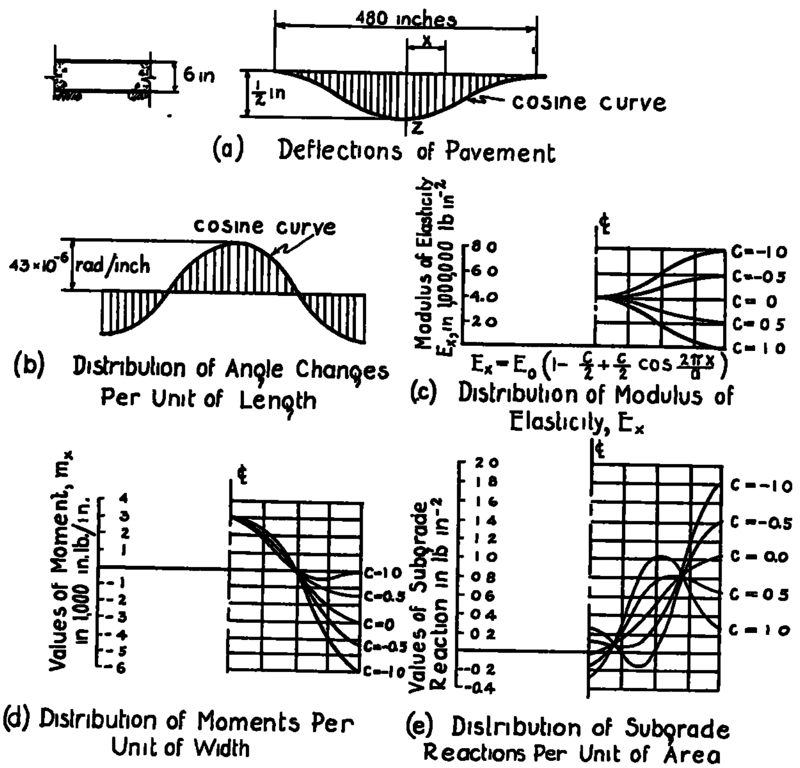

Figure 6. Effects of Variations Along the Pavement in Properties of Material on the Distribution of Moments and Subgrade Reactions: Dimensions as Shown in Figure. Moment Needed to Produce a unit Change in Angle Per Unit of Length was Assumed Equal to $\frac{E_x h^3}{12(1-\mu^2)}$, where μ = 0 and h = 6 inches.

A comparison of the differential settlements of a subgrade loaded with a pavement and those of the same subgrade with the pavement removed may be of some interest. If the subgrade reactions, physical properties, and deflections of a loaded subgrade are completely known, then the deflections of the unloaded

Studied, then, may be the sensitivity of differential settlements of pavements deflected as in Figure 5 to uncertainties in subgrade conditions along the pavement.

Figure 7 compares the deflections of an unloaded subgrade and those of a pavement with deflections and subgrade reactions distributed as follows

$$z = \frac{z_0}{2}\left(1 + \cos\frac{2\pi x}{a}\right) \qquad (13)$$

$$r_x = w - \frac{8\pi^4}{12}\cdot\frac{E_0}{(1-\mu^2)}\cdot\left(\frac{h}{a}\right)^3\cdot\frac{z_0}{a}\cos\frac{2\pi x}{a} \qquad (14)$$

where w is the weight of pavement per unit of area. Considered is the special case of a uniform distribution of modulus of elasticity, E_o, and of subgrade modulus, k_o, along the pavement If the possibility of the subgrade reaction being a tension anywhere is precluded, the diagrams and equations of Figure 7 are of interest only when

$$\frac{z_0}{a} \leq \frac{12}{8\pi^4}\cdot\frac{1-\mu^2}{E_0}\left(\frac{a}{h}\right)^3 w \qquad (15)$$

For the limiting case of contact along the entire length of pavement but $(r_x)_{x=0}$ equal to zero, then

$$z_0 = 8\cdot\frac{1-\mu^2}{E_0}\cdot\sigma_{xm}^2 \qquad (16)$$

$$a = \pi\cdot\sqrt{8h\sigma_{xm}} \qquad (17)$$

where the units are expressed in pounds and inches and σ_{xm} defines the maximum tensile stress. The weight of the pavement was assumed to be 144 pounds foot^{-3} Equations 16 and 17 are expressed in terms of stresses rather than strains because of the particular conditions assumed for the distribution of subgrade reactions along the pavement It is to be noted that for these conditions the maximum deflection defined by Equation 16 is independent of the thickness of the pavement. The thickness of the pavement, however, appears in Equation 17. Figure 8 compares the values of z_o and a computed from equations 16 and 17 for different values of h, E_o, and σ_{xm}

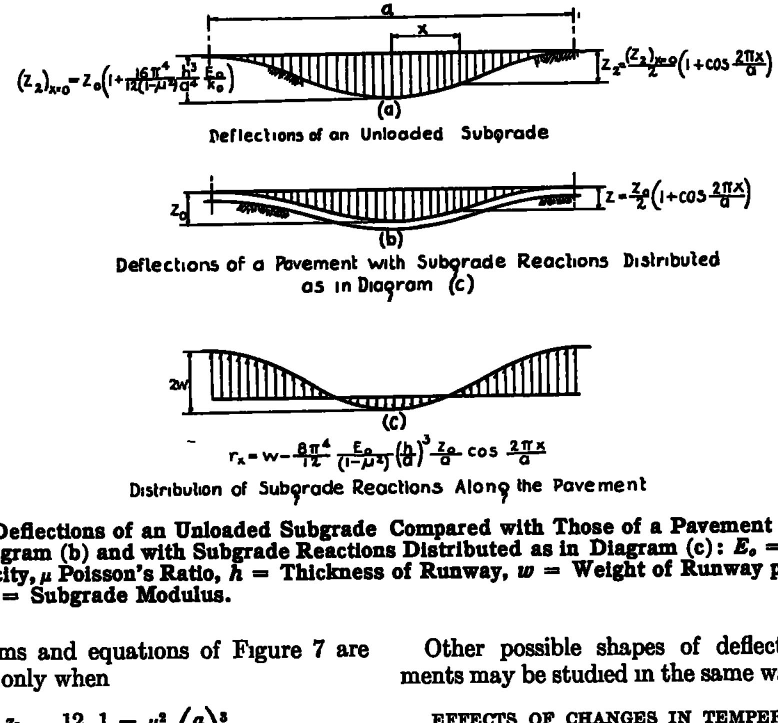

Figure 7. Deflections of an Unloaded Subgrade Compared with Those of a Pavement Deflected as in Diagram (b) and with Subgrade Reactions Distributed as in Diagram (c): E_o = Modulus of Elasticity, μ Poisson's Ratio, h = Thickness of Runway, w = Weight of Runway per Unit of Area, k_o = Subgrade Modulus.

Other possible shapes of deflected pavements may be studied in the same way

EFFECTS OF CHANGES IN TEMPERATURE OR MOISTURE CONDITIONS

Important sources of strains in concrete runways may be changes in moisture conditions or temperatures as controlled by weather and climate Volume changes of concrete pavements may be the result of shrinkage during setting, seasonal or uniform temperature changes, temperature gradients through the thickness of the pavement, temperature gradients along the horizontal dimensions of the pavement, and changes of moisture content produced by a wetting or drying. Much uncertainty exists in the frequency and the possible range of these changes at various airport sites Depending on conditions of load, pavement, subgrade, and history of weather, strains resulting from weather may be a primary source of objectionable cracking

Little is known of the change in length per unit of length that may occur as the result of moisture changes due to weather Reported by Teller and Sutherland[6] are changes in length due to moisture that are the same in magnitude as those produced by a temperature change of 20 to 40 deg F Thus a change in length of 150 millionths of an inch per inch, if completely restrained, gives resulting strains comparable to those reported for the extensibility of concrete. It seems possible that in some regions the effects of alternate drying and wetting may be of equal or greater importance than those of temperature changes

Uncertainty exists as to the value of the thermal coefficient of expansion for concrete. Included among factors apparently affecting the thermal coefficient of expansion are richness of mix, type of aggregate, and water content. In general, a higher value may be expected for a concrete made with siliceous aggregates than for a concrete made with granite or limestone aggregates Reported in the literature is a range of the thermal coefficient from 3.75 to 6 8 millionths of an inch per inch per deg F for a 1 4 5 mix of cement to total aggregate by weight, and a range from 3 6 to 5 5 millionths of an inch per inch per deg F for a 1 9 5 mix of cement to total aggregate by weight[7]

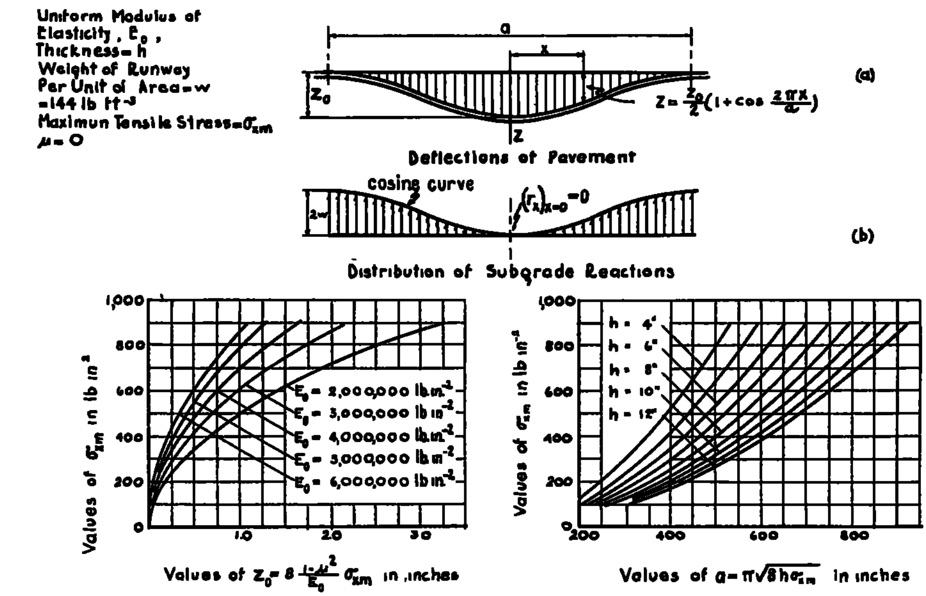

Figure 8. Values of z_o and a Computed From Equations 16 and 17, Respectively, For Different Values of E_o, h, and σ_{xm}. The Pavement is Deflected as in Diagram (a) and the Subgrade Reactions are Distributed as in Diagram (b)

The restraints that may occur to volumetric changes are highly uncertain External restraints to expansion, contraction, or warping may be friction forces or bond between subgrade and pavement, structural features at joints, weight of pavement, and resistance of subgrade to vertical deflection. An example of internal restraint favorable to surface cracking is that of a mass of concrete whose surfaces are drying or cooling as compared with the interior of the mass. A crude but important

[6] L. W Teller and E. C. Sutherland, "The Structural Design of Concrete Pavements," *Public Roads*, November, 1935

[7] For a summary of reported values see, "Concrete Manual," United States Department of the Interior, Bureau of Reclamation, fourth edition, October, 1942, Figure 4, p. 27.

scale of the magnitude of these strains may be obtained from geometry

Consider a large panel of concrete as in Figure 9 whose surfaces are drying or cooling. Diagrams 9(a) to 9(d) represent some possible distributions of temperature changes or changes in moisture content through the thickness *h* of a panel. Let each longitudinal fiber first contract freely as a result of these changes: It is assumed that these shortenings are directly proportional to the temperature changes or changes in moisture content Diagrams 9(a′) to 9(d′), represent the unrestrained shortenings per unit of length, ϵ_o. If cracking is precluded, longitudinal fibers remain parallel and plane sections before drying or cooling remain plane sections after drying or cooling. Diagrams 9(a″) to 9(d″), thus represent the final unit strains ϵ_f, and the restrained unit strains, ϵ_r. Assuming a constant ratio of stress to strain through the thickness of the section, we know from statics that the shaded areas in Diagrams 9(a″) to 9(d″) must be zero. However, it is emphasized that the problem is essentially one of strain and not of stress.

Pictures such as shown in Figure 9 may be drawn to study progressive strains produced by different rates of cooling or drying, provided something is known of the temperature changes or changes in moisture content through the thickness of the pavement. Comparison of Diagram 9(a″) and 9(b″) shows the restrained strains, ϵ_r, are expected to be greater for a rapid rate of cooling than for a slow rate.

In general, the maximum restrained strains may be written in the form

$$\epsilon_r = c_r \cdot \epsilon_o \qquad (18)$$

where c_r is a coefficient defining the degree of restraint. The coefficient, c_r, is $\frac{1}{2}$ and $\frac{2}{3}$, respectively, for a triangular and parabolic distribution of moisture or thermal changes through the thickness of a pavement. For a parabolic distribution of temperature changes, a temperature change of 30°F at the surfaces, and a coefficient of thermal expansion equal to 5 millionths of an inch per inch per deg. F, the maximum restrained strain is 100 millionths of an inch per inch

It is important to note that plastic flow here has no effect on the restrained strains, ϵ_r, as they are a matter of geometry: This assumes a uniform value of the ratio, $\frac{\sigma_r}{\epsilon_r}$, through the thickness. However, if plastic flow can occur, the stresses may be relieved materially as the ratio, $\frac{\sigma_r}{\epsilon_r}$, is thus reduced.

Much uncertainty exists as to the probable distribution of moisture or thermal changes through the thickness of a pavement The distribution is apparently controlled by the

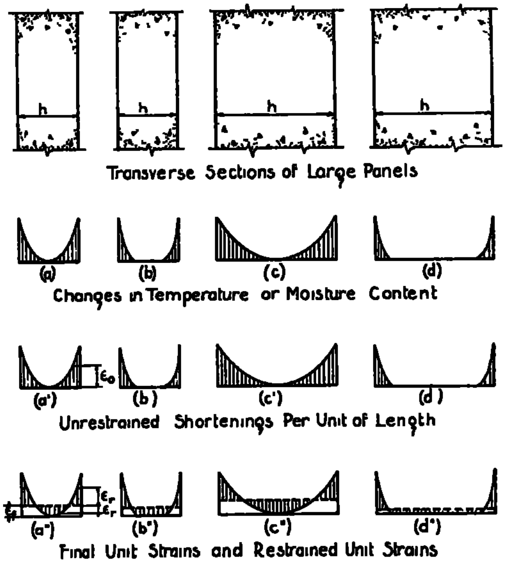

Figure 9. Effects of Changes in Temperature or Moisture Conditions Through the Thickness of a Concrete Panel on Unit Strains.

temperature and humidity of the air, previous temperature and moisture conditions of the pavement, the intensity and angle of incidence of the sun's rays, temperature and moisture conditions of subgrade, and absorption and conductivity of the pavement It is to be noted that many of these factors depend on season, weather, and climate

Figure 10(a) represents but several possible distributions of changes that may occur through the thickness of a pavement. Crude but important pictures of the resulting strains may be obtained from geometry Figure 10(b) represents the unrestrained changes in length per unit of length, ϵ_o, assuming that these changes are directly proportional to the moisture or thermal changes. If plane sections remain plane, Figure 10(c) represents possible distributions of final unit strains, ϵ_f, and restrained unit strains, ϵ_r. The internal or external restraints to the volumetric changes are highly uncertain and remain to be inspected.

Little is known of the differences that may occur between the temperature and humidity of the atmosphere and the average temperature or moisture conditions of a pavement. The temperature and humidity of the atmosphere may be sometime lower than the average temperature or moisture conditions of a pavement. Little is also known of the differences that may occur between the temperature of the air and the surfaces of a pavement. Reported by Teller and Sutherland[8] for tests conducted near Washington, D. C , are temperatures of the air from 10 to 20 deg F higher than those of the top surface of pavements. Reported also were temperatures of the air from 5 to 10 deg. F lower than those of the top surface of pavements. It seems reasonable that bigger differences may occur at some airport sites.

Possible differences between the temperature of the air and the surfaces of a pavement may be an important source of objectionable strains. Little is known of the maximum gradient in temperature and moisture conditions that may occur through a boundary layer of air and the top surface fibers of the pavement: The same may be said of the conditions at the bottom surface of the pavement. Consider for example possible distributions of temperature changes as shown in Figure 11(a) and 11(b). It is assumed that the temperature of the pavement is T deg. F except at the surfaces where a gradient in temperature of t deg. occurs through the boundary layers of air or subgrade and the surface fibers of the pavement. The gradient that occurs through the surface fibers of the pavement is not

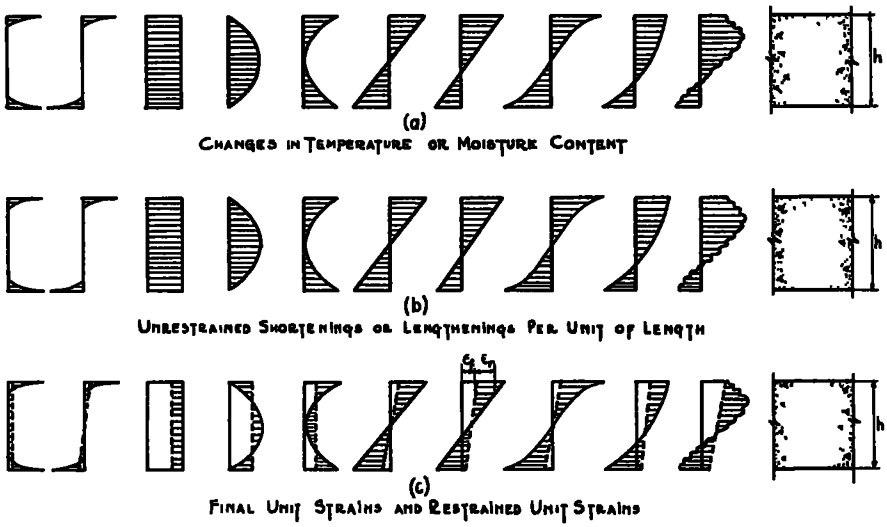

Figure 10. Possible Effects of Different Distributions of Changes in Temperature or Moisture Conditions Through the Thickness of a Concrete Runway on Unit Srains

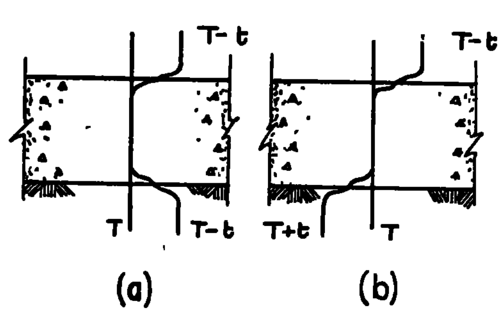

Figure 11. Gradient in Temperature Through a Boundary Layer of 'Air or Subgrade and the Surface Fibers of a Pavement.

[8] L. W. Teller and E. C. Sutherland, "The Structural Design of Concrete Pavements," *Public Roads*, November, 1935, pp. 169–197.

known. For purposes of illustration only, it is assumed that the temperature difference between the surface fibers and the interior of the pavement is $\frac{1}{2}t$ If the pavement remains flat, then as a consequence of geometry plane sections remain plane and the maximum restrained strains are $\frac{1}{2}\,\epsilon_t t$. Values of t equal to 40 deg. F and ϵ_t = 5 millionths of an inch per inch per deg. F give a computed maximum restrained strain of 100 millionths of an inch per inch Little is known of the strain at which surface cracks would appear for conditions as in Figure 11.

It seems possible that a gradient in moisture conditions at the surfaces may result in restrained strains of the same magnitude as those computed for the assumed temperature gradient The frequency and range of alternate drying and wetting or cooling and baking of a pavement may be important sources of scaling of the surfaces.

Uncertainties in strains resulting from a uniform change in moisture condition or temperature are consequent upon the restraints to the volumetric changes If the runway is free to contract, with no curling, the change in length per unit of length due to a uniform drop in temperature is then $\epsilon_t t$. An uncertainty exists as to the total change in length of a runway between expansion joints as a result of uncertainties in ϵ_t and t along the length of a runway. If the average value of $\epsilon_t \cdot t$ is known, then the change in length, δa, of a runway free to contract is accordingly

$$\delta a = (\epsilon_t \cdot t)_{aver}\ a \qquad (19)$$

where a is the distance between expansion joints. Values of a = 80 ft, ϵ_t along the pavement between 3 and 7 millionths of an inch per inch per deg F, and the drop in temperature along the pavement between 40 and 80 deg F, give computed values for the change in length between 0 12 and 0 54 in.

If the volumetric changes are completely restrained along the pavement, the uncertainty in restrained strains depends then upon the uncertainties in $\epsilon_t\ t$ along the pavement. Assuming complete restraint at each section, possible values of ϵ_t along the pavement between 3 and 7 millionths of an inch per inch per deg F, and a drop in temperature along the pavement between 40 and 80 deg F, the restrained strains are then between 120 and 560 millionths of an inch per inch. A comparison of these values with those reported by Hatt, Mills, Savage, and Davis for the extensibility of concrete shows that resultant cracking in this case is inevitable It yet remains to be inspected as to the degree of restraint that is possible.

Although it is recognized that the number, size, and distribution of cracks cannot be predicted, it is of interest to put some quantitative scale on cracking that would relieve completely these restrained strains For the above conditions it is possible for cracks less than 1 ft apart to relieve the restrained strains and not be visible to the naked eye· Cracks open about 1½ thousandths of an inch were assumed for purposes of computation. This does not necessarily mean that the pavement would have no resistance to bending for loads as the cracks might be staggered and of short length

The resistance of a subgrade to horizontal movement of a pavement is highly uncertain. It is doubtful whether the movement at the ends of runway sections can be predicted because of the uncertainties in the restraint that can be offered to the movement The resistance offered by a subgrade to a horizontal movement of runway may be made up of two parts, namely, a resistance to horizontal strains within the subgrade and a resistance to sliding of the runway on subgrade

Included among factors apparently affecting the resistance to horizontal strains within a subgrade are type of subgrade, preparation of subgrade, and history of subgrade and weather Little is known of the possible magnitude of this restraint For clays it seems probable that this resistance depends on the moisture content, magnitude and duration of strain, and rate of application of strain For granular materials it seems probable that this resistance depends on the looseness of the material, the weight of the pavement, on moisture content, and also on the history of load A drying or a freezing of a part of a subgrade may result in a bond between the subgrade and pavement offering then much restraint to volumetric changes of pavement. In this case cracking is inevitable for a drop in temperature of 40 deg F or an equivalent change in moisture condition. However, little or no restraint may be offered to horizontal movement by a "soft" or a "loose" subgrade.

The coefficient of resistance to sliding apparently depends on the roughness of the bottom of the pavement, character of subgrade surface, weight of pavement, number and frequency of movements, and magnitude of displacements Reported in the literature are values of the coefficient of resistance to displacement ranging from 0.8 to about 4 0 for different thicknesses of pavement and for a range of displacement of 0 01 in to 0.10 in.

Uncertain also is the distribution of friction forces along the pavement. If the pavement remains flat, then the change in length, δa, of a runway section may be stated as

$$\delta a = (\epsilon_t t)_{\text{aver}} \cdot a - (c_r \epsilon_t t)_{\text{aver}} \, a \qquad (20)$$

where c_r is a coefficient defining the degree of restraint at each section. At each section, if the ratio of stress to strain, E, is known, the unit stress is then

$$\sigma = Ec_r\epsilon_t t \qquad (21)$$

and the total force per unit of width is

$$F = Ec_r\epsilon_t th \qquad (22)$$

Equation 22 also defines the sum of all the horizontal forces on one side of the section. The expansion and contraction of a pavement is not necessarily symmetrical with respect to its mid-section If the distribution of the horizontal forces is known, the magnitudes can then be computed, provided the variables are known at each section Uncertainties in these variables precludes any attempt to correlate results of tests with analytical results. Of interest, however, are the possible magnitudes of the horizontal forces.

For illustrative purposes only, consider a runway section of length a with horizontal forces distributed symmetrically with respect to the midsection. The sum of all the horizontal forces on one side of the mid-section may be stated as

$$F = f_{\text{aver}} \cdot \frac{a}{2} \qquad (23)$$

where f defines the intensity of the horizontal forces along the pavement. Assuming bond between pavement and subgrade resulting in complete restraint of the volumetric changes at the midsection, the value of the average bond is then

$$f_{\text{aver}} = 2E \cdot 1 \cdot \epsilon_t t \, \frac{h}{a} \qquad (24)$$

For values of E = 4,000,000 lb in.$^{-2}$, h = 8 in , a = 800 in , and $\epsilon_t \cdot t$ between 80 and 200 millionths of an inch per inch, the computed value of the average bond is then between 6 4 and 16 lb in $^{-2}$ Thus considerable bond would be necessary for the restrained strains to be within the values reported for the extensibility of concrete Conversely, if the pavement slides over the subgrade then the restrained strain at the mid-section is

$$c_r \epsilon_t t = \frac{f_{\text{av}}}{2E} \frac{a}{h} \qquad (25)$$

If the coefficient of friction is constant and the frictional forces are proportional to h, the restrained strains are then independent of the thickness of runway. For values of E = 4,000,000 lb in $^{-2}$, a = 800 in , weight of concrete at 144 lb ft $^{-3}$, the coefficient of friction between 0.8 and 4 0, and frictional forces proportional to h, the value of the restrained strain at the mid-section is between 6 and 33 millionths of an inch per inch. Assuming a parabolic distribution of restrained strains along the length of pavement, the total restraint to a change in the length of pavement is then approximately between 4 and 10 thousandths of an inch. The restraint due to friction forces for a temperature drop between 40 and 80 deg F is, in this case, negligible.

The effects of a straight-line temperature or moisture gradient through the thickness of a pavement have been much discussed in the technical literature. Westergaard presented in 1926[9] a theoretical analysis of warping stresses in slabs of certain dimensions and suggested a procedure to be followed in slabs of finite dimensions On the basis of this analysis Bradbury[10] developed general equations for the computation of warping stresses in pavements of finite dimensions In general, the restrained strains at the surfaces of

[9] H. M Westergaard, "Analysis of Stresses in Concrete Pavements Due to Variations of Temperature," *Proceedings*, Highway Research Board, Vol 16, (1926).

[10] R D Bradbury, "Reinforced Concrete Pavements," published by the Wire Reinforcement Institute, Washington, D. C , 1938

a pavement may be stated as follows

$$\epsilon_{rx} = c_{rx} \cdot \epsilon_t \cdot \frac{t}{2} \quad (26)$$

$$\epsilon_{ry} = c_{ry} \cdot \epsilon_t \cdot \frac{t}{2} \quad (27)$$

where

ϵ_{rx}, ϵ_{ry} = restrained strains at the surfaces of a pavement,
c_{rx}, c_{ry} = coefficients defining the degree of restraint at each section,
ϵ_t = thermal coefficient of expansion per degree Fahrenheit,
t = difference in temperature between top and bottom of slab, in degrees Fahrenheit

Much uncertainty exists as to the values of the coefficients of restraint, c_{rx} and c_{ry}. The variables considered by Westergaard and Bradbury include length and width of pavement, and uniform values of modulus of elasticity of the pavement, Poisson's ratio of the material of the pavement, thickness of pavement, coefficient of thermal expansion, change in temperature, and modulus of subgrade reaction. Not considered were possible variations in the above variables along the horizontal dimensions of the pavement. As so much uncertainty exists in these variables it is suggested that for design purposes the maximum coefficients of restraint computed by Bradbury be approximated by

$$c = 0 \text{ for } a \leqq 2l \quad (28)$$

$$c = \frac{a}{4l} - \frac{1}{2} \text{ for } 2l \leqq a \leqq 6l \quad (29)$$

$$c = 1 \text{ for } a \geqq 6l \quad (30)$$

where a = corresponding horizontal dimension of the pavement and

$$l = \sqrt[4]{\frac{Eh^3}{12(1 - \mu^2)k}} \quad (31)$$

For values of E between 3,000,000 and 6,000,000 lb in.$^{-2}$, k between 50 and 400 lb. in.$^{-3}$, μ between 0 and $\frac{1}{4}$, and the thickness h between 6 and 12 in., the value of l is between 19 and 65 in. Table 1 shows the effect of uncertainties in E and in k on the coefficients of restraint for different thicknesses and horizontal dimensions of runways. An inspection of Table 1 shows that for runway lengths of 20 ft or over the maximum coefficient of restraint is 1.00 except for high values of E and low values of k for the 9 and 12-in. thick slabs. However, uncertainties in E and in k within the range of values considered make values of c less than 1 highly unpredictable. For runway sections 10 ft. or less in length and width the coefficients of restraint are sensitive to uncertainties in E and in k.

The maximum difference that may occur at different airport sites between the temperatures of the two surfaces of a pavement is highly uncertain. Reported by Teller and Sutherland[11] for tests conducted near Washington, D. C., are maximum differentials of 23 and 33 deg. F for a 6-in. and 9-in. thickness of pavement, respectively. On the basis of these observations and others, Bradbury[12] suggested for general design purposes a 3 deg. F maximum differential per inch of slab thickness. However, it is recognized that in certain climates the maximum differential may be higher than those just indicated.

Table 2 shows the effect of uncertainties in E and in k on computed strains for different thicknesses and horizontal dimensions of runways: Assumed were temperature differentials of 4 deg. F per inch of slab thickness, ϵ_t = 5 millionths of an inch per inch per °F, and μ = 0.15. Table 2 shows that for runway lengths of 20 ft or over the maximum restrained strain may be approximated for purposes of scale as $10h$ millionths of an inch per inch. It is recognized that uncertainties exist in the coefficient 10 as a result of uncertainties in the value of the maximum temperature differential and in the coefficient of thermal expansion. Thus, other conditions equal, for runway lengths of 20 ft or over the thicker the pavement the bigger the restrained strain, in general, as a result of a temperature gradient. On the other hand, thickening of the pavement does not appreciably reduce the restrained strains due to a temperature gradient until the horizontal dimensions of the pavement are reduced below 7 ft. Depending on

[11] L. W. Teller and E. C. Sutherland, "The Structural Design of Concrete Pavements," *Public Roads*, November, 1935, especially p. 196.

[12] R. D. Bradbury, "Reinforced Concrete Pavements," published by the Wire Reinforcement Institute, Washington, D. C., 1938, see especially p. 21.

the horizontal dimensions of a runway and other factors, the design of a pavement for a temperature gradient may thus be in an uncertain stage. Thickening of a pavement may increase, decrease, or keep constant the resulting unit strains.

Variations in temperature or moisture conditions along the horizontal dimensions of the pavement may make the range of uncertainty in the maximum coefficients of restraint even greater than just indicated. Consider for example a gradient in temperature or moisture through the thickness of a pavement occurring over an area small in comparison to the horizontal dimensions of the pavement. Included among factors making this condition possible

TABLE 1

EFFECT OF UNCERTAINTIES IN E AND IN k ON MAXIMUM COEFFICIENTS OF RESTRAINT TO WARPING FOR DIFFERENT THICKNESSES AND HORIZONTAL DIMENSIONS OF RUNWAYS μ = 0 15

Values of E in lb in^{-2}	Values of k in lb in^{-3}	a_x, a_y — Values of a_x or a_y in feet	Thickness of Pavement in Inches: 6	9	12
			Computed Values of the Coefficients of Restraint, c_x or c_y*		
3,000,000	400	7	0 59	0 32	0 15
		10	1 00	0 65	0 43
		20	1 00	1 00	1 00
		40	1 00	1 00	1 00
4,000,000	100	7	0 12	0 03	0 00
		10	0 53	0 26	0 11
		20	1 00	1 00	0 72
		40	1 00	1 00	1 00
6,000,000	50	7	0 05	0 00	0 00
		10	0 28	0 08	0 00
		20	1 00	0 65	0 43
		40	1 00	1 00	1 00

$c = 0$ for $a \leqq 2l$, $c = \frac{a}{4l} - \frac{1}{2}$ for $2l \leqq a \leqq 6l$, $c = 1$ for $a \geqq 6l$ where $l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$

* See below table 2.

TABLE 2

EFFECT OF UNCERTAINTIES IN E AND IN k ON COMPUTED* MAXIMUM STRAINS FOR DIFFERENT THICKNESSES AND HORIZONTAL DIMENSIONS OF RUNWAYS ASSUMED WERE TEMPERATURE DIFFERENTIALS OF 4°F PER INCH OF SLAB THICKNESS. e_t = 5 MILLIONTHS OF AN IN. PER IN PER °F, μ = 0 15

Values of E in lb. in.$^{-2}$	Values of k in lb in^{-3}	a_x, a_y — Values of a_x or a_y in feet	Thickness of Pavement in Inches: 6	9	12
			Computed Values of Maximum Restrained Strains in Millionths of an inch per inch		
3,000,000	400	7	35	29	18
		10	60	39	52
		20	60	90	120
		40	60	90	120
4,000,000	100	7	7	3	0
		10	32	23	13
		20	60	90	90
		40	60	90	120
6,000,000	50	7	3	0	0
		10	17	7	0
		20	60	59	52
		40	60	90	120

* The coefficients of restraint are the same as those in Table 1

are variations in the properties of concrete, water collected over a small area, uneven drying, and shadows In this case the restraint due to the weight of the pavement and its resistance to bending may then result in a maximum value of the coefficients of restraint, c_{rx} and c_{ry}, approximately 1.

Among sources of strain conducive to transverse cracking at an edge of a runway is a distribution of temperature or moisture changes uniform through the thickness but varying transversely to the longitudinal axis of the runway.

REPORT OF COMMITTEE ON FLEXIBLE PAVEMENT DESIGN

METHODS OF DESIGNING THICKNESS OF FLIGHT STRIPS AND AIRPORT RUNWAYS FOR WHEEL LOADS EXCEEDING 10,000 LB.

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SYNOPSIS

Present construction practices disclose that there is a wide divergence of opinion among engineers concerning the problem of design of airport runways.

This paper describes the methods of design that are at present in use and outlines a method based on large-scale loading tests of trial pavement sections

Three sections of pavement are constructed on a prepared section of the subgrade. The thickness of one of the sections is that estimated to be necessary for the wheel loads in question by any desired method, such as used by the Army, the Navy or by the use of soil bearing test data. The thickness of the second section is 50 per cent greater and of the third 30 per cent less than that estimated. These are tested using a repetitional method of loading The thickness necessary to support the design load for a specified deflection is obtained by plotting the thicknesses of the trial sections against the recorded deflections.

The report stresses the fact that the success of the method is largely dependent upon the correct evaluation of the load bearing test data. It is pointed out that in many cases, due to inadequate construction compaction of the subgrade and pavement courses, a large portion of the initial load settlement may be due to mere consolidation of the component parts of the structure The method makes it possible to consider the total settlement or only that portion of the settlement which is of primary importance as far as the ultimate load carrying capacity of the structure is concerned.

For runways and flight strips designed for wheel loads of not over 10,000 lb. the thickness values published in Wartime Road Problem No. 8 "Thickness of Flexible Pavements for Highway Loads" are recommended. However, wheel loads far greater than those accommodated by highways must often be considered. Records of experience with flexible pavements under such wheel loads are limited. Although many airports have been built recently it will be some time before the lessons they have to teach will become apparent. In the meantime it is necessary to build as best we may in the light of highway experience, research and theoretical considerations.

Three methods are described in this report.

1. The Office of the Chief of Engineers of