ANALYSIS OF STRESSES IN CUTS AND EMBANKMENTS

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SYNOPSIS

The procedure followed in analyzing the stability of slopes will vary, depending upon whether the slope is in cohesive or in non-cohesive soil, and depending upon whether the slope has been formed by embankment or by excavation. Where the natural water table lies above the toe of the slope, the determination of its position, both before and after construction, becomes a factor of importance comparable to the necessity for adequate soil sampling and testing. Except in the simpler cases, a graphical method of analysis is preferable. The Swedish method, as modified to permit the scaling of total normal and tangential forces from corresponding stress diagrams, provides the most satisfactory procedure. A somewhat similar graphical procedure will locate the zone of tension at the crest of the slope. In contrast, the design of embankment often results in a problem of sufficient simplicity to permit stability analysis by means of standard curves. If a fill rests upon a weaker material, the analysis of the stability of the foundation according to the theory of plasticity is necessary. In such cases the settlement resulting from the consolidation of underlying strata also becomes worthy of investigation.

1. *Cuts through Sand.*[1] In cohesionless sand the stability of the cut bank is independent of the depth of cut and of the unit weight of the earth. Under normal conditions a cut through sand is stable if the angle of internal friction, ϕ, exceeds the angle of repose (or angle of slope). The simplicity of this criterion eliminates need for theoretical analyses and, generally speaking, for laboratory testing.

Sometimes secondary factors such as quicksand or internal erosion (piping) require more detailed study. These possibilities fall outside the scope of routine investigation for highway earthwork, and are adequately treated elsewhere.

2. *Cuts through Cohesive Soil.* Highway cuts through cohesive soil ordinarily will encounter the water table at moderate depths, and the method of analysis should be made general enough to include this factor. Before starting the office work, preliminary field and laboratory investigations are necessary. This preliminary effort must include measurement of the depth and thickness of each stratum intersected by the cut, and determination of the friction angle, unit cohesion (c), and unit weight (γ), of the soil. Also essential are groundwater data, including the depth and the slope (S) of the water table, the depth of flow (d), and the direction angle of groundwater flow (δ), which are discussed in item 5.

3. *Sampling and Testing.* No attempt at accurate analysis is justified except on the basis of reliable data from tests made on undisturbed samples taken from sufficient depth to be representative of conditions along potential slip-surfaces. Considerations of cost generally will dictate that such samples be obtained from drillholes rather than from test-pits, and that the drillholes be of a diameter not larger than strictly necessary for compression tests. If a triaxial compression apparatus is available, the test can be started with both lateral and axial pressures equal to the weight of the natural overburden at the point from which the sample was secured. Then, the lateral pressure is reduced until failure occurs. This test procedure increases the reliability of test data by simulating conditions in the prototype, where also failure results from the removal of lateral support, and not from an increase of the vertical load. Simple compression tests on samples taken as explained can be analyzed by assuming the lateral pressure to be the weight of the overburden at the depth of the sample. Triaxial compression is preferable, however, because of the limitations thus imposed on the range of lateral pressure in weakly cohesive soils, and because the actual capil-

[1] References pertaining to each item will be found in the list at the end of the paper.

lary (lateral) pressure is likely to be much less than the foregoing assumed value. The error is conservative, although excessive for many purposes.

Direct shear tests are satisfactory if representative undisturbed samples can be obtained. Because of the relatively small dimensions of the usual shear box, and because the plane of failure is fixed by the apparatus, the presence of pebbles or stone in the sample is more troublesome than in compression tests. As a rule, soil containing particles larger than ¼-in. should not be tested in direct shear boxes of conventional size. Rapid shear tests introduce a viscous resistance to deformation which may not be equally present under field conditions; and, on the other hand, may undervalue the shear strength of the specimen if it has been allowed to absorb water in sampling or in storage. Except

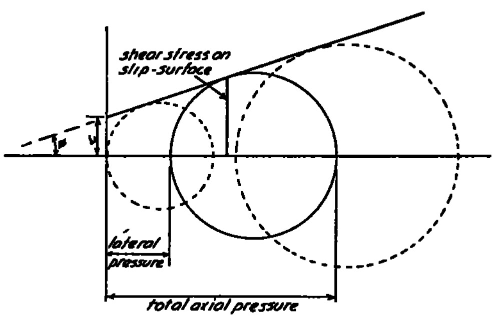

Figure 1

where the natural soil in the field has not fully consolidated under the weight of its overburden, slow shear tests are preferable. Supplementary data taken during test such as stress-strain relationships, and water content or volumetric changes during test are all essential to an understanding of the shearing properties of the soil. Their neglect may introduce some hazard; therefore, in the case of exceedingly important cuts the advice of a soil specialist is to be sought.

Specific gravity, water content, and natural density of undisturbed samples can be measured without much difficulty using ordinary laboratory methods.

4. *Friction and Cohesion from the Tests.* If triaxial compression tests have been performed on a sample, friction and cohesion can be evaluated most easily by graphical means, as in Figure 1. For each test the lateral pressure and the total vertical pressure (principal stresses), at which the sample failed, are laid off on a horizontal axis. A circle is drawn through the two points thus determined, its diameter being equal to the pressure difference (sometimes called "deviator stress"). A series of tests with various lateral pressures provides a family of circles, whose common tangent intercepts the value of the unit cohesion on the Y-axis, and whose slope equals the tangent of the friction angle of the soil tested.

5. *Groundwater.* The location of the water table is a more potent factor in the stability of slopes than any ordinary variation in the physical properties of the soil itself, simply because it is so indefinite. This situation makes it important to determine approximately the position of the groundwater sur-

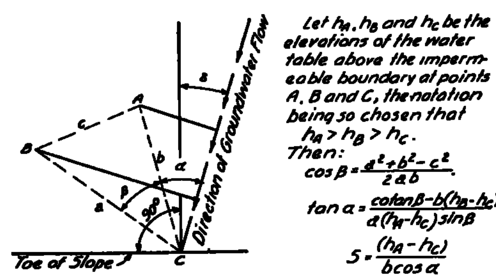

Figure 2

face as it is expected to exist after completion of the excavation.

For a preliminary field investigation a minimum of three test-holes is required, and it is convenient to locate one of these at the toe of the proposed cut, as in Figure 2 (plan). All three holes must reach a source of water supply, however small; and the one nearest the toe of the slope (*C* in Fig. 2) should extend to the full depth of the proposed cut. The direction of ground water flow (angle δ) and its slope (S) are determined as shown in Figure 2. Coefficients of permeability of the soil strata to be encountered in the cut may be found by laboratory tests upon undisturbed samples, especially where the soil is fairly homogeneous; but an approximate value may be obtained directly in the field. Referring again to Figure 2, let water be pumped from hole *C* at some constant rate, Q, in cubic feet per second, until there is no further lowering of levels in the other two holes *A* and *B*.

Let D_A and D_B be the measured drawdowns at these two locations. Then the mean coefficient of permeability of the soil strata involved is of the order of magnitude

$$k = \frac{Q \log_e \frac{b}{a}}{2\pi d(D_A - D_B)} \tag{1}$$

where d is the mean depth of flow in the region of holes A and B. For homogeneous soils, one can assume $d = h_B - \frac{1}{2}(D_A + D_B)$. For artesian flow d is the thickness of the aquifer. For more complex conditions the value of d must be estimated from natural conditions. Equation (1) is based upon the assumption of horizontal flow, which is but an approximation. The proper basis for selecting the location of the test holes is not apparent until the holes have already been made and pumping is under way; but it is advisable to make a and b large while still yielding considerable values of $(D_A - D_B)$.

6. *Trial Design.* When a slope cuts through several different soil strata, or when it intercepts groundwater flow, the various factors upon which stability depends cannot be related in a simple equation. A trial slope angle must be selected either from experience or by simplifying the actual conditions, and analyzed, preferably by graphic methods. The Taylor stability curves (see item 12) can be used advantageously for this purpose if average values of ϕ, c, and γ are selected. In less general use are the various empirical formulas to be found in the literature.

7. *Phreatic Line.* Upon a cross-section of the cut, drawn to some convenient scale, the water table or the upper surface of the groundwater should be located as it will appear after construction. This water table, sometimes called the "phreatic line," represents the locus of zero hydrostatic pressure. Strictly speaking, it does not mark the extent of saturation, as voids in the soil above the phreatic line may be filled with capillary water.

The location of the phreatic line depends upon the extent and permeability of the aquifer, upon the location and nature of the water supply, and upon the drainage to be installed. In natural soil formation only fragmentary knowledge of many of these items may be possible, and only a crude approximation of the phreatic line can be expected. As a guide to judgment, Equations (2) and (3) establish approximate limits within which most practical cases will fall.

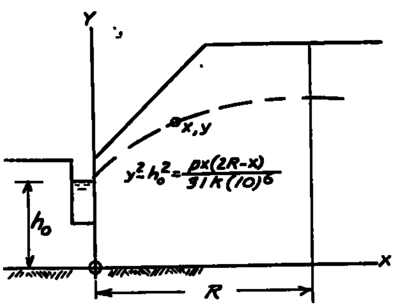

Figure 3

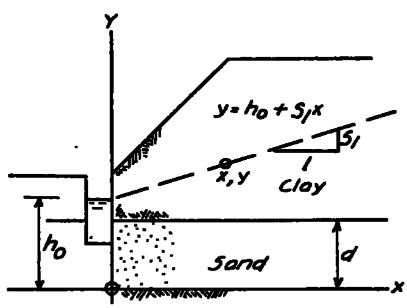

Figure 4

Case 1 (Fig. 3). Homogeneous soil on a horizontal impermeable base; water supply obtained entirely from percolation within the drainage area.

$$y^2 - h_o^2 = \frac{px(2R - x)}{31(10)^6 k} \tag{2}$$

where y = elevation of phreatic line above base in feet

x = horizontal distance normal to direction of drain, ft.

h_o = elevation of water surface in drain above base, ft.

p = maximum rate of percolation in inches per month.

R = width of watershed, assumed to equal $5280 \sqrt{A}$, ft.

A = area of watershed above cut in square miles.

k = coefficient of permeability, ft. per sec.

Case 2 (Fig. 4): Artesian flow from remote source.

$$y = h_o + S_1 x \tag{3}$$

Where $S_1 = S \cos \delta$
and δ = angle between direction of flow and a normal to the drain, as in Figure 2.
S = true slope of water table and will be unchanged by the excavation if the source is sufficiently remote. It can be measured in the field by the methods described in item 5.

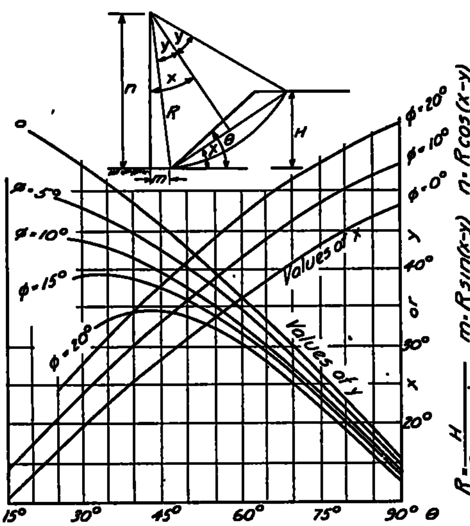

Figure 5

In most practical cases the engineer will find the foregoing formulas to be of only limited value, and the position of the phreatic line must be estimated by individual judgment of the available information.

8. *Location of Trial Slipsurface.* The stability of the slope is studied by applying the conditions of equilibrium to the mass of earth lying above some interface upon which sliding is assumed to impend. For homogeneous material it is generally assumed that a vertical section of the slip-surface (termed also "failure line") is a circular arc. The center of this arc (critical center) may be found using the curves of Figure 5 (based upon data in Reference 4). The position of the critical center is determined by the coordinates m and n, the radius of the circle being R. The values of x and y (in degrees) for a given angle of slope θ and the angle of friction ϕ are introduced into formulas for R, m, and n (see Fig. 5, right). From the point thus located swing an arc through the toe of the slope and to the crest. The mass of earth above this cylindrical surface is to be considered as a free body under the action of its own weight and the pressure of the percolating water, in equilibrium with the normal and

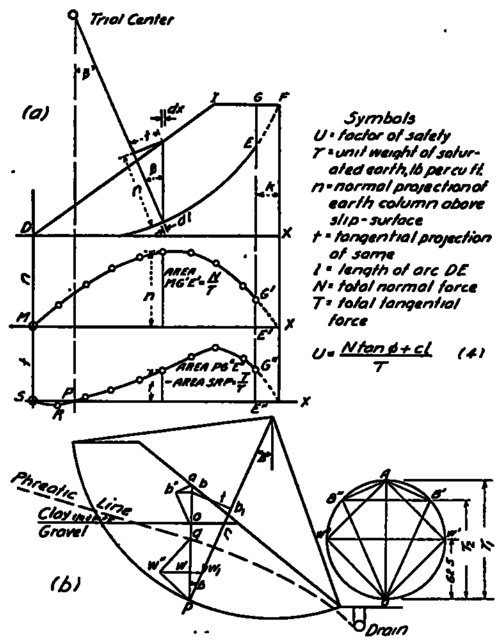

Figure 6

tangential components of the reaction of the supporting soil.

9. *Equilibrium of the Trial Section.* The stability of this free body is determined by the summation of tangential forces along the slip-surface, most easily done graphically. The vertical pressure on any area of the slip-surface is assumed equal to the weight of the column of earth directly above, (which is not exactly correct) and the corresponding scaled distance, as in Figure 6a, can be composed into its normal and tangential components as shown. When strata of different densities are encountered above the selected element it becomes desirable to replace the

actual depth of cover by an equivalent column of uniform density. As this procedure must be repeated a number of times the graphical method illustrated in Figure 6b will be found convenient. In this construction the circle simplifies drawing the mutually perpendicular chords, which in turn make it possible to intercept the height of the desired equivalent column with a single orientation of the triangle. Here, γ_1 represents the greater, and γ_2 the lesser soil density; also, h is the groundwater pressure head. Draw ab'' parallel to AB'', and ob'' parallel to OB''. Project ob'' on oa; then ob is an equivalent column of density γ_1. Also pb is the equivalent height of an entire column of density γ_1, and pb_1 is its normal component.

The effective earth pressure on the elementary area is computed by subtracting the hydrostatic pressure of the groundwater from the normal stress previously obtained. Considering the uncertainty of the true location of the phreatic line it is sufficiently accurate to take the height of phreatic line above the slip-surface as equal to the pressure head at the slip-surface. This value should be reduced to an equivalent earth column by the method already explained. Draw qw'' parallel to AW'', and pw'' parallel to OW''. Project pw'' on pq. Then pw is the groundwater pressure head, expressed in feet of earth, acting on dl. Since the vertical earth pressure was expressed per unit horizontal area, and since the water pressure acts per unit area of the slip-surface, the latter pressure must be divided by cos β before subtraction, β being the angle made by the normal to the slip-surface with the vertical. This is done graphically by subtracting its projection on the radial line from the original normal earth pressure, as in Figure 6b, where pw_1, is the equivalent head corresponding to the horizontal distance, dx ($62.5\ hdl = 62.5\ h \sec \beta\ dx$). The remainder is the effective normal earth pressure, n, and this length is transferred to a normal pressure diagram, as in Figure 6a, where it is plotted against horizontal distance from the toe of the slope.

The hydrostatic pressure has no tangential component, and so the tangential component of earth pressure is transferred without reduction in value to a similar diagram.

The foregoing operations are repeated at enough points along the slip-surface to permit the construction of smooth curves. The net area enclosed between each curve and its horizontal axis is measured with a planimeter or otherwise. Each area in square inches is multiplied by the square of the linear scale and by the chosen specific weight of earth to obtain the total normal or tangential force on the slip-surface, as the case may be (designations N and T in Fig. 6).

The total normal pressure is multiplied by the tangent of the angle of friction to obtain the total frictional resistance. If the slip-surface cuts across strata having different ϕ's, the projection of its intersections with the strata boundaries will divide the n–x diagram into various subareas, each of which must be multiplied by its own coefficient of friction.

With a high water table negative values of n are possible, signifying tension across the slip-surface. As earth has no dependable tensile strength a redistribution of stress must occur, and an open crack is possible. The length of arc, l, should include only that portion of the slip-surface for which n has a positive value, when used in the term cl to compute the total cohesive resistance. As a matter of fact, in an elastic solid horizontal tension must always occur at the crest of an *unstable* slope. This is also true of natural soils where cohesion is a factor. In Figure 7 the arc EF (length $= l'$) represents an upper portion of the slip-surface. The resultant normal force on the section GE may be either tension or compression, depending upon location. If it is tension, then the summation of horizontal forces requires that the horizontal component of cl', the resultant cohesion, must exceed the horizontal component of F_R, the resultant soil reaction.

To find the horizontal projection of force F_R drop perpendicular GJ as in Fig. 7a, and construct angle $KEJ = \phi'$. Then from the plan of forces EKG the reaction F_R for point E equals KE, its horizontal projection being $KK' = m$. Strictly speaking, since distance GE equals z, and not γz, distance KK' is the horizontal projection of force F_R divided by γ. In Figure 7b distances m are plotted for various points along on the arc EF. An intersection of the curve thus obtained with the horizontal line $\frac{c'}{\gamma}$ (Fig. 7b) furnishes point L' which divides arc EF into two parts. There is an excess in cohesion at the right part of the arc $L'F$ which can maintain in equilibrium the portion to the left of LL'. To avoid misunderstandings, express c in lb. per sq. ft. and γ in lb. per cu. ft. Then their ratio $\frac{c}{\gamma}$ will be expressed in feet, and plotted to the scale of the drawing. The horizontal component of the resultant cohesion is simply ck, where k is the horizontal projection of the arc. The boundary of the tensile zone occurs at $GF = k$; i.e., when the area under the horizontal line equals the area under the curve in Figure 7b. It is sufficient to locate this position by balancing the cross-

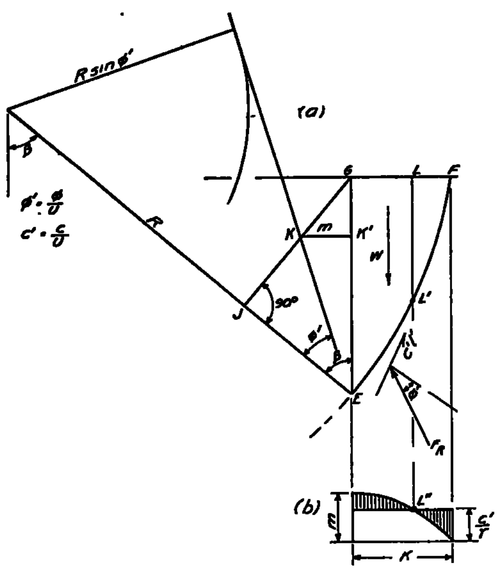

Figure 7

hatched areas by eye. In Figure 6a, the boundary of the potentially unstable earth mass is DEG. The factor of safety, U, is not greatly affected by the crest tension. It is simplest to find U neglecting crest tension, and to modify only the critical value of U by the analysis just discussed.

The factor of safety against sliding along the assumed trial slip-surface is taken to be the ratio of the total resisting forces to the total driving forces, as in Equation (4), Figure 6. This computation must be repeated for a number of trial centers to find that slip-surface along which sliding is most likely to occur. This process may be facilitated by recording each safety factor at the location of the corresponding trial center, and drawing the loci of equal safety factors. It is suggested that for highway purposes the slope be designed to provide a factor of safety between 1.6 and 2.0.

10. *Limitations of the Graphical Procedure.* The assumption of a simple circular arc is in error for stratified soils, and in such cases it may be well to try some compound curves after having selected the critical simple curve by the method of the previous article. In so doing it should be noted that the radius of curvature is greater with higher angles of friction; nor should the possibility of translation in combination with rotation be overlooked, especially where the dip of the strata favors such action. A perched water table will introduce additional seepage forces, and evidence of seepage in the cut face after construction should lead to further analysis to insure that the factor of safety is still ample, or to determine the need for additional drainage.

The method also assumes that at failure the specified friction and cohesion is fully active along the entire slip-surface. This presupposes equality of strain along the entire arc, unless ultimate rather than maximum values of ϕ and c had been used in design. Thus, a compact silt under a loose gravel overburden may fail before the shearing resistance of the overlying material is fully mobilized, unless ultimate resistances had been selected for the analysis.

The method is concerned solely with the resistance of the earth mass as a whole. Progressive failure may occur if an underlying plastic clay is squeezed out before the stability of the integral slope itself is threatened. This must be prevented either by limiting the pressure at the toe to the value of the compressive strength of the local stratum, or by providing local support at the toe by bulkheads, piling, or rock fill. If the thickness of the underlying clay layer, 2a, is relatively small, plastic flow may occur if its shearing strength $c < (\gamma\ a\ \tan\theta)$; where γ is the unit weight of the overlying soil.

11. *Design of Embankment.* The selection of suitable soil for embankment is simplified by the results of experience of the Public Roads Administration and the U. S. Engineer Department. Their recommendations are presented here in references. For anything

other than highly organic soils (which should be avoided) and clean sands, control of the water content at which the soil is placed is highly important. The optimum moisture can be found from standard laboratory tests (AASHO T 99–38). At present the general tendency is to design for the condition of saturation, regardless of the water content at placement. Consequently the shearing resistance of soils for embankment should be obtained from samples that have been compacted at optimum moisture to the density expected in construction, and then allowed to swell or consolidate under a specified normal load (in contact with water) before being sheared. The direct shear test will generally be more convenient than compression tests for this purpose, wherever the maximum particle size permits its use.

If the fill material is reasonably homogeneous, and if seepage forces are eliminated by proper drainage of the base of the fill, the slope angle may be obtained directly through the use of Taylor's Stability Charts, reproduced in Figure 8.

12. *Taylor's Stability Charts.* D. W. Taylor has shown that for any specific values of ϕ and θ the maximum height for a stable slope is given by the equation

$$h = \frac{c}{N\gamma} \text{ or } N = c/\gamma h \qquad (5)$$

where N is the stability number, or Taylor's Number. The factor of safety, U, is introduced into the analysis by using values of c and ϕ equal to their test values divided by U. For γ it is best to use the weight of the saturated soil at maximum dry density. Entering Figure 8 with the computed value of N and the working value of θ, the value of ϕ can be read directly.

Rather than investigate the opportunity for tension at the crest it is more convenient to use only 80 per cent of the otherwise allowable value of c, afterwards applying the safety factor to both c and θ.

If the fill is not drained at the base, if the soil is not free-draining, and if a state of capillary saturation is expected eventually, a prolonged period of rainfall may cause the stress in the porewater to change from tension to pressure. A flooded ground surface is not in itself sufficient to produce this development; in addition there must be time enough and water enough to permit the elastic expansion of the soil mass, for the change in stress cannot be separated from the change in strain. Usually, however, available data will be insufficient for a theoretical treatment of the situation, and where base drainage is lacking it becomes necessary to design for the possibility of saturated soil with a temporary water table coincident with the surface of the fill. The Taylor method permits analysis of this case by using the same chart as before, but with a modified value of ϕ.

$$\phi = \phi\left(\frac{\gamma - 62.5}{\gamma}\right) \qquad (6)$$

where, as before, γ is the weight of a cubic foot of saturated soil at maximum dry density. A few trial computations would emphasize the economy of providing base drainage.

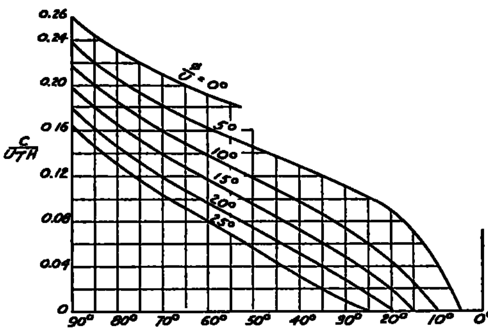

Figure 8

The nature of the safety factor in Taylor's method is somewhat less conservative than in the previously presented graphical analysis. Being applied to ϕ, it becomes based upon the equilibrium of a slip-surface other than the one upon which the ultimate resistance of the soil would be overcome at failure. The true value of ϕ would call for an arc of less curvature. The distinction is not of great importance, being mainly a matter of definition of terms, but the definition used in item 9 appears to be more in accord with structural engineering practice.

It is suggested that the safety factor be selected in the range 1.5 to 1.7. The presence of fewer unknown elements in the embankment problem more than offsets the more conservative definition used in the problem of cut slopes.

13. *Settlement.* Settlement of the embankment itself should be nearly completed during construction, and the analysis of its magnitude is not ordinarily a matter of practical importance.

The consolidation of soil underlying the embankment caused by the weight of the fill may often be the source of important and prolonged settlement. The analysis of this case requires data obtained from consolidation tests made on undisturbed samples of the underlying soil, and will not be treated here.

14. *Stability of Foundations.* If the embankment is placed upon a relatively weak non-plastic base the bearing capacity of the foundation may be computed by the modified Prandtl formula.

$$q = (c \cot \phi + \gamma_f B \tan \alpha)(\epsilon^{\pi \tan \phi} \tan^2 \alpha - 1) \quad (7)$$

where γ_f = unit weight of the supporting soil
$2B$ = crest width plus horizontal projection of one slope
$\alpha = 45 + \dfrac{\phi}{2}$, ϕ being the friction angle of the supporting soil
c = unit cohesion of the supporting soil
ϵ = the base of natural logarithms

If the fill is placed upon a thin, plastic base (thickness of plastic stratum, 2a, less than one-fourth the width of the fill at its base, 2B) the bearing capacity of the foundation is approximately

$$q = \frac{cB}{a} \quad (8)$$

For a thicker plastic base the bearing capacity becomes

$$q = 4c \quad (9)$$

In each of these three cases, a factor of safety may be found by dividing the computed bearing capacity by the average vertical pressure at the base of the fill. 1.5 is a suggested minimum value.

All three cases are concerned with a subsoil relatively weaker than the fill. If the contrary is true, no separate analysis of foundation stability is necessary.

The subject of foundation stability is introduced here only lest it be overlooked. The engineer should also study the more comprehensive treatment of foundations in one of several standard treatises on soil mechanics.

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14 21. Palmer, "Design of a Fill ...," *Public Roads*, Oct., 1939.

22. Barber & Mershon, "Graphical Analyses of Stability ...," *Public Roads*, Oct., 1940.

Note: The graphical procedure presented in item 9 is developed through Ref. 14–17, but includes some original details. The graphical location of the place of zero tension is believed new in principle and in technique.

Equation 8 was given in different form in section 10. It is well presented in Ref. 21, and was first applied to earth fill by Jurgenson.

DISCUSSION ON ANALYSIS OF STRESSES IN CUTS AND EMBANKMENTS

Dr. Jacob Feld, *Consulting Engineer, New York:* The excellent summary of available information on this subject, with a great deal of the published theoretical material which is not directly usable screened out, leaves little necessity for adding to this report. However, I believe that the first paragraph on the subject "Cuts Through Sand" should be somewhat elaborated. The paragraph starts off on the subject of cohesionless sand and it could be assumed that other sands with some cohesive properties should be considered as part of "Cohesive Soil."

In recent years I have come across three peculiar conditions in cuts and embankments all of which were in sands, but where the properties of cohesionless materials were not found. On the northern shore of Long Island near Wading River, there is a large deposit of almost pure white silica sand which appears to have no cohesive properties under usual tests; but which, when exposed on surface of embankments or on surface of cuts, forms an almost impervious hard surface. It has been determined that the cause of the change in material upon exposure to the air is a cementing action from silicic acid which coats the grains of the sand. In embankments subject to considerable ground water flow, it is quite possible that this altered surface will act as a dam to free percolation and when pressures are built up sufficiently, sudden failure of the slope may occur.

A second example was found in glacial sand deposits about 10 miles north of Geneva, N. Y. where the sand pockets had acted as drainage lenses for adjacent soils heavy in limestone leechings. This material when excavated looked like perfectly clean sand and acted as such until the lime solution covering the grains started to set up and it was found that balls of sand and gravel would form quite easily and disrupt the continuity of the embankment materials. In excavations, the slopes would hold up at unreasonably steep angles and in a few weeks sudden failure would occur, apparently from the leeching out of the cementing lime ingredients from rain exposure. A third and much more serious condition was found last year in the very fine sands, mostly coming from eroded rock formations in the area south of Red Bank, N. J. Apparently these sands overlaid by cranberry bogs were full of humic acid resulting from disintegration of vegetation. Samples taken from pits indicated nothing of unusual conditions but it was found that slopes of embankments and slopes of cuts at very conservative angles failed completely during heavy rains. The failure was not only from erosion but also from internal piping as the humic acid absorbed large volumes of water and caused serious expansion of the material.

Since all of these materials were classified as sands from test pits and other observations, some elaboration on the restriction to

11 18. Middlebrooks & Bertram, "Soil Tests . . ." (Table 1). *Proceedings*, Highway Research Board, Vol. 22, 1942.

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14 21. Palmer, "Design of a Fill . . . ," *Public Roads*, Oct., 1939.

22. Barber & Mershon, "Graphical Analyses of Stability . . . ," *Public Roads*, Oct., 1940.

Note: The graphical procedure presented in item 9 is developed through Ref. 14–17, but includes some original details. The graphical location of the place of zero tension is believed new in principle and in technique.

Equation 8 was given in different form in section 10. It is well presented in Ref. 21, and was first applied to earth fill by Jurgenson.

DISCUSSION ON ANALYSIS OF STRESSES IN CUTS AND EMBANKMENTS

DR. JACOB FELD, *Consulting Engineer, New York:* The excellent summary of available information on this subject, with a great deal of the published theoretical material which is not directly usable screened out, leaves little necessity for adding to this report. However, I believe that the first paragraph on the subject "Cuts Through Sand" should be somewhat elaborated. The paragraph starts off on the subject of cohesionless sand and it could be assumed that other sands with some cohesive properties should be considered as part of "Cohesive Soil."

In recent years I have come across three peculiar conditions in cuts and embankments all of which were in sands, but where the properties of cohesionless materials were not found. On the northern shore of Long Island near Wading River, there is a large deposit of almost pure white silica sand which appears to have no cohesive properties under usual tests; but which, when exposed on surface of embankments or on surface of cuts, forms an almost impervious hard surface. It has been determined that the cause of the change in material upon exposure to the air is a cementing action from silicic acid which coats the grains of the sand. In embankments subject to considerable ground water flow, it is quite possible that this altered surface will act as a dam to free percolation and when pressures are built up sufficiently, sudden failure of the slope may occur.

A second example was found in glacial sand deposits about 10 miles north of Geneva, N. Y. where the sand pockets had acted as drainage lenses for adjacent soils heavy in limestone leechings. This material when excavated looked like perfectly clean sand and acted as such until the lime solution covering the grains started to set up and it was found that balls of sand and gravel would form quite easily and disrupt the continuity of the embankment materials. In excavations, the slopes would hold up at unreasonably steep angles and in a few weeks sudden failure would occur, apparently from the leeching out of the cementing lime ingredients from rain exposure. A third and much more serious condition was found last year in the very fine sands, mostly coming from eroded rock formations in the area south of Red Bank, N. J. Apparently these sands overlaid by cranberry bogs were full of humic acid resulting from disintegration of vegetation. Samples taken from pits indicated nothing of unusual conditions but it was found that slopes of embankments and slopes of cuts at very conservative angles failed completely during heavy rains. The failure was not only from erosion but also from internal piping as the humic acid absorbed large volumes of water and caused serious expansion of the material.

Since all of these materials were classified as sands from test pits and other observations, some elaboration on the restriction to the statement made in paragraph 1 of the report should be included.

T. A. Middlebrooks, *Principal Engineer, U. S. Engineer Department, War Department:* Highway cuts in general do not require elaborate stability analysis and the author has done well to emphasize short-cut methods for general use. Where ground water is not a problem, the writer agrees that the stability of slopes in cohesionless material can be determined directly by comparing the angle of internal friction with the angle of the slope (or angle of repose) and that the stability of cohesionless material can be approximated quickly by the use of Taylor's curves.

In connection with the comments concerning the use of triaxial compression and/or direct shear tests, the writer is not in agreement with the author. Whenever the critical circle method (Swedish circle, etc.) is employed for slope stability analysis, the "ultimate" shearing strength as determined by the direct shear test should be used. The strain in the triaxial test is too small to obtain this ultimate value. It must be recognized in the application of the critical circle method of analysis that failure starts at the point of high stress in the lower portion of the embankment or cut slope, and progresses from this point along a circular surface. Therefore, large strains occur in the prototype along the circular surface of failure before complete failure of the slope occurs. If triaxial compression test results are utilized in connection with this method, a conservative value of the shearing strength considerably less than the maximum should be used or a large factor of safety should be included in the analysis.

Where the elastic theory[1,2] is employed to analyze slopes, shearing strengths should be determined from the results of triaxial compression test. This method of analysis presupposes a low percentage of strain and it is not valid if large strains occur. This method is much more involved than the critical circle method of analysis and for that reason is not recommended for analysis of highway cut slopes. However, if it is used, the criteria for design should be that no point in the foundation or embankment should be overstressed, instead of the method using average values as proposed by the writer in 1936.[2]

The procedure outlined for taking into consideration the ground water level in the stability analysis seems to be too involved for practical use. It is believed that the adjusted ground water line after the cut is completed can be sketched in, after a study of the borings, accurately enough for all practical purposes. In any case, it would probably be advisable to assume that seepage will outcrop on the slope at the natural ground water level and to check the stability to be sure that the slope has at least a safety factor slightly greater than one for this case.

Use of Jurgenson's formula $c = \frac{Pa}{L}$ as proposed by the author is not concurred in. This simple relationship has not proved adequate for analyzing this type of foundation.

Prandtl's formula, referred to under "*Stability of Foundations*," is considered applicable to footings, but not to embankment foundations. The stability of the embankment and foundation should be considered jointly when the critical circle method is employed. It might be considered separately, however, if the elastic theory method of analysis is used.

Professor Hennes: Mr. Feld's comments emphasize the futility of hoping to develop any routine procedure in engineering problems that will take the place of professional judgment based upon actual field experience. For this reason the writer has preferred to refer the reader to the specialist in unusual cases, rather than to complicate the paper unnecessarily.

Mr. Middlebrooks rightly calls attention to the difficulty in obtaining ultimate rather than maximum resistances from compressive tests. It was partly in recognition of this fact that the writer advocated direct shear tests for embankment analysis. However, Mr. Middlebrooks' recommendation is less applicable to cut slopes in natural soil deposits.

[1] Jurgenson. "Application of the Theory of Elasticity and Plasticity to Foundation Problems," Boston Society of Civil Engineers, July 1934.

[2] Middlebrooks. "Foundation Investigation of Fort Peck Dam Closure Sections," *Proceedings*, International Conference on Soil Mechanics and Foundation Engineering, Harvard University, 1936.

First. The plane of failure in a direct shear test is predetermined by the apparatus. This is acceptable with homogeneous fill; in heterogeneous natural soil it may yield erroneous high strengths as compared to the compression test where failure can occur along some natural plane of weakness.

Second. The direct shear test is satisfactory with fill, where the grain size of the specimen is subject to control; it is less satisfactory than the compression test for undisturbed samples of till, gravel, and other soils containing large particles of unknown grading.

Third. While the argument for the use of ultimate strengths is unquestionably valid, it can be overemphasized. High intensity of stress, as at the toe of a slope, is not the criterion of failure; the real factor is the ratio of stress to resistance.

In many cases the writer has observed slippage at the crest of the slope before toe failure was equally apparent. If shear strain were limited to a single slip-surface, the strain at the crest should exceed strain at the toe by some component of the volumetric compression in the case of fill, and would be correspondingly less in the case of cut. Actually the situation is complicated by the distribution of shear strain throughout the body of earth. Without supporting evidence, the writer would tentatively suggest that many cases may approach the condition of an incompressible rigid body, with failure induced by substantially equal strain along the slip-surface.

Mr. Middlebrooks' suggestion for taking ground water level into consideration appears to be an attractive simplification for most highway purposes. His comments regarding foundation stability do not seriously conflict with the views expressed in the paper. The Jurgenson and Prandtl formulas were suggested only as supplements to the use of Taylor's curves, and even in that connection the desirability of a more comprehensive treatment was mentioned. Where the critical circle method is used, the inclusion of the foundation in the analysis may be taken as a matter of course.

A RATIONAL APPROACH TO DRAINAGE OF A PERVIOUS SUB-BASE

By Carl F. Izzard, *Senior Highway Engineer, Public Roads Administration*

SYNOPSIS

When a pervious sub-base is placed in a trench section drainage is essential. This paper sets up approximate equations for time of drainage in terms of permeability, slope, and amount of drainable water. Water is assumed to reach the sub-base through joints or cracks in the pavement, or along the edge of the pavement. On flat grades the capacity of the base to carry water longitudinally controls unless drainage is obtained by a continuous longitudinal underdrain at the edge. Otherwise drainage is controlled by the capacity of the "bleeders" or other outlets spaced along the shoulder. The required spacing of these outlets becomes less and less as the grade of the roadway becomes flatter, indicating that uniform spacing regardless of grade is poor design. The theory indicates that continuous longitudinal underdrains should be provided on flat grades, or the sub-base should be extended across the shoulder.

The problem of securing adequate drainage for a pervious sub-base beneath a pavement of any type has not been investigated experimentally to the extent that design principles can be confidently established. In the meanwhile experience has shown the need for such drainage and it is the purpose of this paper to offer a rational approach to the problem based on factors known to affect drainage significantly. Some of these factors are:

(1) cross-section of sub-base
(2) cross-section of roadway