First. The plane of failure in a direct shear test is predetermined by the apparatus. This is acceptable with homogeneous fill; in heterogeneous natural soil it may yield erroneous high strengths as compared to the compression test where failure can occur along some natural plane of weakness.

Second. The direct shear test is satisfactory with fill, where the grain size of the specimen is subject to control; it is less satisfactory than the compression test for undisturbed samples of till, gravel, and other soils containing large particles of unknown grading.

Third. While the argument for the use of ultimate strengths is unquestionably valid, it can be overemphasized. High intensity of stress, as at the toe of a slope, is not the criterion of failure; the real factor is the ratio of stress to resistance.

In many cases the writer has observed slippage at the crest of the slope before toe failure was equally apparent. If shear strain were limited to a single slip-surface, the strain at the crest should exceed strain at the toe by some component of the volumetric compression in the case of fill, and would be correspondingly less in the case of cut. Actually the situation is complicated by the distribution of shear strain throughout the body of earth. Without supporting evidence, the writer would tentatively suggest that many cases may approach the condition of an incompressible rigid body, with failure induced by substantially equal strain along the slip-surface.

Mr. Middlebrooks' suggestion for taking ground water level into consideration appears to be an attractive simplification for most highway purposes. His comments regarding foundation stability do not seriously conflict with the views expressed in the paper. The Jurgenson and Prandtl formulas were suggested only as supplements to the use of Taylor's curves, and even in that connection the desirability of a more comprehensive treatment was mentioned. Where the critical circle method is used, the inclusion of the foundation in the analysis may be taken as a matter of course.

A RATIONAL APPROACH TO DRAINAGE OF A PERVIOUS SUB-BASE

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SYNOPSIS

When a pervious sub-base is placed in a trench section drainage is essential. This paper sets up approximate equations for time of drainage in terms of permeability, slope, and amount of drainable water. Water is assumed to reach the sub-base through joints or cracks in the pavement, or along the edge of the pavement. On flat grades the capacity of the base to carry water longitudinally controls unless drainage is obtained by a continuous longitudinal underdrain at the edge. Otherwise drainage is controlled by the capacity of the "bleeders" or other outlets spaced along the shoulder. The required spacing of these outlets becomes less and less as the grade of the roadway becomes flatter, indicating that uniform spacing regardless of grade is poor design. The theory indicates that continuous longitudinal underdrains should be provided on flat grades, or the sub-base should be extended across the shoulder.

The problem of securing adequate drainage for a pervious sub-base beneath a pavement of any type has not been investigated experimentally to the extent that design principles can be confidently established. In the meanwhile experience has shown the need for such drainage and it is the purpose of this paper to offer a rational approach to the problem based on factors known to affect drainage significantly. Some of these factors are:

(1) cross-section of sub-base
(2) cross-section of roadway

(3) permeability of sub-base
(4) location, kind and size of outlets
(5) longitudinal grade of highway

Taking the simplest problem, consider a sub-base (beneath a concrete pavement) placed in an impervious trench section with outlets of the same material, commonly called "bleeders," spaced at intervals through impermeable shoulders, draining to the face of the embankment slope. Assume that all water to be drained reaches the sub-base from the surface, i.e., through joints or cracks in the pavement, or along its edge.

Hydraulically, the capacity of the system as a whole may be controlled by the capacity of the "bleeder" if the latter is small in relation to the volume of the sub-base, or if may be controlled by the capacity of the sub-base to deliver water to the point of outlet. In the latter case a zero longitudinal grade on the highway reduces the capacity to practically zero, unless pressure is built up by confined water on grade approaching the flat section of road. Even when pressure does exist a time is eventually reached when only the water contained in the sub-base on the flat grade remains, the only available gradient being that afforded by the thickness of the sub-base itself.

Capacity of sub-base

The capacity of the sub-base to carry water longitudinally may be expressed by Darcy's Law

$$Q_b = kA_b s \qquad (1)$$

where k = permeability constant in ft. per sec.
A_b = cross-sectional area of sub-base, sq. ft.
s = longitudinal grade of highway, ft. per ft.

In this equation the hydraulic gradient is assumed to be parallel to the grade of the highway, ignoring possible pressure which might exist, in order to evaluate the most critical situation.

Assuming that a given outlet drains only the volume of sub-base between that point and the outlet next above, the volume of water to be drained out, after cessation of all inflow, is the drainable water in that volume of sub-base. Letting V = volume of drainable water in cubic feet, then

$$V = LA_b m \qquad (2)$$

where L = length of sub-base between outlets, ft.
m = ratio of volume of drainable water to total volume of sub-base.

The drainable water in a cubic foot of the sub-base material as compacted is the total water less that which is held against gravity by capillary forces. The value of m actually varies with gradation and compaction of material but will be taken as one-tenth for illustration.

To measure the effectiveness of drainage the time element must be considered. Studies at Purdue by McClelland[1] afford an insight to the importance of considering the time element, but do not cover the boundary conditions of the present problem. They do indicate, however, that as a rough approximation the average rate of drainage during a substantial portion of the total time during which drainage occurs is about one-half of the maximum initial rate of drainage.

Accordingly, let us assume that average rate of drainage during time T (in seconds) is $Q_b/2$. Then

$$T = \frac{V}{Q_b/2} \qquad (3)$$

Substituting value of V and Q from equations (1) and (2)

$$T = \frac{2LA_b m}{kA_b s} = \frac{2Lm}{ks} \qquad (4)$$

$$\text{or} \qquad L = \frac{Tks}{2m} \qquad (5)$$

Equation (5) indicates that spacing of outlets should be directly proportional to (a) time to drain, (b) permeability, (c) grade of highway and inversely proportional to m. For this case the cross-sectional area of the base drops out since the volume of water to be drained increases as the area increases. Experiment will probably show that m and k are directly related to each other and to the density of the compacted material.[2]

[1] "Large-Scale Model Studies of Highway Subdrainage," T. B. McClelland, *Proceedings*, Highway Research Board, Vol. 23, 1943.

[2] See Baver, "Soil Physics," Wiley 1940, p. 226.

Capacity of bleeder

The capacity of the bleeder (Q_o) may be approximated by the equation

$$Q_o = kA_o s_o \tag{6}$$

Where A_o = cross-sectional area of drain, sq. ft.
s_o = slope of drain, ft. per ft.

Since material is same as that in sub-base (by original assumption), then k is same as in equation (1).

For a balanced design $Q_b = Q_o$ or $kA_o s_o = kA_b s$, which means simply that products of cross-sectional area times slope should be equal.

Application of equations

We now have the basic equations upon which to base our analyses. Taking the case where capacity of sub-base controls, the spacing of bleeders may be computed approximately by equation (5). Transposing we get

$$s = \frac{2mL}{Tk} \tag{7}$$

from which the minimum grade of highway may be computed in terms of spacing of outlets, time to drain and hydraulic characteristics of material in sub-base. For example, if spacing is 80 ft., $m = 0.1$, $T = 86{,}400$ sec. (= 24 hr.) and $k = 1 \times 10^{-2}$ ft. per sec. then

$$s = \frac{2 \times 0.1 \times 80}{86400 \times 10^{-2}} = 0.018$$

In other words, for this example, when grade of highway is less than 1.8 per cent, the sub-base in trench-section will not drain in 24 hr. with a spacing of outlets of 80 ft., if the permeability is only 0.01 ft. per sec. (= 864 ft. per day). This permeability is somewhat greater than that of the fraction between the No. 20 and No. 30 sieves when in a loose condition.[3] Actually a much coarser material would be required because compaction reduces permeability.

Consider now the size of bleeder required to balance the capacity of the sub-base at critical slope just computed. These bleeders are commonly placed on a transverse slope of about ½-inch per foot. Hence slope is 1/24 and $A_o = \frac{Q_o}{k/24}$. For balanced designed

$$Q_o = Q_b = kA_b s$$

from which

$$A_o = \frac{kA_b s}{k/24} = 24A_b s$$

If we take sub-base as being 12 ft. wide (half-width of pavement) by 8 in. thick, then $A_b = 8$ sq. ft. and $A_o = 24 \times 8 \times .018 = 3.5$ sq. ft.

The time to drain a section of base controlled by outlet would be

$$T = \frac{V}{Q_o/2} = \frac{2LA_b m}{kA_o s_o} \tag{8}$$

This equation indicates that size, spacing and slope of bleeders should remain constant in order to drain sub-base efficiently on grades steeper than the critical slope below which sub-base capacity controls.

Grade below which outlet should be continuous longitudinally

If we take the width of bleeder as A_o divided by thickness of sub-base, then width of bleeder, for example above, becomes 3.5 sq. ft. ÷ 0.67 ft. = 5.2 ft. When the spacing becomes equal to this width continuous longitudinal outlet is required. The slope at which this becomes necessary may be found from equation (7)

$$s = \frac{2mL}{Tk} = \frac{2 \times 0.1 \times 5.2}{86{,}400 \times 10^{-2}}$$

= 0.0012 or about 0.1 per cent.

For grades flatter than this, as at bottom or summit of vertical curve, sub-base should extend to the face of the embankment slope, or continuous longitudinal underdrain of adequate capacity should be installed at edge of sub-base. From a practical construction viewpoint the bleeders would probably not be placed closer than several times their width. Also, there is some spacing at which the cost of bleeders exceeds the cost of a continuous longitudinal underdrain. The slope at which continuous outlet becomes necessary could be computed accordingly. For example with

[3] See discussion by Hogentogler and Barber on paper "Soil Moisture and Unsaturated Flow" by Russell and Spangler, *Proceedings*, Highway Research Board, Vol. 21, p. 460 (1941).

minimum spacing of bleeders of 20 ft. this slope would be about 0.5 per cent.

With continuous longitudinal outlet the equations for capacity of sub-base, nos. (1), (3), (4), (5) and (7), no longer apply, since direction of flow is not predominantly longitudinal but rather diagonal to center line, the angle varying with grade of highway.

Capacity of sub-base with continuous longitudinal outlet

The capacity of the sub-base to drain to a continuous longitudinal outlet will depend on the slope (normal to contour lines drawn on the plane of the bottom of the sub-base), the permeability and the cross-section of the sub-base. The design ought to be based on the limiting condition where longitudinal grade is zero and only available gradient is the transverse slope of the sub-base. In this case, the maximum rate at which the drainable water could be removed from the sub-base per foot of roadway is

$$q_b = kcs_b \tag{9}$$

where c = average thickness of sub-base in feet and s_b is transverse slope of bottom of sub-base.

For example, with $k = 10^{-2}$, $c = 0.67$ feet and $s_b = \frac{1}{4}$ in. per ft.

$$q_b = 10^{-2} \times 2/3 \times 1/48 = 1.39 \times 10^{-4} \text{ ft per sec. per foot of length}$$

Longitudinal underdrain designed for this capacity will be adequate where grade of highway is other than zero, since the gradient of the underdrain increases as the maximum slope of the sub-base increases.

Consider the case where the grade of the roadway is zero and the sub-base is crowned to drain to longitudinal underdrains at each edge. The time to drain out the drainable water after inflow has ceased would be

$$T = \frac{V}{q_b/2} = \frac{cmw/2}{kcs_b/2} = \frac{mw}{ks_b} \tag{10}$$

in which w is the width in feet of the sub-base between underdrains.

For illustration, assume a sub-base of any reasonable thickness having a transverse slope from center line of $\frac{1}{4}$ in. per foot, a width of 24 ft. between drains, and permeability $k = 10^{-2}$ ft. per sec.

Assuming $m = 0.1$ as in previous examples, then

$$T = \frac{0.1 \times 24}{10^{-2} \times 1/48} = 11552 \text{ sec., or about } 3.2 \text{ hr.}$$

With continuous longitudinal underdrain the time to drain the sub-base is thus about 3 hr. (for the assumed conditions) as compared with theoretical infinity for the example where drainage was obtained thru bleeders spaced 80 ft. apart on a roadway having a zero gradient, k and m being the same in both cases.

From equations (4) and (10) it may be shown that in order for the bleeders to drain the sub-base in the same time as a continuous longitudinal underdrain it would be necessary for the roadway to have a gradient of about 14 per cent when the bleeders are spaced 80 ft. apart. As a general statement it may be said that the continuous longitudinal underdrain will always provide more effective drainage than will the bleeders (which is self-evident from examination in the first place).

Consideration should be given to the "piping ratio" in designing backfill for underdrains so as to eliminate the possibility of the sub-base material washing out as water drains away.[4] This ratio is determined from the grain-size accumulation curves of the two materials by picking off the 15 per cent passing size of the backfill and the 85 per cent passing size of the sub-base, in millimeters. Their ratio should be not more than 5, that is

$$\frac{15\% \text{ backfill size}}{85\% \text{ sub-base size}} = \text{not more than } 5$$

Discussion

The method of analysis here evolved should be used only with full knowledge of its limitations. Actually, the dimensional values derived from any of the equations are relative and are useful principally as yardsticks to compare one system of drainage with another. They do, however, provide a simple means of evaluating, qualitatively and approximately, the effect of varying spacing, size and slope of outlets, type of pervious material, and cross-sectional area of sub-base.

[4] "Investigation of Filter Requirements for Underdrains," Tech. Memo. 183-1 U. S. Waterways Exp. Sta., War Department, Corp of Engrs., U. S. Army.

The foregoing analysis is by no means rigorous as the field conditions may vary widely from the assumptions necessarily made to simplify the solution. In practice a granular sub-base is frequently placed on a silty soil in which the capillary action may be rapid enough to capture an appreciable proportion of the water flowing through the sub-base laterally. In such cases it would appear logical to drain away water accumulated in the sub-base as rapidly as possible by providing high permeability in the sub-base, and the most efficient outlet, i.e., a longitudinal underdrain at the edge of the sub-base. However, the more permeable (or coarse-grained) the sub-base material becomes, the more likely is intrusion of the silty soil into the sub-base. One solution might be to provide an impermeable bituminous membrane sealing off the bottom of the sub-base from loss of water and from intrusion.

Nothing has been said about the most commonly approved design which provides for extension of the sub-base to the full crown-width of the roadway, permitting drainage to open channel in cut or to slope in embankment. In this design drainage is limited by the permeability and slope of the sub-base, and the time required to drain will be roughly proportional to the distance from the center-line of the highway to the point of outlet. Consequently, from the hydraulic viewpoint, the full width construction is less efficient than that providing a continuous longitudinal underdrain near the edge of the pavement. It would also appear logical that the latter design ought to be less susceptible to blocking of drainage by freezing, since the underdrain pipe would usually be below frost depth, except at the outlet.

Attention is again called to the initial assumption that the need for drainage arises from the entrance of surface water to the sub-base through or alongside the pavement. The extent to which surface water can be successfully and permanently excluded from the sub-base lessens the need for elaborate underdrainage systems designed solely for removing such water. This study specifically does not include design of underdrainage for the purpose of lowering the groundwater table. However, the advantage of utilizing underdrains designed for the latter purpose to serve as outlets for subgrade drainage is obvious. The wide occurrence of mud-pumping, which requires water, usually derived from surface runoff, suggests that subgrade drainage is still a prime requirement if mud-pumping is to be eliminated.

Experimental data are needed especially with reference to permeability of field-compacted granular sub-bases and the drainable water contained in such sub-bases.