METHODS OF ANALYSIS OF FLOW PROBLEMS FOR HIGHWAY SUBDRAINAGE

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SYNOPSIS

In hydrodynamics, the term "flow net" is used to describe a graphical representation of the distribution of hydraulic head and the direction of flow. It is composed of equipotential lines, which connect all points of equal head, and flow lines, which indicate various paths of flow. This valuable theoretical tool, which has had wide application to seepage problems pertaining to various earth structures—particularly dams—has had very limited use in the study of subdrainage problems.

Flow nets have many practical uses, not the least of which is the presentation of a visual picture of the flow conditions. They are also useful in that seepage forces and their effect upon stability can be determined. The safety against "piping" or "boils" can be found from the hydraulic gradients indicated by the flow net at outflow surfaces. If either the quantity of water flowing or the true soil permeability is known, the other can be readily computed using values taken from the flow net. This last application has proven particularly useful in sand-model studies of drainage problems for which it was necessary to establish within narrow limits the soil permeability existing during the tests.

Various methods have been found useful in determining the flow net for a given set of conditions. Each method has its own advantages and limitations, and each is particularly applicable to certain problems.

The graphical method consists of assuming a reasonable set of equipotential and flow lines and revising these lines by trial and error until all necessary conditions have been met. This method, applicable only to steady-flow and two-dimensional problems, requires no equipment but does require considerable skill and experience. Determination of the free surface is particularly difficult.

Electrical models of flow problems are also used in determining flow nets and are based on the analogy between the flow of electricity and the seepage of water. This method permits rapid determination of the free surface and the complete set of equipotential lines. The flow net is completed by graphically plotting the flow lines. This method is also limited to steady-flow problems and can be easily applied only to homogeneous soil conditions. It requires some specialized equipment but can be successfully applied with very little previous experience or skill.

Sand models of fairly large scale provide the most versatile although the most cumbersome method of determining flow nets and studying flow problems. They provide the only means for obtaining solution to both steady-flow and non-equilibrium problems. Therefore, it is particularly useful in drainage research, inasmuch as most drainage problems are of the non-equilibrium type. Piezometer tubes inserted in the model provide the hydraulic head at various points at any selected time. From these values, equipotential lines may be accurately determined, and the flow net can be completed by graphical determination of the flow lines. Principal advantage of this method is that it provides the variation of the free surface with time. Principal limitation of the method is the time and equipment required to perform the tests and to investigate properly all variables, making it useful only in fairly extensive research programs.

PURPOSE AND SCOPE

The important subject of highway subdrainage is still very much in the "rule-of-the thumb" stage. More direct and more analytical methods of approaching subdrainage problems are greatly needed and could do much toward making subdrainage installations more effective and more economical.

The flow net is a valuable theoretical tool for studying seepage problems, and though widely used in other fields—particularly in dams—it has had but very little application to subdrainage. This is probably true because most highway engineers, though aware of its possibilities, are not sufficiently familiar with its details to apply it to their own problems.

This paper is intended, therefore, to define the flow net, to explain its properties, to show its applications to drainage problems, and to describe methods for its determination.

THE FLOW NET AND ITS PROPERTIES

In a body of soil through which water is flowing, the path traced by any individual particle of water is called a "flow line." There are, of course, as many of these paths as there are particles of water.

A line through that body of soil connecting all points which have the same hydraulic head or potential is called an "equipotential line." There are also an infinite number of these equipotential lines.

A "flow net" for an area in which seepage occurs is a graphical representation of the distribution of hydraulic head and the direction of flow in that area. It is made up of a chosen group of flow lines and equipotential lines. The flow lines are so chosen that the same fraction Δq of the total seepage flows between each pair of adjacent lines or in each "flow channel" (Fig. 1a), and the equipotential lines are so chosen that the same difference in head Δh exists between each adjacent pair (Fig. 1b). Furthermore, the increments of seepage Δq and of head Δh are so chosen with respect to each other that the figures formed by the intersection of the flow lines and two equipotential lines are curvilinear squares (Fig. 1c).

There are a number of important properties of such a flow net which are useful in their construction and in their interpretation. The following apply to flow nets representing seepage through homogeneous isotropic soils:

1. All intersections of flow lines with equipotential lines are at right angles (Fig. 1c, Fig. 2a).
2. Flow lines cannot intersect other flow lines, nor can equipotential lines intersect other equipotential lines.
3. All impervious boundaries are flow lines and all boundaries exposed to open water are equipotential lines. Thus in Figure 1c, the bottom boundary which is impervious is a flow line. In an earth dam, the upstream face is an equipotential line.

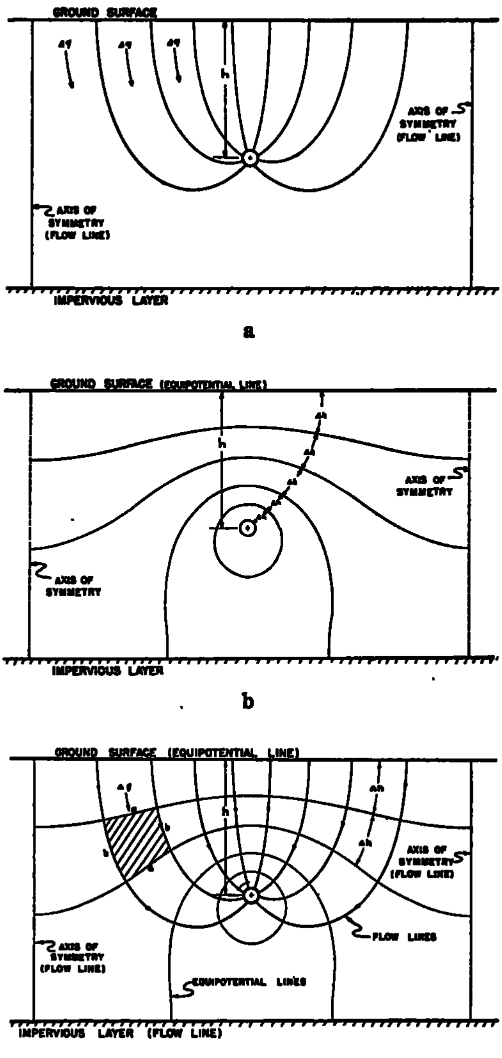

Figure 1. Flow Net Construction for Flow into one of a Group of Parallel Drains. a. Flow Lines. b. Equipotential Lines. c. Completed Flow Net.

4. Where the uppermost boundary is exposed to atmospheric pressure, it is known as the phreatic line or free surface. If seepage is not at equilibrium, this surface is neither an equipotential line nor a flow line but is intersected by both groups of lines. An example of this condition exists when a subdrain is lowering the ground-water table as in Figure 2a. Water in such a case is being removed from the soil at the free surface and that surface is being continually lowered.

   If seepage is at equilibrium, the free surface will be a flow line and will be intersected at right angles by equipotential lines. Such a condition is illustrated in Fig. 2b in which the ground-water surface has been lowered until it is at equilibrium, with as much water entering the section from the sides as is being removed by the drains.

   In either case it is a necessary condition that each equipotential line which intersects the free surface must do so at an elevation equivalent to the head represented by that line. Therefore, since there is a constant difference of head between the equipotential lines, there must also be a constant vertical difference between the intersections of these lines with the free surface (Fig. 3).

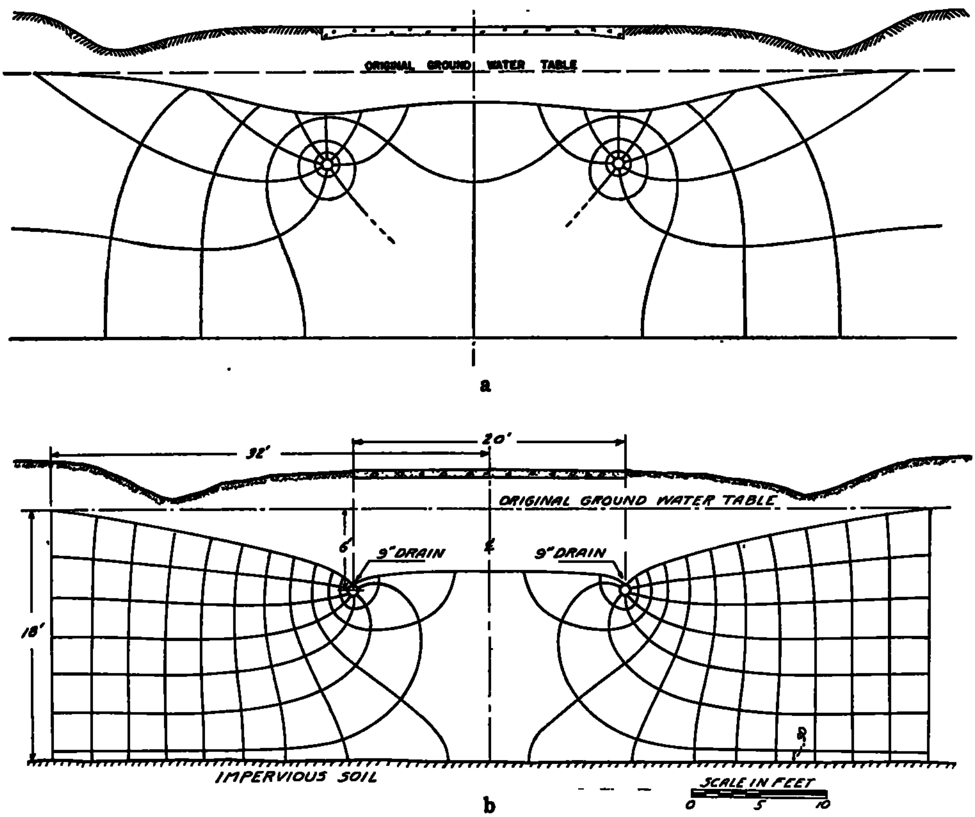

Figure 2. Flow Nets for Installation with Two Parallel Subdrains, Before and After Equilibrium is Established. a. Non Equilibrium Condition. b. Equilibrium Condition.

5. A sloping discharge face, in contact with air, is neither an equipotential line nor a flow line. The intersections of equipotential lines with such a surface must fulfill the same conditions as intersections with a free surface, described above.

6. Irregular sections which do not have the appearance of "squares" can usually be subdivided into three squares and one similarly irregular section, and this subdivision can be continued until the remaining irregular section is infinitesimal (Fig. 4).
7. Lines dividing all boundaries symmetrically also divide flow lines and equipotential lines symmetrically. Thus the center-line of the road in Figure 5 is a line of symmetry for the boundaries and for the flow net lines as well.

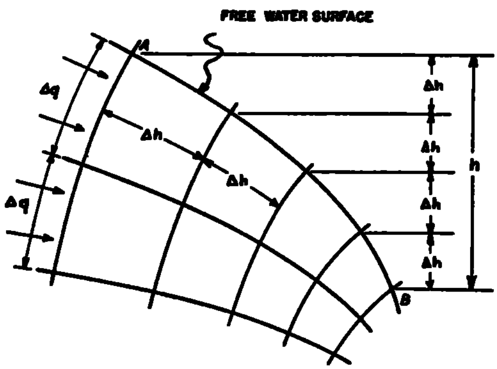

Figure 3. Diagram Illustrating the General Condition for a Free Surface.

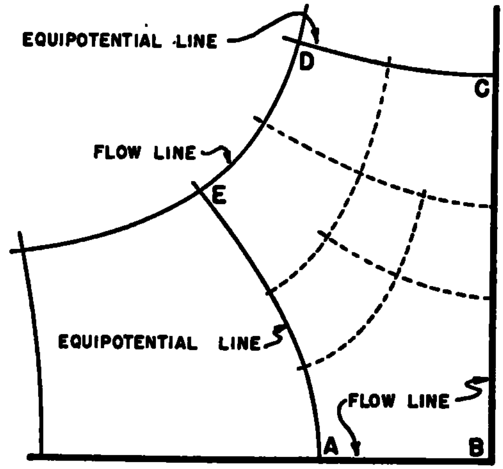

Figure 4. An Irregular Section of a Flow Net Subdivided into Squares.

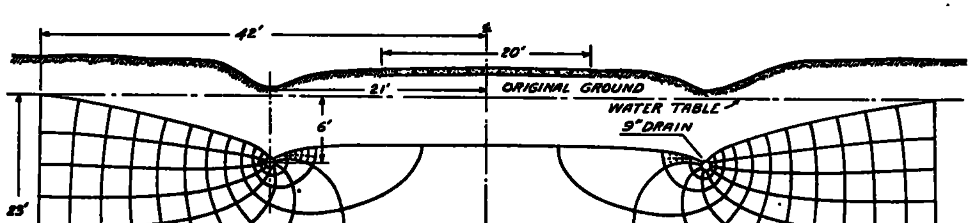

Figure 5. Flow-Net Representing Parallel Drains Beneath the Side Ditches (Symmetrical Section) Homogeneous Soil.

8. Where flow lines cross a boundary between two soils of different permeabilities, their direction is changed similarly to the refraction of light rays passing through two translucent materials. The flow lines are deflected such that the tangents made by any flow line with the boundary are inversely proportional to the respective coefficients of permeability. Furthermore, the squares on one side of the boundary change to rectangles on the other side with the ratio of their sides being equal to the inverse ratio of the coefficients of permeability (1, p. 160).[1] Thus, referring to Figure 6:

$$\frac{\tan \beta}{\tan \alpha} = \frac{k_1}{k_2} = \frac{c}{b} \qquad \text{(I)}$$

[1] Numerals in parentheses refer to the list of references at the end of the paper.

In order to retain squares on both sides of the boundary, the potential interval may be changed such that the ratio of

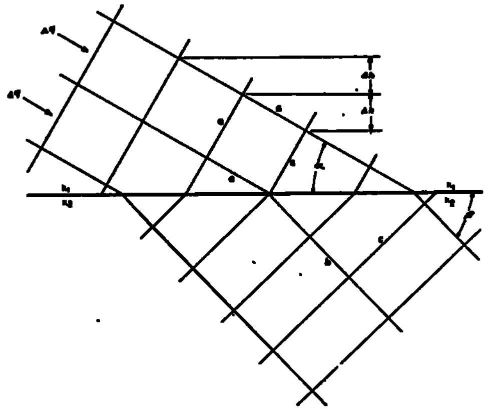

Figure 6. Diagram Illustrating with Magnification the Deflection of Flow Lines at a Boundary Between Soils of Different Permeabilities.

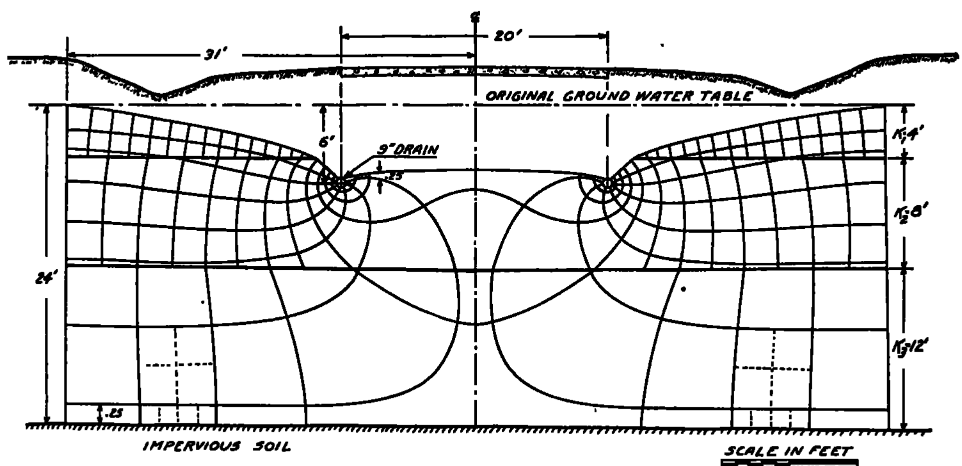

Figure 7. Flow-Net Representing a Pattern of Drainage in Stratified Soil

the intervals is equal to the inverse ratio of the coefficients of permeability:

$$\frac{k_1}{k_2} = \frac{\Delta h_2}{\Delta h_1} \qquad \text{(II)}$$

This was done in the flow net shown in Figure 7. There the potential interval in the lowest layer is four times that in the top layer.

9. Predetermined boundary conditions may sometimes introduce points of singularity at which some of the above conditions may not necessarily hold. Such points seldom occur in highway subdrainage problems and will not be discussed here. For an explanation of singular points, see Ref. (1 p. 165).

USE OF FLOW NETS

To one familiar with the formation and engineering characteristics of soils, it is obvious that drainage analyses by flow nets are inexact; yet, they are not inept. Proper evaluation of such analyses requires recognition of limitations first with regard to the soil and its assumed isotropic and homeogenous properties, and second with respect to the transitory nature of ground-water elevations as they are effected by drainage and by additional supply. Rarely if ever are there actual situations where any of these conditions correspond exactly with those assumed, but when thoroughly acknowledged, these become mere qualifications of the efficacy of this approach.

There are three outstanding and immediate uses which can be made of flow nets in the solution of subdrainage problems. The first is concerned simply with the pattern which is obtained by the construction of the net—a visual aid to the study of points of concern to the engineer. Thus, for a given location where there are several combinations of size, depth and spacing of drains under consideration, the drawing of flow nets representing all of these combinations will aid materially in the determination of the one set of conditions most advantageous from the standpoint of ground-water elevations. The net in Figure 5, for example, would be changed by even minor variations of the two drains, and it would be vastly different if one of the drains were removed. Further than these variations, there are some features of flow which are difficult to comprehend without recourse to models or to a graphical illustration such as a flow net. One such feature may be illustrated by the location represented in Figure 5, where water flowing from the right at depths in the vicinity of the impervious layer reaches the right-hand drain only by a circuitous route leading beneath and to the left of that drain. Peculiarities of this nature, while not of particular importance here, may be of primary concern in other problems of flow.

The second principal use of flow nets pertains to the equipotential lines as means for determining potential gradients—actually a determination of the head acting at any place within the system. Since the head or potential is a measure of the force causing flow, it is a factor governing pressures which are of consequence in stability analyses. Already much use has been made of this technique in analyzing the safety of water impounding structures against "piping" (2) and in computing the stability of such structures, especially when they are subjected to "rapid drawndown" (3). Possibilities of applying the same procedure to highway embankments and cut slopes appear promising, while a similar application to the silting of drains is, at the least, intriguing.

A third application of flow nets is one concerned with the quantities and rates of flow of water through soil. Here the computations are based on Darcy's Law, an empirical formula widely known and considered to be valid by all but a few of the students of soil drainage. This fundamental concept, often stated as "the rate or amount of flow is directly proportional to the hydraulic gradient" has few limitations, those being concerned mainly with laminar flow and saturation or filling of the voids. Mathematically the law is an equation,

$$Q = kiA \qquad \text{(III)}$$

in which:

Q = total quantity of discharge per unit of time
k = coefficient of permeability of the soil
i = hydraulic gradient
A = total cross sectional area

Applied to the flow net, this law provides a solution whereby there are two unknowns either of which may be an object of experimental study.

Referring again to Figure 1c, for a simplified illustration, each channel of flow represented by that diagram may be assumed to be discharging at the drain a quantity of water Δq per unit of time (and per unit of distance perpendicular to the plane of the flow net). There being a total of nine such channels—usually denoted as Nf in the general solution—all by definition carrying equal amounts of water, the total discharge in unit time is $Q = \Delta qNf$. Likewise, the entire potential or head h is lost in five equal potential drops, the total number of these drops being commonly referred to as Nd in the general solution. So, each drop Δh is equal to $\frac{h}{Nd}$. Finally, assuming a division of the net into an infinite number of squares, the hydraulic gradient is equal to $\frac{\Delta h}{b}$ and the cross sectional area is a (both a and b being variable but bearing a constant relationship which by definition of the flow net is unity), and the quantity discharged per channel is:

$$\Delta q = k\frac{h}{Nd}\left(\frac{a}{b}\right) = k\frac{h}{Nd}$$

Collectively, the discharge from all the channels is represented by the expression

$$Q = \Delta qNf = kh\frac{Nf}{Nd} \qquad \text{(IV)}$$

a modern hydraulic neologism peculiar to the flow net. Thus, in any situation represented by a flow net, either the quantity of discharge or the coefficient of permeability may be determined if the value of the other is known.

FLOW NET CONSTRUCTION

Methods for determining flow nets are of three general types—the graphic, the electrical analogy, and the flow model or flume. Each has advantages and limitations. The first requires no equipment, but skill and experience are essential; the second, while not so much dependent upon skill, is feasible only with much preparation and equipment; and the third is highly requisite of both skill and equipment with the added detriment of being cumbersome. However, the last of the three methods is the only one applicable to both equilibrium and non-equilibrium flow and to two or three dimensional flow; hence, it is the most versatile.

The graphic procedure is wholly dependent upon the geometric properties of the flow net as they have been defined and upon the boundaries which are known or can be determined. Unfortunately, in most of the problems common to highway and airport drainage, not all of the boundaries are known. This is particularly true of the free surface. Even here the position is not completely indefinite because the hydraulic properties of this surface demand that certain requirements be fulfilled, namely: perpendicular (but not necessarily vertical) intersections with equipotential lines—since the method is limited to equilibrium flow, and equal vertical distances between these points of intersection. (See Fig. 3.) Furthermore, there are some solutions, such as that for the so-called "basic parabola" (1, p. 146) which are apposite to preliminary locations of the phreatic line for highway drainage installations though not specifically designed for nor strictly correct in those situations.

Usually a graphical solution is obtained by trial and error, the initial trial being simply one of reasonable assumptions for the groups of lines and the error being the common result of such assumptions. Then, through persistence, numerous adjustments, and sometimes several revisions of the entire net, a pattern which will satisfy all of the defined properties of a net is established. Consequently, it is obvious that there is no substitute for practice and experience in the use of this method, although there are some principles worthy of consideration by beginners and the experienced as well. These are concerned mainly with opportunities to observe well constructed nets representing situations similar to those in question, and with the advisability of limiting the net to only a few lines on the first draft. Later, subdivisions can be made as a measure for determining the accuracy of the pattern.

A method of determining flow nets which has frequently proven useful is that of the electrical model. Using an electrically conducting sheet to represent the section in which water is seeping, and placing electrodes at the points where water enters or leaves the section, simple electrical measurements permit the rapid determination of the free surface and any desired set of equipotential lines. With that information, the flow net can be completed rather easily by plotting the flow lines at right angles to the equipotential lines, thus fulfilling the flow-net conditions previously discussed.

The basis for electrical models lies in the analogy between the flow of electric current and the seepage of water through soil. It can be shown theoretically (4) that if an electrically conducting medium should be given boundaries geometrically similar to those of a seepage section, and if current is made to enter and leave that conductor in a manner similar to the entrance and exit of water from the seepage section, then the distribution of potential and the direction of flow of current in the former will be geometrically similar to the distribution of head and the direction of flow of water in the latter. Thus any flow net characterizing the flow in one would be equally applicable to the other.

In applying this principle, it is first necessary to choose a satisfactory conducting medium. To permit measurements with a simple electrical circuit, the medium should be of relatively high resistance, and to produce results of sufficient accuracy, it should be reasonably uniform. One medium which has been found very satisfactory is made by spraying a mixture of powdered graphite and shellac on heavy cardboard. It has been found that about eight coats of this mixture should be applied to produce a uniformly resistant sheet. To produce the effect of different layers in the same cross section, those layers where greater permeability is desired may be sprayed with extra coats of graphite until the thickness of each layer is directly proportional to its assigned permeability. A graphite sheet made up of eight coats usually has a resistance in the order of 20 to 50 ohms per unit width and unit length.

When a satisfactory conducting medium has been determined, a sheet of that material is cut to the shape of the seepage section, using any convenient scale, and electrodes are connected to the sheet at the points where water enters and leaves the section. In cutting the sheet and placing the electrodes, it is sometimes necessary to assume certain unknown boundary conditions. For example, in recent tests of drainage sections by this method, made at the Joint Highway Research Project, Purdue University, it was found necessary to assume that at some point to the side of the drain, the elevation of the ground-water surface was constant. The distance to such point is theoretically infinite, but under field conditions it of course has some finite value. A section typical to those that were studied is diagrammed in Figure 8a. Having chosen the point, *A*, where the elevation of the ground-water surface was assumed to be constant, a sheet of the conducting medium was cut in the shape of a rectangle, A B C D. All boundaries other than AB—which was assumed—were definite properties of the section studied, AD being the original ground-water surface, CD a line of symmetry, and BC an impervious boundary. A bar electrode was then placed along the line AB and a circular electrode of diameter equivalent to the drain size was placed at the drain location. There again an assumption was necessary, namely that the entire circumference of the drain would be at the same potential. Although this could be possible only when the drain was flowing full, any error produced by the assumption was small.

The electrical circuit used for such models is a simple Wheatstone bridge, using dry cell batteries to produce direct current for the model. Application of this circuit to the models just mentioned is shown in Figure 8b. The two resistances, R_1 and R_2, were variable and could be set at any desired ratio. To find a point on the model of any desired potential, the resistances R_1 and R_2 were set so that the connection, *E*, between them was at that potential. Then the potential probe, *P*, was moved over the model sheet until a point was found for which the galvanometer, *G*, did not deflect. That point then was also at the desired potential since no current was flowing through the galvanometer. In this manner, all points of a given potential or, conversely, the potential of all points on the sheet could be found.

When a model simulating seepage with a free surface is connected in a circuit as just described, one condition remains to be satisfied. This is the condition imposed by the force of gravity which causes the original ground-water surface to drop to a new free surface, curving toward the point of lowest potential. Electric current has no body-force analagous to gravity, and therefore flows through the entire conducting medium. As suggested by Wyckoff and Reid (5), flow through the model can be made to correspond to seepage simply by cutting away the upper

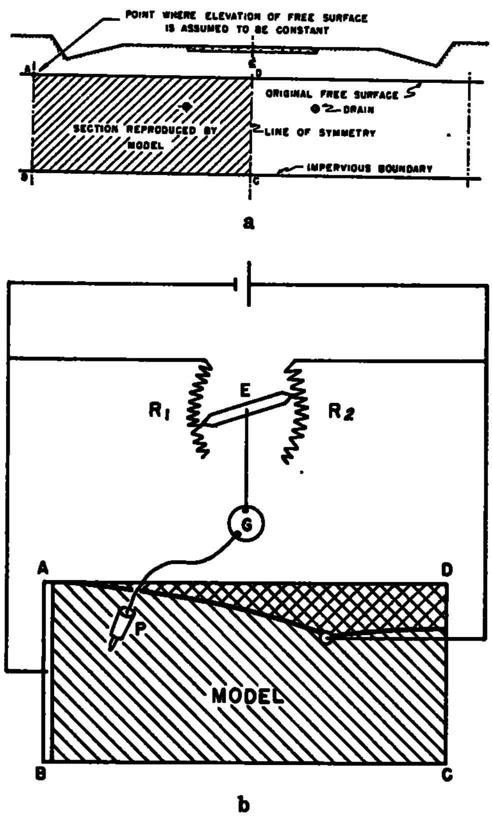

Figure 8. Electrical Model for Determining Drainage Flow. a. Typical Cross Section Studied By Means of Electrical Models. b. Wheatstone-Bridge Circuit Used in Model Studies.

section of the conducting medium until every point along the upper boundary is at a potential directly proportioned to its elevation. In Figure 8b, the portion of the model which has been cross-hatched twice is approximately the portion of that model which would be cut away as a result of the above procedure.

After the position of the free surface has been established and all other boundary conditions have been satisfied, the Wheatstone bridge may then be used to determine the equipotential lines for the flow net. Any convenient number of potential increments, Δe, of the total potential, $e_1 - e_2$, may be chosen. The resistances R_1 and R_2 are then set so that their connector, E, has a potential of $e_1 - \Delta e$. The potential probe can then be moved over the model, and by keeping the galvanometer reading at zero, the equipotential line representing that potential will be traced out. Setting the resistances then such that the connector has a potential of $e_1 - 2\Delta e$ the next equipotential may be traced, and so on until the complete set of equipotential lines has been found. When testing models made of graphite sheets, it was found convenient to draw the equipotential lines on the sheet as they were traced, using a white pencil. When the complete set had been found, they were transferred to tracing paper and the flow net was completed by drawing in the flow lines.

This method of determining flow nets can be easily applied to homogeneous sections or sections where the differences in permeability between layers is not large. It requires some special electrical equipment, but has the particular advantage that it can be applied with a little practice by persons having no previous experience with flow nets. It provides flow nets only for equilibrium conditions, and where certain boundaries must be assumed the solution provided by it should be considered as only qualitative. Figures 1c, 2b, 5 and 7 of this report are flow nets which were determined by this method and were taken from a report by G. W. McAlpin (6).

Electrical models with a conducting medium of very low resistance have been used (7) but they require a more specialized technique and are more limited in application than the models just described. The conducting medium employed was a thin metallic paper having a resistance of 0.02 ohms per unit width and unit length. Measurement of the small resistances involved in such tests required the use of a Kelvin bridge. This circuit eliminated the effect of contact resistances which made the Wheatstone bridge inadequate for such application.

The principal advantage of this type of electrical model is the uniform resistance of the conducting medium making possible the electrical determination of the ratio $\frac{Nf}{Nd}$ appearing in equation IV. This ratio is found, by a comparison of Ohm's Law with Darcy's Law, to be equal to the resistance per unit width and unit length of the conducting medium divided by the total resistance of the electrical model. Thus, the rate of flow through the seepage section duplicated by the electrical model may be found without the drawing of a flow net. The serious limitation of this type of model is the difficulty encountered in producing the desired boundary conditions at the various electrodes.

For seepage under equilibrium conditions, a flow net, as determined by either of the methods previously discussed, provides a complete description of the distribution of flow. However, for those cases in which the flow is non-equilibrium, that is, those in which the free surface varies with time, neither the rate at which the free surface changes at any point nor the rate of change of flow can be determined by either graphical or electrical means. In subdrainage, for instance, the free surface is continually lowered from some high position, produced by heavy rains, to some lower position at which equilibrium is established. In evaluating a given drainage installation, therefore, it is desirable to know the time required for the free surface to be lowered a given distance.

Sand models of non-equilibrium flow problems have proven one of the most successful means for investigating the change of flow conditions with time. They can be used to determine both the rate of draw-down of the free surface and the changing rate of discharge from the section. While there are undoubtedly many variations of model technique which are worthy of note, only one general procedure will be given here for purposes of illustration.

The proper scale for sand models is not critical. By means of a model law, to be given later, the results of a model may be applied with reasonable accuracy to a prototype of much larger scale. Economy and convenience will always limit the size of the model. On the other hand, if the model scale is too small, accurate measurements become difficult and capillary forces introduce unknown variations.

The ideal soil for such a model is a fine sand, having a coefficient of permeability in the order of 1×10^{-4} ft. per sec. Flow through such a sand will be sufficiently fast that tests will not require too long to perform. On the other hand, velocities through it will not be high enough to produce turbulence, even at points of flow concentration and high gradients. Such turbulence might occur if a coarse sand or a sand and gravel is used. If the coefficient of permeability and the non-capillary porosity of the sand used is known, the model law can also be used to extend the model results to other soil types.

The model may be constructed of any convenient material, such as wood or steel. In smaller models, one side of the model is frequently made of glass. Such models are particularly good for demonstration purposes. It is doubtful, however, whether the advantage of visibility makes the glass side worthwhile in larger models. Provisions in the model for entrance and exit of water depend of course on the section to be investigated. In determining the rate of flow through the model, it is convenient to carry the outflow either to a weir box for continuous measurement or to a small tank where the rate can be determined at any chosen time either by weight or by volume, depending on the magnitude of flow. To determine the distribution of flow, piezometer tubes are introduced through a number of holes at selected points in the side of the model. During a test, the water level in any tube indicates the total head at the point where that tube enters the model. Having thus provided for the proper boundary conditions and for measurement of the rate and distribution of flow, the sand is placed in the model. Extreme care should be taken to place the sand as nearly as possible at a uniform density, without stratification and without segregation.

In performing a test on a non-equilibrium model, the free surface is first brought to its highest position. Then, allowing the outflow to begin, readings of the rate of discharge and of the head indicated by each piezometer tube are taken simultaneously at logarithmic time intervals throughout the test until equilibrium is established.

On a plot of the cross section of the model, showing all piezometer readings at some selected time, the free surface existing at that time and any desired set of equipotential lines may be interpolated from the readings. The flow net for that instant may then be completed by graphically constructing flow lines at right angles to the equipotential lines, fulfilling the necessary conditions of a flow net as previously discussed. If the time selected for this plot was before equilibrium was established, then some of the flow lines will originate at the free surface as in Figure 2a. Using such a flow net and the corresponding rate of discharge as determined during the test, the coefficient of permeability of the sand may be determined from equation IV. In performing a series of model tests with the same soil, the coefficient of permeability should be determined in this manner for each test. This is necessary because, for various reasons, the coefficient of permeability of one soil will vary appreciably from time to time, and in extending the results of a test by means of the model law it is necessary to know the true permeability existing during that test.

If the elevation of the free surface is critical at some particular point or points, the elevations existing at those points may be found from the piezometer readings for each time interval during the test. From the data, curves may be drawn showing the draw-down at each of the selected points from the start of the test until equilibrium was established. Variation in the rate of discharge from the beginning to the end of the test may be plotted in a similar manner.

From those curves, similar curves expressing the rate of draw-down and the rate of discharge may be determined for the full-scale prototype and its proper soil by means of a model law developed for this purpose (8). In general terms, the model law states that for seepage sections of geometrically similar boundary conditions, the time required to produce the same proportional draw-down in each section varies inversely with the coefficient of permeability of the soil and directly with some typical dimension and the non-capillary porosity of the soil. Thus, if d_1 and

d_2 are the depths the free surface has been lowered at some particular point in the model and prototype respectively, and D_1 and D_2 are some typical dimension such as the drain depth, the same proportional draw-down would exist in both model and prototype if:

$$\frac{d_1}{D_1} = \frac{d_2}{D_2} \qquad \text{(V)}$$

For such a condition, according to the model law:

$$\frac{t_1 k_1}{m_1 D_1} = \frac{t_2 k_2}{m_2 D_2} \qquad \text{(VI)}$$

in which t_1 and t_2 are total elapsed times,
k_1 and k_2 are coefficients of permeability of the soils,
m_1 and m_2 are non-capillary porosities of the soils.

It follows further from the model law that:

$$\frac{q_1}{k_1 D_1} = \frac{q_2}{k_2 D_2} \qquad \text{(VII)}$$

in which q_1 and q_2 are the rates of flow per unit thickness of the section.

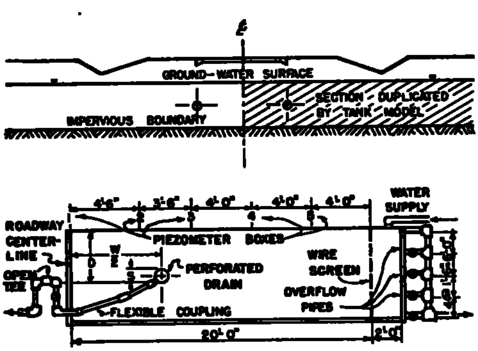

Figure 9. Details of Tank for Drainage Model Studies.

In recent tests conducted by the Joint Highway Research Project and reported to the Highway Research Board last year (8), this model procedure was used in studying a typical series of drainage installations. The model set-up used is shown in Figure 9. As shown, the model was 20 ft. long and 6 ft. deep. It had five sets of piezometers at the positions indicated, each containing nine tubes spaced at 6-in. intervals in a vertical line.

In determining the position of the free surface within the model at any desired time, a plot using the piezometer readings was made for each set of tubes. One such plot is shown in Figure 10. In this figure, the pressure at each piezometer connection was plotted against the depth of the connection below the original position of the free surface. By drawing and extending a line through those points, the depth from the original free surface to the point of zero pressure was found. That point of zero pressure was, of course, the new

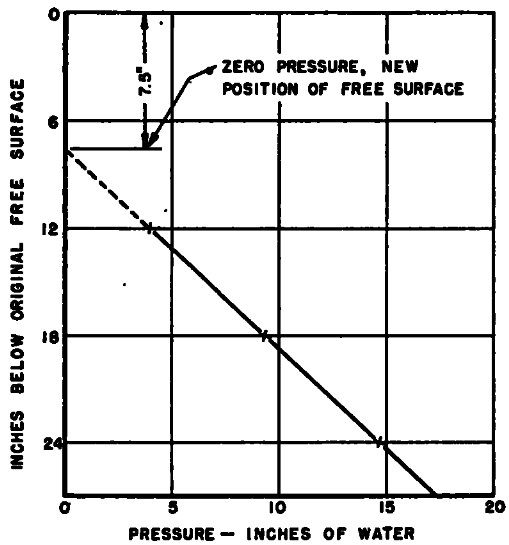

Figure 10. Plot of Piezometer Pressures to Determine New Position of the Free Surface in a Sand Model.

position of the free surface. By determining such points at each set of piezometer tubes, the free surface for the entire model was established for that time interval.

To illustrate the method of determining the coefficient of permeability for a test, as previously described, a typical cross-sectional plot of one of the models is shown in Figure 11.[2] All of the piezometer readings at a certain elapsed time, expressed in inches of water above the model floor, are shown in this plot. The free surface shown was found by the

[2] Due to the three-dimensional flow of water as it enters the perforations of the drain, the 39-in. equipotential line lies very close to the drain and does not appear in either Figure 11 or Figure 12.

method just illustrated. The dashed lines in the figure represent equipotential lines which were interpolated between the various piezometer readings. The flow net was completed graphically by drawing in the flow lines. In Figure 12[3] is shown the completed flow net. From the flow net, the ratio of flow channels to potential increments is seen to be $\frac{2.9}{12}$. The corresponding discharge measured during the test was 7.25×10^{-5} c. f. s. per foot. The depth to the drain, which corresponds to ploys a viscous fluid confined between closely spaced parallel transparent plates. For proper spacing of the plates, the flow of the viscous fluid would be comparable to the flow of water through soil. Howland (9), who recently proposed this method as a possible means for drainage investigations, derived a model law for the application of model results to the soil-and-water prototype. So far as is known, this method has not been applied to drainage studies, but doubtlessly it is worthy of consideration.

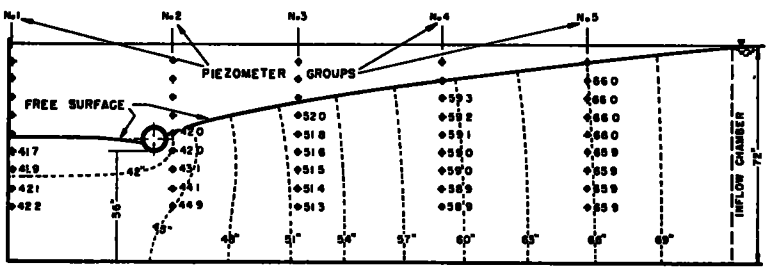

Figure 11. Plot of Piezometer Heads to Determine Equipotential Lines

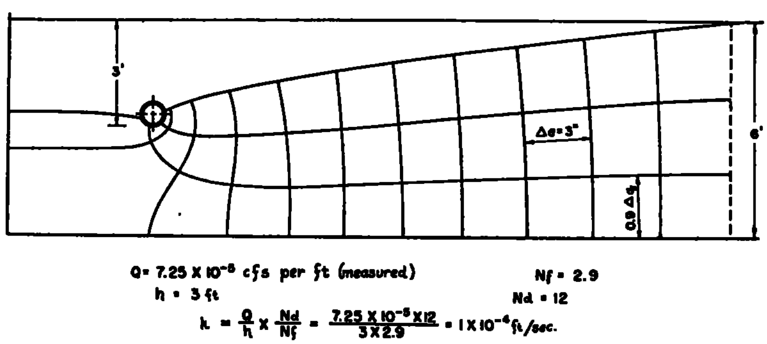

Figure 12. Flow Net for Sand Model to Determine the Coefficient of Permeability

the total head acting, was 3 ft. From equation IV, it follows then that the coefficient of permeability existing during that test was:

$$k = \frac{7.25 \times 10^{-5} \times 12}{3 \times 2.9}$$

$$= 1 \times 10^{-4} \text{ feet per second}$$

Sand models as just discussed are drastically limited by their size and cost, the space required for their construction and operation, and the time required to perform an appreciable number of tests. A method for making similar investigations of non-equilibrium problems, which appears to be much simpler, em-

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DISCUSSION

Mr. Philip Keene, *Connecticut Highway Department:* Probably the best use of the flow net in highway subdrainage today is the first one given by the authors—the visual picture of the flow and equipotential lines. Not only is the flow pattern of great aid to the soil mechanics engineer but it helps him in educating the engineers of design and construction with whom he must cooperate, who frequently have confused ideas regarding seepage, both gravitational and capillary.

As stated by the authors in their list of important properties of the flow net, flow lines change direction when crossing a boundary between two soils of different permeabilities. When the permeabilities have a ratio of 100, such as for coarse sand and fine sand, this change has a very pronounced effect on the flow net, as the flow lines will change their direction by about 80 deg. when crossing the boundary. Due to the rather small depths involved in highway subdrainage problems, one or two thin layers, 4 inches or less thick, will often be of great importance when constructing a flow net, if their permeability differs greatly from the main stratum or strata. In glacial soils, such as found in New England, such thin layers are often encountered; for example, two 2-in. coarse sand layers or two or three ½-in clayey layers in an 8-ft. deposit of fine sand. Their importance is known from the practical experience and they should be watched for when sampling in borings and test pits.

It is noted that in Figures 2(b), 5, 7 and 8 the point where the flow line is tangent to the constant ground-water surface was assumed to be about 21 ft. from the drain pipe. It would be interesting to know whether this was chosen simply to effect solutions to illustrate the paper or whether it is based on observations of actual installations having certain conditions of rainfall, soil permeability, etc.