

TRANSFER OF EARTH PRESSURES BY SHEARING STRESSES

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SYNOPSIS

The earth mass under consideration is assumed to be homogeneous and to possess a certain shearing strength. No complicated mathematics or elastic formulas are used in the paper, but solutions are accepted or rejected on the basis of their possibility and probability. The discussion is based on two statements: (a) The total lateral pressure on a vertical plane is constant and may be expressed by diagrams of constant area, though in the proximity of a "disturbance" such as a tunnel or a retaining wall, the shape of a pressure diagram is distorted, and (b) The influence of a "disturbance" is stronger close to the latter than far away from it. The paper consists of two parts. Part One deals with the stress distribution around a tunnel without lining whereas in Part Two different types of retaining walls and pressure on them are considered. In both cases earth pressures are transferred by systems of shearing stresses.

Limitations and Terminology of the Paper. A semi-infinite mass, homogeneous, i.e. made throughout of the same material, but not necessarily elastically isotropic is considered in this paper. No assumptions as to the applicability of Hooke's Law or to the continuity of strains are made. It is understood, however, that under certain circumstances the mass may fail, and that a failure consists in a separation of a part of the mass from the rest of it, or in plastic flow. The only characteristics of the material constituting the mass which will be needed in the discussion which follows is its shearing strength. If the mass is cohesive and its angle of internal friction approximates zero, the shearing strength s may be assumed equal to the unit cohesion of the material c .

In the semi-infinite mass under consideration circular holes (tunnels) may be pierced, or a part of the mass may be cut out by a vertical and a horizontal plane, and the vertical slope thus formed provided with a wall. Hereafter, such holes or slopes will be termed "disturbances."

Method of Attack In this paper attempts are made to reach solutions by simple reasoning free as far as possible from assumptions and complicated mathematical symbolism. The basis for accepting or rejecting solutions as used in this paper is their possibility and probability under the given circumstances.

Shearing Stresses as Caused by "Disturbances." In a semi-infinite mass without "disturbances" all vertical and horizontal

(lateral) pressures are balanced. Hence in such a mass there are no shears along vertical and horizontal planes. A "disturbance" changes the values of the vertical and horizontal pressures within the mass and hence sets up shearing stresses along vertical and horizontal planes, as along other planes as well.

Assume that the diagram in Figure 1a represents the excess of horizontal pressure on some vertical plane within a mass, this excess being due to a disturbance. Using Figure 1a construct the diagram of total shear (Fig 1b) caused by the disturbance, in the same way as similar diagrams are constructed for beams. Each horizontal ordinate E of the total shear diagram (Fig 1b) equals numerically the area in Figure 1a above the corresponding horizontal plane. Designate with z and $z + dz$ the depths of the planes MN and $M'N'$, respectively. Each ordinate E of the diagram of total shear (Fig. 1b) is balanced by the sum S of shearing stresses along plane MN from the given vertical plane to infinity so that $E = S$. The ordinate of the excess pressure diagram (Fig 1a) being the derivative of the total shear $\frac{\partial E}{\partial z}$, the condition of equilibrium of the horizontal layer between the planes MN and $M'N'$ would be:

$$\frac{\partial E}{\partial z} \cdot dz = \Delta S \quad (1)$$

In equation 1 ΔS is the change in total shear when passing from plane MN to plane $M'N'$. This change will be considered positive if directed toward the disturbance as in Figure 1, and negative if directed from the disturbance. These changes of total shear will be termed "shears ΔS " hereafter, the designations used for these shears in the figures being "+ shears ΔS " and "- shears ΔS " for positive and negative directions of these shears respectively. If the value of E is at a maximum, the diagram of total shear E possesses a vertical tangent at that level (Fig. 1b). It is obvious that when the value of E reaches a maximum, the change in shear ΔS equals zero (equation 1); in other words, shears ΔS change their signs at those horizontal planes where the total shear E is at a maximum.

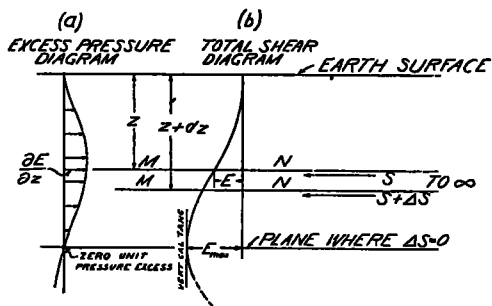


Figure 1. Shearing Stresses Balance Excess Pressure

Equally, the unit pressure excess $\frac{\partial E}{\partial z}$ (Fig. 1a) equals zero at those planes where the total shear E is at a maximum and $\Delta S = 0$. Since the vertical plane in Figure 1 is chosen arbitrarily, it follows, that along the horizontal plane where $\Delta S = 0$, the horizontal pressure is the same as in a mass without disturbance.

Transfer of Pressures. What is meant by the term "transfer of pressures" may be seen from the simple analogy, Figure 2. If a body A (weight W) is pressed between two plates B , it will not fall, provided the force of friction between body A and plates B balances the weight W . It is also obvious that the force of friction sets up shearing stresses both in body A and in plates B . In this case what is meant by the term "disturbance" corresponds to the hollow below

body A (Fig. 2). This analogy corresponds to the case of a tunnel in rock or stiff clay which may be in equilibrium without lining, or to the case of holes pierced in a masonry wall. If the overburden over the tunnel

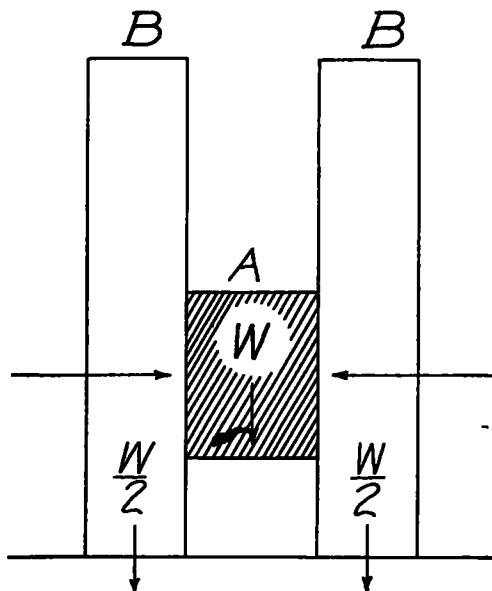


Figure 2. Shearing Stresses Transfer Pressure

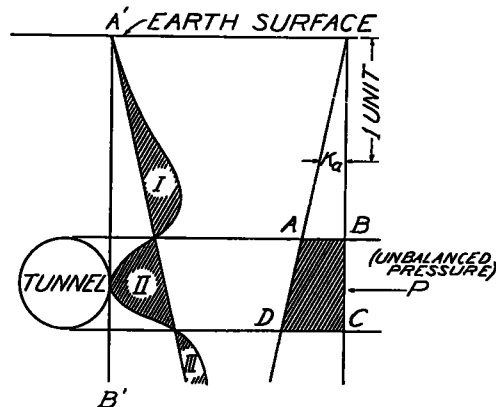


Figure 3: Lateral Pressure Diagram for Plane A'B'

does not fall down, it means that its weight is taken up by the shearing stresses and deposited at some other part of the mass. This transfer of pressures by shearing stresses is generally defined as "arching."

Basic Statements The discussion hereafter is based on the following two statements.

Statement 1. The total lateral pressure on a vertical plane within the mass is constant and may be expressed graphically by a diagram of constant area. Assuming that the diagram of lateral pressures in a mass without "disturbance" is triangular (Fig. 3, right), it should be concluded that the diagram of lateral pressures on a vertical plane tangent to a circular hole in Figure 3, left, must have the same area. Theoretically speaking, both diagrams in Figure 3 should be extended down to infinity. In reality, however, the "disturbance" practically has no influence on the stress distribution in the mass at a rather short distance. Therefore, in equalizing both diagrams in Figure 3 only a limited part of the infinite diagram (Fig. 3, right) may be considered. Statement 1 is evidently based on simple statics.

Statement 2. The influence of a "disturbance" on the stress distribution in the mass is stronger close to the "disturbance" than far away from it. At an infinite distance from the "disturbance" the latter has no influence whatsoever so far as the stress distribution in the mass is concerned. In the opinion of the writer, Statement 2 is self-evident. The vertical plane AB tangent to the circular hole in Figure 3 is the closest to the disturbance and so are the horizontal planes tangent to that hole; hence the maximum unit shearing stresses developed along these planes should be larger than the maximum unit shearing stresses at some other vertical or horizontal planes. It should be remembered in this connection, that the unit shearing stresses at the axis of symmetry of a "disturbance" (for instance, circular hole in Fig. 3) vanish.

Self-Balanced Systems of Shearing Stresses. Since the total pressure on different vertical planes within the mass is constant, pressure excess I + III in Figure 3, left, must equal pressure deficiency II. The sum of the vertical shears on any vertical plane from the earth surface to infinity adds to zero. Note, however, that the unit horizontal shears at each and every point along that vertical plane are equal to the vertical shears; hence their sum also adds to zero.

Lateral Pressure Diagram. It is often assumed that the diagram of lateral pressure

in a natural earth mass is triangular (as in Fig. 3, right). This assumption is no more than a possibility since very little is known about the value of the ratio of the horizontal to the vertical pressure in the earth. The reason is obvious: stresses within a body cannot be measured, and attempts to do so fail. Displacements caused by stresses can be measured, however, and stresses estimated from them; but in a natural earth mass these displacements have been already completed long ago and cannot be measured. It is assumed in this paper that the ratio of the lateral pressure to the vertical pressure in a

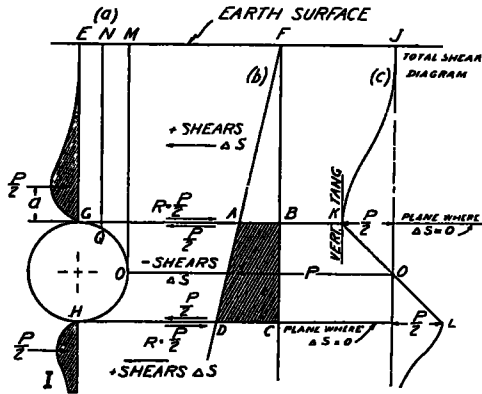


Figure 4. Shearing Stresses Around the Tunnel

semi-infinite earth mass (designation K_0) may be expressed by the Rankine formula:

$$K_0 = \tan^2 \left(45^\circ - \frac{\phi}{2} \right) \quad (2)$$

where ϕ is the angle of internal friction of the material of the mass. The value of K_0 as given by equation 2 is possibly slightly less than the actual ratio in question, and to correct it a somewhat larger value of the angle of repose instead of the angle of friction could be used in equation 2.

In the following Part 1 and Part 2 of this paper all problems are conceived as two dimensional.

Part I

CIRCULAR TUNNEL WITHOUT LINING

Figure 4 represents conditions of equilibrium of the earth mass surrounding a tunnel with-

out lining. Diagram formed by oblique line FD and the vertical FC (Fig. 4b) is the lateral pressure diagram at an infinite distance from the tunnel. The upper part of it FAB is balanced by the corresponding part of the symmetrical earth pressure diagram (on the other side of the center line of the tunnel) In the same way all pressures below level CD are balanced. Trapezoid $ABCD$ (cross hatched in Fig. 4b) corresponds to the unbalanced earth pressure; its resultant P is applied somewhat below the middle of the altitude of trapezoid $ABCD$. For the purposes of the present paper it is accurate enough to assume that the resultant P acts at the middle of the altitude BC . Its value equals:

$$P = [K_a \gamma h] [2r] = 2K_a \gamma hr \quad (3)$$

where h is the depth of the horizontal diameter of the tunnel and γ the unit weight of the material of the mass.

Separate area $GBCH$ from the rest of the mass and consider its equilibrium. Force P would be then balanced by equal reactions $R = \frac{1}{2}P$. In the case of a deep tunnel this approximation is accurate enough. Consider now conditions of equilibrium of part $EFBG$ which is acted upon by the horizontal force $\frac{P}{2}$ acting towards the tunnel. Its equilibrium requires the existence of a pressure $\frac{P}{2}$ distributed along face EG . The position of the resultant of this pressure cannot be determined statically. Its approximate diagram based on statement 2 is shown in Figure 4a.

In an analogous way considering the equilibrium of the mass below level HC , it may be concluded that there is distributed pressure $\frac{P}{2}$ acting on the vertical face HI extended down to infinity. Since length HI thus extended is larger than length EG , the maximum unit pressure below the tunnel (maximum ordinate of the pressure diagram) is possibly larger above the tunnel than below it.

On the basis of these considerations, the total shear diagram has been constructed (Fig. 4c). As stated in discussing Figure 1, each horizontal ordinate in Figure 4c equals numerically the area of the pressure diagram

in Figure 4a above a given horizontal plane. The value of the total shear equals zero at the level of the horizontal diameter of the tunnel. In reality, since area $ABCD$ is trapezoidal, the zero total shear is located somewhat below this level.

Above plane GK and below plane HL (horizontal planes tangent to the tunnel) the shears ΔS are directed toward the disturbance and are positive (+ shears ΔS). Between planes GK and HL the shears ΔS are negative (- shears ΔS). Thus the unbalanced pressure P as expressed by the trapezoid $ABCD$ is transferred by shearing stresses

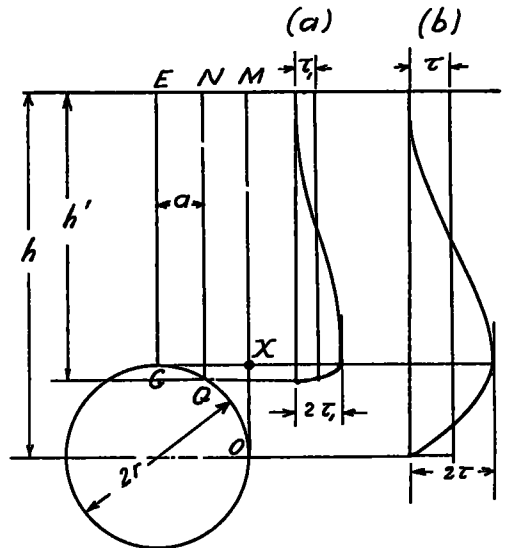


Figure 5. Vertical Shears along planes OM and NQ

to the faces EG and HI , the latter being extended indefinitely downwards. The total pressure on the vertical plane EI (centerline of the tunnel) equals the total pressure in the undisturbed mass as expressed by lines FD and FC in Figure 4(b).

Vertical Shears Supporting the Weight of the Overburden. In the case of a deep tunnel the weight W of the overburden $EGOM$ (Fig. 5) approximately equals:

$$W = hr\gamma \quad (4)$$

where h is the depth of the horizontal diameter of the tunnel and γ the unit weight of the material in which the tunnel is built. This

weight is supported by the vertical shear along face *OM*, the average unit shearing stress along this face being.

$$\tau = \frac{W}{h} = \frac{hr\gamma}{h} = r\gamma \quad (5)$$

Designating with *a* the distance of the vertical plane *NQ* (length *h*, Fig. 5) from the centerline of the tunnel, it may be written that the average unit shearing stress τ_1 , along that plane is.

$$\tau_1 = \frac{ah'\gamma}{h} = a\gamma \quad (6)$$

which means that for a rather deep tunnel the value of the average unit shearing stress increases from the centerline of the tunnel towards the plane *OM* tangent to the tunnel following approximately a straight line law.

Two questions are to be answered now. (a) what is the value of the maximum unit shearing stress τ_{max} along plane *OM*, and (b) at what depth from the earth surface is this maximum unit shearing stress applied?

Value of the Maximum Shear Along a Vertical Plane. If the shear distribution along a vertical plane were triangular, the maximum unit shear would be twice the average shear. The probable actual shapes of the diagram in question are shown in Figure 5 in curved lines. It is probable that the value of the maximum unit shear is close to twice the average unit shear. Reverting to equation 5 it may be seen that for deep tunnels the value of the average unit shear practically does not depend on the depth of the tunnel; and it is probable (but not perfectly sure) that the same is true of the value of the maximum unit shear.

The shear diagrams as in Figure 5(a) and (b) may be obtained first by transforming areas of the overburden or a part thereof into rectangles of equal area and afterwards transforming these rectangles into shear diagrams of equal areas. The horizontal ordinates of these shear diagrams may be increased or decreased according to the scale of stresses used. As to the location of the maximum unit shearing stress along vertical plane *OM* it may be expected to be close to point *X*, which is the point of intersection of the vertical and horizontal planes tangent to the tunnel. This follows from statement 2.

Maximum Possible Shear Around the Tun-

nel Reverting to Figure 5, it is obvious that there are four points *X* close to which the maximum possible shearing stress takes place.. These points are located symmetrically with respect to the tunnel.

The triangular pressure distribution and the value of K_a as determined by equation 2 may hold for an undisturbed semi-infinite mass; but it is not known how the value of K_a may change next to a disturbance. If it does not change or changes but a little, the value of the maximum unit shearing stress in the vicinity of points *X* may be estimated. In fact, the vertical pressure at point *X* being (*h-r*) and the vertical (or horizontal) shear $2\gamma r$, the radius of the corresponding Mohr's circle (Fig. 6) is the maximum possible unit shear. For purely cohesive materials when the value of K_a tends to unity, the value of

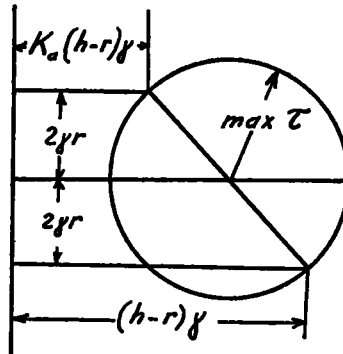


Figure 6. Mohr's Circle for points *X*

the maximum possible unit shear around the tunnel tends to $2\gamma r$. Placing:

$$2\gamma r = c \quad \dots \quad (7)$$

the maximum radius of a tunnel safe against plastic flow in cohesive material (unit cohesion *c*) is:

$$r = \frac{1}{2} \cdot \frac{c}{\gamma} \quad \dots \quad (8)$$

It should be remembered that the ratio $\frac{c}{\gamma}$ is very much used in the theory of stability of slopes.

Reverting to equation 5, it may be stated that if a tunnel without lining has a radius twice as large as furnished by equation 8, the overburden probably will not fall down into

the tunnel as a unit; but to prevent plastic flow at points *X* the radius of the tunnel should be estimated using equation 8.

Transfer of the Weight of the Overburden. Taking moments of the forces acting on that part of the mass between the vertical and horizontal planes passing through the center of the tunnel, we have (Fig. 7):

$$\frac{P}{2} \left(a + \frac{r}{2} \right) - W \left(x + \frac{r}{2} \right) = 0 \quad (9)$$

where: *a* is the vertical distance from the top *G* of the tunnel to the resultant of the pressure $\frac{P}{2}$ distributed along face *EG* (Fig. 7), and *x* is the horizontal distance from the side

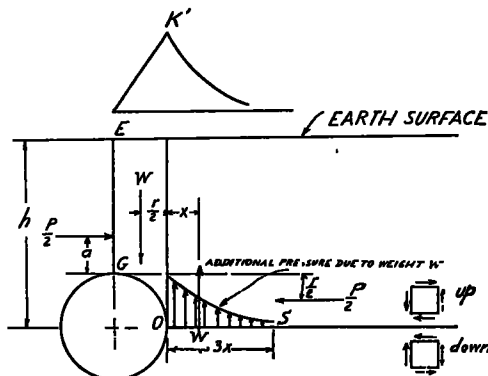


Figure 7. Transfer of the Weight of the Overburden

(haunch) of the tunnel *O* to the resultant of the additional reaction (pressure) acting on the horizontal plane *OS* due to the weight of the overburden *W*. (Fig. 7). The pressure *P* (as shown in Fig. 4) was considered uniformly distributed along face *BC* when writing the condition of equilibrium (9).

The distance *x* can then be defined as:

$$x = \frac{P}{2W} \left(a + \frac{r}{2} \right) - \frac{r}{2} \quad (10)$$

Placing now:

$$P = 2K_a \gamma hr \quad \text{and} \quad W = \gamma hr,$$

the value of *x* would be:

$$x = K_a \left(a + \frac{r}{2} \right) - \frac{r}{2} \quad (11)$$

The value of *K_a* in a semi-infinite mass cannot be larger than 1. Hence for this limiting case

$$x = a \quad (12)$$

In virtue of statement 2 the value of *a* and hence of *x* cannot be large. Placing, for example, $a = \frac{2}{3} r$ and considering that appreciable additional pressures on plane *OS* cannot be large beyond a distance about $3x$ from the tunnel, it may be concluded that at a short horizontal distance from the tunnel, somewhat about twice its diameter, the influence of the disturbance on the stress distribution in the mass probably is negligible.

Compressive Stress around the Tunnel

The value of the compressive stress at the top of the tunnel due to "natural" triangular pressure distribution which existed in the mass before the construction of the tunnel, is:

$$\sigma = K_a (h - r) \gamma \quad (13)$$

and that at the sides (point *O* in Figs. 4 and 7):

$$\sigma = h \gamma \quad (14)$$

In addition to these stresses compression due to the disturbance of the mass should be estimated

Provided $R = \frac{P}{2}$ (Fig. 4b), the shearing

stress distribution above plane *GB* in a given material probably depends little on the distribution of pressure *P* along face *BC*; the same may be said of the tangent to the total shear diagram at point *K* (Fig. 4c). Hence it is always possible to visualize such a pressure distribution along face *BC* as to furnish a continuous curve instead of straight line *KOL*, this new continuous curve being a continuation of the continuous curve *JK* (Fig. 4c). The term "continuous curve" as used here defines a curve having at each point one tangent only. The composite curve thus constructed must have a vertical tangent at point *K* (Fig. 4c) because the total horizontal shear across the mass is at a maximum at that point. This result is in accordance with the statement already advanced that planes *GK* and *HL* (Fig. 4) are planes where shears $\Delta S = 0$. In its turn, a vertical tangent to the total shear diagram at point *O* (Fig. 4c) means that the additional unit pressure at the top

of the tunnel due to "disturbance" equals zero. The compression stress at point 0 (Fig. 4a) equals AB as in Figure 4(b), and the maximum compression stress acts somewhere above the top of the tunnel.

The point where the maximum unit compression σ_{max} takes place, can be located graphically in an approximate way by tracing freehand a curve from point B up (Fig. 8) to bound an area equal to $\frac{P}{2}$, P being the force expressed by area $ABCD$ (consult also Fig. 4). A vertical tangent to this curve (point T in Fig. 8) defines the location of the maximum unit compression stress above the tunnel.

The ordinates of the sketch at the top of Figure 7 are sums of the vertical shears acting on the vertical planes between the earth surface and the horizontal plane passing through the center of the tunnel. Point K' is analogous to point K in Figure 4(c), the tangent at point K' is not horizontal, however. This means that the vertical unit pressure at point 0 (Fig. 7) is larger than at any other point of the horizontal plane passing through the center of the tunnel. To estimate this pressure, let us assume as was done previously in discussing Figure 7, that

$$x = \frac{3}{2} r,$$

and a straight-line pressure distribution, then $OS = 3x = 4.5r$. Designating the additional unit compression stress at point 0 with σ_0 :

$$\frac{1}{2} \cdot 3x \cdot \sigma_0 = W = h\gamma r \quad (15)$$

from which:

$$\sigma_0 = 0.45\gamma h \quad (16)$$

This value is probably somewhat exaggerated because of the curvature and infinite length of the pressure diagram. A value of $\sigma_0 = \frac{1}{3} \gamma h$, i.e. about one third of the basic unit compression stress probably would be satisfactory. The additional compressive stress at point 0 (Fig. 7) increases, however, as the value of K_a decreases.

The unit compression stress at the bottom H of the tunnel (for letter H , see Fig. 4) is defined by the ordinate CD in Figure 4b. The additional unit compression stress due to disturbance, acts somewhat below point H .

Danger of Tension Failure Line marked γz in Figure 8 shows the vertical tension stress due to the weight of the overhanging material above the tunnel. There will be a tension failure and falling down of the overhanging material at the point where the sum of the absolute values of the compression stress σ and the tension stress γz_0 (Fig. 8) is more than approximately twice the shearing strength. The corresponding Mohr's circle for plastic, equilibrium of a cohesive material

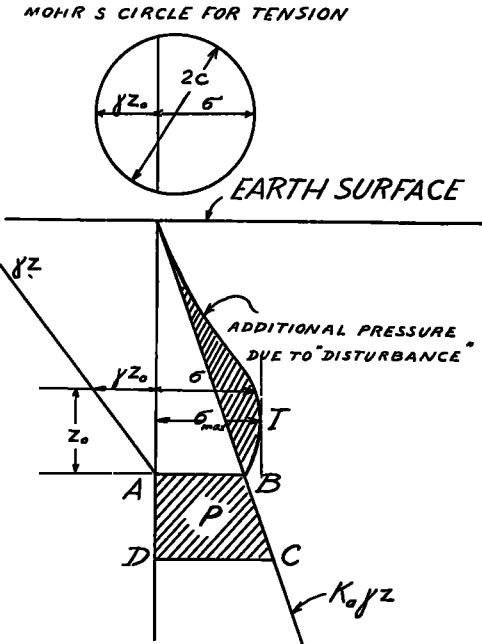


Figure 8. Compression above the Tunnel; Danger of Tension Failure

is shown in Figure 8, top. Obviously, after a tension failure occurs, there may be still a possibility of recurrence.

Part 2

PRESSURE ON RETAINING WALLS

In an earth mass without disturbances there are neither horizontal nor vertical shearing stresses. When a tunnel without lining is built, a part of the mass is pushed away from the tunnel, whereas the other is pushed toward the tunnel. This motion "mobilizes" the shearing stresses in the mass in

two opposite directions. In an analogous way, when a retaining wall does not move at all, there are neither vertical nor horizontal shearing stresses in the backfill. A small horizontal translation of a vertical retaining wall is required to produce the conventional triangular Rankine pressure distribution. Some horizontal and vertical shears which necessarily appear in this connection, may be neglected; and the lateral pressure on an unmovable retaining wall (which corresponds to an undisturbed earth mass, (equation 2) may be assumed to be triangular but without small horizontal and vertical shears in the backfill.

Translating Retaining Wall. If a retaining wall does not move, the value and the point of application of the thrust *T* (Fig. 9) are controlled by the "natural" triangular pres-

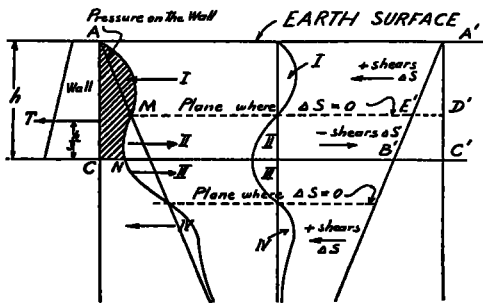


Figure 9. Lateral Pressure on a Translating Wall

sure distribution, i.e. the distribution which exists in a semi-infinite mass made of the same material as the backfill. For lack of information about this "natural" pressure distribution, the Rankine triangular pressure distribution has been conventionally substituted for it (equation 2). If the wall translates, the lower part of the backfill pushed by larger lateral pressure follows the wall rather closely, leaving the upper part behind. This lag in motion creates horizontal shears: from the wall,—in its lower part and toward the wall in its upper part. The shears above plane *E'D'* (where the shears $\Delta S = 0$) have to be balanced by the corresponding increase in pressure I, whereas below that plane there is a decrease in pressure II. If the wall stops in its translation, areas I and II are equal, provided the frictional resistance at the base of the wall re-

mains constant. The resultants of the pressures expressed by areas I and II create a couple balanced by the resistance of the wall. Areas I and II upon being superimposed on the triangular pressure diagram, furnish a pressure diagram *AMNC* similar to that obtained in corresponding experiments and observations.

Thus the wall is in equilibrium; for the equilibrium of the mass a couple III-IV balancing the couple I-II is needed, however, The final pressure distribution in the mass is shown in Figure 9, left.

Pressure on the Bracing of an Excavation. In this case the transfer of pressure by shearing stresses takes place in a way analogous to that in the case of a translating wall (Figs. 9 and 10). It should be noticed that in the case of a rigid translating wall there is gen-

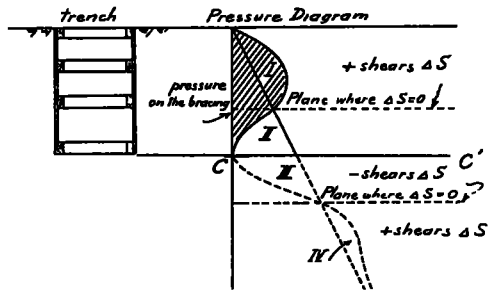


Figure 10. Lateral Pressure on the Bracing of an Excavation

erally some unit pressure *CN* at the base of the wall (Fig. 9). This is because of insufficient flexibility or insufficient translation of the wall which does not permit shearing deformations in the upper part of the wall to be fully developed; hence the corresponding shearing stresses and the increase in pressure I do not reach a possible maximum. If, however, the wall is flexible enough, as is the case of bracing, pressure I (Fig. 10) is fully developed, its limit being reached when unit pressure at point *C* becomes zero.

It should be noticed that the solutions given in Figs. 9 and 10 are qualitative rather than quantitative since the true shape of the curves shown in these figures depends on the properties of the earth material.

Pressure on Steel Sheet Piling. The active pressure on steel sheet piling bulkhead provided with a tie fixed to anchor

piles or anchor plates will be discussed (Fig. 11). In this case there are two "disturbances": (a) the tie and (b) the passive resistance of the earth mass. Both of them act against the "natural" earth pressure and hence cause active positive shears in the mass (+ shears ΔS in Fig. 11). To locate the planes where $\Delta S = 0$, the deflected structure should be constructed as in Figure 11b. Its points of inflection (marked P.I. in Fig. 11b) control the location of the planes where $\Delta S = 0$. The values of ΔS are negative in the

lating wall (above plane CC' , Fig 9) The lower part of the diagram of active pressure on the bulkhead (below some plane CC' , Fig. 11) resembles the diagram of pressures in the earth mass below plane CC' (Fig. 9).

The translating retaining walls and bracings of excavations can be analyzed using the Rankine formula for the determination of the total thrust acting on the wall or bracing. The point of application of this thrust is higher than the third point of the height, however, and corresponding corrections should be introduced. Notice the analogy in location of areas I, II, III, and IV in Figures 9, 10, and 11 (and in Fig. 3 as well).

CONCLUSION

In an unloaded undisturbed semi-infinite mass there are neither vertical nor horizontal shearing stresses. A "disturbance" causes such stresses. In an earth mass thus disturbed the zones of action of positive and negative shearing stresses (or more accurate, of their changes ΔS) are to be located. The negative shears ΔS decrease the lateral unit pressure existing in an undisturbed earth mass whereas the positive increase it. In this paper the case of a tunnel without lining and three cases of retaining walls have been discussed. It was found that qualitatively all cases discussed are particular cases of one and the same problem as it may be seen from the pressure distribution curves (Figs. 3, 9, 10, 11). The approach to the solution of the problem as used in this paper probably may be applied to the study of the redistribution of pressures in earth as caused by all kinds of retaining walls, deep excavations; cuts; embankments; and their slopes.

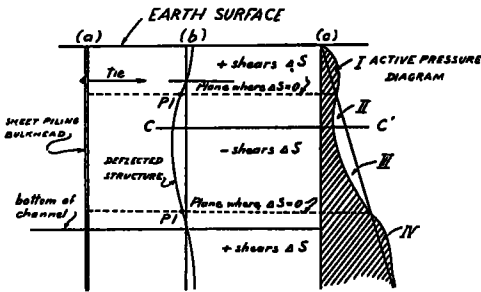


Figure 11. Active Earth Pressure on a Bulkhead

span between the tie and the bottom of the channel. These negative shears decrease the pressure in the span in question and transfer it to the tie and the bottom of the channel. Accordingly the bending moment in the span is considerably decreased.

Simple examination of Figure 11c shows that the analysis of a sheet piling bulkhead using the Rankine triangular pressure distribution is entirely on the safe side. At the same time this examination shows that the upper part of the bulkhead (above some plane CC' , Fig 11) behaves qualitatively as a trans-