# THEORETICAL ASPECTS OF WATER ACCUMULATION IN COHESIVE SUBGRADE SOILS

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## SYNOPSIS

A scientific attack on the water accumulation problem must concern itself with the following questions

1 What is the ultimate water sorbing capacity of a soil and how does it depend on its intrinsic surface chemical and physical properties, on its past history, and on its present temperature and pressure conditions?

2. What means are available to nature for movement of water in soil, and which of the available means play the greatest role under certain conditions of soil, soil moisture, atmosphere, and temperature?

3. By what mechanism can water under tension (capillary) and water under adsorption stresses (solid and plastic films) be changed into free water, the form in which it affects bearing power most?

A discussion of these theoretical aspects of non-gravitational water movement in soils is presented with the hope that it will stimulate comparison with general observational and experimental data

The accumulation of water in subgrades and bases and the concomitant decrease or eventual loss of bearing power is a fact often observed and always dreaded in airport and highway engineering This water accumulation is not restricted to regions of humid and subhumid climates nor to the presence of high ground water tables; rather, some of its most impressive examples have been observed in deserts and at considerable distances from the ground water table. Of course, water accumulation presumes the availability of water in one form or other; the more easily water is available the greater a rate of accumulation can be expected. It is important for any theoretical analysis and practical evaluation that "available water" include not only the "visible" capillary and ground water, but also the "invisible" film water and water vapor which are always present in soils even under and conditions

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THE WATER AFFINITY OF SOIL MATERIALS

The water affinity of soil material can be measured in a number of different ways A few typical methods in use at the present time are.

1. Water intake and swelling of a soil sample placed on a porous plate and kept in capillary contact with free water;

2 Water absorption from an atmosphere held at a predetermined temperature and water vapor pressure (hygroscopicity).

3. Viscosity determination especially of dilute clay suspensions and calculation of the adsorbed water hull by means of theoretical formulae of the type developed by Einstein;

4 Field-, vacuum-, centrifuge-, and other moisture equivalents,

5. Consistency tests.

6. Heat of wetting.

Each of these tests gives different numerical data; however, the data obtained by means of any or all available tests for water affinity of different soil materials, prove that this property depends on.

1. The internal surface area per unit weight of the soil material, in other words, on its fineness and gradation,

2 The physical and chemical character of this internal surface, which is a function of the mineral and non-mineral soil constituents their adsorbed ions and their surface coatings.

3 The state of aggregation and the density of the soil system.

Since every one of the methods named gives only a portion of the general picture of the water affinity, it is good practice to employ several conjointly if the general picture is required as a basis for intional soil stabilization.

For engineering purposes the most suitable type of test is one in which the soil system is permitted to saturate itself in capillary contact with free water A good example is the California bearing ratio test For such purposes it is of greater importance to ascertain what the water does to the volume and the bearing power, than how much water is taken in; hence, the swelling and bearing capacity determinations

The presence of water in a soil system is in itself not objectionable so long as it does not affect the total volume or the bearing power. In fact, water might even be beneficial, such as capillary water in sand, which adds cohesive properties Water intake studies have, therefore, greatest engineering value only if they are correlated with resulting changes in volume and stability.

Soils normally do not lose their stability if they do not swell; swelling is an expression of the water affinity of a soil material, which depends on the factors enumerated By emphasizing one or another of these factors, or group of factors, several swelling theories have been proposed. These have been discussed in detail in other publications and need not be repeated here Soil swelling can normally be traced to either or both: michanical factors, such as resiliency of mica and of certain types of organic matter, and surface-chemical factors; of which the latter seem to be of greater interest for our problem. The simplest concept of surface-chemical swelling is to consider it as a combination of:

a) Hydration, or strong binding of a number of water molecules especially on the exchangeable cations surrounding the soil particles and to some extent on the soil particles themselves, which hydration is responsible for the heat of wetting, and

b) osmotic pressure of the adsorbed ions, which is an expression of their kinetic forces balanced by the electrostatic attraction of the charged particle surface.

The energy with which soil materials adsorb water indicates what work can be performed by the soil to satisfy its water affinity.

## ENERGIES AND MECHANISMS INVOLVED IN THE MOVEMENT OF WATER THROUGH SOILS

Energy is equivalent to work; it is force times distance The energy required to move an object from one energy level to a higher one equals the difference between the two energy levels and the resistance overcome on the way. The latter is responsible for energy dissipation in the form of heat.

What energies are available to perform the work of moving water into a soil, aside from or in addition to gravitational movement? At the present time only hydration and osmotic energy are known to exist for this purpose. What is the order of magnitude of these energies?

a) The Hydration Energy: Hydration energies usually are expressed as heat of wetting, if we assume an intermediate heat of wetting of 10 cal perg for a clay having a surface area of 600  $\times$  10<sup>3</sup> sq cm.per g. we obtain a wetting energy of  $\frac{10}{6 \times 10^5} = 1.67 \times 10^{-5}$  cal per sq cm. = 698 erg per sq. cm ; the surface energy of water at 20° C. 1s = 72.8 erg per sq cm If a water-air surface is formed in the process of spreading water over the soil particles under consideration only 698 - 728 = 625 erg per sq. cm. of energy are available for water attraction per sq. cm. If the latter energy can be utilized completely for spreading water in a direction opposite to that of gravitational pull, we can write the mechanical energy as:  $625 \operatorname{erg} \operatorname{per} \operatorname{sq} \operatorname{cm} = 6.25 \times 10^{-5} \operatorname{Joule}$ 

# $= 6.25 \times 10^{-5} \times 101.97$ g meters

$$= 6.37 \times 10^{-3}$$
 g meters

If the thickness of the water film to be lifted is  $10^{-5}$  cm., then the height to which this film could be lifted, if there were no frictional resistance, is:  $6.37 \times 10^2 = 637$  meters or 2170 ft. in the case that a new water surface is formed; and 711 meters or 2335 ft. in the case that no new water surface is formed. In hydraulic language we may then say that we have 2170 and 2335 ft, respectively, of a water head available for overcoming friction and for lifting water in the case of a soilmineral composition possessing a wateraffinity of  $1.67 \times 10^{-3}$  cal. per sq. cm Experimental determination of the heat of wetting requires a certain amount of skill and apparatus which may not always be available in soil testing laboratories. It is fortunate, therefore, that a good average value of this property may be calculated according to Zunker from the hygroscopic moisture of a soil over a 10 per cent sulfuric acid solution. The relationship is.

Heat of wetting =  $50 \times \text{hygroscopicity cal}$ per g (1)<sup>1</sup>

b) Osmotic Energy: The osmotic energy of a dispersed system is usually represented as:  $\pi_0 V = n R T$ , where

- $\pi_0$  = osmotic pressure, usually expressed in atmospheres
- V = volume of the system, usually expressed in liters
- n = number of particles expressed as multiples of the Avogadro number dispersed in V
- R = gas constant
- T = absolute temperature

A formula also exists for cases in which the particles can move in two directions only (on a surface), instead of being able to move in three directions (volume) as is considered in the general case. Since water films in soils are generally polymolecular the volume formula will be used in the present derivation, although we must start by expressing concentration as a function of surface In non-saline soils, the osmotic pressure is usually due to the exchangeable ions located on the surfaces of the soil particles

According to Baver and Whitt (2) a clay possessing an exchange capacity of 60 milliequivalents per 100 g has a surface area of about  $600 \times 10^3$  sq cm. per g. This gives  $10^{-9}$ equivalents per sq cm. If the thickness of the water film under consideration is  $10^{-5}$ cm then

<sup>1</sup> Numbers in parentheses refer to the list of references at the end of the paper

# SOILS

$$\pi_0 = \frac{10^{-9} \times 82.06 \times 293}{10^{-5}}$$

 $= 82 \times 293 \times 10^{-3} = 24$  atmospheres

Since 1 atmosphere equals about 1000 cm. of water head,

$$\pi_0 \approx 2400 \text{ cm. or } 80 \text{ ft}$$

The osmotic pressure created is inversely proportional to the thickness of the water film Therefore, if the thickness of the film is  $10^{-6}$  cm then  $\pi_0$  equals about 800 ft The osmotic head is to be added to the hydration head.

c) The So-called Capillary Potential: When water is adsorbed into the internal surface of a porous material from a contacting liquid water reservoir, the surface tension of the water resists an increase of the total air-water interface of the system. As a result, a water column is carried along with the advancing wetting sheet to a height which is a function of the surface tension of the water, the geometry and dimensions of the pores and the affinity of the water for the pore wall or for the water film covering the pore wall.

For the case of a capillary, these relationships are shown by the formula.

$$H=\frac{2T}{r\rho g}\cos\alpha$$

where

H =capillary rise or depression

- T =surface tension of the liquid
- r = radius of capillary
- $\alpha$  = angle of contact between hquid and capillary wall

 $\rho = \text{density of liquid}$ 

g =gravitational constant

The angle of contact is a measure of the affinity of the liquid for the material of the capillary; if this affinity is great, then the angle of contact is zero and its cosine equals unity. As this affinity decreases, the cosine of  $\alpha$  passes through zero at  $\alpha = 90$  deg and becomes -1 at  $\alpha = 180$  deg A zero angle of contact indicates that the affinity of the liquid for a particular solid is equal to or greater than the affinity of the liquid molecules to their own kind Proper admittance of the primary role of the affinity of water for the

pore wall still leaves "capillarity" as an important water supplying mechanism, by determining how much of this affinity may be utilized for the upward movement of a water column.

The formula given above for the height of capillary rise has been developed from the static conditions obtaining after capillary rise has been completed; at that time a water column of the length H is held by the surface tension force  $2r\pi T$  hence

$$r^{2}\pi H\rho g = 2r\pi T \text{ dynes}$$
$$H = \frac{2}{r} \cdot \frac{T}{\rho g};$$

It should be noticed that  $\frac{2}{r} = \frac{2 r \pi H}{r^2 \pi H}$ ; this

means that the capillary height H is proportional to the ratio of wetted capillary surface sto the volume v of capillary hquid. This relationship which permits writing of a more general formula for capillary rise, viz

$$H = \frac{S}{v} \cdot \frac{T}{\rho g}$$

seems first to have been recognized by Mitscherlich; it was further developed by Zunker (3), who, under assumption of spherical soil particles, arrived at the formula

$$H = 60 \frac{T}{\rho g} \cdot \frac{1-p}{po} U \,\mathrm{cm},$$

where

U = specific surface

p = porosity

po = stress and air-free portion of porosity, i.e., that part of the pore space which is filled with free water, excluding the strongly adsorbed water zone closest to the soil particles, and also air spaces.

If the capillary rise phenomenon is considered from an energy instead of a force point of view, we see that during the rise a surface energy of  $2 r \pi TH$  ergs has been utilized, this expenditure resulting in an increase of the potential energy of the raised water column of  $r^2 \pi \frac{H}{2} g_{\rho}$ . If all

surface energy involved had been changed into potential energy of the water column then:  $2r\pi TH$  would equal  $r^2\pi \frac{H^2}{2}\rho g$  or H would equal  $\frac{4T}{r\rho g}$ .

Since it actually equals  $\frac{2T}{r\rho g}$ , one half of the total surface energy involved must have been used to overcome the viscous friction This is a very interesting example of an equipartition of energy It would be of practical importance if this equipartition could be shown to hold for other forms of antigravitational water movement.

# Rate of Water Movement

It is important to know not only that a certain reaction occurs, but also how fast it occurs. In fact, the rate of reaction is often its most significant feature from a practical point of view The speed of water movement into a soil is a function of the existing potential (difference between the energy levels of water at different positions in the system), the type of flow (capillary, sheet, vapor) and the distance from the water source.

a) Capillary Upward Flow: If we consider the so-called capillary potential h as that portion of the affinity potential which is available for saturated pore flow and employ it in the Darcy formula for flow of water through soils, we obtain the following rate formula for anti-gravitational flow.

$$\frac{dz}{dt} = \frac{k}{n} \left( \frac{h-z}{z} \right)$$

in which

- dz = distance differential in vertical direction
- dt = time differential
- k = coefficient of permeability of soil
- n = porosity of soil
- h =capillary potential
- z = distance of capillary meniscus from ground water level.

By integration we obtain:

$$t = \frac{nh}{k} \left( \ln \frac{h}{h-z} - \frac{z}{h} \right)$$

where t is the time required for the meniscus of the capillary water to rise to the height z above the free water level. For the purpose of an illustrative calculation the following parameters derived from experimental data on a K-Putnam soil are employed (4):

> $k = 1 \cdot 10^{-7} \text{ cm per sec}$ n = 0.56 $r = 1.9 \cdot 10^{-4} \text{ cm; with}$ T = 70 dyne per cm,h = 753 cm and $\frac{nh}{k} = 264 \cdot 10^7 \text{ sec} = 84 \text{ years,}$



Figure 1. Rate of capillary rise for K - Putnam soil

for different values of z, t then has the following values:

In ft.	/ in years
$\frac{h}{8} = 3.1$	0.76
$\frac{h}{4} = 62$	2 99
$\frac{h}{2} = 124$	16 1
$\frac{1}{2}h = 186$	53.7
h = 247	90

These data are also given in Figure 1.

.

Consequently, for a soil of this type it takes almost one year for capillary water to rise to a height of 3 ft, about two years for a rise of 5 ft., and about three years for a rise of 6 ft. Rise to the height h would require an infinite time period.

b) Film or Sheet Flow: It was calculated that 'a surface with a wetting energy of 625 erg per sq. cm. should be able to lift a film of the thickness of  $10^{-5}$  cm to a height of 637 meters if this lifting involves the formation of an arwater interface. While very few data are available for the permeability of a soil with respect to film flow, extrapolation of data obtained by Winterkorn and Moorman gives a value of k for the soil and the film thickness under consideration of about  $10^{-9}$  cm. per sec. Inserting these values in the previously used formula, taking.

$$\frac{nh}{k} = 6.37 \times 10^{13} \text{ seconds}$$
$$= 2.07 \times 10^6 \text{ years}$$
$$t = \frac{nh}{k} \left( ln \frac{h}{h-z} - \frac{z}{h} \right)$$

we obtain the following times required for definite z values:

-		In ít.	f in years		
h 10,000	-	0 197	01 (3.65 days)		
h 1,000	-	1 97	1 035		
$\frac{h}{100}$	=	19 <b>7</b>	$21 \times 10^2$		
h 8	=	248	$219  imes 10^4$		
h 4	-	496	$7.8  imes 10^4$		
ћ 2	-	985	$4.0  imes 10^{5}$		
ą,	-	1477	$1.27 \times 10^{6}$		
h	-	1970	<b>00</b>		

It is interesting that for film flow the total amount of water transported within a certain time is proportional to the amount of surface per unit volume available for such flow, i e, it increases with increasing bulk density of the soil system

d) Vapor Pressure Potentials: Water vapor flows from regions of high to low pressures The vapor pressure in soil increases with increasing moisture content and temperature; it decreases with increasing water affinity of the soil system. The respective relationships are easily determined by hygroscopicity experiments over solutions of known vapor pressure. In the lower soil strata where temperature and pressure conditions are more or less stationary, water vapor flow takes place by diffusion which is a very slow process.

In the upper soil strata, temperature and pressure fluctuations result in convectional exchanges between soil air and outside atmosphere, which may bring about water vapor movement and moisture absorption by the soil with a greater speed than obtainable by diffusion alone. The mechanism involved is quite complicated, however, data obtained by Lebedeff and others show that a considerable amount of water may be supplied to surface soil layers by this process. This phenomenon deserves further study.

# MECHANISM INVOLVED IN CHANGING WATER UNDER TENSION AND WATER UNDER COMPRESSION INTO FREE WATER

Water exists in soils under three conditions of stress

1. Compression: hygroscopic or strongly adsorbed water existing nearest to the solid surfaces, the compression being evidenced by increased density of the water often expressed as an apparent increase of the density  $\gamma$  of the clay minerals This latter phenomenon is a function of the surface activity of the clay and can be related to the heat of wetting and the hygroscopicity (10 per cent Sulfuric acid) by the following formulae:

 $\gamma = 2.652 + 0.01167 \times \text{hygroscopicity, or}$  $\gamma = 2.652 + 0.000233 \times \text{heat of wetting};$ 

2. Tension. capillary water hanging on the wetted perimeter of a concave meniscus, the viscosity of the water is practically that of free water;

3. Stressless: water in large pores, and in the transition zone between compressed and tension water, also, osmotic swelling water is relatively free from stress

The effect of these stress conditions on bearing power of a soil is as follows.

1. Pure capillary water increases soil stability as evidenced in sand, the internal tension acting similarly to an equivalent external compression,

2. Small amounts of compression water are essential for cohesion in a soil; however, larger

amounts decrease cohesional as well as frictional stability;

3. Free water and osmotic swelling water decrease the stability.

It is of greatest practical importance that capillary and compression water may be changed into free water and vice versa by the normal temperature fluctuations occurring in the surface layer of the earth The factors responsible for such change and the quantitative aspect can be easily deduced from the formula for capillary rise,

$$H=\frac{60T}{\rho g}\frac{1-p}{p_0}\cdot U$$

Taking the following data which are conservative for a K-Putnam soil and the respective constants for water at temperatures of 20 deg. C. and 30 deg C respectively.

		20 = 056		Difference in Property	
Temperature, °C Porosity, p Stress free porosity.	°C =		30 0 56	Δ	Δ%
po Surface tension, T density. o	0 8 0	028 728 099	035 712 823 099568	0 07 1 6 0 00255	20 22 095

we see that:

- 1. The effect of density change with temperature is sufficiently small to be neglected;
- 2. The effect of the temperature change on the surface tension is about 2 per cent;
- 3. The greatest change in capillary potential with temperature is due to change in the amount of stress free porosity, the latter changes by 20 per cent decreasing the capillary potential and therefore the height of capillary rise by the same percentage In other words if such a soil is capillarily saturated at 20 deg C., sudden increase in temperature by 10 deg. C affects the stability of the soil similarly to the addition of about 4 per cent of water without temperature increase Obviously this is a very serious effect, however, the proper evaluation of temperature effects must also take into account the effect of temperature on the rate of capillary flow

As shown above 
$$t = \frac{nh}{K} \left[ \ln \frac{h}{h-z} - \frac{z}{h} \right];$$

as  $\rho_0$  becomes larger with increasing temperature *h* becomes proportionately smaller, 1 e, nh = constant for different temperatures which leaves *K* for further consideration, if related to the stress free porosity K becomes  $K_0$  which latter, should for air-free water, be directly proportional to the viscosity of free water At 20 and 30 deg. C. the viscosity of free water is 10 and 8 millipoises, respectively Accordingly, the increase in permeability is 20 per cent as is that

of the rate of rise It so happens that  $\frac{K}{n}$ 

is a constant for the case considered, and

since  $h_{20}$  is smaller than  $h_{20}$ ;  $\frac{dz}{dt}$  dz is

smaller at 30 deg. than at 20 deg C.

These considerations show the great importance of temperature fluctuations in the soil-water relationships obtaining under natural conditions of exposure.

#### SUMMARY

Theoretical aspects of non-gravitational water movement in soils have been presented in a form whith it is hoped will stimulate comparison of the theoretical deductions with general observational and experimental data. The importance of the rate of water movement has been emphasized. The need for more experimental data concerning the effect of temperature on the stress-free pore volume and on other soil-water relationships has been indicated. Because of the importance of temperature effects an appendix has been prepared containing a condensed statement of commonly available knowledge concerning the theory of temperature fluctuations occuring in the surface layer of the earth for which the authors claim no credit

### APPENDIX

## Temperature conditions in the surface layer of the earth

Science differentiates between three types of heat transfer:

- Conduction, in which the excitation of an atom or molecule is transmitted to its neighbor by direct contact, oscillationlike in solids and liquids and by impact and exchange of momentum in gases;
- 2. Convection in liquids and gases, where a portion of matter at a higher temperature is mechanically mixed with matter at a lower temperature,
- 3. Radiation by means of waves traversing

space from one body to another without affecting the interlying space.

All three types are of importance with respect to the temperature condition of the surface layer of the earth. Irradiation of the earth from the sun and radiation from the earth into space govern the temperature conditions on the uppermost earth surface layer.

Heat conduction appears to be mainly responsible for transmitting the surface fluctuations downward into the soil Heat convection plays a role when air is pumped into or sucked out of the soil pores as a result of sudden changes in atmospheric pressure and of differences in the temperature of the pore air and external atmosphere, respectively.

In the following, attention will be paid mainly to heat conduction. It is hoped that opportunity will be found at a later date to treat the other phenomena

The fundamental differential equation for heat conduction as derived by Fourier is as follows

$$\frac{dQ}{dt} = -kA \frac{d\vartheta}{dL},$$

where

1

dQ = amount of heat transferred

dt = the 'time during which dQ is transferred, therefore

 $\frac{dQ}{dt}$  = the time rate of heat transfer

- A = the area at right angle to the direction of heat flow
- $\frac{d\vartheta}{dL}$  = the rate of temperature change in
  - the flow direction or the thermal gradient
  - k = a material constant, the numerical value of which depends on the system of measurement used. It is called the thermal conductivity constant.

Fourier's equation expresses a general scientific relationship which holds irrespective of the measuring system employed—whether British, or cgs. Foi the purpose at hand the metric, or better, the cgs system is adopted. In this system the respective units are. L in cm.; A in sq. cm.,  $\vartheta =$  in deg C., Q in small or gram calories— cal, k in calories per square centimeter per sec. per gradient  $\frac{\text{degree centigrade}}{2}$  or

cm

2

3

$$k = \frac{\text{cal}}{\text{cm}^2. \text{ sec.}} \frac{\text{degree}}{\text{cm}}$$
 .Fourier's equation

may also be written as:

1a. 
$$dQ = -k \, d\sigma \cdot dt \cdot \text{grad } \vartheta$$

 $d\sigma$  representing an element of area

In heat conduction problems one differentiates between steady and non-steady flow. In steady flow as much heat enters a volume element as flows out of it; consequently the condition of the volume element is not changed. The only specific body constant entering in this process is its heat conductivity

In non-steady heat conduction the temperature of the body itself changes. In this case, heat may be drawn off from or added to parts of the body. This heat subtraction or addition results in temperature changes which are inversely proportional to the heat capacity per volume element of the body. This heat capacity is a product of mass per volume element times heat capacity per mass. In the cgs system the product is expressed as density  $\rho$ 

$$\ln \frac{g}{cc} \times \text{specific heat} \frac{cal}{g} \text{ or } \frac{cal}{cc}$$

The temperature conditions in surface soils and in pavement-soil systems are results of non-steady heat flow. For the sake of simplicity and as a first practical approximation, the specific problem is considered as the movement of heat or cold from a surface of practically infinite dimensions into the interior of the body, the direction of movement being at a right angle to the surface. The simplified problem involves, therefore, only flow in one direction. If the flow is assumed to be in the x- direction, and if  $\frac{dQ}{dt}$  at a point x is abbreviated  $\phi x =$  the heat flux per unit of time, then  $\phi x = -k \frac{d\vartheta}{dx}$  At a distance dxfrom x an amount of heat equal to  $\phi_{x+dx}$ may leave the volume element per unit of time. If the second term is smaller than the first, heat in the amount of  $\frac{\partial \phi}{\partial x} dx dy dz$  per unit of time is retained in the volume element The heat retained per time element dxdudz dt is accordingly  $\frac{\partial \phi}{\partial x} dx dy dz dt$ ; this heat raises the temperature in the volume element

by  $d\vartheta$ . Consequently  $\frac{\partial \phi}{\partial x} dx dy dz dt = c\rho$  $d\vartheta dx dy dz$  and since  $\phi = -k \frac{\partial \vartheta}{\partial x}$ .

$$k \frac{\partial^2 \vartheta}{\partial x^2} = c \rho \frac{\partial \vartheta}{\partial t}$$

This equation describes the heat conduction at any particular moment on the earth surface or at any interior point. However, it must be solved for the specific conditions involved in the present problem.

The temperature on the earth surface is a function of time, it reaches a maximum in daytime and a minimum at nighttime. The temperature at any distance x from the surface is a function of both time and distance from the surface. It is possible to express this functional relationship as.

$$\vartheta = Ae^{(\alpha z + \beta t)}$$

If x and t are both equal to zero then  $\vartheta$  becomes A; according to equation 3,  $\frac{\partial \vartheta}{\partial t} = \beta \vartheta$ ;

 $\frac{\partial\vartheta}{\partial x} = \alpha\vartheta$ , by substitution in equation 2 we obtain  $k\alpha^2 = c_{\rho\beta}$ .

Several solutions of this equation are possible. Because of the known fluctuating character of the surface temperature which possess a daily maximum and a daily minimum value, we assume that  $\beta$  is imaginary, accordingly  $\vartheta$ is represented by a complex function and.

$$k\alpha^{2} = c\rho\beta_{0}\imath$$
, where  $\imath = \sqrt{-1}$ ,  
 $\alpha^{2} = \frac{c\rho\beta_{0}2i}{2k}$  and  
 $\alpha = \pm \sqrt{\frac{c\rho\beta_{0}}{2k}} \cdot \sqrt{2\imath}$   
 $= \pm m(1 + \imath)$  where  $m = \sqrt{\frac{c\rho\beta_{0}}{2k}}$ 

Substituting in equation 3 we obtain

5  $\vartheta = Ae^{\pm mx} e^{i(\pm mx + \beta_0 t)}$ ; at the surface where x = 0 $\vartheta_{x=0} = Ae^{i\beta_0 t}$ 

By definition of the imaginary power, it can be divided into sine and cosine functions. Accordingly:

$$\delta \qquad \vartheta_{x=o} = A(\cos\beta_0 t + \imath \sin\beta_0 t)$$

This equation, as previously indicated, represents periodic temperature changes, on the earth surface, both daily and yearly periodic changes occur.

For the problem under investigation only that solution of equation 5 possesses a physical meaning in which m is negative. Equation 5 may now be rewritten as follows

7. 
$$\vartheta = Ae^{-mx} \{\cos (-mx + \beta_0 t) + i \sin (-mx + \beta_0 t)\}$$

This equation shows that temperature fluctuations occur not only on the surface but also in the interior of the earth crust. If we consider only the imaginary part of equation 7 and define A as the amplitude of the fluctuation around the average temperature  $\delta av$  we can write the temperature function as

8. 
$$\vartheta = Ae^{-mx} \cdot \sin(-mx + \beta_0 t) + \vartheta av$$

The physical picture expressed by equation 8 is that of a damped sinusoidal heat wave penetrating from the surface into the interior of the earth.

How far does this heat wave progress in the time of one period T, or how large is the wave length  $\lambda$ ?

After one period the sine function must have returned to its original value, accordingly:

$$-mx + \beta_0 t = -m(x + \lambda) + \beta_0(t + T)$$

 $m(x + \lambda) - mx = \beta_0(t + T) - \beta_0 t$  $m\lambda = \beta_0 T$  $\lambda = \frac{\beta_0 T}{m},$ 

9.

since  $\vartheta$  is periodic in T,  $\beta_0$  must be  $\frac{2\pi}{T}$ , there-

$$m\lambda = 2\pi$$

The speed of propagation of the heat wave is:

10. 
$$v = \frac{\lambda}{T} = \frac{\beta_0}{m}$$

If we introduce the material properties into equations 9 and 10 respectively, we obtain:

11. 
$$\lambda = 2\pi \sqrt{\frac{2k}{c\rho\beta_0}} = 2\sqrt{\frac{\pi kT}{c\rho}}$$

12. 
$$v = \frac{\lambda}{T} = 2\sqrt{\frac{\pi k}{c\rho T}}$$

Equation 12 shows that the speed of progression of the heat wave is inversely proportional to the square root of the period, while the wave length is directly proportional to the square root of the period. In other words, a long period has a long wave with a small speed of propagation, while a short period has a short wave with a great speed of propagation. Since the square root of 365 is about 19 it follows that the daily heat wave is about one nineteenth as long as the yearly heat wave.

Both the daily and yearly heat waves decrease in amplitude with depth of penetiation into the soil. The rate of damping of the heat waves is perhaps even more important than the speed of penetration The damping effect is usually expressed by the ratio of the amplitudes at distances of one half wave length. If

$$\vartheta_{(x=0)} = C \sin \beta_0 t + \gamma \text{ and}$$

$$\vartheta = C e^{-mx} \cdot \sin(-mx + \beta_0 t) + \gamma$$

for

for

 $\sin(-mx+\beta_0 t)=1;$ 

$$\vartheta = Ce^{-mx} + \gamma, \text{ since } m = \frac{2\pi}{\lambda}$$
$$\vartheta = Ce^{\frac{-2\pi}{\lambda} \cdot x} + \gamma$$
$$x = \lambda$$
$$\vartheta_{\lambda} = Ce^{-2\pi} \text{ and } \vartheta_{\lambda} = Ce^{-\pi}$$
$$e^{-2\pi} = \frac{1}{535}$$
$$e^{-\pi} = \frac{1}{23}$$

The damping ratio is therefore 23; this means that an amplitude of 23 deg. of the heat wave on the surface results in an amplitude of 1 deg. at a depth of one half wave length. At a depth of one wave length the amplitude has fallen to  $\frac{1}{535}$  of the value at the surface.

**430** 

The numerical value for the wave length  $\lambda$  depends upon the material constants for the specific heat (c), the density ( $\rho$ ) and the heat conductivity constant (k) of the soil under consideration.

Average values of these constants for moist cohesive soils expressed in the respective units of the cgs system are:

$$k = 0.004$$
$$\rho = 1.6$$
$$c = 0.4$$

Using these values we obtain for the yearly wave

$$\lambda_y = 2 \sqrt{\frac{3 \ 14 \times 0 \ 004 \times 3600 \times 24 \times 365}{1 \ 6 \times 0.4}}$$
  
$$\lambda_y = 1560 \ \text{cm or } 15.6 \ \text{m}$$
  
$$\frac{\lambda_y}{2} = 780 \ \text{cm or } 7.8 \ \text{m}$$

for the daily wave we obtain:

$$\lambda_d = 2 \sqrt{\frac{3.14 \times 0.004 \times 3600 \times 24}{1.6 \times 0.4}}$$
$$\lambda_d = 82.4 \text{ cm}$$
$$\frac{\lambda_d}{2} = 41.2 \text{ cm}$$

According to the chosen material constants a variation in surface temperature of 23 deg. during the daily cycle will show itself as one of 1 deg. at a depth of 41 cm

Similarly, a variation of surface temperature of the same amount in the yearly cycle will have been damped to 1 deg. for a depth of 78 m.

Of course at these depths of one half wave length the direction of the temperature change is opposite to that on the surface, that means the lowest absolute temperature at a depth of  $\frac{1}{2}\lambda$  coincides with the maximum surface temperature, and vice versa This fact has been made use of to the greatest extent by cultures not possessing the advantages of modern refrigeration and heating.

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- (3) Zunker, F see reference (1) page 99
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# DISCUSSION

CHAIRMAN HOGENTOGLER. Doctor Winterkorn has mentioned a school of thought that considers that soil actually breathes, that under conditions of high atmospheric pressure and sunshiny weather, air moves into the soil portion and when the soil cools and atmospheric pressure drops it comes out again. I believe we have some definite evidence of moisture actually being taken out of the air and wetting materials under certain conditions.

MR R. E. BOLLEN, Nebraska Department of Roads & Irrigation: At one time during the drouth (1933-1938) in Nebraska, we made some observations and experiments relating to the movement of soil moisture in a vapor state Observations were made of a bituminous mat located on Highway No 30 in Cheyenne County, Nebraska A section of this road was covered by a 2-in. bituminous mat constructed directly on the soil subgrade. During

15 to 20 days of hot, dry weather in July the bituminous mat corrugated, shoved and wrinkled under the action of traffic Investigation disclosed that the subgrade was firm and stable and that its moisture content varied from 8 to 12 per cent, however, the top  $\frac{1}{2}$  to 1 in. of the subgrade, directly below the bituminous mat In some areas free water existed was wet directly under the mat The surface of the mat was in very good condition for preventing the entrance of moisture from the top. This observation was made after a period of three or four months of no rainfall We were amused at that, because we had been led to believe that the water always came from the lower part of the subgrade by capillarity. In this instance the highest moisture content existed at the top of the subgrade and decreased with depth. We had no explanation for this phenomena We talked to a number of engineers about it and they all told us that

the only way water could get to the top of the subgrade was by capillarity. We ceased thinking about it for a time but a little later made some experiments in the laboratory in which we took the same soil and compacted it in a column, 6 in. in diameter, at the same density and moisture content that existed in the subgrade. We enclosed the column to prevent moisture evaporation, cooled the top of the column, and maintained the bottom of the column at a uniform temperature At the end of about 60 days the soil in the top of the column had about 3 per cent more moisture than the soil in the bottom of the column. The experiment was repeated except that the bottom of the column was maintained at a lower temperature than the top. Again a higher moisture content was found in the soil which had been maintained at the lower temperature. Since there was no outside source of water, this indicated that the moisture transfer had taken place by some means other than by capillarity. This is the only evidence I have seen demonstrating the movement of moisture in soil in a vapor state. However there are several examples of movement of soil water in the vapor state quoted in the literature.1

Since the range and changes of temperature affect the upper part of the subgrade the temperature cycle of the air, the mat and the subgrade should be considered at the location where the field observations were made There were 6 to 8 hours of the day when the black surface of the mat absorbed considerable solar heat, consequently the top 3 in. of the subgrade was much hotter than the lower part of the subgrade This set up a favorable condition for moisture vapor movement downward Several hours before and after this period the subgrade and surface may have had the same temperature and there would have been no moisture vapor movement. During the evening the mat was rapidly cooled and the upper 2 in of the subgrade was consider-

<sup>1</sup> "The Movement of Ground and Soil Water by A F Lebedeff", *Proceedings*, First International Soil Congress, Vol 1, p. 459, 1927.

"The conservation of Soil Moisture and the Theory of Vapor Movement", by W W Burr, J C Russell and H. E Weakly, Unpublished paper of Department of Agronomy, Nebraska Agricultural Experiment Station, Lincoln, Nebraska. ably colder than the soil below. This permitted the upward movement of moisture vapor. Since the bituminous mat was colder than the soil, moisture vapor could condense directly under the mat for 10 to 12 hours, thus, due to the longer period during which the vapor movement was upward, accumulation of moisture directly under the mat could occur. No other proof has been given or seen that the preceding explanation correctly describes the accumulation of moisture under the mat However, there is no question that the moisture content of the upper inch of the subgrade was considerably higher than lower depths in the subgrade during the period when this investigation was made

It is obvious that the movement of moisture vapor can occur only under certain conditions and that a differential of either temperature or pressure is necessary for this movement. As Dr. Winterkorn has pointed out, a certain type of soil is necessary and some chemical activity or potential energy is necessary to raise the moisture against gravity This limits the occurrence to certain localities having a definite set of climatic conditions.

GENERAL ARMAND MAYER, French Air Ministry. We had the same experience in an old field near Paris. The first time we saw it we wondered how the water came to be below the bituminous surface This water could not have come from the water table which was very low and also, between the water table and this surface, were much drier particles than the ones we found under the bituminous surface We had to attribute the phenomenon to condensation exactly as Doctor Winterkorn mentioned.

DR. JACOB FELD, Consulting Engineer. If we consider moisture as being of three different states, liquid moisture or water, gaseous moisture which is made by adding energy to liquid moisture and solid moisture which is made by taking energy out of liquid moisture, then I think we can understand a great number of these phenomena. For instance, on one of the slides, which shows the channels of flow through soil, the actual volume through which the liquid moisture can flow is not the gross void volume of the soil, but that volume less the solid moisture volume Now the solid moisture is made solid by being included or adsorbed by soil grains and that is done by the loss of energy in the moisture. Adding energy or heat to the soil causes the moisture to liquify again The apparent water content is increased, and the apparent void content is also increased, with no exterior moisture addition or any total volume change

I was in the Sinai Peninsula east of the Suez Canal late in February 1932 In the desolate desert lands I noticed that the Arabs were scratching out furrows with camel power and wood ploughs where there was no sign of any water. Upon inquiry of how often it rained, one of the men said he wasn't sure, but he remembered about two years ago that it was a pretty cloudy day and that to his best knowledge the last time he remembered having seen rain was about five years ago Yet some crops are harvested. The planting is done on slopes or flat areas just below sand dunes facing north and west, that is, the Mediterranean Sea exposure Sharp drop in air temperature at night precipitates the moisture in the warm vapor laden winds coming off the sea. Sufficient moisture accumulates in the sand dunes to nourish the growth of hardy wheats, barley, and other cereals Sımılar phenomena cause the moisture accumulation in exposed road and base surfaces in arid regions

DR WINTERKORN The field of solid water is complicated as shown by the phase diagram for water (Fig A). According to this, there are five known solid modifications of water, each one of them stable within a certain range of pressure and temperature conditions

The following conclusions may be drawn from the diagram.

- (a) The melting point of water decreases with an increase in pressure only up to 2050 kgm per sq cm. At all higher pressures, the melting point increases with increasing pressure
- (b) The maximum expansion pressure obtainable in the freezing of water is 2050 kgm per sq. cm, and cooling below -22 deg. C does not increase the expansion pressure.

From the concept of equality of effect of external and internal pressures, we may add:

(c) The liquid state is an unstable condition for water below -22 deg. C., and if water does not freeze under expansion at this temperature, this is because it is already solidified or is being solidified as a result of external or internal pressures as ice, III, V, or VI, respectively. According to Freundlich and others adsorption pressures of 10,000 and more atmospheres are not at all unreasonable. If the effect of adsorption pressures is comparable to that of externally applied pressures it can easily be seen why soil film water can range from solid





to liquid condition at temperatures above or below the normal freezing point of water.

Another important consequence is that phase and therefore volume changes are not confined to the freezing point curve for ice I, but occur over a large temperature range in accordance with the range of adsorption pressure acting on the soil moisture This means that almost every temperature change results in a phase change in the adsorbed moisture, having a similar if smaller loosening effect on a soil system as the freezing of free water. MR W. H. CAMPEN, Omaha Testing Laboratories: On one project we had to do considerable compacting during the night when the temperature was cool Apparently we got fair density and stability during the night, but the next day, when the temperature rose, the stability was less than it was before and tests disclosed higher moisture contents than was expected; that is, higher than the stability would indicate at low temperatures. This incident indicates that more water acts as free water as the temperature rises.

# SOME PROBLEMS IN SUBGRADE MOISTURE CONTROL

# BY M. G. SPANGLER

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#### SYNOPSIS

The phenomenon of moisture accumulation and accompanying decrease in bearing capacity in soil subgrades beneath impervious pavements has long been recognized by highway engineers, but very little progress has been made toward quantitative analysis of the problem Rather, highway and airport engineers have leaned toward the assumption that all subgrades will eventually become saturated by capillarity and that pavements should be designed to withstand this extremely unfavorable subgrade condition

A recent survey of actual subgrade moisture conditions under both highway and airport pavements has revealed that some subgrades do approach saturation moisture content after a period of years of service, but that many subgrades do not approach this limiting condition. Rather, they seem to increase in moisture content subsequent to pavement construction and approach a fairly constant terminal value which may be appreciably less than saturation. Also the facts revealed in this survey are compatible in many respects with the implications of the Buckingham capillary potential concept, which has been utilized to a considerable extent by soil physicists in the field of agronomy, and the author is led to suggest that this concept of capillary flow and equilibrium may be a valuable tool for use in subgrade moisture studies and predictions

This paper gives a brief discussion of the Buckingham capillary potential hypothesis, of the relationship between capillary potential and various properties of soil, of the methods which have been developed to measure the relationship between capillary potential and soil moisture content and concludes with the suggestion that the terminal moisture content of a pavement subgrade may be predicted if the sorption characteristics of the soil and the distance to a free water table are determinable. The author also makes a plea for more complete long time measurements of subgrade moisture contents, soil temperature fluctuations, and capillary potentials under actual pavements in order to enhance our knowledge of this problem.

The very thorough compilation and digest of subgrade soil moisture data made by the Highway Research Board and reported by Mr Kersten  $(5)^1$  a year ago provides highway and airport engineers with an excellent reservoir of factual information concerning this very important subject Among the many facts revealed by this study it is of particular

<sup>1</sup> Figures in parentheses refer to the list of references at the end of the paper

nterest to note the number of clay and silty clay subgrades in which the humidity of the soil, that is, the moisture content expressed as a percentage of the saturation value, was very high extending well up into the nineties in many instances Although Kersten's study does not deal with the condition or the probable life of the pavements resting upon these wet subgrades and does not offer a clue as to a limiting amount of moisture which can be