were established. These are now in use at Capital Transit Company. For the sake of clarity these might be stated as:

(a) Men whose score is 65 or more furnish the most suitable candidates for employment.
(b) Men whose score is between 50 and 64 should be employed if other factors indicate that they will be suitable.
(c) Men whose score is 49 or less should be rejected unless there is strong evidence from other factors that they will be suitable.

6. Based on the evidence of this study, the use of the American Transit Motor Ability Test will materially aid in reducing the number of men who must be eliminated after a training and trial period.

Another evaluation of this test is planned. The population for that study will be drawn from the group which is being tested prior to employment. At that time we shall also investigate the relationship between test score and the ability of the candidate to complete training.

Acknowledgements

This report is based on an extensive validation study which has been in progress at the Capital Transit Company during the past three years. The author is greatly indebted to the following men: Alexander Shapiro, Director of Personnel, Capital Transit Company; Glen U. Cleeton, Consultant, American Transit Association; Merwyn A. Kraft, Accident Prevention Engineer, American Transit Association; George W. Cramer, Supt. of Transportation Personnel, Capital Transit Company. This experiment was designed and completed as a cooperative effort of the American Transit Association and the Capital Transit Company. Each step that was taken was carefully considered in extensive conferences by those listed. The author has served as the executor of these joint decisions and as the reporter of the findings.

TOLL BRIDGE INFLUENCE ON HIGHWAY TRAFFIC OPERATION[1]

By M. Earl Campbell

State Planning Engineer, State Road Commission of West Virginia[2]

SYNOPSIS

The diversity of opinions and policies with respect to the toll system is leading to serious inquiry as to the relative effects and values of free and toll bridges. A rational approach to the problem is attempted in this report.

This paper presents the development of a mathematical method for measuring objectively the restraint imposed upon highway traffic operation by toll bridges.

A formula is developed for measuring the monetary value placed upon time by the composite motorist. The application of the formula depends upon a measurement of sphere of influence of adjacent free and toll bridges, toll rate, and rates of vehicular speeds in the area studied.

Conversely, the formula may be used to determine the sphere of influence of the toll facility when the monetary value of time is given.

The method presented comprehends the analysis of time and delay studies, correlated to origin and destination surveys made before and after freeing toll facilities.

A number of specific cases were studied during 1946 by the Planning Division of The State Road Commission of West Virginia to develop a reservoir of experience. An interpretation of the analyses of cases studied (of which this paper presents but one case) indicates that:

1. Each toll facility has its own peculiar sphere of influence as determined by

[1] This paper is a brief of Parts I, IV, V, and VI, of a thesis bearing the same title submitted by the writer in partial fulfillment of requirements in Highway Traffic Engineering at the Bureau of Highway Traffic, Yale University, 1946.

[2] Mr. Campbell is now Engineer of Traffic and Operations, Highway Research Board.

factors which include toll rate and speeds of travel in the immediate area, together with its economic and social environment.

2. Each toll facility exerts its own peculiar influence upon highway travel with respect to diverted, barred, stranded and user traffic.

3. The perpetuation of tolls on a fully amortized facility to protect the financial stability of an adjacent toll structure may not be necessary. Each toll bridge is inherently protected if sufficient distance and travel time separates it from the next adjacent bridge. Each situation presents a special study in cause and effect.

4. The individual motorist does not place a precise, uniform, monetary value on his time, but selects a route which more nearly accommodates the urgency of immediate demand. Time cost, it appears, is a variable dependent upon individual driver judgment, time of trip (hour and day), together with economic demands upon and economic environment of motorist.

5. Origin and destination surveys, in conjunction with time delay studies, furnish very precise instruments for measuring sphere of influence, lines of equity, values of vehicle time, and diversion of traffic. They also serve a useful purpose in estimating the anticipated change in facility use occasioned by a change in toll rate.

A very notable revival of interest in the toll system is making itself evident today as a means for more rapidly expanding our transportation system. From national level to state level, from state level to community level, and from community level to the individual there is a serious inquiry into this subject. A diversity of opinions and policies on this vital problem are demonstrated.

Whereas on the one hand federal and state agencies are pledged to a program of purchasing and freeing toll bridges (*1*)[3], on the other hand the impetus for building more new toll facilities brought construction of these to record levels by 1941 (*2*) (at which time World War II halted all construction except that necessary to the war effort).

During 1945 there were an estimated total of 205 toll bridges and seven toll tunnels operating in the United States. Of the toll bridges 103 were publicly owned and 102 were privately owned. Since 1940, six new toll bridges have been opened to traffic, 41 toll bridges were eliminated and 11 toll bridges changed from private to public ownership (*3*).

An opportunity to conduct original research on the question of the relative effect of free and toll facilities on highway traffic by way of objective measurements was presented in a most tangible form. The legislature of the State of West Virginia in regular session in 1945 authorized the purchase of two toll bridges on the Kanawha River for the purpose of making them toll free. At the same time the State Road Commissioner authorized the planning division to make studies of traffic transfer across the Kanawha River to determine the need for and the proper location of additional bridges.

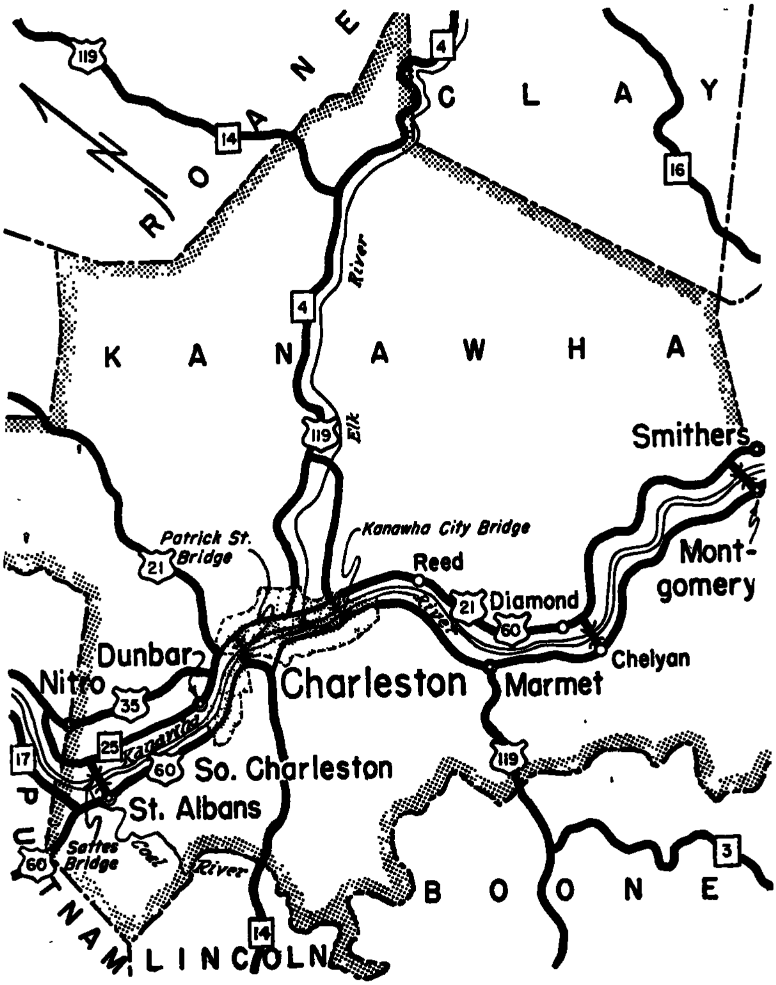
The potentialities of making time and delay studies together with origin and destination surveys both before and after the bridges were freed was recognized with respect to answering certain pertinent questions relative to toll bridge influence. Proposals were submitted to the Public Roads Administration and approval obtained for making this study a joint project.

The specific objectives of the study are enumerated as follows:

1. The effective sphere of influence of toll bridges in terms of travel time and distance
2. Amount of community traffic diverted, barred, and stranded by tolls
3. The minimum separation required in terms of time and distance from the next adjacent free bridge to protect the financial stability of a toll structure
4. A monetary evaluation of vehicle time
5. An evaluation of the possibilities of the origin and destination survey as a method for measuring the influence of tolls on traffic.

Bridge traffic analyzed in this study includes that using the Sattes and Chelyan toll bridges, and the Patrick Street (Charleston), Kanawha City, and Montgomery free bridges. The toll bridges were freed while the study was in progress and both before and after studies were made at these bridges. The Key Map shows the location of the bridges studied.

[3] Italicized figures in parentheses refer to list of references at the end of the paper.



Key Map

THE PROBLEM ANALYZED

Interpretation of "Sphere of Influence"—Prerequisite to its measurement "sphere of influence" must be defined. To begin, and for purpose of illustration only, assume a series of three free bridges to be serving a certain geographical area. It is desired to determine the sphere of influence of the central structure.

An origin and destination survey shows trips across the central bridge with origins and destinations beyond the other two bridges. It also shows trips across the other two bridges with origins or destinations further from the two bridges than from the central bridge. In effect, vehicles have passed by the near bridge to cross one further distant, and all in all, a great amount of trip overlap is revealed. In the over-all picture the sphere of influence of the central bridge appears to comprehend a substantial part of the spheres of influence of the adjacent free bridges as indicated in Figure 1. Inasmuch as the central bridge may be used by vehicles whose origins and destinations are beyond the two other bridges, it becomes necessary to analyze the usage, or trip type, in order to delimit the sphere of influence and eliminate the extensive overlap of adjacent spheres, and apportion to each bridge its contingent share of influence.

An analysis of transfers across a river disclose the general types of trips, namely:

1. The "U" type, with "J" type as a variant
2. The "S" type, with the "L" type as a variant

The trip with origin and destination beyond the adjacent bridge is usually of the S type, whereas the U type trip usually has both origin and destination between the bridge crossed and the adjacent bridge. A comparison of these two trip types reveals fundamental concepts for evaluating the contingent influence of adjacent bridges, namely:

1. The S trip, with its variant, may include no out-of-way travel distance between origin and destination, therefore it may make no

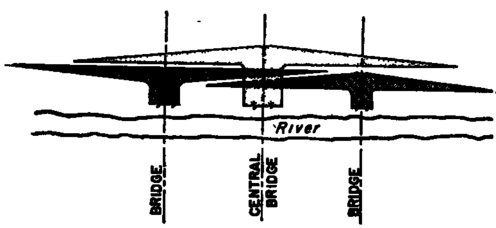


Figure 1. Typical Traffic Flow Diagram Trans River Traffic

difference (in distance) which bridge a vehicle crosses as long as the trip maintains its characteristic S shape (i.e. it does not change to the U type). Therefore, as there is no difference with respect to which bridge is crossed by the S type trip, we may at once eliminate this type of trip from the place of first emphasis in determining the sphere of influence of free bridges.

2. The U trip, with its variant, always includes out-of-way travel distance between origin and destination, therefore it always makes a difference in distance with respect to which bridge a vehicle crosses, except in certain cases where the out-of-way distance is equal regardless of bridge crossed. This exception will occur when the centroid of the trip is at the midpoint in distance and time between bridges. The U type trip, then, must receive the first emphasis in determining the sphere of influence.

Before proceeding further in the development of the method of analysis it would be well to assign a name to that direction of travel, every part of which contributes toward reducing the initial relative distance between origin and destination (i.e. that travel which includes no out-of-way distance). The term "apposite" has been chosen to designate this type of travel. Apposite travel may be defined as that travel which is an attribute of relative position and is independent of constraint. Conversely, "inapposite" was chosen to represent out-of-way or irrelevant travel, and may be defined as travel resulting from constraint of fixed facilities, and is, therefore, dissociated to relative positions.

As applied to the problem at hand the S trips represent apposite travel. Whereas U trips (with variant J clipped down to a U) represent the inapposite (Fig. 2).

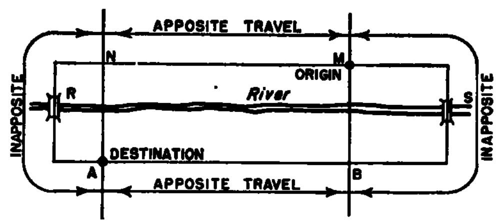


Figure 2. Diagram Showing Apposite and Inapposite Travel Distance—In the figure, travel between M and N, or B and A is apposite, whereas travel via NRA and MSB is inapposite.

Pursuing still further the illustration in which free bridge spheres of influence are subjected to analysis, it may be assumed as axiomatic, all other things being equal, that the natural sphere of influence of a free bridge located between two other free bridges will extend out to a division line midway in distance between the adjacent bridges. Hereinafter this division line will be referred to as the "isometric line." Actually, however, equal adequacy and attractiveness of bridges and facilities is seldom realized. In particular, the time of travel varies between bridges so that the division line midway in time between bridges (hereinafter referred to as the "isochronic line") is seldom coincident with the isometric line. Since the sphere of influence of a bridge is determined both by time and distance factors (supplemented also by intangible psychological factors), the sphere will be fixed by a line (disregarding intangibles) of equal economy, or a natural line of equity (for free bridges) which would fall between the isometric and isochronic lines (Fig. 3).

Now, assume that one of these free bridges becomes a toll bridge. Proceeding with the development of method of analysis, there is now, in addition to items of time and distance, a third tangible item of economy; that of tolls. Immediately, the line of equity is transferred from its original position to a new position nearer to the toll bridge, thus developing assymetry between the trip centroids related to the toll bridge and the free bridge. This new position of the line establishes the actual sphere of influence of the toll facility and for convenience will be referred to hereinafter as the actual line of equity (Fig 4).

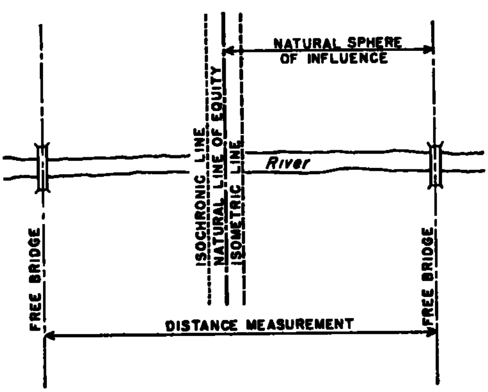


Figure 3. Diagram Showing Natural Sphere of Bridge Influence

The sphere of influence may then be interpreted as the actual line of equity enveloping a facility.

Influence of Constraint—The general theorem relative to influence of constraint may now be stated—The measurement of assymetry between the natural line of equity and the actual line of equity evaluates the influence (in magnitude) of the constraint. Referred to the free bridge this assymetry may be called the excess inapposite travel (time and distance).

Defining Differential Measurements—The measurement of excess inapposite travel is a differential measurement in that it is a measurement in terms of distance of the composite value of the time and distance differentials equated against the toll (or other constraint) cost. The composite differential value may be represented, then by the out-of-pocket (toll) cost which is assumed to be equal in amount to the aggregate value of total operating costs plus value of total travel time saved; or additional cost (Toll ± operating cost) equals value of total travel time saved.

In order to evaluate time and distance differentials separate measurements may be made for each. The time differential may be obtained by measuring from the actual line of equity to the isochronic line. The distance differential may be obtained by measuring from the actual line of equity to the isometric line.

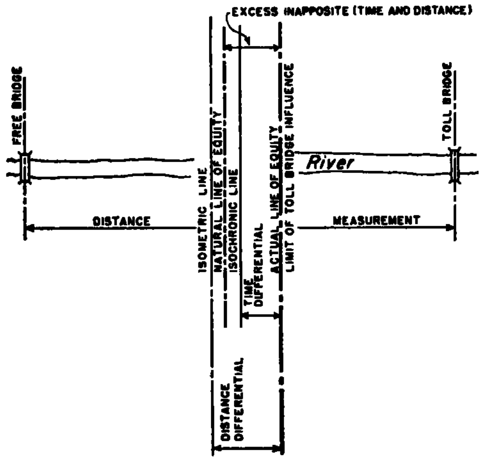


Figure 4. Diagram Showing Natural and Actual Lines of Equity

Correlation of differentials is established on a common monetary basis. Toll costs are known. Distance costs (commonly referred as vehicle operating costs per mi) can be closely calculated. Time value is determined from the formula developed in following pages.

Locus of Actual Line of Equity—Before proceeding further with the method of translating the time differential into terms of monetary cost per min, the method of determining the location of the actual line of equity will be developed. The mechanics will be presented in detail in the section, MECHANICS OF ANALYSIS. An understanding of underlying principles, however, is in order in this development of method of analysis.

Reference is made to Figure 5 entitled "Graph Showing Method of Locating the

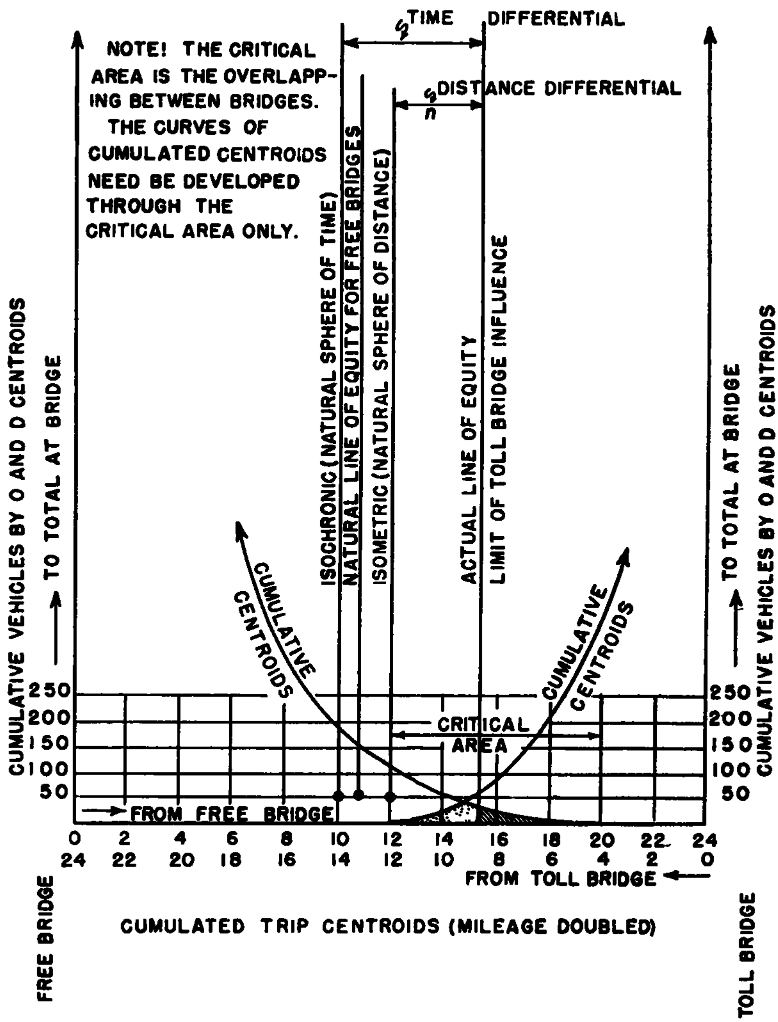


Figure 5. Graph Showing Method of Locating Actual Line of Equity

Actual Line of Equity." That point in the overlapping area of travel between the bridges (critical area) where the vehicular miles of travel, in this area, by way of the toll bridge equals the vehicular miles of travel by way of the free bridge establishes the actual line of equity between the bridges (See Item 12, *Basic Assumptions* under MECHANICS OF ANALYSIS). Hence, the area representing equal cumulative travel by way of each bridge establishes the point fixing the actual line of equity.

Diverted Traffic—Traffic crossing the free bridge and with centroid of origin and destination further distant from the free bridge than the natural line of equity is diverted from the toll bridge for reason of cost or convenience (Fig. 6). When toll charge is removed it is assumed that this traffic will return to the free structure. Therefore, those trips crossing the natural line of equity to use the free bridge with centroid of origin and destination beyond natural line of equity may be combined with the trips crossing the toll structure when it is made free.

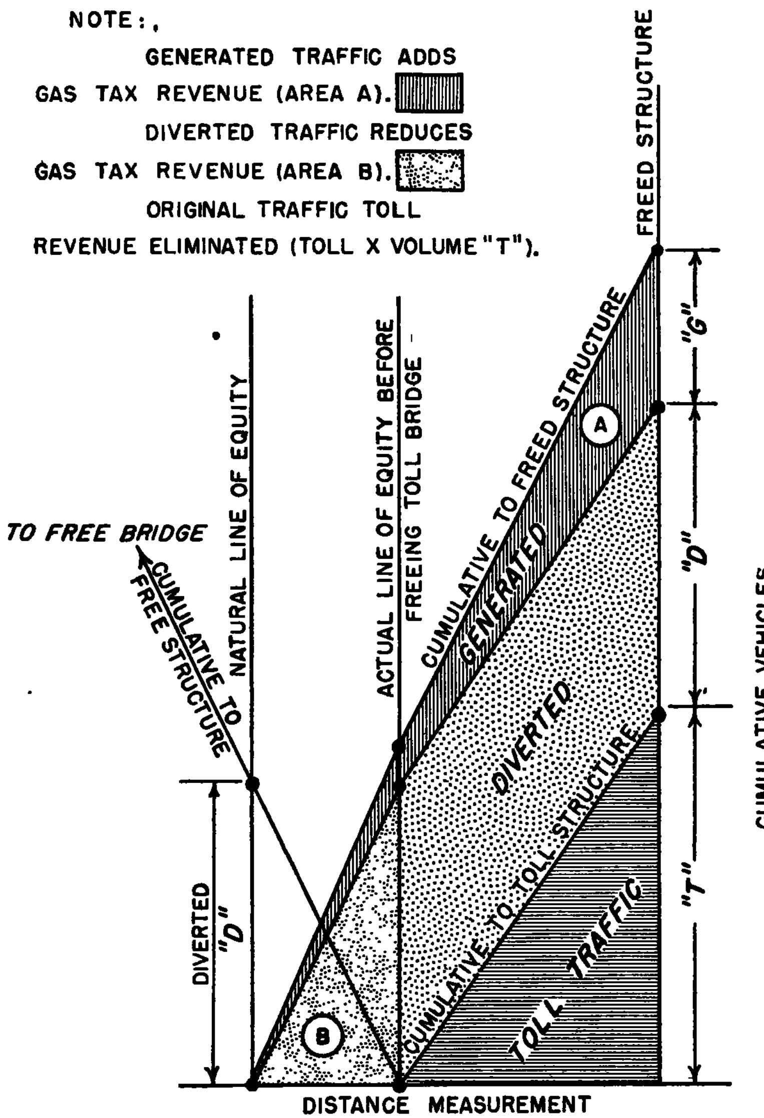


Figure 6. Illustration of Accrued Traffic Upon Freeing a Toll Structure

Locus of Natural Line of Equity—Using the general formula (see CORRELARY STUDY):

$$x = \frac{T - 4cn}{d(k - 1) + 2n(k + 1)} \cdot \frac{V}{60},$$

we may now substitute for x the value of time in terms of cost per min, and substitute 0 for value of T. The average speed values may now be taken to the isochronic line (approximately correct). Solving for n the differential distance is obtained. A positive value is directed toward the toll bridge, a negative value toward the free. In any event the locus should fall between the isochronic and isometric lines (Fig. 14).

Induced Traffic—This traffic is variously referred to as induced, generated, stranded, or potential traffic. It is traffic with centroid of origin and destination within the natural sphere of influence of the facility considered, but which is dormant or stranded because the total cost of use of the facility (including toll, time and vehicle operation) is greater than the return (economic or social) justifies. A decrease or increase in toll rate or travel time or travel distance will be reflected in a respective increase or decrease in induced traffic. Induced traffic can be measured as that residual of traffic after the original and diverted traffic is subtracted from the total traffic pursuant to an economic change resulting from toll, time or distance change.

Diverted traffic volume can be computed closely, but induced traffic can only be estimated on the basis of previous experience of comparable situations. It is likely that the magnitude of induced traffic will be greater in the area nearer the freed facility, since the population density generally increases as a bridge is approached; and the use of the bridge in its immediate environs does not involve substantial time and operation costs.

FIELD PROCEDURES

The previous discussion indicates that; (1) the trip centroid loci, (2) average speed of travel, (3) operating costs per mile, and (4) toll rates per vehicle must be determined. The usual method of procuring this data is from (1) origin and destination surveys; (2) time-delay studies; (3) route inspection and operating cost anlaysis; and (4) published toll rates. Each procedure is treated herewith.

Origin and Destination Survey—Origin and destination surveys were made of traffic traveling both directions across each bridge during a 24-hr. week-day period. An approximate 100 percent sample was obtained for each hour period in order to obtain as true a portrayal of travel as practicable. Each trip origin and destination was precisely determined by interviewer so that pin-pointing to the nearest tenth mile could be obtained. Standard procedure was used in outlining and conducting procedures.

Time-Delay Study—This study was made by the floating car method. Runs were made over the courses to locate control points, which were selected at frequent intervals. Trips were run to insure an average of traffic conditions. From 7:00 AM until midnight not less than one round trip was made in each hour, and as many as three round trips were secured in the hours of heavy traffic. The hours from midnight to 7:00 AM were considered to be represented by the other trips which had been accomplished without delay, under similar light traffic conditions.

Route Inspection and Operating Cost Computation—Fairly level grades with little traffic congestion, and few required stops led to the assignment of the commonly accepted 3 cent per mi. operating cost as appropriate for passenger vehicles operating in the areas studied (*4*).

Published Toll Rates—Toll rates as published by the bridge companies and on file in the office of the Public Service Commission of the State of West Virginia were adopted for use in this study.

MECHANICS OF ANALYSIS

Basic Assumptions in Analyzing Origin and Destination Study

1. The measurement of excess inapposite travel evaluates the constraint.
2. Apposite travel will always be the same regardless of bridge crossed.
3. Without the influence of constraint trips will follow route with lease inapposite travel.
4. A vehicle will not cross a toll facility when it can cross a free facility and by so doing avoid both toll and inapposite travel.
5. Conversely, only those trips which have less inapposite travel and time by way of toll bridge can find it profitable to use toll facility.
6. A vehicle will cross a toll facility to avoid extra inapposite travel as long as the cost of toll is less than the cost of extra inapposite travel and extra time by way of a free facility.
7. A vehicle will cross a toll facility and include inapposite travel in the trip up to the point where toll cost plus inapposite travel cost plus time cost balances the inapposite travel cost plus excess time cost in traveling by way of the free bridge.
8. Inapposite distance is equal at the isometric line regardless of travel direction.
9. Excess time is equal at the isochronic line regardless of travel direction.
10. When trip centroids from the free bridge extend beyond the natural line of

equity to overlap with trip centroids from the toll bridge they enter the critical area.

11. When trip centroids from the toll bridge overlap the trip centroids from the free bridge they enter the critical area.

12. That point within the critical area where the cumulative travel by way of the toll bridge equals the cumulative travel by way of the free bridge establishes the actual line of equity between the bridges, which is the limit of their respective spheres of influence and equals the point of maximum inappoite travel with respect to each bridge.

13. Drivers whose daily trips have origin within or near the critical area on one side of the river and destination within or near the critical area on the opposite side of the river are assumed to have evaluated the cost of inapposite travel and excess time as applicable to their individual circumstances, and then to travel by the most economical route.

14. Toll paid is measurement of monetary value of tangible and intangible benefits.

15. When a toll bridge is freed the sphere of its influence will ultimately coincide with the natural line of equity.

16. The differential measured between natural sphere of influence and actual sphere of influence after a toll bridge is freed (and traffic pattern stabilized) may be due to force of habit, or other psychological constraint.

17. Where centroid of trip falls in the line of equal equity it is as cheap to travel one direction as the other.

Selection of Trips for Analysis—Predicated upon the *Basic Assumptions* the trips selected from the O and D study for analysis may be limited to:

1. Free Bridge Interviews
   (a) U turns having at least one crossing of the isometric line
2. Toll Bridge Interviews
   (a) U turns with trip centroids overlapping or approaching the centroids of trips selected from free bridge interviews
   (b) S turns, with one terminus overlapping inapposite travel of trips selected from free bridge interview

Actually all U and S trips on toll bridge must be reviewed to see whether they meet these conditions, and centroids of each trip calculated and compared for overlapping possibilities.

Elimination of Irrational Trips—If the driver passes by a toll bridge to use a free bridge and then repasses the same toll bridge again on the other side of the river, and in so doing the distance and time costs exceed the cost of travel via toll bridge, this trip is not used in analysis.

If driver passes free bridge to use toll bridge and then repasses the same free bridge again on the opposite side of the river, he had other reasons than cost and convenience in using toll bridge, and this trip is deleted.

S Trip Analysis—If a driver makes an S trip across the toll bridge, having one terminus between toll bridge and free bridge the trip is included as an L type, with the other terminus at the opposite end of the toll bridge.

The basis for this is the fact that as the driver approaches a toll bridge with destination between the toll bridge and the adjacent free bridge, he can take no choice of bridges until he reaches the toll bridge, which becomes in effect the origin of route choice.

Expansion Factors, O and D Study—Since the development of the critical area depends upon the number of trips overlapping, the "Before" and "After" studies must be expanded to a common basis. This may be done by eliminating the seasonal influence, or by expanding to a known normal ADT for a specific year. This study is expanded to the normal 1940 ADT. Expansion was performed by using a fixed recorder in the same area for deriving factors of expansion.

Zoning for Study—Reference is made to Figure 11 showing zoning layout. In the analysis each trip is considered individually, and for that reason the interviews were pin-pointed for origin and destination. Zones, however, of 1 mi in length along the river were resorted to for convenience in grouping interviews for analysis. Nevertheless, travel distances were measured to the nearest 0.1 mi, and thusly, coded.

Time-Delay Analysis—The hourly average travel time for each zone was tabulated and weighted according to the proportional density of the traffic for each hour, thus obtaining average speed of travel between control points applicable to the 24-hr traffic density.

Speeds via the several bridges to lines of natural and actual equity were obtained by interpolation and accumulation.

CORRELARY STUDY

In the following study, which translates additional monetary cost for amount of time saved into monetary value of time per min, the development is based on the method in analytical geometry of determining the locus of a point at the intersection of lines of given slopes (comparable to a problem in intersection of grades).

The slopes in the immediate problem are the rates of cost per mi for travel via a toll bridge and via the adjacent free bridge. The rate of cost per mi includes the vehicle operating cost per mi of travel plus the vehicle time cost per mi of travel. The offset distance of the locus of the point of intersection from the isometric line is the distance differential, n.

In the case at hand the locus is known, the relative rates of slope per mi are known and their points of intercept on the Y axis are known. The solution consists of finding the absolute rates of slope (cost) per mi of travel. With these values determined the vehicle cost per mi of travel is found by simply deducting the operating cost per mi. The residual values are translated into cost per min by multiplying by speed of travel, in mi per hour, and dividing by 60.

In the development of the correlary study specific values of operating cost, travel time, travel distance, differential distances and toll charge are assumed and a graph drawn in accordance with these assumptions; then the analysis is developed and proved by using the assumed values. This procedure is used in order to provide a concrete illustration of a somewhat abstract theory.

Translation of Time and Distance Differentials into Cost per Minute—Two cases are analyzed for passenger vehicles.

Case 1

Hypothesis: Isometric and Isochronic Lines Coincident

Based on an assumed toll charge of \$0.25 per passenger vehicle, operating cost of \$0.03 per vehicle mi, cost of \$0.01 per min for time value of passenger vehicle, 12-mi travel distance between bridges, and average speed of 30 mi per hr between bridges, Figure 7 was drawn to indicate influence of toll facility. It will be noted that the differential measurement between actual line of equity and natural line of equity is 1.25 mi (equals assymetry of inapposite travel).

The formula for intersection of slopes (when speeds and operating costs are equal in each sphere of influence) may be expressed as follows (Fig. 7):

$$\frac{y}{2s} = 2n$$

or transposing

$$\frac{y}{2n} = 2s$$

where:

y = toll charge (also equals T)
n = assymetry of bridge influence
$2n$ = distance differential (total excess inapposite travel on both sides of river)
s = sum of operating and time cost per mi (Slope)

Now let:

x = time cost per min
c = operating cost per vehicle mi
t = time cost per vehicle mi
V = average speed in mi per hr

then $2s = 2c - 2t$

Whence:

$$\frac{y}{2n} - 2c = 2t$$

$$t = \frac{y}{4n} - c$$

where t = time cost per mi

Since $x = t \cdot \frac{V}{60}$ and $t = \frac{y}{4n} - c$

by substituting, and letting $y = T$:

$$x = \frac{V}{60}\left(\frac{T}{4n} - c\right) \quad \text{.......... (1)}$$

Proof (use numerical values assumed and solve for x):

$$x = \frac{30}{60}\left(\frac{25}{5} - 3\right) = \$0.01 \text{ per min}$$

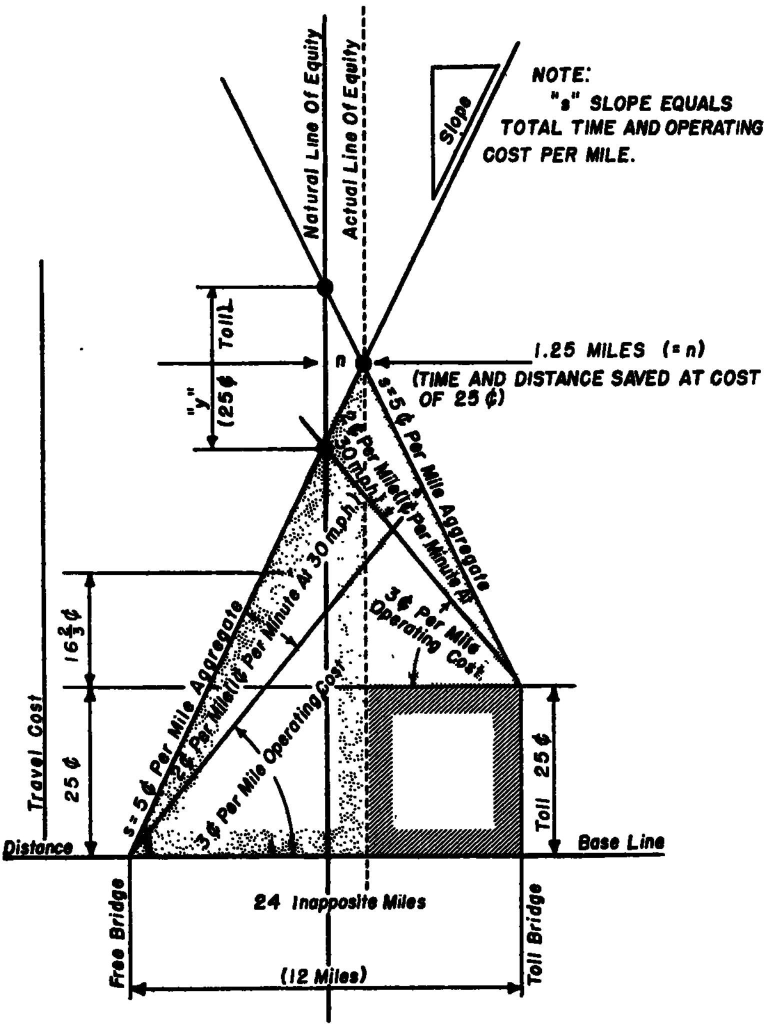


Figure 7. Case No. 1—Graphical Analysis with Equivalent Speeds—Isometric and Isochronic Lines Co-incident (Speed = 30 mph)

Case 2

Hypothesis: Isometric and Isochronic Lines Non-coincident

Assumption: Average speed of travel via free bridge is 20 mi per hr and via toll bridge 30 mi per hr. Other conditions same as in Case 1, namely: Average toll paid by vehicles in critical area \$0.25, operating cost of vehicle \$0.03 per mi, \$0.01 per min for time value, distance of 12 mi between toll and free facility.

It will be noted (Fig. 8) that the distance between the actual line of equity and

(a) Isometric line is 0.59 mi (1.18 inapposite mi)

(b) Isochronic line is 1.79 mi (3.58 inapposite mi)

Let X = time cost per mi

Then from relations shown in Figure 8,

$y = 2 \cdot 1.79 \left(2 \cdot 3 + \frac{5X}{2}\right)$; in which $1.79 = m$

(by method of intersection of slopes).

$b = [12 - 2 \cdot 0.59(3 - x)]$ in which $0.59 = n$

$$\text{and } b = 32.46 + 10.82X$$

$$y = y_1 + y_2$$

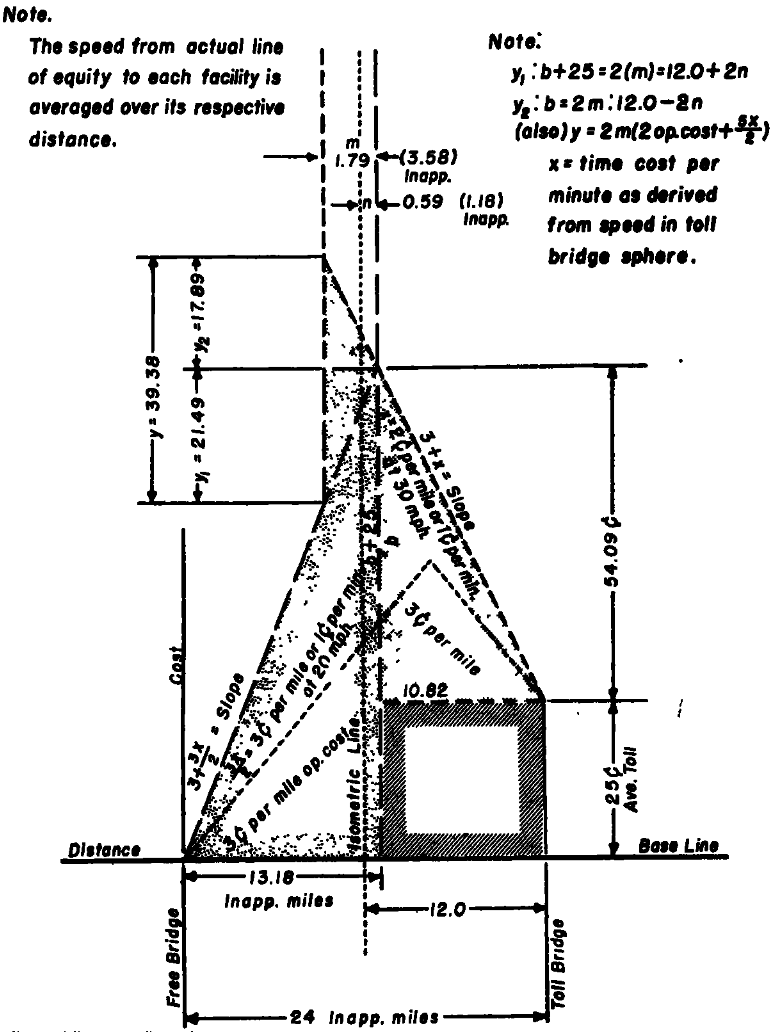


Figure 8. Case No. 2—Graphical Analysis with Variable Speeds—Isometric and Isochronic Lines Non Co-incident

$y_1 : b + 25 = 2\cdot 1.79 : 12 + 2\cdot 0.59$ (similar triangles)

$$y_1 = \frac{2\cdot 1.79(32.46 + 10.82X + 25)}{13.18}$$

Where $32.46 + 10.82X = b$

$y_2 : b = 2\cdot 1.79 : 12 - 2\cdot 0.59$ (similar triangles)

$$y_2 = \frac{2\cdot 1.79\cdot 32.46 + 10.82X}{10.82}$$

Equating values of $y = y_1 + y_2$:

$$2\cdot 1.79\left(2\cdot 3 + \frac{5X}{2}\right) = 2\cdot 1.79\left(\frac{57.46 - 10.82X}{13.18}\right) + 2\cdot 1.79\left(\frac{32.46\cdot 10.82X}{10.82}\right)$$

Simplifying: dividing through by $2\cdot 1.79$

$$2\cdot 3 + \frac{5X}{2} = \frac{57.46 + 10.82X}{13.18} + \frac{32.46 + 10.82X}{10.82} \qquad \text{(A)}$$

$$35.6519(12 + 5X) = 5.41(57.46 + 10.82X) + 6.59(32.46 + 10.82X)$$

$$48.4195X = 96.9482$$

$X = 2$ = Time cost per mi for the average speed via toll bridge

Since the speed of this group is given as 30 mi per hr then,

$$\frac{30}{60} \times 2 = \$0.01 \text{ per min}$$

Now let:

c = average vehicle operating cost per mi

$k = \frac{V_T}{V_F}$, in which

V_T = average speed in toll bridge sphere of influence

V_F = average speed in free bridge sphere of influence

d = distance between bridges (or one-half complete fixed circuit)

$2n$ = distance differential

T = toll cost

Translating equation (A) in terms of data given:

$$2c + (X + kX) = \frac{(d - 2n)(c + X) + T}{d + 2n} + \frac{(d - 2n)(c + X)}{d - 2n}$$

$$X(1 + k) = \frac{2d(c + X) + T}{d + 2n} - 2c$$

$$X = \frac{2dX - 4cn + T}{(d + 2n)(1 + k)}$$

Multiply by denominator: and assemble factors in X to left of equality sign:

$$X(d + 2n)(1 + k) - 2dX = -4cn + T$$

$$dX + dkX + 2nX + 2knX - 2dX = -4cn + T$$

$$X(dk + 2n + 2kn - d) = -4cn + T$$

$$X = \frac{-4cn + T}{dk + 2n + 2kn - d}$$

$$X = \frac{T - 4cn}{d(k - 1) + 2n(k + 1)}$$

Now, if we let x equal time cost per min, we must multiply by $\frac{V_T}{60}$ to reduce to cost per min, and

$$X = \frac{T - 4cn}{[d(k - 1) + 2n(k + 1)]\frac{60}{V_T}} \quad \ldots (2)$$

which becomes $x = \frac{VT}{60}\left[\frac{T}{4n} - c\right] \quad \ldots\ldots\ldots (3)$

$$\text{or } n = \frac{T}{4\left(\frac{60}{VT}x - c\right)} \quad \ldots\ldots (4)$$

when speeds are equal and $K = 1$ (See Case 1)

Interpreting and Investigating Formula

1. When speeds are equal via both toll bridge and free bridge and there is no positive differential measurement, the value of x becomes infinitely large—the driver "has more dollars than sense."

2. When value of x equals 0, speed of travel is cancelled out, and differential distance equals $\frac{T}{4c}$, equals maximum value of n.

3. When speeds are equal via both toll and free bridges, n is directly proportional to T for any specific value of x.

4. When k equals 1, T equals 0, and n equals 0, x is indeterminate.

5. When k is greater than 1, or less than 1, x is a function of d (distance between bridges).

6. When k equals 1, x is independent of d.

Applying Formula

1. The value of x is determined for Patrick Street-Sattes area, in the section, APPLICATION OF ANALYTICAL METHOD.

2. By supplying values for x, c, and T, the value of n may be found which measures the influence of, or delimits the sphere of influence of a toll bridge.

3. By platting families of curves and correlating with O and D data, and stranded and diverted traffic data, adjustments may be made in toll schedules to secure optimum traffic maximum rates. Classification by type must be made in this analysis (Fig. 9).

4. By solving for value of n for a given set of conditions it may be determined how close a toll bridge may be placed to a free bridge and still maintain required traffic density to pay amortization costs. Here d approaches $2n$ as a limit.

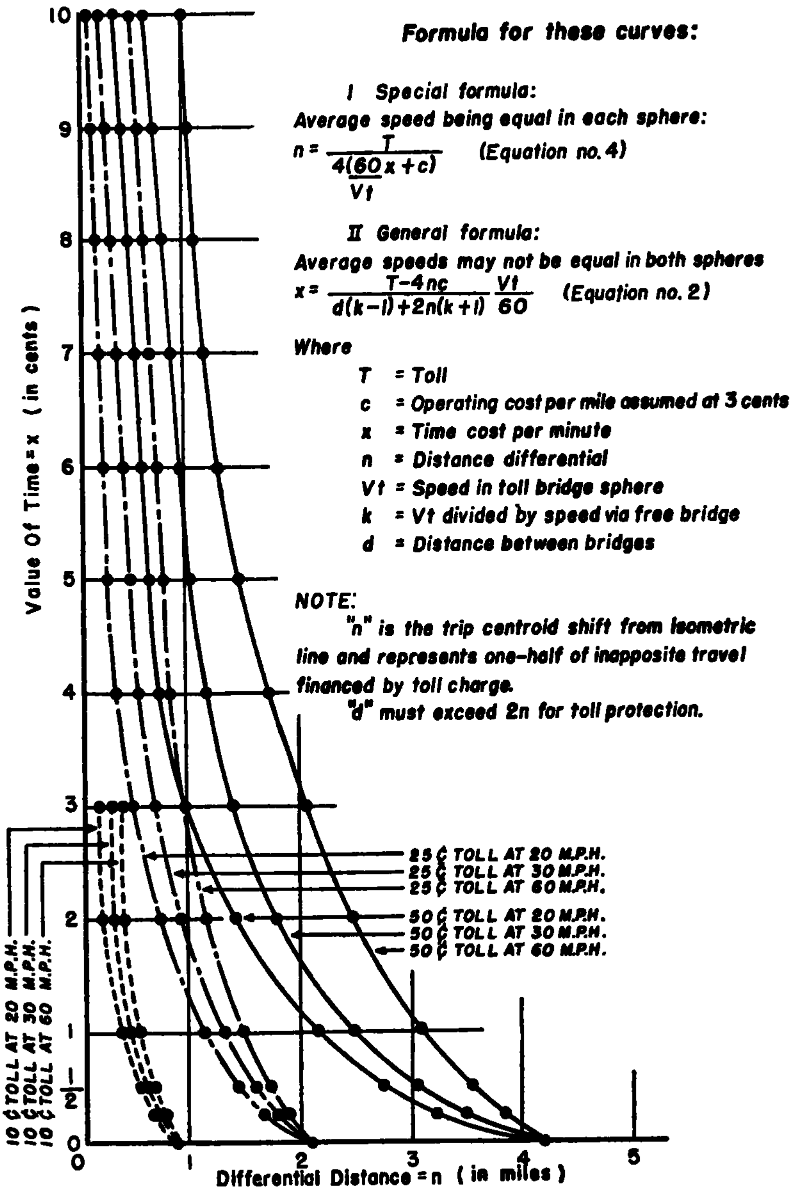


Figure 9. Families of Curves for Specific Toll Rates and Speeds Related to Time Value and Distance Differential

APPLICATION OF ANALYTICAL METHOD

An analysis of the sphere of influence and differential measurements with resultant effects is treated for the Patrick Street-Sattes Bridge couple.[4] The amount of diverted, barred, and stranded traffic is developed for the Sattes Bridge, where an origin and destination survey was made both before and after it was freed of tolls. Value of time per passenger vehicle minute is also determined.

[4] In the original thesis analyses are developed for three bridge couples. One couple is treated here to establish the propriety of the method.

After the actual line of equity was determined the speeds via each bridge to this line were calculated from the time-delay data (Fig. 14). The time-delay data were obtained by the floating car method in which sufficient trips were made during each of the 24 hours of the day to provide a stable speed pattern for each hour. The average speed was obtained by weighting the hourly average speeds by the respective hourly density of traffic.

Only passenger vehicle traffic is included in this study, in order to bring out the essential development with simplicity. This type of traffic paid the same toll per vehicle, regardless of occupancy, at the Sattes Bridge, thus enabling the study of all passenger vehicles in a unified group. Commercial vehicle behavior and costs can be studied by grouping vehicles in brackets with respect to speeds, operating costs and toll charge.

The Patrick Street Bridge (Fig. 16) crosses Kanawha River at the west corporate limits of Charleston, carrying US 60, W.VA. 4, and W.VA. 13 traffic, and providing a connecting link between the City of Charleston (Pop. 67,914 in 1940) and City of South Charleston (Pop. 10,377 in 1940). The ADT on this bridge amounted to 19,500 vehicles in 1940.

The Sattes Bridge (Fig. 17) crosses the Kanawha River to connect Saint Albans (Pop. 3,558 in 1940) with Sattes (unincorporated) and Nitro (Pop. 2,983 in 1940). The 1940 ADT was 1162 vehicles.

This study includes the trip with centroids of origins and destinations between the Patrick Street Bridge and Sattes. The Sattes-Winfield couple is not included for the reason that the Winfield Ferry (about 14 miles west) serves less than 50 vehicles a day—such a small volume as to be relatively insignificant.

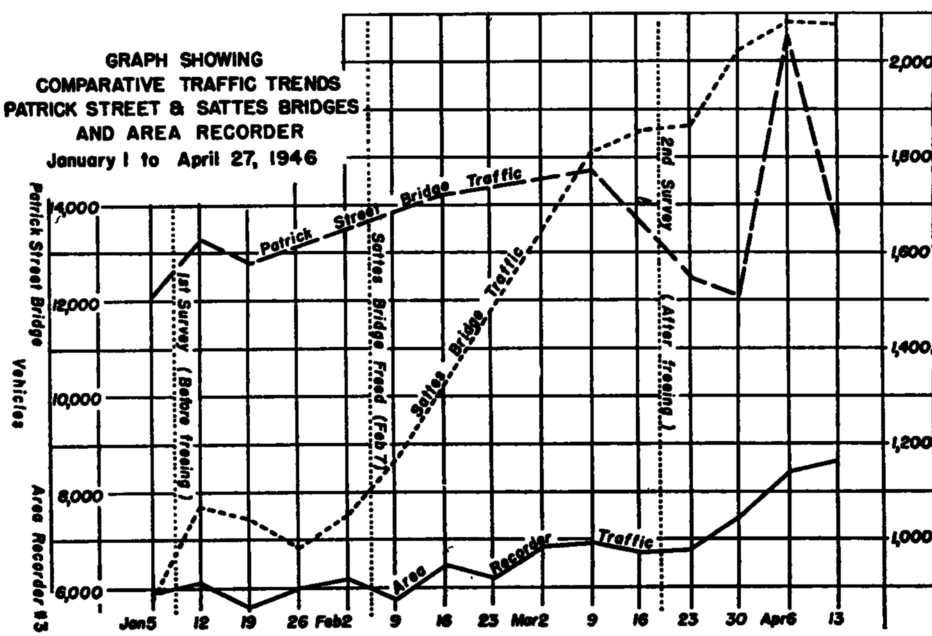


Figure 10

A force of inexperienced interviewers were employed on this project and, although under excellent supervision, it was difficult to obtain perfection in answers, particularly with reference to pin-pointing of the origin and destination of each trip. The repeat survey was delayed until 6 weeks after the bridge was freed (from Feb. 7 until Mar. 19) to allow the traffic to stabilize in its use of the bridge couple at the optimum usage of the Sattes Bridge. Referring to Figure 10, it will be noted that the percentage increase on the Sattes Bridge after the repeat survey is less than the seasonal increase at the area (automatic) recorder. Portable recorder tubing was frequently cut on both bridges, making an accurate day by day density count almost impossible. The erratic fluctuations in the Patrick Street Bridge graph possibly reflects inefficiency in the machine counts.

Figure 11 shows the comparable before and after traffic by origin and destination for each zone between the Patrick Street Bridge and Winfield Ferry. Trucks are included in the total traffic in this portrayal.

troid between east terminus and Sattes Bridge east of actual line of equity would probably be diverted over Patrick Street Bridge and are, therefore, included in the 150 diverted vehicles enumerated above. It is not believed that any diversion of "S" type trip occurs via Winfield Ferry since the Sattes Bridge is probably the more attractive from time standpoint. The freed bridge should

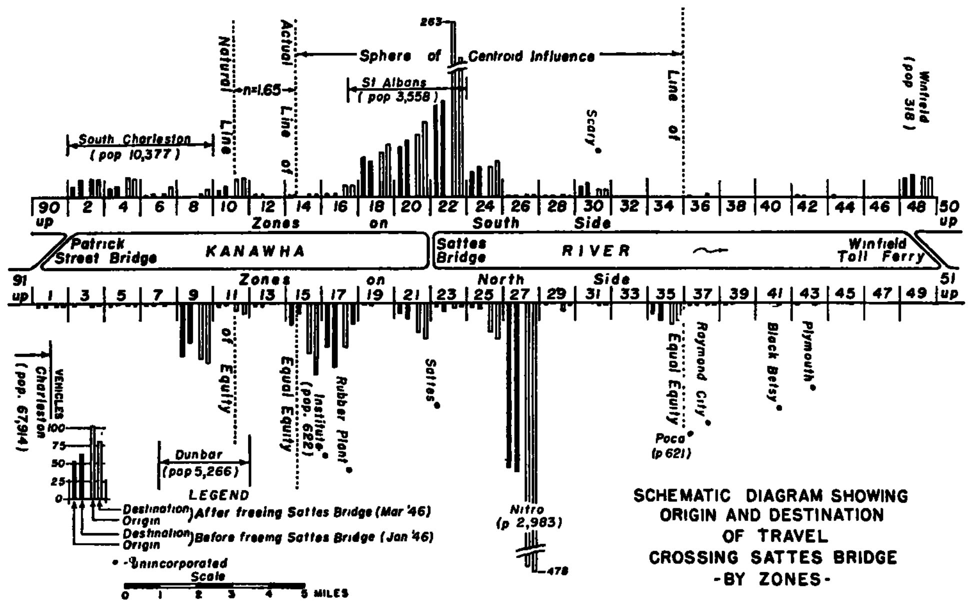


Figure 11

Analysis of Before and After Origin and Destination Study—The analysis is treated in 6 divisions.

1. *Diverted Traffic.* The estimated east to east traffic that would use the Sattes Bridge after tolls were eliminated was based on the amount of traffic that travelled out of the way to avoid a toll charge (Tab. 1, 2, and 3). This traffic, with centroid of origin and destination west of the natural line of equity, amounted to 150 vehicles (Figs. 12 and 14).

With respect to west to west traffic no origin and destination study was made on the toll ferry 14 mi to the west of Sattes for the reason that it carries less than 50 vehicles per day, and having a $0.25 toll charge, and adding an appreciable delay in travel time it was believed to divert but little traffic from the Sattes Bridge. With respect to S trips having terminus east of Sattes those trips with cen-

divert a few cars from the Ferry, but there are but very few that can be diverted.

Thus, probably all told, there were between 150 and 200 passenger vehicles diverted from the Sattes Bridge by way of other facilities. For purpose of further analysis the diverted traffic is assumed to be 175 vehicles or 15 percent of the total 1940 ADT toll traffic, but about 9 percent of hypothetical 1940 ADT freed traffic.

2. *Barred Traffic.* This traffic consists of the foreign vehicle or vehicle having terminii beyond the sphere of influence of the toll bridge, which did not use the bridge until it was freed of toll. It is closely allied to stranded traffic, the limitation being that barred traffic has centroid of terminii beyond natural line of equity. Computing this by a zone to zone transfer by IBM and excluding trips previously used in computing diverted

traffic (practically all of the barred traffic is found with terminii in the S trips) about 35 new trips were found, in addition to 14 trips previously using the toll facility from areas entirely outside sphere of influence.

TABLE 1
ESTIMATED EAST TO EAST TRANSFER PLUS DIVERTED S TRAFFIC VIA SATTES BRIDGE AFTER FREEING (ESTIMATED FROM THE BEFORE STUDY)

Centroid[b] Dist.	No. Veh.	Cum.	Centroid Dist.	No. Veh	Cum.
mi.			*mi.*		
0.20	1.2	277.2	3.36[a]	4 3[a]	174.2
0.40	2.4	276.0	3.40	1.2	169.9
0.45	3.3	273.6	3.41[a]	10.3[a]	168.7
0.50	2.4	270.3	3.50	3.7	158.4
0.70	1.4	267.9	3.60	1.2	154.7
0.75	1.2	266.5	3.61[a]	1.5[a]	153.5
0.80	1.2	265.3	3.65	1.2	152.0
0.85	1.2	264.1	3.66[a]	3 4[a]	150.8
1.01[a]	1.2[a]	262.9	3.70	1.2	147.4
1.05	1.2	261.7	3.71[a]	1.6[a]	146.2
1.10	1.2	260.5	3.80	2.4	144.6
1.31[a]	4.3[a]	259.3	3.95	2.4	142.2
1.35	2.5	255 0	3.96[a]	3.8[a]	139 8
1.40	2.5	252 5	4.05	4.8	136.0
1.50	1.2	250.0	4.15	6.8	131.2
1.55	3.6	248 8	4.16[a]	6.5[a]	124 4
1.60	1.5	245.2	4 20	1.3	117.9
1.70	1.2	243.7	4.21[a]	6 5[a]	116.6
1.75	4.9	242.5	4.25	3.7	110.1
1.80	2.5	237.6	4.26[a]	6.7[a]	106.4
1.85	3.2	235.1	4.31[a]	3.7[a]	99.7
1.86[a]	1.3[a]	231.9	4.36[a]	4.6[a]	96.0
1.90	3.6	230.6	4.50	1.2	91.4
2.00	1.2	227.0	4 51[a]	6.6[a]	90.2
2.05	4.8	225.8	4.60	1.2	83.6
2.06[a]	1.8[a]	221.0	4.61[a]	7.0[a]	82.4
2 10	3.6	219.2	4.66[a]	5.5[a]	75.4
2.15	6.1	215.6	4.71[a]	1.2[a]	69.9
2.20	1.2	209.5	4.76[a]	2.0[a]	68.7
2.26[a]	3.4[a]	208.3	4.81[a]	15.2[a]	66.7
2.30	2.4	204.9	4.85	2.4	51.5
2.35	1.2	202.5	4.86[a]	4.6[a]	49.1
2.40	2.4	201.3	5.01[a]	8.2[a]	44.5
2.45	2.4	198.9	5.06[a]	1.9[a]	36.3
2.60	1.2	196.5	5.11[a]	3.6[a]	34.4
2.70	1.2	195.3	5.21[a]	5.3[a]	30.8
2.85	1.2	194.1	5.26[a]	3.1[a]	25.5
2.90	3.6	192.9	5.41[a]	15.0[a]	22.4
2.96[a]	1.4[a]	189.3	5.45	1.2	7.4
3.00	2.4	187.9	6.05	1.3	6.2
3.06[a]	1.5[a]	185.5	7.00	2.4	4.9
3.15	1.2	184.0	7.50	1.3	2.5
3.25	4.0	182.8	8.25	1.2	1.2
3.26[a]	3.4[a]	178.8			
3.35	1.2	175.4			

[a] Traffic via Patrick Street Bridge diverted from Sattes Bridge to avoid toll payment (apparently) consisting both of diverted U and S trips.
[b] The term Centroid Distance as used herein is the distance from the subject bridge to the centroid of apposite travel.

3. *Stranded Traffic.* (See Tab. 3) Traffic which previously did not exist except as a static reservoir is referred to as stranded traffic. As previously defined all new, induced, or generated traffic with centroid of origin and destination (or one terminus of an S trip) within the natural sphere of influence of the freed facility may be termed toll stranded traffic; or total traffic in this area less original, diverted and barred traffic equals stranded traffic.

TABLE 2
ACTUAL TOTAL EAST TO EAST TRANSFER VIA SATTES BRIDGE AFTER FREEING
(O and D Survey—March 19, 1946)

Centroid Dist.	No. Veh.	Cum.	Centroid Dist.	No. Veh.	Cum.
mi.			*mi.*		
0.10	1.1	237.0	3.85	2.3	119.4
0.15	5.1	235.9	4.00	1.2	117.1
0.20	3.9	230.8	4.05	5.0	115.9
0.40	1.2	226.9	4.15	10.7	110.9
0.55	1.1	225.7	4.25	2.2	100.2
1.00	1.2	224.6	4.30	2.4	98.0
1.25	2.3	223.4	4.40	5.0	95.6
1.50	4.5	221.1	4.45	1.2	90.6
1.65	2.2	216.6	4.50	2.3	89.4
1.70	3.5	214.4	4.55	3.5	87.1
1.80	3.4	210.9	4.60	8.3	83.6
1.90	1.2	207.5	4.75	3.5	75.3
2.00	5.8	206.3	4.85	2.2	71.8
2.05	6.8	200.5	5.00	1.1	69.6
2.10	1.3	193.7	5.10	1.2	68.5
2.15	13.9	192.4	5.20	1.1	67.3
2.20	7.8	178.5	5.25	11.7	66.2
2.25	2.3	170.7	5.30	2.3	54.5
2.35	1.2	168.4	5.50	1.2	52.2
2.60	1.7	167.2	5.65	1.1	51.0
2.65	19.4	165.5	5.75	7.0	49.9
2.90	1.2	146.1	5.90	1.2	42.9
2.95	1.2	144.9	6.00	5.0	41.7
3.00	1.1	143.7	6.05	1.1	36.7
3.15	1.1	142.6	6.10	2.2	35.6
3.30	2.5	141.5	6.15	1.1	33.4
3.35	5.0	139.0	6.25	4.5	32.3
3.40	2.7	134.0	6.30	3.4	27.8
3.45	2.4	131.3	6.35	2.2	24.4
3.50	3.6	128.9	6.45	1.1	22.2
3.55	1.1	125.3	6 50	5.1	21.1
3.60	1 4	124.2	6.60	6.2	16.0
3.70	1.1	122.8	6.90	2.6	9.8
3.75	1.2	121.7	7.50	3.7	7.2
3.80	1.1	120.5	7.65	1.1	3.5
			7.75	1 2	2.4
			7.85	1.2	1.2

The total passenger vehicle traffic expanded to 1940 ADT under tolls was 845. The expansion after freeing the bridge to the 1940 ADT raises this total to about 1500, or an increase of about 655 vehicles per day. Commercial vehicles are not included. The aggregate ADT was raised from 1162 vehicles to approximately 1800 vehicles.

The diverted traffic (175 trips) plus the barred traffic (35 trips) plus the original

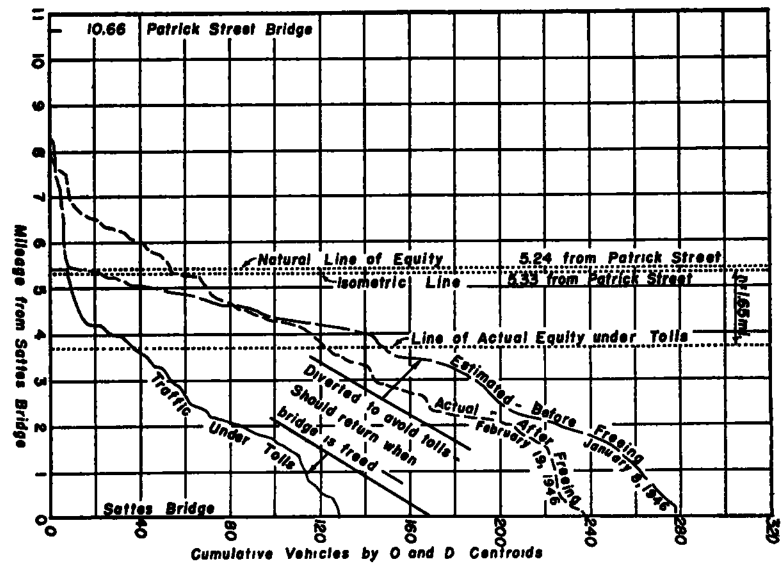


Figure 12. Accrued Traffic—East to East U Turns Estimated and Actual After Freeing Sattes Bridge

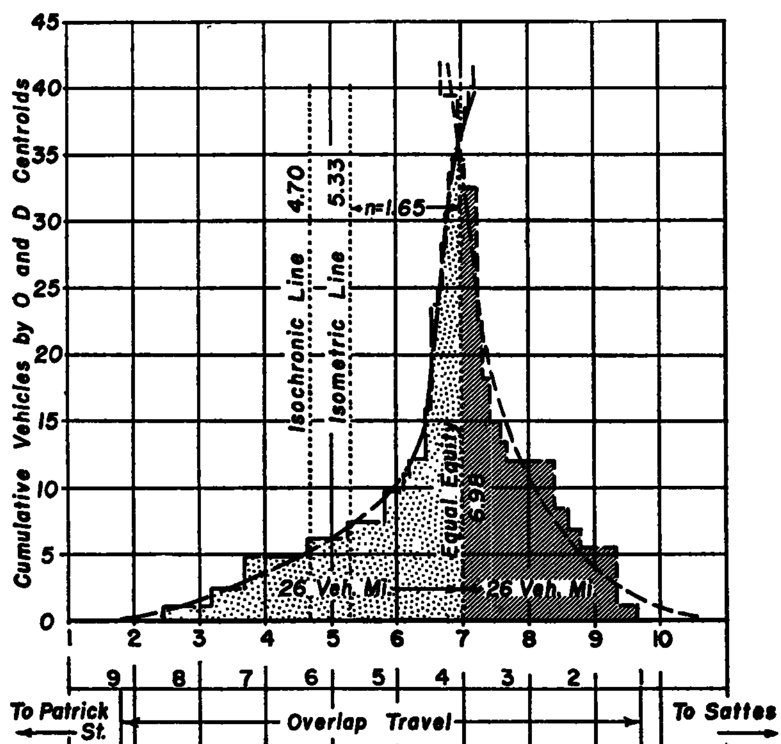


Figure 13. Graph Showing Patrick Street-Sattes Critical Area and Line of Equity

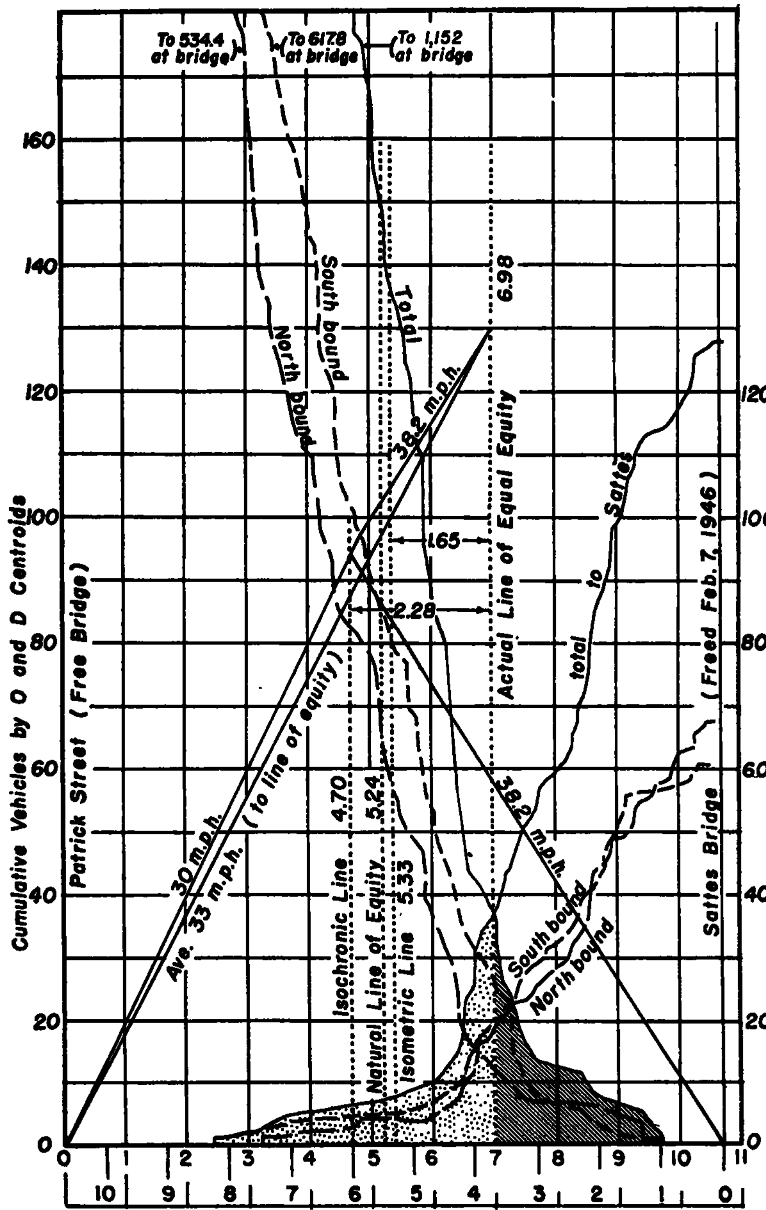


Figure 14. Graph Showing Patrick Street-Sattes Trip Centroid Distribution

traffic (845 trips) makes a total of 1055 trips, which subtracted from 1500 trips, leaves 445 trips per day as the stranded traffic, which is 53 percent of the (passenger) toll traffic (Tab. 4). It will be noted that most of the stranded traffic increase is close in.

4. *Sphere of Toll Bridge Influence.* The method of calculation of line of equal equity (Fig. 13) is shown (See Tab. 5). Vehicle volume and distance are taken from Tables 6 and 7.

Thus, from Table 5, a distance of 7.00

TABLE 3
SATTES BRIDGE USAGE BY O AND D CENTROIDS BEFORE AND AFTER FREEING
(Approximate 100 percent samples unexpanded—Use expansion factor of 1.23 for before values and expansion factor of 1.18 for after values to translate to 1940 ADT)

O and D Centroid Distance	U Trips				S Trips			
	East-East Travel *Before*	East-East Travel *After*	West-West Travel *Before*	West-West Travel *After*	East Terminus *Before*	East Terminus *After*	West Terminus *Before*	West Terminus *After*
mi.	*veh.*	*veh.*	*veh.*	*veh.*	*veh.*	*veh.*	*veh.*	*veh.*
0.25					61	116	69	168
0.50	6	10	6	6				
0.75					25	54	17	58
1.00	3	1	2	21				
1.25					62	31	160	330
1.50	16	9	72	196				
1.75					8	60	2	8
2.00	26	37	27	39				
2.25					11		10	7
2.50	12	19		10				
2.75					11	40		
3.00	7	13	1	2				
3.25					56	71	46	66
3.50	13	15	19	17				
	Line of Actual Equity—(3.68) (Sattes—Patrick Street)							
3.75					3	10	3	3
4.00	17	17	1	4				
4.25					30	19		
4.50	5	23	21	24				
4.75					4	31		
					Natural Equity North Side River (5.57) (Sattes—Patrick Street)			
5.00	3	8			48	183		
5.25		6			3	4		
	Line of Natural Equity—(5.33) (Sattes—Patrick Street)				Natural Equity South Side River (5.09) (Sattes—Patrick Street)			
5.50	1	8		2	17	64		
5.75	8	5				1		
6.00	1	9						
6.25	9	5					2	
6.50	1	4	8	4				
6.75	2	2					17	17
7.00		1						
7.25							13	27
7.50	1	5	12	14				
	Line of Natural Equity—(7.50) (Sattes—Winfield)				Line of Natural Equity (Sattes—Winfield)			
7.75	1	1						
8.00	2		20	27				
8.25	5	7						
8.50				3				
8.75								
9 00		1						
9.25								
9 50	1	0	1					
9.75		6						
10.00			1	2				
10.25	1	1						
10.50			4	1				
10.75								
11.00								
12.00			1					
13.00			1					
14.00			10	1				
Total Trips	141	213	207	373	339	684	339	684

−0.025 = 6.975, say 6.98 mi from Patrick Street Bridge locates the limit of spheres of centroid influence of the two bridges. But, this is an imaginary line derived by statistical process. Few individual motorists place a precise value upon their time, apparently, for the extreme values indicated on the graph vary from an extreme negative value to an extreme positive value (Fig. 13).

TABLE 4
PATRICK STREET—SATTES BRIDGES: SUMMATION OF ACCRUED PASSENGER TRAFFIC

Type	Amount	Percentage of Passenger Toll Traffic
	veh.	%
Diverted	175	20
Barred	35	4
Stranded	445	53
Total Accrual	655	77

TABLE 5
CUMULATIVE VEHICULAR MILEAGE (USING 7.0 MI FROM PATRICK ST. AS REFERENCE LINE)

Via Toll Bridge			Via Free Bridge		
Vehicles	Miles	Veh. Mi	Vehicles	Miles	Veh. Mi.
1.2	4.55	5.46	1 2	2.65	3.18
1.3	3.80	4.94	4 3	2.35	10.10
2.4	3.30	1.92	1.3	1.80	2.34
1.3	2.35	3.05	1.8	1.60	2.88
1.2	1.75	2.10	3.4	1.40	4.76
2.4	1.15	2.76	1.4	0.70	0.98
1.2	0.90	1.08	1.5	0.60	0.30
1.2	0.80	0.96	3 4	0.40	1.36
3.7	0.55	2.03	4.3	0.30	1.29
1.3	0.50	0.65	10.3	0 25	2 51
6.6	0.45	2.97	1.5	0.05	0.07
1.2	0.35	0.42	34.4		29.77
3.6	0.30	1.08			
2.4	0.25	0.60			
2.0	0.20	0.40			
1.2	0.10	0.12			
1.2	0.05	0.06			
35.4		30.60			
plus 35.0	0 03	1.05	Difference		1 88
		31.65			
Adjusting-minus 35.0	0.025	0.88	plus 39	0 025	0.97
		30.77			30.74

7.00 − 0 025 = 6.975. Say 6.98

5. *Corrolary: Monetary Evaluation of Passenger Vehicle Time per Minute.* Equation (2) may be expressed:

$$x = \left[\frac{T - 4cn}{d(k - 1) + 2n(k + 1)}\right]\frac{V}{60}$$

Substituting values determined, assuming operating cost at \$0.03 per mi (Fig. 14):

$$x = \frac{25 - 4\cdot3\cdot1.65}{10.66\cdot0.16 + 2\cdot1.65(1.16 + 1)}\cdot\frac{38.2}{60}$$

$$= \frac{5.20}{8.83}\cdot 0.637$$

$x = 0.375$ cents per min

Distribution of costs per vehicle: Before Freeing (Fig. 15).

Via Sattes Toll Bridge

TABLE 6
SATTES BRIDGE (BEFORE STUDY)—TOTAL CUMULATIVE TRIP CENTROIDS EAST TO EAST TRANSFER

Centroid Dist.	No. Veh.	Cum.	Centroid Dist.	No. Veh.	Cum.
mi.			*mi.*		
0.20	1.2	127.4	2.70	1.2	59.5
0.40	2.4	126.2	2.85	1.2	58.3
0.45	3 3	123.8	2.90	3.6	57.1
0.50	2.4	120.5	3.00	2.4	53.5
0.70	1.4	118.1	3.15	1.2	51.1
0.75	1.2	116 7	3.25	4 0	49.9
0.80	1.2	115.5	3.35	1.2	45.9
0.85	1.2	114.3	3 40	1.2	44.7
1.05	1.2	113.1	3.50	3.7	43.5
1.10	1.2	111.9	3.60	1.2	39.8
1.35	2.5	110.7	3.65	1.2	38.6
1.40	2.5	108.2	3.70	1.2	37.4
1.50	1.2	105.7	3 75	1.2	36.2
1.55	3.6	104.5	3.80	1.2	35.0
1.60	1.5	100.9	3.90	2.8	33.8
1.70	1.2	99.4	3.95	2.4	31.0
1.75	4.9	98.2	4.00	3.6	28.6
1.80	2.5	93.3	4 05	1.2	25.0
1.85	1.2	90.8	4.15	6.6	23.8
1.90	3.6	89.6	4.20	1.3	17.2
2.00	1.2	86.0	4.25	3 7	15.9
2.05	4.8	84.8	4.50	1.2	12.2
2.10	3.6	80.0	4.60	1.2	11.0
2.15	6 1	77 4	4.85	2.4	9.8
2.20	1 2	71 3	5.45	1.2	7.4
2.30	2 4	70.1	6.05	1.3	6.2
2.35	1 2	67.1	7.00	2 4	4.9
2.40	2 4	65.5	7.50	1.3	2.5
2.45	2.4	63.1	8.25	1.2	1.2
2.60	1.2	60.7			

Time cost per mi

$$= \frac{60}{38.2}\cdot0.375 = 0.589 \text{ cents}$$

Total time cost

$$= 0.589\cdot3.68\cdot2 = 4.33 \text{ cents}$$

Total operating costs

$$= 3.0\cdot3.68\cdot2 = 22.08 \text{ cents}$$

Toll cost

$$= \underline{25.00} \text{ cents}$$

Total costs to line of equity

$$= 51.41 \text{ cents}$$

Via Patrick Street Free Bridge

TABLE 7
PATRICK STREET BRIDGE—TOTAL CUMULATIVE WEST TO WEST TRANSFER

Centroid Dist.	No. Veh.	Cum.	Centroid Dist.	No. Veh.	Cum.
mi.			*mi.*		
0.10	1.8	1152.0	3.40	7.8	309.7
0.25	10.2	1150.2	3.45	6.7	301.9
0.30	114.6	1140.0	3.50	2.4	295.2
0.40	9.1	1025.4	3.55	7.0	292.8
0.45	1.6	1016.3	3.60	4.9	285.8
0.50	25.9	1014.7	3.65	1.2	280.9
0.55	11.6	988.8	3.70	5.3	279.7
0.60	23.3	977.2	3.75	3.0	274.4
0.65	16.9	953.9	3.80	9.5	271.4
0.70	4.2	937.0	3.85	5 0	261.9
0.75	27.0	932.8	4.00	3.0	256.9
0.80	44.1	905.8	4.05	1.5	253.9
0.85	16.9	261.7	4.10	6.0	252.4
0.90	1.4	844.8	4.15	17.1	246.4
0.95	17.4	843.4	4.20	7.6	229.3
1.00	9.3	826.0	4.25	5.6	221.7
1.05	7.0	816.7	4.35	1.2	216.1
1.10	7.4	809.7	4.40	5.2	214.9
1.15	7.3	802.3	4.45	14.2	209.7
1.20	7.2	795.0	4.50	10.3	195.5
1.25	17.4	787.8	4.55	3.4	185.2
1.30	2.7	770.4	4.65	1.2	181.8
1.35	1.2	767.7	4.75	2.9	180.6
1.40	1.5	766.5	4.80	2.4	177.7
1.45	11.1	765.0	4.90	3.4	175.3
1.50	9.4	753.9	4.95	3.1	171.9
1.55	16.1	744.5	5.00	1.8	168.8
1.60	47.9	728.4	5.05	14.9	167.0
1.65	3.2	680.5	5.15	15.0	152.1
1.75	8.0	677.3	5.35	1.9	137.1
1.80	6.1	669.3	5.40	1.2	135.2
1.85	3.4	663.2	5.45	5.3	134 0
1.95	13.8	659.8	5.55	3.6	128.7
2.00	5.5	646.0	5.60	1.9	125.1
2.05	4.8	640.5	5.65	8.2	123.2
2.10	9.6	635.7	5.80	4.6	115.0
2.20	8.4	626.1	5.85	15.2	110.4
2.25	11.5	617.7	5.90	2.0	95.2
2.30	18.1	606.2	5.95	1.2	93.2
2.35	5.9	588.1	6.00	5.5	92.0
2.40	5.6	582.2	6.05	7.0	86.5
2.45	9.1	576.6	6.15	6.6	79.5
2.50	57.9	567.5	6.30	4.6	72.9
2.55	22.4	509.6	6.35	3.7	68.3
2.60	12.6	487.2	6.40	6.6	64.6
2.65	6.9	474.6	6.45	6.5	58.0
2.70	13.8	467.7	6.50	8.3	51.5
2.75	13.2	453.9	6.70	3.8	43.2
2.80	19.8	440.7	6.95	1.6	39.4
2.85	23.1	420.9	7.00	3.4	37.8
2.90	9.1	397.8	7.05	1.5	34.4
2.95	10.3	388.7	7.25	10.3	32.9
3.00	5.8	378.4	7.30	4.3	22.6
3.05	25.1	372.6	7.40	3.4	18.3
3.10	7.9	347.5	7.60	1.5	14.9
3.15	9.3	339.6	7.70	1.4	13.4
3.20	9.9	330.3	8.40	3.4	12.0
3.25	5.3	320.4	8.60	1.8	8.6
3.30	1.9	315.1	8.80	1.3	6.8
3.35	3.5	313.2	9.35	4.3	5.5
			9.65	1.2	1.2

Time cost per mi

$$= \frac{60}{33}\cdot 3.75 \quad = 0.682 \text{ cents}$$

Total time cost

$$= 0.682\cdot 6.98\cdot 2 \ = 9.52 \text{ cents}$$

Total operating cost

$$= 3.0\cdot 6.98\cdot 2 \ = \underline{41.88} \text{ cents}$$

Total costs to line of equity

$$= 51.40 \text{ cents}$$

6. *Solution*[5] *for Natural Line of Equity* (Fig. 12).

$$x = \frac{T - 4cn}{d(k-1) + 2n(k+1)}\cdot\frac{V}{60} \quad \ldots\ldots\ldots (2)$$

$$x = \frac{0 - 4cn}{10.66\cdot 0.27 + 2n(1.27 + 1)}\cdot\frac{38.2}{60}$$

Substituting value of x (Division 5. *Correlary*)

$$0.375 = \frac{0 - 4\cdot 3n}{2.88 + 4.54n}\cdot 0.637$$

$$1.08 - 1.70n = -7.64n$$

$$9.34n = -1.08$$

$$n = -0.09$$

Natural line of equity, after freeing

$$= 5.33 - 0.09 = 5.24$$

Distribution of costs: After Freeing

Via Patrick Street

Time cost

$$= \frac{60}{30}\cdot 0.375 \ = 0.75 \text{ cents per mi}$$

Total time cost

$$= 5.24\cdot 2\cdot 0.75 = \ 7.86 \text{ cents}$$

Operating cost

$$= 5.24\cdot 2\cdot 3.0 \ = \underline{31.24} \text{ cents}$$

Total cost to line of equity

$$= 39.00 \text{ cents}$$

Via Sattes

$$\frac{60}{38.2}\cdot 3.75 = 0.6 \text{ cents per mile (Time cost)}$$

Total time cost

$$= 5.42\cdot 2\cdot 0.6 \ = \ 6.50 \text{ cents}$$

Operating cost

$$= 5.42\cdot 2\cdot 3.0 \ = \underline{32.52} \text{ cents}$$

Total cost to line of equity

$$= 39.02 \text{ cents}$$

OBSERVATION

This study raises the question: "How much is time worth to induced (or stranded) traffic?" Apparently their time is worth less, or their ability to purchase use of the

[5] This is a sample procedure—the same method would be followed for any other bridge couple.

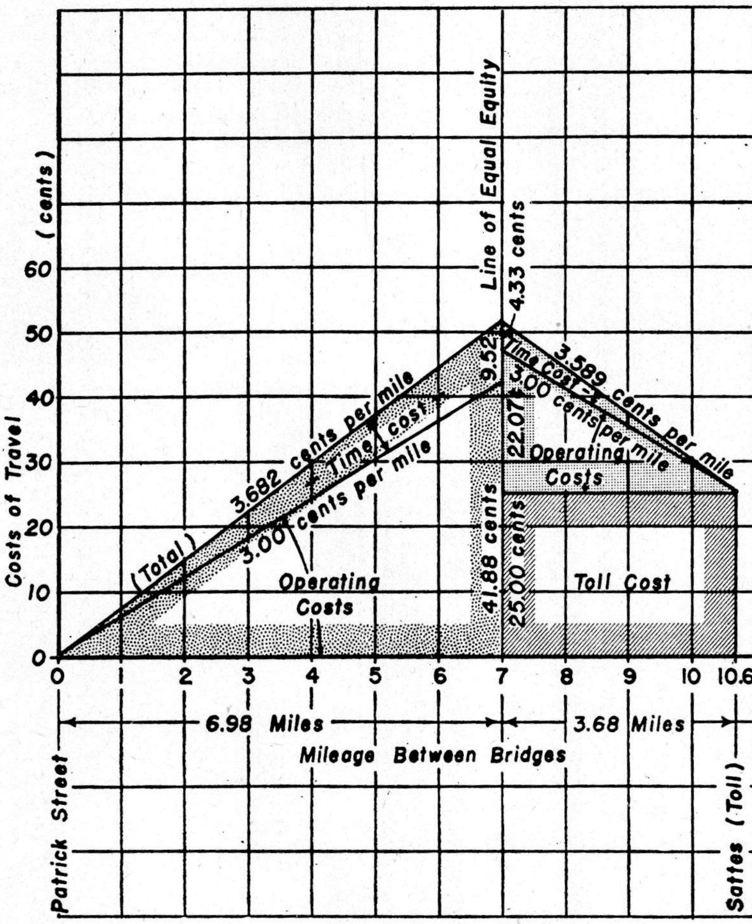


Figure 15. Graph Showing Distribution of Costs of Travel via Patrick Street, Charleston, and Sattes Toll Bridge

Figure 16. Patrick Street Bridge—From North Side of River Looking West

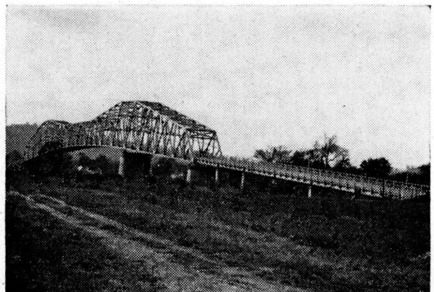


Figure 17. Sattes Bridge—From North Side of River Looking South—Freed Feb. 7, 1946

facility is less than that of the original toll traffic.

The ability to pay for a purchase of time must be coupled with the desire to make the purchase. An addition of induced traffic over the freed Sattes Bridge equal to 53 percent of the original toll traffic resulted from removal of the purchase price on time. Again: "How much induced traffic can be secured with a reduction of 50 percent in toll charge?" The line of equal equity balances the value of time with the rate of toll for those who have the desire to purchase time and can afford to do so at the prevailing rate. Now, suppose the rate reduced. "Will the change in position in the line of equity be proportional to the change in toll rate, or will it move so as to indicate a new, lower, value for time, as more traffic is induced?"

These questions can only be answered factually, it would seem, by making an O and D study before and after changes are made in toll rates.

Another question that presents itself, to remain unanswered in this study, is this: "To what extent will induced traffic on a freed bridge operate to oppose diversion of traffic from an adjacent toll bridge thus affording protection to the toll bridge financial structure?"

CONCLUSIONS[6]

An interpretation of the mathematical analysis of cause and effect lead to the following conclusions:

1. The motorist does not place a precise, uniform, monetary value on his time, but selects a route which more nearly accommodates the urgency of immediate demand. Time cost, it appears, is a variable dependent upon individual driver judgment, time of trip (hour and day), together with economic demands upon and economic environment of motorist.

[6] These conclusions, taken from the original thesis, were based on the study of three couples of bridges.

2. Each toll facility has its own peculiar sphere of influence as determined by factors which include toll rate and speeds of travel in the immediate area, together with its economic and social environment.

3. Each toll facility exerts its own peculiar influence upon highway traffic in the nature of diverted, barred, stranded, and user traffic.

4. The perpetuation of tolls on a fully amortized facility to protect the financial stability of an adjacent toll structure may not be necessary. Each toll bridge is inherently protected if sufficient distance and travel time separates it from the next adjacent bridge. Each situation should be the subject of a special study in cause and effect.

5. Origin and destination surveys, in conjunction with time-delay studies furnish very precise instruments for measuring sphere of influence, lines of equity, values of vehicle time, and diversion of traffic. They also serve a useful purpose in estimating the anticipated change in facility use occasioned by a change in toll rate.

REFERENCES

1. Tucker and Leager, "Highway Economics," pp. 84–86, Scranton, Pennsylvania: International Textbook Co., (1942)
2. Byron W. Shimp, "Financing Public Improvements," p. 1 and 62, New York: Van Ingen and Co.
3. H. S. Fairbank, Deputy Commissioner, Public Roads Administration, Washington, D.C. Mr. Fairbank states "The above data have not been verified and the Public Roads Administration cannot at the present time take responsibility for their accuracy."
4. Coverdale and Colpitts, "Report on Traffic and Revenues, Lake Washington Bridge Project," New York—Coverdale and Colpitts, p. 27, (1938)
   See also Jorgensen's "Origins and Destinations of Highway Traffic, the Basis for Connecticut Planning" *Proceedings*, Highway Research Board, Vol. 23, p. 372, (1943).