VIBRATORY AND SLOW REPETITIONAL LOADING OF SOILS

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SYNOPSIS

The paper forms a continuation of the publication, "Effect of Vibrations on the Bearing Properties of Soils", by G. P. Tschebotarioff, which appeared in Vol. 24, *Proceedings* of the Highway Research Board (1944). It reports further findings made since then during a study carried out for the Technical Development Service of the Civil Aeronautics Administration by the Soil Mechanics Laboratory of Princeton University.

Controlled stress small scale plunger tests on two types of clay and unconfined compressive strength tests performed on one clay under both static and vibratory conditions indicated that for the soils tested the deformation producing capacity of a vibratory force was no greater than that of an equivalent static force so long as there was no resonance of the vibrator-soil system. Comparison of static and vibratory consolidation test results on three types of clay supported this finding.

Previously reported plunger tests on a well graded sand had shown that the sand was very sensitive to the action of vibratory forces. Further test results obtained on a uniform poorly graded beach sand are now presented. This sand was found to be even more sensitive to vibratory forces which produced up to 140 times greater plunger penetrations than the penetrations produced by equivalent static forces. This was still the case when the vibratory force was very small and did not exceed ± 2.0 percent of the original static load, even though this load was first reduced by some 15 percent to simulate load redistribution between wheels such as takes place during engine warm-up of airplanes in the field.

Tests with sand-clay mixtures indicated that the resistance to vibratory forces of sands could well be improved by the addition of clay.

A series of slow repetitional plunger load tests was performed on sands and on a clay with a specially designed machine. The results showed that, so long as no resonance of the vibrator-soil system occurred, the frequency of load repetition was of no importance. Identical plunger penetrations were obtained for the same number of load repetitions irrespective of whether the test was performed at one cycle per min or at 1500 cycles per min. For the same number of load repetitions on sand, plunger penetrations increased with the intensity of the slow repetitional or vibratory load. For the same fractional value of the slow repetitions naturally increased as a fraction of the original load, the plunger penetrations naturally increased also with the value of the original load. However, a slow repetitional force of the same intensity was found to produce somewhat higher penetrations with a decrease of the intensity of the original static load.

A previously observed rapid increase of both static and dynamic plunger penetrations when the sand was below a certain density, was now related by means of triaxial shear tests to the so-called "critical density."

Mention is made of the results of two supplementary investigations performed for the project.

The purpose of this investigation, sponsored by the Technical Development Service of the Civil Aeronautics Administration and

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conducted under the direction of the senior author by the Soil Mechanics Laboratory of the School of Engineering, Princeton University, was to obtain some insight into the relative importance of the various factors affecting the bearing properties of soils in airport subgrades and base courses when subjected to vibratory and to slow repetitional loading by warming-up and by taxiing aircraft.

The results of the first year's experiments were reported in 1944 to the Highway Re-



Figure 1

Curve 1. Max. shear values developed by vertical pressure N (Case 1 of Prof. Baron) corresponds to condition when airplane motors are not running.

Curve 2. Max. shear values developed by a vertical pressure V only (V = 0.857 N) corresponds to condition when airplane motors are running at 2200 rpm, but the effect of the horizontal force is ignored.

Curve 3. Max. shear values developed by the original vertical pressure N plus a horizontal force $H = \frac{N}{3}$ (Case 3 of Prof. Baron). Condition cannot actually occur.

Curve 4. Max. shear values developed by the vertical pressure V = 0.857 N plus a horizontal force H = $\frac{V}{3}$. Actual condition when motors are running at 2200 rpm.

search Board in a paper by G. P. Tschebotarioff $(1)^3$. This paper reports results obtained since then. The main soil tests were supplemented by two additional investigations.

² Italicized figures in parentheses refer to list of references at the end of the paper.

Prof. Frank Baron of Yale University, acting as special consultant to the Princeton Soil Mechanics Laboratory, was requested to undertake a mathematical study of the redistribution of shearing stresses in a pavement beneath an airplane wheel during warmup of the motors. The results of this study are described in detail elsewhere (7). Figure 1 shows an adaptation of Prof. Baron's findings to likely actual conditions. Curve 1 shows the distribution with depth of maximum shearing stresses developed by a vertical force N and curve 3 refers to the distribution of maximum shearing stresses developed by a horizontal force $H = \frac{N}{3}$ added to the original normal force N. However, during the warm-

normal force N. However, during the warmup of motors the vertical load on the main tires is reduced by some 15 percent (4). Curves 2 and 4 correspond to this condition. By comparing curve 4 to curve 1 (the latter corresponding to the condition when motors are not running) it can be seen that the application of a horizontal force to the pavement surface creates an increase of shearing stresses only at a depth smaller than 17 percent of the major diameter of the tire imprint, e.g., only within the depth of pavement surfacing.

A second special investigation was performed for the purpose of determining the characteristics of vibratory forces transmitted to pavements by airplane tires. Dr. R. K. Bernhard, acting as special consultant to the Princeton Soil Mechanics Laboratory, designed equipment for the purpose (8). Trial field measurements with this equipment showed that for the type of plane tested with properly adjusted engines the vibratory force transmitted to the pavement had the order of magnitude of ± 1 to ± 2 percent of the original static load. This result agreed with the results of the Marietta tests performed by the U.S. Engineers at approximately the same time (4).

The LaGuardia trial tests, however, disclosed an additional important fact. It was found that the vibratory forces transmitted to the pavement could reach a value of ± 4.3 percent of the original static load, whereby this did not correspond to the highest speed of the motors, indicating possibilities of resonance of certain component parts of the plane at intermediate speeds. Studies previously performed at Princeton indicated that below a certain density range both static and dynamic plunger penetrations increased rapidly. Referring to Figure 2, it may be seen that the break in the curves occurs at a void ratio of approximately 0.65. For the soil used in these tests (Sand A), standard AASHO compaction procedure gives a void ratio at maximum density of 0.575; at 95 percent of maximum density a void



Figure 2. Area of plunger = 3 sq in—Diam.of Container = 6 in.—Surcharge = 12.3 lb.(Note: Incremental load was applied in 10-lb increments. Gradual load was applied by filling tank with water over a 1-hr period.

ratio of 0.67; and, at 90 percent of maximum density a void ratio of 0.76. From these figures, it may be seen that, for the sand employed in these tests, densities less than 95 percent of maximum resulted in very large penetrations. By the use of triaxial compression tests, the critical density of Sand A was found to be at a density greater than 95 percent of AASHO maximum (Fig. 3). The significance of these data in terms of field application is obvious.

The first series of vibratory soil experiments

already reported (1) were performed by means of plunger tests of the controlledstress type. The testing set-up is shown by Figures 2 and 3 of Reference 1.

The results indicated that the bearing properties of the one type of clay tested were not unfavorably affected by vibrations so long as the frequency of the vibrations lay outside the resonance range of the vibratorsoil system. Additional plunger tests have been performed with the same type of silty clay (referred to for purposes of brevity as the "red clay") and another type (referred



to as "blue clay"). The properties of both types of clay (grain size distribution, LL, PL, and PI) are summarized on Figure 4.

It can be seen from Figure 5 and from Figure 6 that for both types of clay the deformation producing capacity of a vibratory force is approximately equal to that of an equivalent static force.

Several consolidation tests were then performed with standard equipment both in the usual static manner and on a vibration table designed for the Princeton Soil Mechanics Laboratory by Dr. R. K. Bernhard and Dr. J. G. Barry (5). A typical test result is shown by Figure 7 which gives the pressurevertical deformation curve (at a semi-log scale) of a Wyoming bentonite during consolidation tests performed both under static and under vibrated conditions. The initial water content approximately corresponded to the liquid limit. The type of bentonite used was known to have high thixotropic







Figure 5. Material; Red clay—Compaction Method; 2000 psi static load—Water content determined at end of test.-Diameter of Container = 6 in.—Area of plunger = 3 sq in. Symbol 2 3 Surcharge-lb.... 18.6 12.3 12.3 Centrifugal force-23.2 40.4 10.3 lb.. 1500 1000 1500 Frequency-rpm... 31.7 31.7 Static load-psi 55

properties, yet there is practically no difference between the static and the vibrated consolidation curves. Similar results were obtained during comparative consolidation tests performed with red clay and with undisturbed samples of Flushing Meadows clay as shown by Figure 8 and 9. The latter clay



Figure 6. Material; Blue Leaf clay—Compaction Method; 2000 psi static load—Diameter of Container = 6 in.—Area of Plunger 3 sq in.— Surcharge 18.61b—Centrifugal Force = 10.31b— Frequency = 1000 rpm—Static Load = 31.7 psi.



Figure 7. Pressure - Vertical Deformation Curves—Wyoming Bentonite—Vibratory Test Performed on the Vibration Table—Frequency = 1080 cycles per min.—Amplitude = 0.001 in.— In each case a 1-in. thick sample of 2.5-in. diameter was used. The initial water content wi was 57.2 percent. The final water content wf was 32.7 percent. The consistency corresponded to 15 blows on the liquid limit device at start of test.

had an initial water content of 95 percent and was very sensitive to remolding.

Some doubts arose as to the suitability of



Figure 8. Pressure - Vertical Deformation Curves—Red Clay—Dynamic and Static Consolidation—Vibratory Test Performed on the Vibration Table—Frequency = 1080 cycles per min.—Amplitude = 0.001 in.—In each case a 1-in. thick sample of 2.5-in. diameter was used. The initial water contents wi were 35.1 percent in the static tests and 34.2 percent in the vibrated tests. The soil constants were; LL = 31; PI = 7.



Figure 9. Pressure - Vertical Deformation Curves — Flushing Meadow Clay — Vibratory Test Performed on Vibration Table—Frequency = 1080 cycles permin.—Amplitude = 0.001 in.—Initial Water Content wi = 95 percent— LL=88—PI=30—Both Samples Undisturbed— Note: The one point which falls below the curve for the vibrated test shows the accidental effect of torsional deformation during application of that load increment.

vibration tables for this type of tests. On the other hand, plunger tests could not well be performed at water contents above the

plastic limit. For that reason comparative static and vibratory unconfined compressive strength tests were performed on cylinders of red clay 3.4 in. in diameter and 7 in. high. The stress-strain curves obtained are shown on Figure 10. It can be seen that the vibratory force did not produce any decrease of the shearing strength of the specimen in excess of the strain corresponding to the static value of the vibratory stress increment.



Figure 10. Unconfined Compression Test | Red Clay—Compaction = 500 psi static load— Water Content W (see figure)—Diameter of Sample = 3.36 in. (static), 3.39 in. (vibrated)— Height of Sample = 7 in.—Centrifugal Force = 3.31 lb—Frequency = 1500 rpm.

Thus, all three types of vibratory tests performed on clay soils led to the same conclusion concerning the absence of detrimental effects of vibrations on the compressibility or the shearing strength of cohesive soils.

On the other hand, the preliminary tests on one type of sand (referred to as "Sand A" (1)) showed the considerable effect of vibrations on the bearing properties of cohesionless soils. Plunger tests indicated that a vibratory force could produce on Sand A up to 40 times greater deformations than those produced by an equivalent static force. It was therefore decided to perform one more series of tests on a different type of sand. Sand from the famous racetrack at Daytona Beach, Florida was selected for the purpose. It will be referred to as Sand B. Figure 11 gives the grain-size distribution curves of



Figure 11. Granular Analysis of Sands A and B



Figure 12

both Sands A and B. Figure 12 shows a microphotograph of Sand B. Figure 13 shows a microphotograph of Sand A.

Plunger tests performed in a manner identical to the one previously described (1) indicated that the bearing properties of the uniform Daytona Beach Sand B were



Figure 13



Figure 14. Sand B—Compacted by Static Load as Noted Adjacent to Plotted Points on Figure—Surcharge = 0.49 psi—Diameter of Container = 6 in.—Centrifugal Force = 23.2 lb—Frequency = 1500 rpm—Area of Plunger = 3 sq in.—Static Load = 31.7 psi.

affected by vibrations to an even greater extent than those of the better graded Sand A. It can be seen from Figure 14 that a vibratory



Figure 15. Sand B—Compacted by Static Load as Noted in psi Adjacent to Plotted Points on Figure—Surcharge = 0.49 psi—Diameter of Container = 6 in.—Area of Plunger = 3 sq in.



Figure 16. Sand B—Compacted by Static Load as Noted in psi Adjacent to Plotted Points on Figure—Surcharge = 0.49 psi— Diameter of Container = 6 in.—Area of Plunger = 3 sq in.—Centrifugal Force = 23.2 lb—Frequency = 1500 rpm—Static Load = 31.7 psi.

force could produce on Sand B up to 140 times greater deformations than the deformations produced by an equivalent static force.

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Figure 15 and 16 show the effects of density on the static and the dynamic plunger penetrations on Sand B.

So far all of the plunger tests were performed with vibratory forces equal to ± 25.0 , ± 12.5 , and ± 6.25 percent of the original static load. In order to simulate field con-



Figure 17. Sand A—Submerged—Compacted by 1500 psi Static Load— Void Ratio, e = 0.590— Diameter of Container = 6 in.—Area of Plunger = 3 sq in.— Surcharge = 0.49 psi—Centrifugal Force = 3.31 lb—Frequency = 1500 rpm.

ditions during airplane engine warm-up a further series of plunger tests was performed during which the static load was first reduced by approximately 15 percent whereupon a vibratory load equal to only ± 2.0 percent of the original static load was applied to the plunger.

Figures 17 and 18 show typical results of this type of test. It can be seen that in the case of the well graded Sand A the slight vibration produced approximately 15 percent additional plunger penetration as compared to the penetration produced by the original



Figure 18. Sand B—Dry—Compacted by 1000 psi Static Load—Void Ratio, e = 0.847—6-in. Container, 3-sq in. Plunger, 0.49-psi Surcharge —Centrifugal Force = 3.31 lb—Frequency = 1500 rpm.



additional 138 percent plunger penetration as compared to the penetration caused by the original static load, although the vibratory force was only ± 2.0 percent of that load. This demonstrates the fallacy of the reasoning which is sometimes advanced (Ref. 4, p. 30, Par 67) to the effect that since the reduction of the static load on the main wheels of a plane may amount to as much as 15 percent, it may compensate for the effect of vibratory forces amounting to about ± 3 percent of the static load. In the case of cohesionless soils the deformation producing capacity of static and of vibratory forces cannot be directly compared on the basis of their relative magnitude.

In order to estimate the effect, if any, of variations in the frequency of load repetition (outside of the resonance range) the following experiment was performed. A slow (1 cycle per min) repetitional plunger load test was run under conditions otherwise identical to the ones of similar vibratory tests performed at a frequency of 1,500 cycles per min. The results are shown on Figure 19. It can be seen that for the same soil of identical density range the frequency of load repetition was found to be of no importance. The only thing that mattered was the number of load repetitions so long as other conditions remained the same.

This finding was confirmed by a series of slow repetitional plunger tests performed on a machine shown by Figure 20. It was specially designed for the purpose, at the senior author's request, by Mr. Edward R. Ward, research associate at the Princeton Soil Mechanics Laboratory.

When plunger penetrations were plotted against the number of load repetitions (up to 10,000) at a semi-log scale, the curves obtained on cohesive soils had the shape shown on Figure 21 which was in substantial agreement with the results of previous similar slow-repetitional loading experiments reported for cohesive soils by Mr. Kersten (3). The curves obtained on sands, however, had an entirely different shape (Fig. 21) which indicated the considerable and continued sensitivity of sands to any type of repetitional loading, whether vibratory or slow repetitional. Figures 22 to 26 are graphical test records which illustrate the above statement.

Figures 27 and 28 give the summary of slow repetitional load test results on the Daytona Beach Sand B and Figures 29 and 30 give similar data for Sand A. It can be seen that for the same number of load repeti-

static load. In the case of medium density (45%) Sand B, 10 min vibration produced an

tions the effect of a slow repetitional load increases with its own intensity. For the same fractional value of the slow repetitional or vibratory force, expressed as a fraction



Figure 20



Figure 21

of the original load, the plunger penetrations naturally also increased with the value of the original load. However, a slow repetitional force of the same intensity was found to produce somewhat higher penetrations with a decrease of the intensity of the original static load (Fig. 31).

A comparison of data obtained from vibratory and from slow repetitional load tests on the Daytona Beach Sand B showed that 10 full load repetitions increased the plunger penetrations in approximately the same proportion as 10 minutes vibration (that is



Figure 22. Vibratory Test—Sand B—Dry— Static Compaction (noted on curves)—Void Ratio (noted on curves)—6-in. Diameter Container, 3-sq in. Plunger, 0.49-psi Surcharge, 23.2-lb Surcharge, 1500-rpm Frequency, 31.7-psi Static Load.

1,500 repetitions) with a vibratory force equal to only ± 2.0 percent of the same original static load. Any similar comparisons in the field should take into consideration the distribution of traffic on airfields (a point brought out by Mr. L. A. Palmer (2)) as opposed to the usual concentration of coverages during most accelerated traffic tests.

Some tests performed with sand-clay mixtures indicated that an addition of some clay



Figure 23. Vibratory Test—Red Clay—2000lb Static Compaction—Water Content at End of Test (noted on curves)—Other conditions same as for Figure 22.



Figure 24. Slow Repetitional Test—Sand A—Dry—1000 psi Static Compaction, Void Ratio (noted on curves), 6-in. Diameter Container, 3-sq in. Plunger, 0.49-psi Surcharge, 1-rpm Frequency, 165-lb Plunger, Counter Weight (noted on curves).



Figure 25. Slow Repetitional Test—Sand B— 2000 psi Static Compaction—Other conditions same as Figure 24.

to sand, the exact amount depending on the gradation of the soils, was sufficient to make



Figure 26. Slow Repetitional Test—Red Clay —300 psi Static Compaction—Water Content at End of Test (noted on curves)—6-in. Container, 3-sq in. Plunger, 0.49-psi Surcharge, 1-rpm Frequency, 45-lb Plunger, Counter Weight (noted on curves).







the mixture behave more like a clay than like a sand.

It is hoped that the studies described in



Figure 29. Slow Repetitional Test—Sand A—Dry—1000-psi Static Compaction—Void Ratio (see Fig. 24)—6-in. Container, 3-sq in. Plunger, 0.49-psi Surcharge, 1-rpm Frequency, Plunger Weight, T (noted on curves).



Figure 30. Conditions same as for Figure 29



Figure 31. A slow repetitional force of the same intensity produces somewhat higher plunger penetrations with a decrease of the intensity of the original static load except at very high values of plunger penetrations (Sand B).

this paper may contribute to a better understanding of the performance of different types of soils in the presence of vibratory and of slow repetitional loading. The results obtained seem to indicate the need for increased attention to the grading, to the compaction and to the drainage of sand subgrades and base courses beneath pavement sections likely to be subjected to vibratory and to slow repetitional loading.

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NEW GLACIAL FEATURES IDENTIFIED BY AIRPHOTOS IN SOIL MAPPING PROGRAM

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SYNOPSIS

At the outset of the current soil mapping program, from aerial photographs, of the State of Indiana certain glacial airphoto patterns were found to be very complex and as a result an endeavor was made to trace the glacial features which have complicated the patterns.

The first part of the paper discusses the areal soil (regional soil) mapping program and how it is being done from aerial photographs. The common bedrock and glacial patterns are discussed and illustrated so that the reader may better understand the complex patterns. The remainder of the paper discusses many of the complex patterns and the glacial features which cause them to be developed. The complex patterns of these soils are being studied with the object of determining the lateral extent of such soils and their relation to highway engineering. The following are a few of the complex situations being studied: intermixed gravel and drift on glacial drift of an older period; recent terrace gravels on older drift deposits; sand clay and gravel clay deposits associated with older glacial drift; pre-glacial drainage; glacial drift borders; and resorted recent drift on older drift.

This study reveals that these patterns have definite elements which aid in their identification. Materials associated with these patterns are important in both highway and airport engineering. For example, sand-clay and gravelclay materials find use in base course construction and in the surfacing of county roads in areas where other engineering soils are undesirable. The prediction of older drift is important because of its dense, indurated structure and low permeability.

The soil mapping program, which is being initiated, is comprehensive and a large amount of data are now being obtained. When the project is completed, maps will be available which will show the locations of potential sources of aggregates, base course materials, and materials for low cost roads for state and county use. Maps will be available for use in evaluating pavement performance; they can also be used in the design of pavements on an areal or regional basis from the standpoint of soils. The speed with which the program proceeds will depend, largely, upon the accuracy and refinement with which these complex glacial features can be evaluated from aerial photographs.

Purdue University and the State Highway Commission of Indiana are co-operatively undertaking a state-wide program for the purpose of preparing the following county maps: (1) a drainage map, showing major streams, creeks, large gullies, and small field gullies; and (2) an engineering soils map, showing the areal extent of soils, including the