DEPARTMENT OF SOILS INVESTIGATIONS

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SCANDINAVIAN SOIL FROST RESEARCH OF THE PAST DECADE

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SYNOPSIS

Fundamental research work, dealing with conditions and mechanics of frost heave and frost boils and with methods for preventing and counteracting them (base exchange, isolation, deep drainage, etc.) was completed prior to 1935. A formula was written giving frost-heave speed as a function of load pressure, ground water depth, and fundamental soil properties: capillarity, permeability etc.

In the past decade (1937–1946) the principal advance in this field has been the result of the comprehensive Norwegian investigation on frost depth as a function of temperature conditions and material properties. The paper describes this work and contemporary Swedish research in frost action.

Formulas are derived for a given "frost quantity" (freezing index in U. S. A.) by which frost depth may be calculated as a function of the soil material's heat conductivity, heat capacity, and the freezing energy liberation of its water content at freezing. The formulas may be used in homogeneous materials and where several strata of different materials are present.

The effect of soil stabilizing chemicals (calcium chloride, water soluble resins, sulphite leach, etc.) on frost action is reported.

Most of the original roads in Scandinavian countries were very poor because of severe climate, light population, and an exceptionally high frequency of soils very sensitive to frost. The advent of motor traffic on these roads pointed at an early date to the urgent need for study of the soil frost problem. In the middle twenties systematic research was started dealing with the formation of frost heaving and frost boils and with counteraction methods. Mass exchange, isolation, deep drainage, and arctic-alpine frost structuring of ground (solifluction etc.) were included in the studies.

These researches were performed before 1935 in cooperation with the contemporary research work of Stephen M. Taber, P. Kokkonen, and others (see Beskow (1935)—American edition[1] (1947)—with detailed bibliography). They consisted of continual frost research dealing mainly with practical counteraction methods, and some special theoretical detail questions.

The principal works of the past decade have been the comprehensive Norwegian investigations of frost depth in relation to temperature-time conditions and material properties (1937–1946) performed by Brudal, Arne Erikson, Heje, Skaven Haug, Watzinger, Kindem, Michelsen, and others (see Beskow[1], bibliography). These works, giving frost depth as a function of the product of frost temperature and time, called "frost quantity" by the Norwegians and "freezing index" by the Americans, may be of special interest and thus are given major space in this paper.

FROST DEPTH AND FREEZING RESISTANCE CALCULATIONS

Frost Depth Formulas—Frost depth, as a function of time and surface temperature, could be calculated in accordance to Stefan's for-

[1] Gunnar Beskow, "Soil Freezing and Frost Heaving with Special Application to Roads and Railroads" Swedish Geological Society, Series C, No. 375, 26th Yearbook No. 3, Technological Institute, Northwestern University, Evanston, Ill., (Nov. 1947).

mula for ice thickness, through the substitution of proper material coefficients. The formula for frost depth thus will be

$$S = \sqrt{\frac{2t + T\lambda}{Q}} \qquad (1)$$

where

$$Q = 80f + \frac{t}{2}(0.5f + 0.55s) \qquad (2)$$

or more simply

$$Q = 80f + 0.2t \text{ (approx.)} \qquad (3)$$

and

S = frost depth in cm.
T = time in sec.
t = surface temperature in deg. C. below 0
λ = heat conduction coefficient (cal. per cm. sec. deg.)
Q = "frost storing capacity" (cal. per cu. cm.)
f = (freezing) water content, pro volumina (g. per cu. cm.)
s = mineral soil substance, pro volumina (cu. cm. mineral substance per cu. cm.)

In formula (2) the sum, $0.5f + 0.55s$, represents heat capacity of soil, where 0.5 and 0.55 are heat capacity of ice (referred to water volumina) and dry soil mineral substance, respectively. Pure empirical data may be substituted for $0.55s$ in cases of organic soil.

As the total heat capacity of frozen mineral soil shows rather small variations (from about 0.5 to 0.4 for wet clay, less than 0.4 for water saturated and 0.38 for moist sand, to about 0.35 cal. per cu. cm. for dry sand), an approximate figure (0.4) could be used for all mineral soil, resulting in the simple approximate formula (3).

These formulas, dealing with f as water content (although the water exists as ice), presume no total volume increase, i.e. the water is considered as expanding within the pore space, as in moist sand when freezing. In case there is volume increase of the freezing soil, the increase must be regarded in the calculation. Referring to water content of unfrozen soil, heat capacity of ice (72 cal. per cu. cm.) has to be substituted for specific heat (80 cal. per g.) thus:

$$Q = 72f_i + 0.2t \text{ (approx.)} \qquad (3b)$$

where f_i indicates ice content, instead of water content in g. per cu. cm.

This correction is of special importance in case of prominent frost heaving, with considerable water-suction from below toward the frost limit. Here the figure which matters is not the original water content, but the water (ice) content after freezing.

The formulas also presume that all water at the frost border is frozen. This condition is practically realized in case of coarser soils, but in case of colloidal soils (clays) consideration must be given to the complicating matter of water's freezing-point depression, i.e. clay water starting freezing somewhat below zero C. with the percentage of frozen water gradually increasing with lower temperature but a considerable part of the water still unfrozen at a temperature of some degrees below zero.

To correct for this condition:

1. Reduce temperature t by an amount equal to the real temperature at the frost border (in wet fat clays about −0.2 to −0.5 deg. C.).

2. Water content f should be the freezing portion of water, approximately the mean quantity within the soil frost layer. For example, if top temperature = −5 deg. C., frost border temperature = −0.5 deg. C., the mean temperature = −2.75 deg. C.; at this temperature say 10 percent (by weight) of the water is still unfrozen, and therefore must be subtracted from the total water content when calculating f.

And still the formula is somewhat incorrect.

THE CONCEPT OF "FREEZING INDEX" OR "COLD QUANTITY"

In formula (1) frost-depth is given as a function of $t \times T$, i.e. product of temperature and time. Theoretically temperature t is presumed constant; in practice, t = mean temperature.

This fundamental entity called, by the Norwegians, "cold quantity," is calculated from meteorological statistics, as a cumulative product of time and registered air temperature degrees below the freezing point, for a given freezing period. For the technically important case of a winter's ultimate frost depth, frost quantity is given for the whole winter period: either for the coldest winter registered (maximum frost quantity), or as a statistical mean figure (normal frost quan-

tity). The geographical distribution of these entities called, by the Americans, "freezing index," is given on maps showing isometric lines, published for the U. S. A. and Norway in 1944. The freezing index (U. S. A.) is expressed in degree-days below 72 F. and the cold quantity (Norway) in degree-hours below zero C.

The meteorological freezing index or frost quantity deals with air temperature and the frost depth formula with surface temperature.

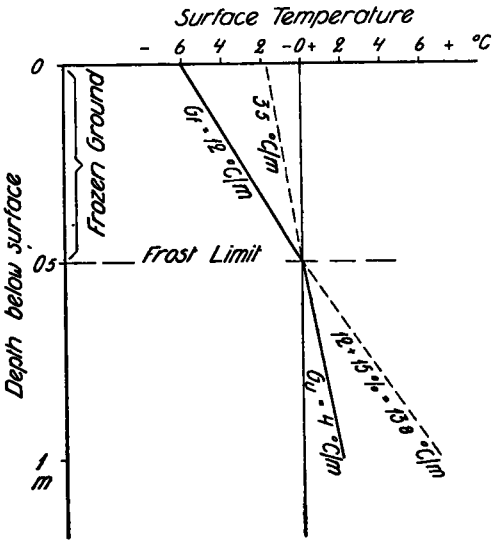

Figure 1. Effect of Heat Conduction from Below—Temperature Gradient Curves: Unbroken Lines = Actual Temperature, when freezing is going on; Broken Lines = Limits for Equivalence, e.g. no freezing going on (no heat alteration at frost border). Ratio of heat conductivity frozen: unfrozen = $\lambda_f : \lambda_u$ = 1.15.

These, of course, are not always identical. The difference mainly depends on the following factors: heat exchange conditions between surface and air; radiation effect; condensation of air moisture on the surface; and evaporation from the surface. Most serious is the effect of heat radiation toward open space. This effect is often limited through air moisture condensation, which starts when the surface temperature deficit has reached the critical point and restrains the temperature difference increase. Also white-frost strongly reduces radiation through its whiteness. In addition the pure isolation effect of white-frost cover is not unimportant.

Thus if freezing index is calculated from air temperature, the values obtained will be low where climatic conditions include frequent cloudless sky and dry air.

But even in subarctic-continental winters surface temperature in open terrain is only a few degrees lower than air temperature; for Sodankylä in northern Finland, winter temperature difference between surface of snow cover and air is about 1 to 2 deg. C. In climates which are not so extreme the difference is still smaller and generally may be neglected.

Sun radiation does little beyond reducing the dark radiation temperature deficit; however in some locations the sun effect late in winter may be rather important. Of course, local terrain conditions do influence practically all of the factors: i.e., the well-known difference between open field and forest, between hills and depressions. Nevertheless meterological freezing index is usable and valuable for approximation if corrected with regard to radiation in more extreme cases.

Where a systematic study or research is contemplated, it is highly desirable to obtain complete records of local air and surface temperatures.

HEAT CONDUCTION FROM BELOW

The role of heat conduction from below, counteracting (reducing effect of) "cold conduction" from above depends upon the quotient between temperature gradient below and above the frost limit.

Assume a frost depth of 0.5 m., surface temperature of −6 deg. C., and a mean gradient in the frozen layer of 12 deg. C. per m. With slight approximation heat liberation (or "cold consumption") is restricted to the frost limit. If the gradient below frost limit is 4 deg. per m., the heat conduction from below toward the frost limit is $\frac{1}{3}$ of the heat conduction from the frost limit toward the surface thus freezing is reduced by $\frac{1}{3}$.

This may be visualized graphically: the gradient line below the frost limit extended to soil surface shows the surface temperature for equilibrium (no freezing nor melting) or the "critical surface temperature"; which in the given example is −2 deg. C. Thus the "active frost excess" is only −6 − (−2) = −4 degrees, or $\frac{2}{3}$ of the surface cold (See Fig. 1).

In most cases this is a slight approximation —heat conduction coefficient often is somewhat higher in frozen than in unfrozen soil. In clay soils the difference is about zero; in sandy soils it may reach 10 to 20 percent.

Thus in the frost depth formula correction for heat conduction from below has to be performed, principally by subtracting from temperature t the product SG

where G = temperature gradient below frozen layer, and

S = depth of frozen layer.

The reduction may be performed on the basis of other assumed simplifications:

1. Assume gradient G constant. Critical surface temperature, SG, will increase in proportion to the growing frost depth S. Also, if active frost excess, $t - SG$, is assumed constant (i.e. surface temperature gradually sinking), the frost depth is a function of the square root of time, and the product of time T and SG would be $\frac{2}{3}$ of the product of time T and SG at the end of the freezing period in question (area of a parabolic figure in proportion to a rectangular figure). Temperature t in the formulas thus should be corrected toward $t - \frac{2}{3}SG$. These assumptions may be approximately applicable to the first part of a winter period, while temperature is sinking, but for long-time periods the gradient decrease must be taken into account.

2. Assume gradient G decreasing in proportion to frost depth S; thus SG is constant, that is the critical surface temperature is constant. Thus of course the reduction would be t changed toward $t - SG$, for which S and G may be figures chosen from the middle or somewhat later period of freezing.

Thus in the frost depth formula (1) instead of temperature t, use $t - qSG$, where q may lay between $\frac{2}{3}$ and 1.

$$S = \sqrt{\frac{2\,(t - qSG)\,T\lambda}{Q}} \qquad (4)$$

It must be emphasized that this correction is but roughly approximative.

The source of heat conduction from below toward the frost limit, for the early freezing period, could be said to be the summer heat stored up in the depth; toward winter's end this summer heat excess is mainly consumed, and the magnitude of gradient G depends mainly upon the year's average soil surface temperature. A depth of some 15 m. (10 to 20 m. depending on ground heat conductivity) is the level where yearly variations are practically eliminated, temperature being practically constant at the average yearly soil surface temperature. Toward late winter and early spring gradient G is nearly proportional to this mean yearly temperature (for details see Beskow 1935–1947, Fig. 86).

Empirical values for gradient G are known from many places in different geographical-climatological situations. For southern-middle Scandinavia G equals about 0.1 – 0.07 deg. C. per cm. (10 – 7 deg. C. per m.) for the beginning of December, dropping to some 0.04 deg. per cm. for January and to 0.02 or 0.03 deg. per cm. for winter's end. Even in northernmost Sweden, at the polar circle latitude, the gradient is rather high, approximately $\frac{2}{3}$ of the above figures. This is due to winter's thick snow cover which keeps the mean soil temperature about 3 deg. C. above mean air temperatures.

TABLE 1

VALUES OF TEMPERATURE GRADIENT G

Locale	Middle Dec.	Dec.–Jan.	Middle Jan.	Jan.–Feb.	Early March
	deg. F. per ft.				
Pierre, So. Dak.					
A		3–4		2.5	2
B				2.5	2
Sp		3–4		3(–2.5)	2
Watertown, So Dak.					
A	3				2
B	3				2
B	6		5		
Fargo, No Dak.	10	9		4	3
Presque Isle, Maine		3–3.3			1.5–2
Presque (turf)		1.8–2			1.2–1.5
Dow Field, Maine..				3.2–4.5	1.9–2.3
Dow Field (turf).				2.3	(1)–2

Note: 1 deg. F. per ft. = 1.82 deg. C. per m. = 0.0182 deg. C. per cm.

In the northern zone of U. S. A. this gradient is of the same magnitude. From the Frost investigation reports of the U. S. Army Corps of Engineers (1944–45) the data in Table 1 may be extracted, for airfields on mineral soil.

The quantitative importance of heat conduction from below may be evident from a simple example: At a frost depth of 0.5 m. and a surface temperature of some –8 to 5 deg. C. and an upper gradient of 10 to 16 deg.

per m.; a subfrost gradient G of some 3 to 5 deg. per m. will cause a reduction of frost penetration speed by one third; at a frost depth of 1 m. the reduction would be two thirds.

The relative effect of this heat conduction from below is thus directly proportional to frost depth. A covering layer of snow strongly increases this effect, as shown below.

EFFECT OF SNOW

The effect of a covering layer of snow on the surface could most simply be taken as an equivalent increase in thickness of frozen soil.

Speed of heat transfer from the freezing border (or freezing speed) is proportional to the temperature gradient in the frozen layer, which is t/S, where S = thickness of frozen layer. When heat has to pass through a top snow layer, temperature gradients in both layers, after equilibrium is reached, are inversely proportional to their heat conductivity. The snow layer produces the same effect as a frozen soil layer of thickness S_f

$$S_f = S_s \times \frac{\lambda_f}{\lambda_s}$$

where S_s = thickness of snow, λ_f and λ_s = heat conductivity of frozen soil and snow respectively.

Thus in the calculation, add to the existent soil frost depth the thickness of snow cover, S_s, multiplied by λ_f/λ_s.[2]

As the heat conductivity of snow λ_s is rather low, mostly about 10 to 15 percent of that of ordinary mineral soil (See Beskow (1935–1947) Fig. 83 and Table 24) the effect of snow is very high, as is well known from praxis—the snow cover corresponding to a soil layer seven to ten times thicker.

It may be emphasized that a moderate snow cover strongly increases the relative effect of heat conduction from below, and that ordinary snow cover usually protects the soil from freezing, or may cause melting from below.

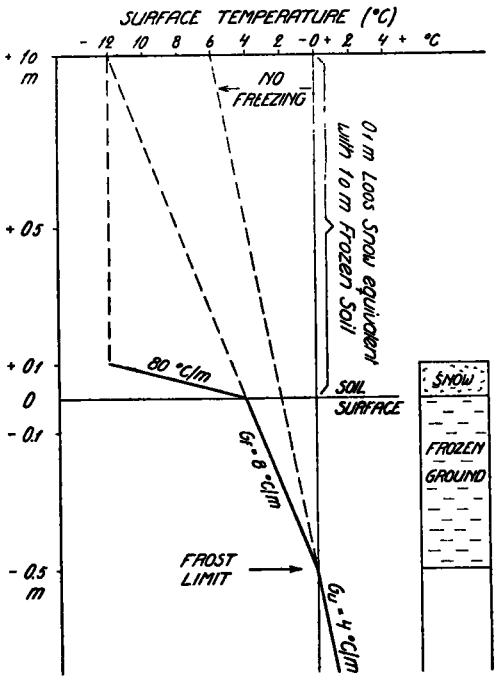

Figure 2. Effect of Snow—Heat conductivity of frozen soil is assumed equal to 10 times that of snow(λ_f/λ_s = 10); thus a 0.1-m. layer of snow corresponds to a 1.0-m. layer of soil. Shown also is the surface temperature required to maintain freezing (in the given example − 6 C.; in case of no snow it would have been − 2 C).

FREEZING RESISTANCE AND FROST DEPTH CALCULATION FOR DIFFERENT LAYERS

For practical purposes it is generally of special value to know the relative frost depth in different materials usable as backfill, or to be able to calculate the necessary total thickness of non-frost-sensitive layers, in cases where security against every frost action is wanted. Especially for excavation and "isolating" backfill, where the excavation depth plays an important economic role, the choice

[2] This approximation, where heat capacity is excluded from calculation (i.e. heat energy liberation restricted to frost limit) always gives results which are higher than the actual effect of snow cover, thus the figure for equivalent soil thickness should be somewhat reduced. In case of wet soils (except clays, see above) the error is moderate, and de facto very small where the calculations deal with moderately thick snow, fallen on ground already frozen to some depth. As long as the temperature of the air does not drop sharply, the heat capacity of the frozen soil layer plays a rather unimportant role in the freezing formula, and the approximation is nearly correct.

of the most frost restraining material is desirable.

In the Norwegian works this matter has been the technical aim. They calculate freezing resistance of any layer, expressed in cold quantity, or minus deg. C. hours.

For the top layer this freezing resistance Ω (product of minus degrees and time) is simply derived from formula (1), thus;

$$\Omega_1 = T \cdot t = \frac{Q_1 S_1^2}{2 \lambda_1} \qquad (5)$$

For a second, subjacent layer Ω would be;

$$\Omega_2 = T \cdot t = Q_2 S_2 \left(\frac{S_2}{2 \lambda_2} + \frac{S_1}{\lambda_2} \right) \qquad (6)$$

For any deeper layer the formula would be;

$$\Omega_n = T \cdot t = Q_n S_n \left(\frac{S_n}{2 \lambda_n} + \Sigma \frac{S_o}{\lambda_o} \right) \qquad (7)$$

where $\Sigma \frac{S_o}{\lambda_o}$ is the sum of S/λ for all the covering layers.

Freezing resistance of the whole series is attained through calculating each separate layer, and summing up.

In the formulas of Watzinger and Kindem heat conduction from below is not included. Including this factor, the formulae for the first and second layers become:

$$\Omega_1 = T\,(t - 2/3\, S_1\, G_1) = \frac{Q_1 S_1^2}{2 \lambda_1} \qquad (8)$$

$$\Omega_2 = T \left[t - G_2 S_2 - S_1 \frac{\lambda_2}{\lambda_1} \right]$$

$$= Q_2 S_2 \left(\frac{S_2}{2\lambda_2} + \frac{S_1}{\lambda_1} \right) \text{(approx)} \qquad (9)$$

Here G_1 and G_2 are the sub-frost-level temperature gradients, in layers 1 and 2.

The practical use of these formulas may be exemplified.

Assume a covering isolating top layer of moor, 30 cm. thick, on fat clay. The frost-storing capacity of the moor is $Q_1 = 20$ cal. per cu. cm. (corresponding to a water content of about 25 percent by volume), its heat conductivity $\lambda_1 = 0.0005$ cal. per cm. sec. deg.

Neglecting heat conduction from below, frost resistance of the top layer thus would be (formula 5):

$$\Omega_1 = T \cdot t = \frac{20 \times 900}{0.0005 \times 2} = 18 \times 10^6 \text{ deg. sec.}$$

$$= 5000 \text{ deg. hr} = 210 \text{ deg. days}$$

Now you want to know the frost quantity necessary to freeze the clay to a 50-cm. depth. The clay is characterized by: $\lambda_2 = 0.003$, $Q_2 = 25$ (freezing water content about 30 percent by volume. According to formula (6) the freezing resistance would be $\Omega_2 = T \cdot t = 25 \times 50\ (8300 + 60000) = 85 \times 10^6$ deg. sec. = 23600 deg. hr. = 985 deg. days

Thus total frost resistance $\Omega_1 + \Omega_2 = 1195$ deg. days.

Regarding heat conduction from below the calculation would be as follows:

Suppose that the sub-frost gradient in the clay ground during the first period (freezing of moor), is, as a mean, $G_2 = 0.03$ deg. C. per cm. In the moor layer gradient $G_1 = \frac{0.003}{0.0005} \times 0.003 = 0.18$ deg. C. per cm.

Thus, according to formula (8)

$\Omega_1 = T(t - 2/3 \times 30 \times 0.18) = T(t - 3.6) = 18 \times 10^6$ deg. sec.

For the following period (freezing of clay to 50-cm. depth) the gradient (in clay) has fallen from a mean of 0.03 to say 0.015 deg. per cm. at the end of the period. According to formula (9) the calculation would be:

$$\Omega_2 = T \left[t - 0.015 \left(50 + \frac{30 \times 0.003}{0.0005} \right) \right] =$$

$$T(t - 3.4) = 85 \times 10^6 \text{ deg. sec.}$$

The importance of heat conduction from below is obvious: if mean frost temperature t is, e.g., -5 deg. C. freezing would take threefold the time calculated with no regard to this factor.

In praxis, calculation for heat conduction from below has to be as practical as possible, adapted to existing special conditions. For computation of frost quantity or freezing index, surface frost temperature must be reduced by the critical temperature, calculated from the subfrost gradient. The formulae above are intended only to show the leading principles.

Concerning the practical use of frost resistance calculations, it may be emphasized that because of the decisive influence of complicating factors, especially heat conduction from below, an attempt should always be made to connect the calculations with some local empirical observations (frost depth, material constants, air temperature curve, snow cover).

Even a single completely elucidated frost depth observation may give a solid basis for comparative calculations of rather high exactness. Where frost depth as a function of time-temperature is known for one material, the calculation for each other material, or combination of materials, may be performed accurately and simply. For all cases geographically well distributed series of soil temperature observations are highly desirable.

A common ideal aim for construction interests is to get maximum frost penetration resistance at minimum layer thickness; whether it be for frost-safe isolation as required for water-pipes or for special foundations or base-sub-base constructions designed to detain, for the longest possible time, frost penetration down to frost active sub-soil. The leading principle consists of combining an upper layer of material with low heat conductivity (e.g. porous slag, charcoal) with a second, lower layer with high frost storing capacity (e.g. wet moor), the upper layer acting as a heat insulating coat, saving for as long as possible the heat storage capacity of the underlying water saturated (but of course not in itself frost-heaving) material.

The Norwegians have developed this principle systematically even in praxis, using moor as a cheap material with low heat conductivity and high water capacity and thus high freezing resistance. The use of moor as a frost preventing backfill or isolation material in roads, has from old times been practiced in northern Scandinavia, surely as a result of empiric observations of moor which is not frost-active, as well as of the low frost depth in moor.

A disadvantage is the considerable settlement and compression in moor material. Anyhow, for many important practical purposes, especially for railway sub-bases, the Norwegians have systematically and with good success improved the use of moor for frost resistant sub-base constructions (see Beskow 1947).

The gain when using selected material for sub-base fill, is of a magnitude of 2 to 4 times higher frost penetration resistance, as compared with ordinary sand-gravel fill to the same depth.

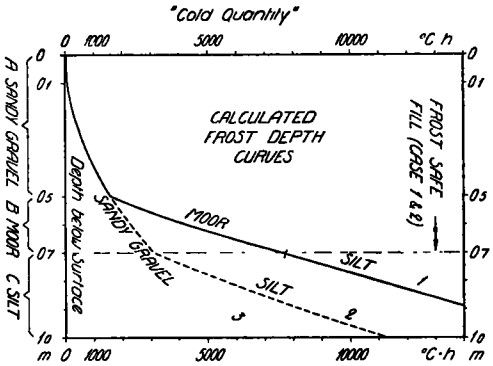

Figure 3. The effect of 20 cm. of Moist Moor on Frost Penetration Depth (Graphical Example of Norwegian Practice)—Material constants (in frozen state):

	Material		
	A Sandy Gravel	B Moor	C Silt
Water (ice) content, f (percent by vol.	10	40	30
Heat capacity (cal. per cu. cm.)	0.36	0.25	0.5
Heat conductivity, λ (cal. per cm. sec. deg. C.) . ..	0.002	0.001	0.0045
Frost storing capacity, Q (cal. per cu. cm.)	9.1	33	25

Curve No. 1 shows the frost depth for sandy gravel—moor—silt; No. 2 for 70 cm. of sandy gravel on silt (without moor layer); No. 3 for sandy gravel alone. For this excavation of 70 cm., the bottom layer of moist moor has extended the period of no frost heaving from 3,100 deg. hrs. (for 70 cm. of sand only) to 7,700 deg. hrs. Notice: No reduction is made for heat conduction from below. For temperate climate the empirical cold quantity ought to be reduced by about 30 percent which is, however, a very rough approximation.

Complications Concerning Effect of Sandy Layers—Since old times sand sub-base has been used as a frost action preventive; as men of practice had found sand to be not frost-heaving nor frost boiling. In some districts even a more complicated arrangement was used, namely coarse sand as a bottom layer, covered with refilled frost heaving soil; the purpose, of course, being the reduction of earthmoving cost.

The mechanics of this arrangement, "sand isolation," was made clear by soil frost action researches in the twenties: the coarse bottom sand layer acted by breaking the capillary connection between the covering soil and the subsoil; a "capillary isolation" effect. As destructive frost heaving (and water accumulation in the frozen layer) is caused through capillary water suction upwards toward the frost limit, ordinarily from a free ground water source, it is important to keep the free water level at a sufficient depth below the top of the sand layers. In case of high free water (e.g. where side ditches are not deep enough) the sand isolation layer would act as a water conduit, intensifying frost action in the covering soil.

This is a very important difference: an ordinary all-through sub-base of sand may stand waterfilled and yet fulfill its function; but where frost sensitive soil is replaced over a bottom sand layer (sand isolation), the depth down to free water must be enough to exceed the capillarity of the sand in use, securing an air-filled sand pore system subjacent to the dangerous covering soil.

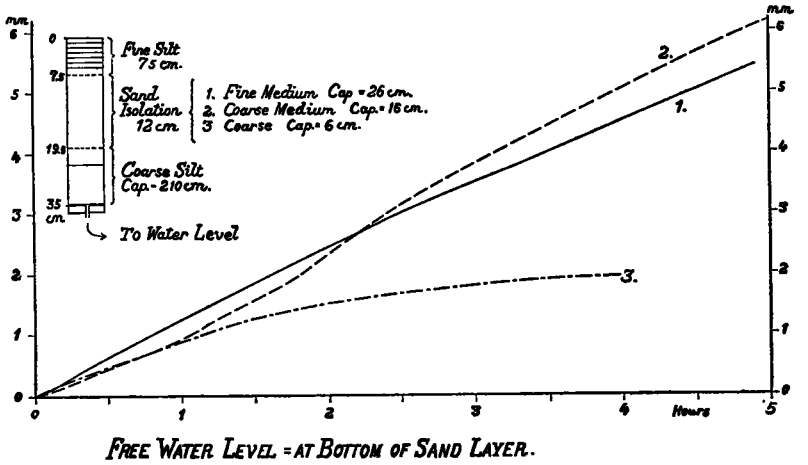

Fig. 4a

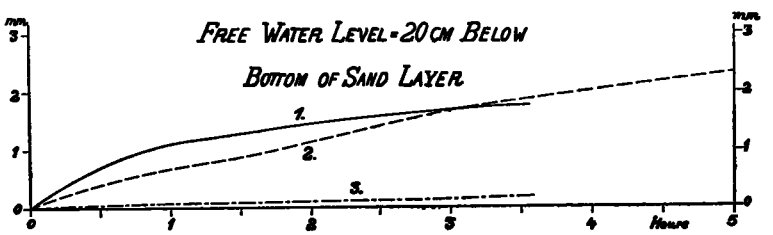

Fig. 4b

Figure 4. Frost Heave Speed of Fine Silt Placed on Sand Base of Different Grain Coarseness—Two Different Stands of Free Water Level—Whole series was water-filled before starting, i.e. before free water was lowered to the given stands, and freezing was started after 24 hours. In case of Figures 4, 1 and 2, capillary water connection is still existing, in other cases, capillary connection is broken more or less totally. Load = 37 g. per sq. cm.

This matter may be exemplified by a laboratory freezing test series (Fig. 4) showing frost heave rate in fine silt refilled above different sand isolation materials, at different free ground water level depths.

Another matter is frost heave water suction from other water sources than free ground water; namely from capillary water in coarser elements of a heterogeneous soil complex. Say you have fine silt above fine sand or

coarse silt; when the fine silt starts freezing, its capillarity is "activated", i.e. an intense pore-water tension develops, down to the bottom, causing a suction from the coarser soil, in which the capillary water is not so strongly held, i.e. the capillary tension limit is comparatively low. This is illustrated in Figures 5 and 6.

As established from practical experience, embankments containing frost active silt may cause trouble, even when rather high and perfectly drained, with free ground water standing very deep below the freezing level. The cause will always be frost active fine silt in contact with medium coarse soil, acting as a capillary water reservoir. Coarser soils of

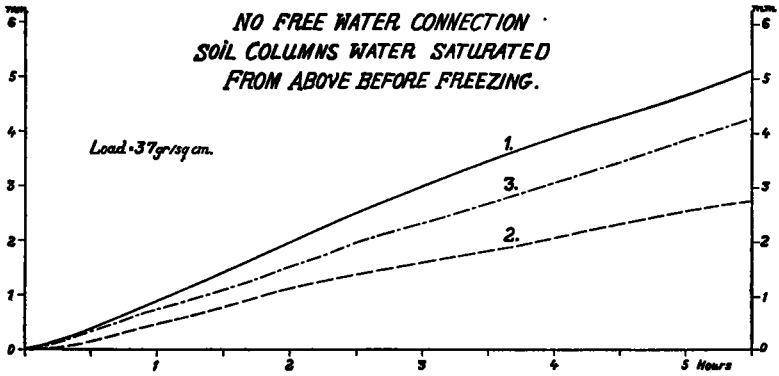

Figure 5. Sand or Coarse Silt Acting as "Secondary Water Magazine," Supplying Frost Heave of Superadjacent Fine Silt

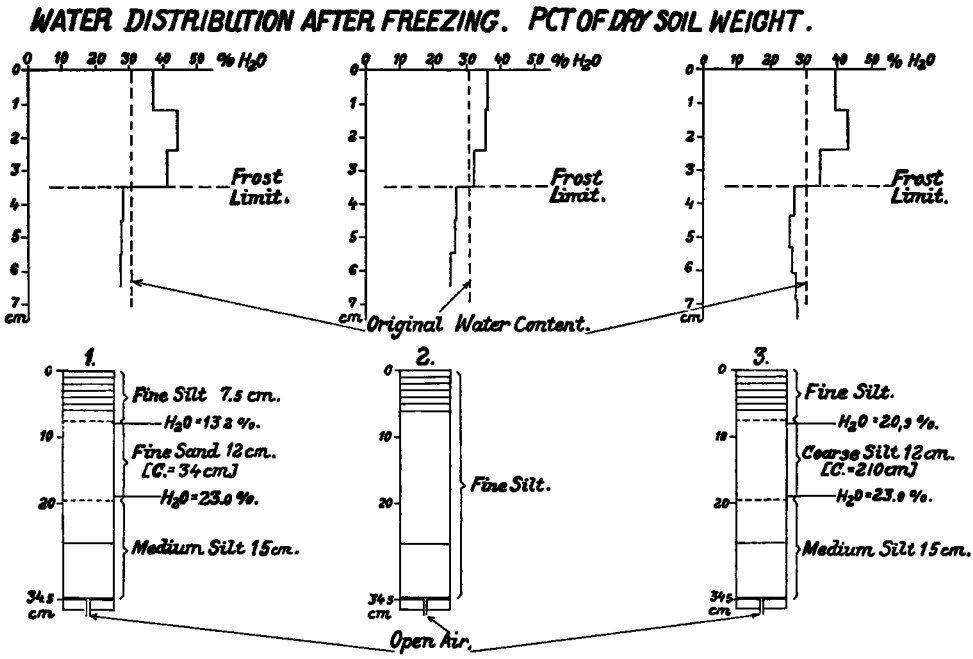

Figure 6. Soil Column Arrangement and Water Distribution in Fine Silt Top After Freezing (See Fig. 5)

very low capillarity, such as pure gravel or coarse sand, do not give that effect. As a consequence two practical rules for using highly frost active silts for embankments are: (1) Avoid jumbling variants of different coarseness, place the finer variants at bottom, the coarser toward top, never inversely; (2) Isolation layers cutting capillary connection are good, but they must consist of rather coarse sand, and be situated high enough above any free water level.

Effect of Resin Treatment on Frost Action—As is well known, chemical agents which are effective in soil stabilization also reduce the rate of frost heave.

For chemical soil stabilization against frost action, particularly in preparation of rather thick soil layers, down to a depth of 2 to 3 ft., the process of application plays a dominating economic role. Agents which have to be thoroughly mixed in with the soil—and especially those which require that the soil be dried out—would be economically inferior to chemicals applicable in genuine (not colloidal) water solution, as the latter require but rough lump disintegration of soil and not all-through mixing, owing to suction. Such water soluble agents as calcium chloride and paper pulp sulfite leach, do produce good effects but only at rather high concentrations (1 to 3 percent

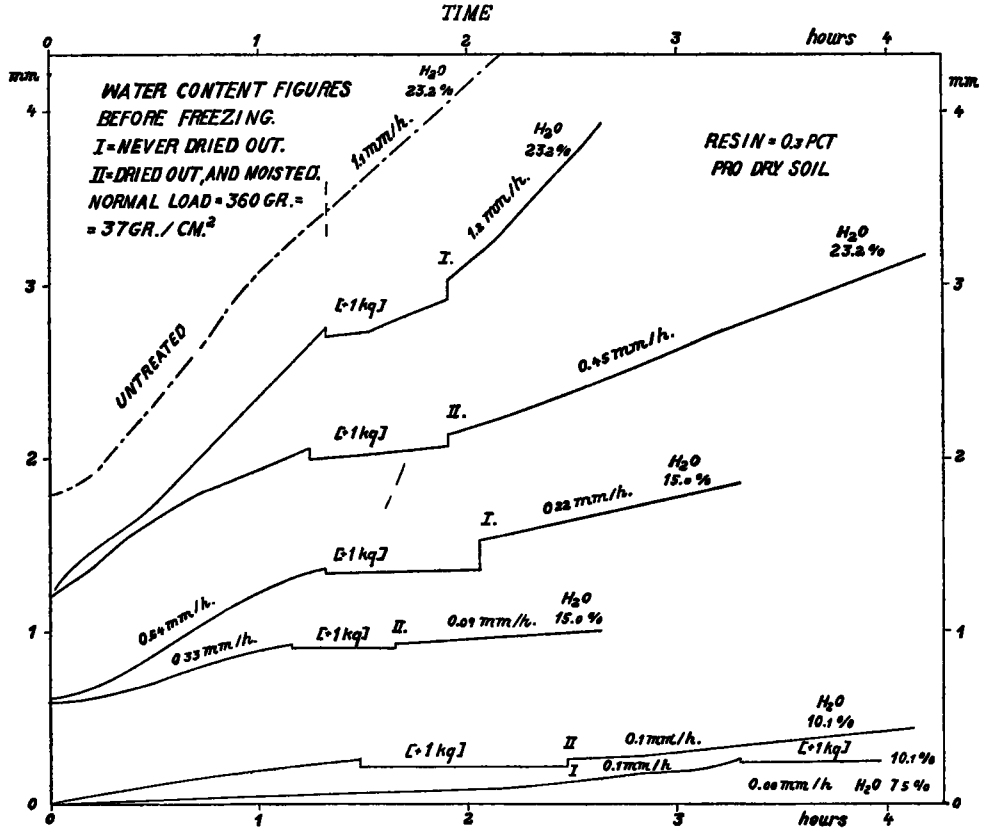

Figure 7. Effect of Resin Preparation on Frost Heave Speed in Coarse Silt—Silt columns 70 mm., placed on sand base, water filled, and in connection with free water level (as in Fig. 4), 10 cm. below bottom of silt column. Each pair of curves represents the same water content before freezing started, in each pair No. II was dried out after treatment and then moistened, No. I was not dried out.

Treatment with alkaline resin solution (water content = 23.2% (full saturation)), resin = 0.3% —all percentages per dry soil weight.

by weight of dry soil) (See Beskow 1935–47, Figs. 51–55).

The use of more highly active agents is preferable; the American and English experience with resin treatment stimulated tests with resin for deep-soil stabilization against frost action.

This research, including both laboratory and field tests on the types of highly frost active silts common in Scandinavia, has given results not too promising from a practical point of view.

Good effects were obtained, using alkaline resin solutions, in a quantity of some 0.1 to 0.5 percent by dry weight of soil. Such treatment would reduce frost-heave rate and frost water suction to a trifle, but only if the soil was dried out after treatment. If once dried out, the soil sample, even after mixing with water to full capillary saturation, and in suction connection with free water, will show but a small part of the frost-heave rate of raw soil (See Fig. 7).

Even simple water-suction tests (without freezing) show analogous results.

This of course may depend upon the way in which resin forms an adsorption layer on the particle surface. The existence of a water film between the growing ice crystal and the soil particle is the necessary condition for the frost heaving process. The dimensions of this film, and the mobility of water within it, determine the frost heave rate. Pressure reduces frost heave rate and water suction to the frost level, through diminishing thickness of adsorption film (in the remaining layer water is also less mobile). In an analogous way surface active substances affect frost heave rate, through altering thickness and even molecular mobility within the adsorption water film.

Where resin stabilization dries out after treatment, evidently a much more effective structural arrangement of surface adsorbed agent is caused affecting the adsorption water film. This arrangement is of permanent character, not reversed through mechanical water addition. Where this is in field surface soil stabilization, the drying out is easily reached; but where deep stabilization of subgrade is attempted, the requisite condition of having soil dried out after preparation would mean a practical disadvantage.

DISCUSSION

W. L. Shannon, *Harvard University*. Dr. Beskow presents a brief resumé of the principal contributions by Scandinavian engineers to frost action research during the past decade. He credits the Norwegian investigators with making a comprehensive study of frost depth as related to the time-temperature conditions and material properties. It is interesting to note that during this same period parallel studies were carried out in the United States, principally by the Corps of Engineers.

In these studies several equations, all similar to equations (1) and (4), were developed by the Frost Effects Laboratory, N. E. Division, Corps of Engineers. In addition, a large number of observations[1] were made of frost depth beneath air-field pavements which were maintained substantially free of snow and ice. Each observation of frost depth was compared with values which were computed using these equations. In the computations two values for thermal conductivity were used within which it was believed the actual average value would lie. The volumetric heat of the moist soil and the latent heat of fusion of the water in the soil were computed from water content and unit weight determinations made before and after freezing. In those cases where all the water in the soil did not freeze, two limiting values were assumed for the amount of water freezing. The two extreme values for frost depth, computed from the several equations, for each location differed by 40 to 90 percent. The observed value for frost depth generally, though not always, was within the extreme range. No one equation gave a value which was more consistently correct than any other.

If the same values for thermal properties and freezing index are entered into each of the several equations, the extreme values for frost depth would differ by about 40 per cent.

For the prediction of frost depth, it is usually necessary to select two limiting values for thermal conductivity, latent heat, and volumetric heat. These values will combine in any of the equations to give a range of frost depth for any value of freezing index or time-

[1] "Report on Frost Investigations," New England Division, Corps of Engineers, Boston, Mass., April 1947.

temperature relationship. If several different equations, based on differing soil temperature conditions, are used, a still wider range for frost depth will result. It appears that the principal utility of all these equations is not in the prediction of frost depth but rather in understanding the effects of the several variables on frost depth.

A simplified equation was developed[2] for the frost depth in non-cohesive soils in which the water content was more or less constant during freezing. In equation (1) the product $t \times T$ is equal to F, the freezing index or frost quantity. Then, letting $\sqrt{\frac{2\lambda}{Q}}$ equal B, we may write:

$$S = B\sqrt{F}$$

An analysis of numerous frost depth observations in non-cohesive soils indicates that B is, for all practical purposes, a constant having a value of 1.5 where F is in degree-days (Farenheit) below freezing and S in inches.

In order to obtain a better idea of the thermal conductivity of soils, two independent investigations were made, one by the Frost Effects Laboratory, N. E. Division[3], and one by the St. Paul District, Corps of Engineers. The tests by the Frost Effects Laboratory indicated the following conclusions for non-cohesive soils:

1. The thermal conductivity in the dry state was the same for soil at above freezing and below freezing temperature.
2. The thermal conductivity increased with increasing water content and also with increasing unit weight.
3. The thermal conductivity for saturated non-cohesive soils was greater in the frozen state than in the unfrozen state.

To my knowledge, A. Casagrande[4] was the first to recognize the importance of the cumulative temperature plot in frost problems. The use of the cumulative temperature value in degree-days below freezing was first introduced in 1929 in connection with frost research conducted by A. Casagrande while in the service of the Bureau of Public Roads.

[2] William L. Shannon, "Prediction of Frost Penetration," *Journal*, N. E. Water Works Association, Vol. LIX, No. 4, pp. 356 to 363.

[3] William L. Shannon and Winthrop A. Wells, "Tests for Thermal Diffusivity of Granular Materials," *Proceedings*, ASTM, Vol. 47.

[4] Discussion by A. Casagrande of Paper by A. C. Benkelman and F. R. Omstead, "A New Theory of Frost Heaving," *Proceedings*, Highway Research Board, Vol. 12 (1932) pp. 168 to 172.