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FURTHER METHODS FOR THE ANALYSIS OF DATA TAKEN IN THE HVEEM STABILITY TEST

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SYNOPSIS

The third and fourth of a series of papers concerning the analysis of data taken in the triaxial test for Hveem stability of bituminous road materials and their application to problems of design. In the first two of these papers[1] certain special methods of analysis and correction of the data have been presented. The first of these papers consisted chiefly of a review of the familiar graphical solution for combined static stress known as the Mohr Circle diagram with some applications to the analysis of data taken in the Hveem test for stability, and the second outlined another somewhat similar solution, with restricted application, the results being equivalent to those obtained in the Mohr diagram method. The third of these papers is given as Part 1 of this article. It describes a graphical method of correction of lateral stress readings for lateral surface voids and for measuring lateral deformation of the specimen at any stage of the test. The fourth, Part 2 herein, describes and illustrates proposed methods for the determination of the physical constants of road materials from data taken in the Hveem stability test.

PART 1

A GRAPHICAL METHOD FOR CORRECTION OF LATERAL STRESS AND MEASUREMENT OF LATERAL DEFORMATION OF A SPECIMEN IN HVEEM'S TEST FOR STABILITY

A scientific method of design utilizing any construction material must include consideration of two factors: the relation between physical properties of the material; and its qualities requisite for good performance. The design method, to be adequate, requires accurate measurement of these properties and qualities and involves statistical analysis of a large body of data taken in laboratory and field tests in order to correlate the two factors.

No such method appears to have been developed for asphaltic concrete and an investigation was undertaken by the writer in an attempt to solve the problem. Data were used from several hundred tests for Hveem stability on various asphaltic concrete mixes.

About seven hundred diagrams have been constructed employing the methods outlined in the previous papers, and the shear resistance on the plane of failure, obtained from the constructions, has been correlated with Hveem percent stability, yielding a correlation coefficient of approximately 0.90.

In the present paper, a graphical method for measuring lateral deformation of the Hveem specimen, and for correcting the lateral stress readings for surface voids is described and illustrated with data taken in the Hveem test for stability.

Method of Taking Data and Construction of Charts—Because of a certain quantity of air always present in the Hveem stabilometer system during a test for Hveem stability, and also due to such miscellaneous factors

[1] Published in *Proceedings* Highway Research Board Vol. 26, (1946) p. 100.

as compression of the water and expansion of the apparatus, the stress registered on the stabilometer gauge is not the true radial stress which would be developed by a given load on the specimen with a rigid incompressible system and the unit deformation calculated from this stress by Hooke's law is not the correct value of radial strain. In most cases the true value is considerably greater than the observed value, and the object of this investigation was to devise a method for the correction of the lateral stresses, as read on the stabilometer gauge, and for measuring the actual lateral deformation of the specimen at any given stage of the test from data taken in the routine test for Hveem stability of asphaltic concrete specimens. By maintaining the quantity of air in the stabilometer water chamber at a constant value, and treating it, together with the effect of miscellaneous expansion of the apparatus, as a constant of the stabilometer, it is sufficient to find a correction for the air trapped in the lateral surface voids of the specimen.

In order to preclude possible errors due to strains occurring in a real specimen, which are variable from specimen to specimen and very difficult to determine accurately, a rigid metal specimen was used for the experiment consisting of a brass cylinder turned down to a diameter of 3.98 in. so as to fit snugly into the barrel of the stabilometer. With this rigid dummy specimen in place and after adjusting the stabilometer to read an "initial displacement" of about 0.152 cu. in. for the usual rapid increase of the pressure gauge reading from 5 psi. (taken as practical zero) to 100 psi., the stress was varied from 5 to 200 psi., in increments of 10 psi., by means of the stabilometer pump, and its value recorded, together with the corresponding displacements, in the first and second columns of Table 1. The metal dummy specimen was then removed, weighed, and four holes about ⅛ to ¼ in. in depth were bored in its surface with a ½-in. bit, after which it was replaced and a similar series of data to that made on the smooth dummy was taken and recorded. The remainder of the experiment duplicated

TABLE 1

RELATION BETWEEN VOLUME OF LATERAL SURFACE VOIDS AND STABILOMETER GAUGE READING (USING DUMMY SPECIMEN WITH VARYING VOLUME OF SURFACE VOIDS)

Weight of Dummy g	1805.8	1805.0	1803.9	1803.1	1799.1	1795.1	1790.9	1786.0	1781.0	1776.4	1771.0
Vol. Surface Voids *cu. in.*	.000	.0056	.0132	.0187	.0464	.0740	.1031	.1369	.1713	.2032	.2406
Radial Stress σ_3	*Displacements of Stabilometer Pump Cubic Inches*										
psi											
5	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
10	.024	.026	.026	.025	.030	.034	.034	.036	.034	.034	.032
20	.054	.056	.058	.C62	.072	.078	.082	.084	.084	.080	.078
30	.074	.078	.082	.088	.100	.110	.114	.116	.116	.116	.118
40	.090	.098	.102	.110	.122	.134	.140	.142	.146	.150	.154
50	.106	.112	.116	.124	.140	.154	.158	.166	.172	.180	.188
60	.118	.126	.130	.140	.158	.170	.178	.190	.202	.208	.218
70	.130	.138	.144	.154	.174	.186	.194	.212	.226	.234	.242
80	.142	.148	.156	.166	.184	.198	.214	.231	.246	.256	.264
90	.152	.160	.168	.178	.196	.212	.230	.250	.268	.276	.284
100	.162	.170	.178	.186	.206	.224	.244	.268	.284	.292	.302
110	.170	.180	.188	.196	.214	.238	.258	.288	.302	.308	.334
120	.180	.188	.194	.204	.226	.250	.274	.302	.314	.324	.354
130	.188	.198	.204	.212	.234	.262	.288	.316	.327	.340	
140	.198	.208	.212	.219	.244	.272	.302	.326	.342	.358	.380
150	.208	.214	.218	.224	.254	.282	.314	.336	.355	.374	.398
160	.214	.220	.225	.232	.262	.294	.324	.346	.370	.386	.413
170	.222	.226	.231	.241	.272	.304	.332	.356	.382	.400	.428
180	.230	.234	.238	.248	.280	.316	.340	.368	.392	.416	.442
190	.234	.240	.244	.256	.288	.324	.346	.374	.402	.428	.450
200	.240	.244	.252	.264	.296	.332	.354	.384		.438	.458
Initial Displacements (5–100 psi)	.152	.158	.167	.172	.192	.208	.228	.248	.262	.278	.290

Weight of Metal Cylinder in Air at end of experiment = 1767.1 grams.
" " " " " Water at 17 C. " = 1566.8 " .
Average Density of Cylinder = 8.63 g. per cc.
Note: Displacements in cubic inches are obtained by multiplying the linear displacement of the pump piston, given by the attached Ames dial, by the factor 2 which is the area of the piston.

this performance, except that after taking the fourth set of readings, holes were bored in increments of twelve instead of four for the remaining sets. The data are listed in Table 1.

The density of the material (yellow brass) was obtained by Archimedes' method of weighing the cylinder in air and in water, and dividing the weight in air by the difference between the weights in air and in water after applying a correction for the temperature of the water. Dividing the loss in weight of the cylinder at various stages of the experiment by its density and reducing to cubic inches gave the volumes of surface voids found in the second row of Table 1.

Curves 3 to 13 in Figure 1 are constructed from the data in Table 1 by plotting stabilometer gauge readings as ordinates against the displacements in the stabilometer water compartment, listed in columns 2 to 11 of Table 1, as abscissae. As indicated on this chart, each curve constitutes a calibration curve for a given volume of surface voids, and the displacement of a specimen corresponding to any gauge reading is given by the abscissa of the point whose ordinate represents the value of the gauge reading. The two dotted curves numbered 1 and 2 represent ideal relations between gauge readings and stabilometer displacements which should exist if we had a rigid system with no occluded air and a non-

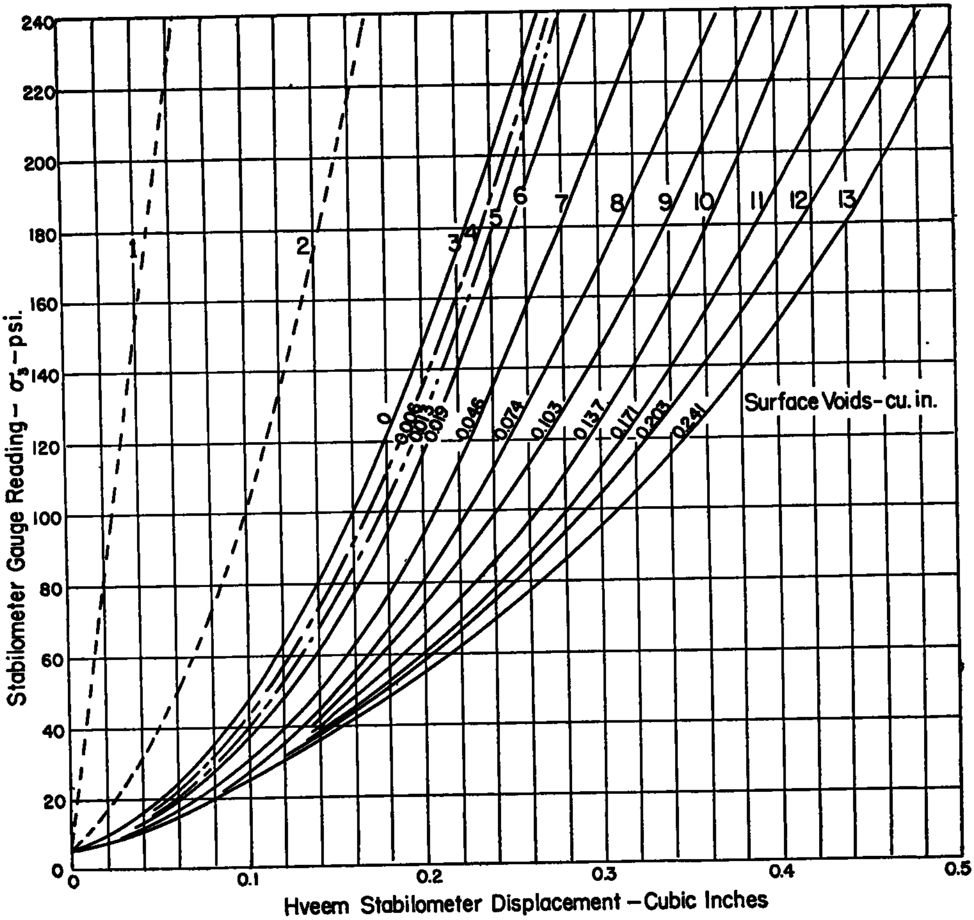

Figure 1. Calibration Curves for Determination of Corrected Values of Lateral Deformation and Stress for Hveem Specimens with Varying Volume of Lateral Surface Voids

rigid system with no occluded air, respectively. They are drawn from data given in Mr. Hveem's instructions for the care and use of the stabilometer. Curve 1 is drawn straight thru the point given in Mr. Hveem's data on the assumption that the gauge reading is proportional to the volume deformation causing it if the system is rigid, and curve 2 is drawn

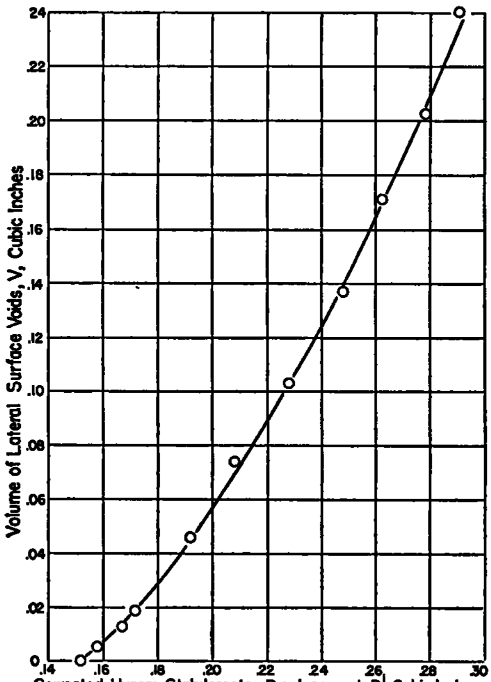

Figure 2. Relation between Volume of Lateral Surface Voids and Hveem Displacement on Brass Dummy Specimen

concave upward in its lower part on the assumption that miscellaneous expansion of the apparatus will decrease asymptotically from a maximum to a constant value in about the same fashion as is shown in the other curves. The exact form of these curves is immaterial for the present since no use is to be made of either of them in finding the corrections.

The graph in Figure 2 was constructed by plotting volumes of lateral surface voids against values of displacement, taken in the customary manner employed in the Hveem stability test, on the metal specimen at each stage of the experiment. From this graph the volume of lateral surface voids contained on a specimen may be obtained from its Hveem displacement after correction for strain resulting from the inward radial stress of 100 psi. applied in taking the displacement. A statistical mean for the part of displacement due to lateral yield or strain may be obtained for any given type of mix by smoothing with a plastic composition, the lateral surfaces of a number of specimens of the selected design type having approximately the same diameter as the rigid dummy specimen, and then taking an initial displacement on each of them. The difference between their average initial displacement and the displacement on the dummy specimen should be the average displacement resulting from yield of specimens of the type tested under the stresses existing during a displacement measurement. Then the portion of total initial displacement on any given specimen due to surface voids alone would be the difference between its initial displacement and the average strain determined in the manner described above. But a better method for correction of the observed displacement would be to multiply it by a factor, calculated from the average difference correction which would yield the same corrected value. Also, it is obvious that the corrected value of the displacement may be obtained from the final displacement normally made in the Hveem test for stability as well as from the initial displacement, by multiplying it by the appropriate correction factor based on calculation from statistical means. A tentative value of the required factor for correction of final displacement based on data from some eighty-three tests on specimens made from a wide range of designs, shown in Table 2, was found to be 0.695. The process of arriving at this factor was, briefly stated, to substitute the average value of Hveem stability (49 percent) in Hveem's stability formula written in the form

$$S = \frac{22.2}{\frac{\sigma_3}{400 - \sigma_3} + 0.222}$$

in which σ_3 represents a corrected average value of lateral stress at a vertical load of 5000 lb. (or 400 psi.) and solve the equation for σ_3, which was found to be 75 psi. Then from Figures 1 and 2, the corrected value of Hveem displacement required to increase the

TABLE 2

DATA TAKEN IN THE HVEEM TEST FOR STABILITY ON SPECIMENS OF ASPHALTIC CONCRETE—INCLUDING AN INITIAL DISPLACEMENT ON THE SPECIMENS AND THE DIFFERENTIAL DISPLACEMENT

Laboratory Number	R.D. No.	Initial Displ. on Spec. (D')	Final Displ. on Spec. (D)	$D' - D$	Radial Stress at 400 psi. Stress Axial (σ_a)	Hveem Stability (S)
		in.	*in.*	*in.*	*psi.*	%
Rs-46-308	158(5)	0.182	0.107	0.75	51	67
309	"	0.149	0.109	0.40	45	53
310	"	0 139	0.104	0.35	50	49
311	"	0.132	0.107	0.25	48	58
312	"	0.175	0.115	0.60	43	52
319	155(2)	0.158	0.117	0.41	68	41
320	"	0.156	0.127	0 29	72	39
321	"	0.164	0.126	0 38	77	38
322	"	0.169	0.120	0.49	78	35
323	"	0.155	0.128	0.27	87	31
324	155(3)	0.156	0.110	0.46	62	49
325	"	0.181	0 128	0.53	74	40
326	"	0.158	0.118	0.40	80	35
327	"	0.178	0.130	0.48	79	31
318	160(6)	0.256	0.213	0.43	85	61
341	158(7)	0.136	0.101	0.35	50	36
342	"	0.152	0.115	0.37	45	48
343	"	0.153	0.115	0.38	57	53
344	157(13)	0.129	0.120	0.09	56	46
345	"	0.162	0.123	0.39	54	59
346	159(4)	0.192	0.125	0.67	64	48
349	153(11)	0.169	0.130	0 39	31	64
350	"	0.181	0.149	0.32	97	61
351	"	0.168	0.138	0.30	30	65
370	153(13)	0.169	0.128	0.41	30	67
371	"	0.185	0.144	0.41	28	61
372	157(16)	0.194	0.132	0.62	40	56
373	"	0.180	0.140	0.40	37	58
376	157(17)	0.200	0.144	0.56	38	61
377	"	0.200	0.157	0.43	32	58
378	"	0.200	0.130	0.70	38	61
367	159(7)	0.200	0.112	0.88	44	53
368	"	0.217	0.128	0.89	47	52
369	"	0.195	0.127	0.68	58	61
364	161(3)		0.128		48	53
365	"	0.175	0.123	0.52	34	53
366	"	0.172	0.112	0.60	34	56
385	165(2)	0.206	0.153	0.53	26	69
387	"	0.184	0.136	0.48	30	47
388	"	0.168	0.139	0.29	64	54
389	"	0.203	0.131	0.72	37	63
390	"	0.182	0.149	0.33	103	49
384	156(4)	0.210	0.126	0.84	51	59
411	157(19)	0.180	0.154	0.26	39	63
412	161(4)	0.178	0.133	0.45	45	56
413	"	0.185	0.136	0.49	42	49
414	"	0.196	0.131	0.65	47	49
415	"	0.195	0.140	0.55	52	45
416	"	0.185	0.154	0.31	58	54
417	"	0.185	0.135	0.50	55	48
419	165(3)	0.170	0.140	0.30	59	54
420	"	0.171	0.130	0.41	40	41
421	"	0.148	0.129	0.19	52	19
423	156(6)	0.216	0.134	0.82	73	45
424	"	0.198	0.133	0.65	41	51
426	165(4)	0.193	0.133	0.60	17	21
427	"	0.152	0.134	0.18	49	51
428	"	0.182	0.132	0.50	53	47
429	"	0.188	0.139	0.49	36	62
450	162(2)	0.170	0.150	0.20	135	34
458	161(5)	0.122	0.090	0.32	44	67
459	"	0.160	0.130	0.30	44	43
460	"	0.155	0.114	0.41	41	45
461	"	0.152	0.143	0.09	40	39
462	"	0.125	0.198	0.27	39	53
463	"	0.162	0.118	0.44	49	46

Table 2—*Continued*

Laboratory Number	R.D. No.	Initial Displ. on Spec. (D')	Final Displ. on Spec. (D)	$D' - D$	Radial Stress at 400 psi. Stress Axial (σ_a)	Hveem Stability (S)
		in.	*in.*	*in.*	*psi.*	%
Rs-46-464	172	0.160	0.083	0.77	84	37
465	"	0.128	0.070	0.58	97	41
466	"	0.133	0.095	0.38	95	34
467	"	0.115	0.092	0.23	92	38
468	"	0.112	0.082	0.30	100	37
469	"	0.100	0.084	0.16	102	36
470	"	0.135	0.090	0.45	103	33
472	"	0.118	0.088	0.30	103	35
473	"	0.118	0.089	0.29	89	36
474	157(21)	0.115	0.103	0 12	48	48
475	"	0.142	0.110	0 32	41	48
476	"	0.148	0.100	0 48	65	37
477	"	0.144	0.103	0.41	40	56
478	165(5)	0.135	0.118	0.17	43	56
479	"	0.127	0.104	0.23	47	52
481	"	0.118	0.098	0 20	31	63
482	"	0.161	0.122	0.39	24	58
Average Values		0.165	0.122	0.43	59	49

lateral stress from the average of 59 psi. to 75 psi. is found to be 0.085 in. and the ratio between this corrected displacement and the average of the measured initial displacements (0.165) is 0.515. Multiplication of this factor by 1.35, which is the calculated ratio of the average initial to the average final displacements, gives 0.695 for the required correction factor for final displacement. This proportionality factor was checked roughly by filling the surface voids of a typical specimen of the same diameter as the dummy with a plastic composition and taking its displacement. Volume of surface voids may also be calculated directly from measurements of specimen diamensions and data on specimen density by the Archimedes experiment when this is available.

Although not used directly in the present investigation it is of interest to note that the maximum and average differences between initial and final displacements shown in the fifth column of Table 2 are relatively large, the average of 0.43 in., or 0.86 cu. in., being 35 percent of the average of the total final displacements. This differential displacement, which is probably almost entirely independent of surface voids and other accidental specimen defects, measures the recovery from lateral deformation of the load. This partial recovery from deformation is evidently due to: (1) back pressure of compressed air occluded in the stabilometer water chamber and trapped in surface voids of the specimen; (2) air voids in the specimen itself; and (3) true elasticity of the material. Since these factors operate in effect in the material when placed in the highway, and to at least as great an extent as in the test, the voids in the stabilometer and the average specimen surface being approximately one percent as contrasted with the voidage of two to five percent for which bituminous road materials are usually designed, it would seem that it is just this practical pseudo resilience in which we are primarily interested. For this reason, it would appear that a practical stability index should include this resiliency factor as well as that which measures rigidity. The effect of this factor on the stability index is illustrated in a numerical example given at the end of the report.

A few discrepencies due to experimental or recording errors are obviously contained in the data shown in Table 2, but it is believed that they have no undue effect on the calculated results. However, the correction factor used in the calculations is merely provisional and it is too general for application to extreme types of bituminous mixes. The direct method of measurement of volume of surface voids from data taken in the density determination is recommended because of its greater simplicity and accuracy. Results will be affected to some extent by variations in shape, size, and distribution of surface voids as well as their total volumes, because of stiffness of the diaphragm which prevents its complete penetration into the cavities, but it is believed that the voids made on the rigid dummy represent a fair average for those found on actual specimens.

Method of Using Graphs and Numerical Illustrations—The final displacement always taken and used in the calculations in the Hveem test for stability is multiplied by an experimentally determined factor in order to obtain a corrected value for the part of its displacement due to lateral surface voids. Then, from the graph in Figure 2, the volume of its surface voids is found. This volume indicates which calibration curve in Figure 1 is to be used in finding the corrected value of its lateral stress. e.g., that developed by an applied load of 5000 lb. (or 400 psi.), and of its lateral deformation in cubic inches. If the volume of lateral surface voids as determined from Figure 2 does not fall on any of the calibration curves in Figure 1, the proper curves may be selected and corrections made by graphical interpolation. In this connection it may be noted that Figures 1 and 2 may be combined by placing a scale of corrected final displacement on the upper margin of Figure 1. Having located the proper calibration curve in Figure 1, the corrected value of the given lateral stress and of the corresponding lateral deformation of the specimen are found by the following method:

A straight horizontal line thru the terminal of the given lateral stress ordinate is drawn to its intersection with the calibration curve just defined, and a vertical straight line thru this intersection is next drawn to its intersection with curve No. 3, which represents the relation between stress and displacement for a rigid specimen with zero surface voids. The co-ordinates of this latter point of intersection represent corrected values of lateral deformation (the abscissa) and of lateral stress (the ordinate), both values being corrected for air trapped in the lateral surface voids of the specimen. That these co-ordinates do represent corrected values of the quantities referred to is evident from the meaning of a calibration curve. The abscissae of points of intersection of a horizontal line with the various curves obviously represent experimentally measured volume deformations of the water-air compartment of the stabilometer required to produce the given gauge reading, at different volumes of surface voids of the specimens. Likewise, the ordinates of intersection points of a given vertical line with the various curves give the lateral stresses produced by a number of specimens with different volumes of lateral surface voids having a constant deformation indicated by the abscissa of the intersections. Thus the ordinate of the intersection of the vertical, erected at the terminal of the abscissa representing the corrected deformation of the specimen, with the calibration curve for zero surface voids will give the corrected value of the stabilometer gauge reading, i.e., lateral stress. This statement involves two assumptions, both of which appear to be approximately correct. They are, first, that leakage of air from the surface voids during the test is the same for the actual specimens as for the metal specimen, and, second, that entrapped air in the surface voids exerts the same effect on staloilometer gauge readings as does air occluded in the water compartment of the stabilometer. The first of these assumptions is believed to be justified, in a qualitative fashion, by the fact that the backward creep of the gauge hand does not appear to be very different in the two cases; and the correctness of the second is illustrated by the close resemblance of the two curves, (a) and (b) shown in Figure 3, which curves give the relation between stabilometer gauge readings and volume changes produced in the stabilometer water compartment by means of the displacement pump, and volume changes in specimen surface voids, respectively.

As a numerical example, data taken on a certain bituminous concrete specimen in Investigational Project No. 11 and designated as C-A_5-R_2-10-3(1), for which the developed radial stress σ_3 at an axial stress of 400 psi. was 59 psi., and the final displacement was 0.167 in. is used for an illustration. Multiplying the final displacement D by the provisional correction factor 0.695 gives 0.116 in. or 0.232 cu. in. for the corrected displacement D'. From the curve in Figure 2 it is seen that the volume of lateral surface voids on the specimen is 0.109 cu. in. From Figure 1, by interpolation between curves 9 and 10, and using the uncorrected value of 59 psi. for radial stress, the corrected displacement is seen to be about 0.180 cu. in. and the corrected value of radial stress to be 120 psi. Substitution of this corrected radial stress in Hveem's stability formula, and omitting the final displacement D, which is apparently a correction of the gauge reading for specimen surface voids, we obtain 34 percent for the stability. This is lower than the value yielded by Hveem's method, which was found to be 41 percent before correction for specimen height and 35 percent after correction. However, if the average difference $D' - D = 0.43$ in., or 0.86 cu. in. (See Table 2), which measures the residual lateral deformation of the specimen after the load is reduced from 11400 to 1000 lb., be substituted for final displacement into the stability formula as a multiplier of the corrected lateral stress, the uncorrected stability is found to be 35 percent which after correction for specimen height is about 33 percent. Since no initial displacement was taken on the specimen in this particular case, the true value of its stability corrected for both resilience (as defined above) and surface voids is indeterminate; but, since the difference $D' - D$ is seen to be less than unity for all the specimens shown in Table 2, the true value after correction for resilience will always be greater than that obtained by neglecting this property.

Summary of Method of Using Graphs.—The necessary procedure in obtaining corrected values of lateral stress and deformation of specimens from data in Hveem's test for stability may be stated briefly as follows:

1. Multiply the "final displacement" taken on the specimen by a factor representing a statistical average resulting from independent tests on a large number of typical specimens, for the purpose of finding that part of the displacement which is due to its lateral surface voids.

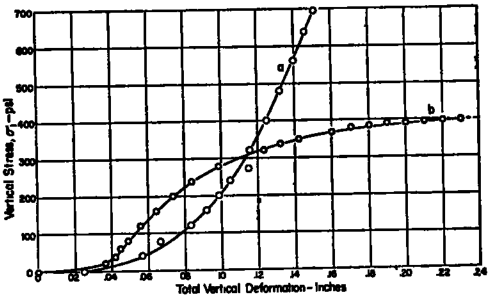

Figure 3. Relation between Stabilometer Gauge Reading and (a) Volume of Lateral Surface Voids at a Constant Stabilometer Displacement of 0.152 Cu. In. and (b) Stabilometer Displacement in Cubic Inches at a Constant Volume of Surface Voids (0.152 Cu. In.)

2. From the curve in Figure 2 determine the actual volume of its surface voids and locate the calibration curve in Figure 1 representing the volume just found, interpolating if necessary for greater accuracy. An alternate method in which volume of surface voids is obtained from data taken in the density determination is suggested as being more direct and accurate, when this data is available.

3. Follow a horizontal line whose ordinate represents the observed (uncorrected) lateral stress developed by an axial stress of 400 psi. to its intersection with the calibration curve previously located, and then follow a vertical line thru this point to its intersection with the

calibration curve, No. 3 for a smooth rigid specimen. The ordinate of this latter intersection gives the corrected value of lateral stress and its abscissa the lateral deformation in cubic inches, both corrected for air trapped in lateral surface voids of the specimen.

In addition to the application of the graphs to the problems outlined and illustrated in this article, they may also be applied to the study of volume changes occurring in the specimen during the test for Hveem stability and of their effect on interpretation of the data and the dimensions in a Mohr diagram constructed from them, and to the calculation of physical constants of road materials, etc., but these more important applications will be reserved for treatment in a later report.

PART II

DETERMINATION OF PHYSICAL CONSTANTS OF ROAD MATERIALS FROM TRIAXIAL COMPRESSION TEST DATA

Theory and Method of Taking Data.—In the triaxial compression test using the Hveem stabilometer, in which lateral pressure is measured with a type of gauge (Bourdon) depending directly or indirectly on the movement or deformation of the material being stressed, the stress is a function of the deformation and, by proper calibration, the gauge dial will read values of the actual deformation. A null method of measuring the lateral stress, in which the force required to correct or prevent the potential deformation is measured might be better from a theoretical standpoint, but would entail practical difficulties. However, for reasons which will be pointed out presently, the measurement of lateral deformation instead of its suppression may possess advantages in the use and interpretation of the triaxial test data. For instance, a knowledge of the relation between applied load and the resulting lateral flow or deformation of a bituminous mixture is of prime importance in evaluating its usefulness as a road material. The ratio of these two quantities, which will be shown to equal the ratio of Young's modulus E to Poisson's ratio μ, for an unconfined compression test, may be determined from the triaxial test by calibrating the lateral pressure gauge to read "displacement" or volume deformation of the water chamber. Such a calibration curve for a specimen with no surface voids shown in Part 1 (Fig. 1, curve 3) was obtained by taking and plotting readings of the gauge corresponding to a number of volume changes calculated from readings of the Ames Dial which measures the motion of the displacement pump piston. The data

TABLE 3
CONFINED AND UNCONFINED COMPRESSION TEST DATA ON ASPHALTIC CONCRETE SPECIMENS AND ON DUMMY SPECIMEN

Lateral Stress (σ_2)	Stabilometer Displacement (on Dummy Specimen) D'			Specimen R-A_6-R_2-12-3 (1) Confined Compression: Vertical Stress (σ_1)	Specimen R-A_6-R_2-12-3 (1) Confined Compression: Linear Vertical Deformation	Specimen X — 18 Unconfined Compression: Vertical Stress (σ_1)	Specimen X — 18 Unconfined Compression: Linear Vertical Deformation
psi.	*turns*	*in.*	*cu. in.*	*psi.*	*in.*	*psi.*	*in.*
11	.13	.013	.026	0	.025	0	.000
21	.32	.032	.064	40	.057	20	.036
30	.40	.040	.080	80	.067	40	.042
41	.49	.049	.098	120	.083	60	.045
51	.55	.055	.110	160	.092	80	.049
61	.61	.061	.122	200	.099	119	.056
71	.65	.065	.130	240	.105	159	.065
80	.70	.070	.140	320	.116	199	.074
90	.74	.074	.148	400	.125	239	.084
101	.79	.079	.158	480	.132	279	.099
110	.815	.081	.163	560	.140	318	.123
120	.84	.084	.168	640	.146	337	.132
130	.88	.088	.176	696	.151	350	.142
140	.91	.091	.182			368	.160
150	.94	.094	.188			377	.170
160	.985	.098	.197			385	.180
170	1.01	.101	.202			388	.190
180	1.05	.105	.210			390	.200
190	1.09	.109	.218			393	.210
200	1.13	.113	.226			395	.220
						396	.230

Note: One turn of stabilometer displacement pump handle is equivalent to 0.2 cu. in. change in volume of the water chamber, with the rigid dummy specimen in place.

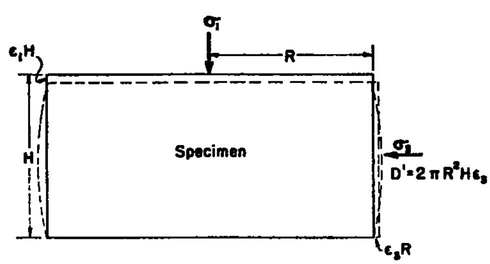

Figure 4

from which this calibration curve is plotted were obtained by decreasing the volume of the water chamber with the pump handle and reading the Ames dial and stabilometer pressure gauge simultaneously. It is exhibited in Table 3, columns 1 to 4. The average radial unit extension of a specimen corresponding to any gauge reading may be obtained from this calibration curve by simple calculation (See Fig. 4).

Derivation of Equations—The form assumed by Hooke's law in the case of an elastic solid acted on by the combined stresses in the triaxial compression test for stability is

$$\epsilon_3 = \frac{\sigma_3}{E} - \frac{\mu}{E}(\sigma_3 + \sigma_1) \qquad (1)$$

in which ϵ_3 = average lateral unit (radial) strain
σ_1 = vertical (axial) applied stress
σ_3 = lateral (radial) stress developed in the test.

Dividing (1) by σ_1 gives

$$\frac{\epsilon_3}{\sigma_1} = \frac{\sigma_3}{E\sigma_1} - \frac{\mu}{E}\left(\frac{\sigma_3}{\sigma_1} + 1\right) \qquad (2)$$

which gives the ratio of lateral (radial) strain to applied unit load in terms of Young's modulus E, the ratio of Poisson's ratio μ to Young's modulus, and the normal stresses σ_1 and σ_3.

For an unconfined test, in which the lateral supporting stress σ_3 becomes zero, equation (2) reduces to

$$\frac{\epsilon_3}{\sigma_1} = -\frac{\mu}{E} \qquad (3)$$

where σ_1 is the compression machine dial reading divided by the area of the top of specimen and ϵ_3 is given by

$$\epsilon_3 = \frac{D'}{2\pi H R^2} \qquad (3a)$$

in which D' is the total lateral volume change of the specimen in cubic inches produced by σ_1 and H and R are its height and radius, respectively, in inches (See Fig. 4). The negative sign indicates the relative directions of the displacement and the stress and will be neglected. Substitution of this expression for ϵ_3 in equation (3) gives

$$\frac{\mu}{E} = \frac{D'}{2\pi H R^2 \sigma_1} \qquad (4)$$

Discussion of Results.—In deriving the equations it is tacitly assumed that Hooke's Law is valid, although the material may be semiplastic. That the assumption is partially justified over much of the range of data is indicated by the straight, (or approximately straight) portions of the stress-deformation curves shown in Figure 5 plotted from data in columns 5, 6, 7, and 8 of Table 3. One of these curves (a) was plotted from data taken in Investigational Project No. I. P. 11 on a flint aggregate design No. R-A_6-R_2-12–3(1) in the ordinary triaxial compression test for Hveem stability in which a lateral stress of σ_3 = 111 psi. was developed at σ_1 = 400 psi.; and the other curve (b) was plotted from data in an unconfined compression test on a specimen designated as X-18, Special Investigational Project No. 7. Both specimens con-

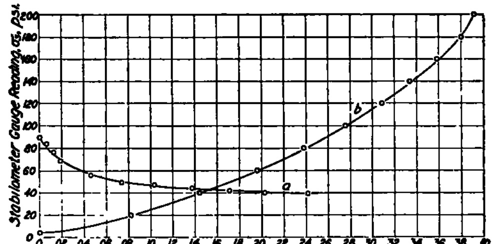

Figure 5. Vertical Stress-Vertical Deformation Curves for (a) Asphaltic Concrete Specimen R-A_6-R_2-12-3(1), (I.P. No. 11) in Confined Compression and (b) Specimen X-18 (S.I. No. 7) in Unconfined Compression

tained a relatively great amount of cut-back asphalt (12 and 18 percent by weight, respectively) and the one having 12 percent asphalt was made from flint aggregate. Hence both should have been above the average in plasticity; and yet they apparently obeyed Hooke's Law over a considerable range, viz., from about σ_1 = 20 psi. to σ_1 = 200 psi. for the specimen tested in unconfined compression and above σ_1 = 200 psi. for the other. The relatively high rate of load application used (0.05 in. per min.) contributes to the observed elastic behavior of the semiplastic material. However, the departure of actual road materials from the ideal elastic condition here assumed is sufficiently great to introduce discrepancies of considerable magnitude in some of the results derived from the elastic equations. Since the ratio between lateral flow and applied load appears to have an important bearing in the determination of the usefulness of a material when placed on the highway, it is suggested that this simple ratio ϵ_3/σ_1, which is easily measured from the triaxial test and in situ, might be a better measure of the true practical value of the material than other measures of stability now used. As may be seen from the approximately linear form of the upper portion of the calibration curve (Figure 1) lateral stress, σ_3, as given directly by the stabilometer gauge reading, might be substituted for the corresponding strain ϵ_3, when the lateral stress is above 80 psi., for purposes of relative measurements.

In this connection, it is of interest to recall that the ratio σ_1/σ_3, defines the angle of shear α and so the angle of internal friction ϕ, since $\tan^2 \alpha = \tan^2\left(45 + \frac{\phi}{2}\right) = \frac{\sigma_1}{\sigma_3}$(for a cohesionless material), and that the triaxial test is primarily a test for the measurement of this factor of internal friction in asphaltic concrete. It may also be noted that the complement of this ratio, viz., $1 - \frac{\sigma_3}{\sigma_1} = \frac{\sigma_1 - \sigma_3}{\sigma_1}$ is proportional to maximum shear stress. These facts provide a theoretical basis for the proposed use of the ratio ϵ_3/σ_1, which has been shown by typical examples to be proportional to the ratio σ_3/σ_1 over a considerable interval as an index of stability for road materials.

The result expressed by equation (4) enables one to replace the two important physical constants E and μ by a single constant or modulus M which occurs in the elastic strain equations and which is easily measured from data taken in the triaxial compression test. The lateral deformation corresponding to any applied load may be obtained for this case by raising the load to the given value (e.g. 2500 or 5000 lb.), and then reducing the pressure in the water chamber to a reading of 5 psi. (the practical zero) by backing off the displacement pump piston and noting the amount of increase in volume registered by the Ames dial. The Ames dial reading, when reduced to cubic inches by multiplying it by two, should be the total lateral volume deformation of the specimen, for zero lateral stress, from which the average unit radial strain is easily calculated in terms of this deformation and the specimen dimensions using equation (3a).

As is evident from equation (2) the quantity ϵ_3/σ_1 is a physical constant characteristic of the material if σ_1 is proportional to σ_3, i.e., for a cohesionless material whose properties are related by Coulomb's equation, which reduces to the constant $M = \mu/E$ when σ_3 becomes zero. If it is desired to obtain absolute values of Young's modulus and Poisson's ratio, or to calculate the change in total volume of the specimen from data in the routine test, it is necessary to record the vertical deformation of the specimen during the test. It is possible, however, by taking a series of pairs of principal stresses to calculate these physical constants by the following simple method which does not require the direct measurement of strain:

Substitute two pairs of principal stresses within the range for which the elastic condition may be assumed to hold, together with the corresponding values of radial strain ϵ_3, derived from the radial stresses by means of the calibration curves (Figure 1), into equations (1) or (2), thus forming a pair of simultaneous equations in μ and E. Solve these equations for μ and E. The same procedure may be followed for determining these physical constants from axial stress-strain data, using the strain equation $\epsilon_1 = \frac{\sigma_1 - 2\mu\sigma_3}{E}$ with an observed value of the strain ϵ_1 provided the conditions for elastic deformation obtain.

Numerical Examples—In order to illustrate the methods described in the preceding sections the following data taken in a test on a typical specimen made from flint aggregate and designated as R-A$_{13}$-R$_2$-16–3(1) was substituted in equation (1):

Height $= H = 2.02$ in.
Radius $= R = 2.00$ in.
Radial stress at an axial stress of 400 psi. $= \sigma_3 = 83$ psi.
Radial stress at an axial stress of 560 psi. $= \sigma_3 = 119$ psi.

Volume of surface voids was calculated from the dimensions of the specimen and from data taken in the routine density determination by the Archimedes experiment. This gave $\pi R^2 H - 25.387 = (3.142)\ (4)\ (2.02) - 25.387 = 0.172$ cu. in.

From Figure 1, it is seen that the lateral deformation corresponding to $\sigma_3 = 83$ psi. is approximately 0.236 cu. in. and the radial unit extension ϵ_3 as obtained from equation (3a) is

$$\epsilon_3 = \frac{D'}{2\pi HR^2} = \frac{0.236}{2(3.142)(2.02)(4)} = 0.00465$$

Also the value of radial unit extension at σ_3 = 119 psi. calculated in the same manner is found to be ϵ_3' = 0.00614.

Substitution in equation (1) yields the pair of simultaneous equations in μ and E, viz,

$$0.00465 = \frac{83}{E} - \frac{\mu}{E}(483)$$

$$0.00614 = \frac{119}{E}\frac{\mu}{E}(679)$$

Solution of these equations gives

μ = 0.228 and E = − 5832.84 psi.

for the physical constants required, the negative value of E being due to the fact that the radius increased (outward) while the radial stress was directed inward.

Then $M = \mu/E = -0.000039$ and the average ratio ϵ_3/σ_1 obtained from the given values of σ_1 and derived values of ϵ_3 is 0.0000115.

Data from the test on another specimen made from limestone aggregate designated C-A_{24}-R_2-20–3(2), with a height of 2.05 in., radius of 2.00 in., σ_3 (at σ_1 = 400 psi.) = 125 psi. and σ_3 (at σ_1 = 560 psi.) = 195 psi. when treated by the same method gave the following intermediate and final results:

Volume of surface voids = 0.220 cu. in. Lateral deformation (from Fig. 1) are found to be 0.345 and 0.450 cu. in. at σ_3 = 125 psi. and 195 psi., respectively, and the corresponding unit extensions are 0.00670 and 0.00873. Poisson's ratio = μ = 0.453 and Young's modulus = E = − 16789.88 psi. $M = \mu/E$ = 0.0000270 and ϵ_3/σ_1 = 0.0000161.

For obvious reasons a different method must be used for measuring lateral deformation in the case of the unconfined compression test than that used for the ordinary triaxial test. Two or three methods might be employed. The most direct and simple way would be to compute the deformation from measurements made with calipers. Another would be to measure the over flow of water by means of a graduated manometer tube. The displaced water due to the deformation may also be measured quite easily, however, as previously stated, with the displacement pump attachment on the stabilometer in the following manner:

With the specimen loaded to a given value, open the valve between the water chamber and displacement pump cylinder, set the attached Ames dial at zero and back the piston off until the stabilometer gauge reading is reduced to 5 psi. The Ames dial reading, reduced to cubic inches displacement by means of the factor 2, is the required lateral deformation of the specimen corresponding to zero lateral pressure. By way of illustration, the data from one such test, substituted in equation (4) gives the following result:

$$\frac{\epsilon_3}{\sigma_1} = \frac{\mu}{E} = \frac{D'}{2\pi\sigma_1 HR^2}$$

$$= \frac{(0.2)(0.86)}{2(3.14)(400)(1.96)(2)^2} = 0.0000087$$

If the lateral deformation corresponding to the ordinary triaxial test conditions, for which σ_3 was found to be 60 psi., had been used for the calculation of D', i.e. D' = 0.122 (obtained from the calibration curve), then the value of ϵ_3/σ_1 would have been 0.0000062; but this would not have been the correct value of μ/E, because $\sigma_3 \neq 0$.

Because of changes in total volume, with accompanying compaction and induced anisotropy, occurring during compression of specimens containing aggregate particles, due to rearrangement of the particles, it is not believed that the value of μ/E obtained is as accurate or constant as it would be for a more homogeneous material. While the foregoing results show considerable variation in the values of the ratio ϵ_3/σ_1, which equals μ/E in the unconfined compression test, it is quite probable that the method of measurement here presented gives as accurare results as do other experimental methods for which recorded results on a relatively homogeneous material such as cast iron have been found to differ by as much as 33 percent of their average for μ and 44 percent for E (See Mark's Handbook for Mechanical Engineers). The values of ϵ_3/σ_1 for the two widely differing specimens of asphaltic concrete in the preceding examples are seen to differ from their average by 33 percent (with $\sigma_3 \neq 0$) and for μ/E by 36 percent. After all there is no reason for expecting any constancy of ϵ_3/σ_1 or μ/E as between specimens of different design, or of the same specimen at different degrees of compaction, because they are essentially different materials. Smaller stresses and deformations, as well as a different rate of load application, might be expected to yield somewhat different values for these physical constants.

Summary of Results—Results of the preceding study may be stated briefly as follows:

1. The lateral volume deformation of a specimen tested in the triaxial compression test is read from the appropriate calibration curve (Figure 1) constructed by plotting stabilometer gauge readings as ordinates against volume changes in the stabilometer water chamber produced by the stabilometer pump and registered on the attached Ames dial, as abscissae.

2. From this lateral volume deformation the average radial unit extension, or strain ϵ_3, is computed by formula (3a) and substituted, together with the corresponding axial and radial stresses, σ_1 and σ_3, in the strain equation representing Hooke's Law for combined stress (equation (1)) which reduces to $\epsilon_3/\sigma_1 = \mu/E$ when $\sigma_3 = 0$.

3. From data taken in the unconfined and confined compression tests on two typical asphaltic concrete specimens, for which $\sigma_3 = 0$ and $\sigma_3 \neq 0$, respectively, it is shown that the deformation is nearly elastic over considerable ranges of stress (Figure 5) so that Hooke's Law may be used throughout this range.

4. The ratio of the lateral strain ϵ_3 to the applied unit load σ_1 is suggested as a useful index for evaluating the practical quality (or stability) of the material for highway construction because the value of any mixture as a road material depends on its resistance to lateral flow under an applied vertical load, and also because of the ease of measurement of this simple ratio by means of the ordinary triaxial compression test.

5. This ratio ϵ_3/σ_1 is shown to be approximately proportional to the ratio σ_3/σ_1 at lateral stresses above 80 psi., and to equal the ratio of Poisson's ratio to Young's modulus (i.e. μ/E) for the unconfined compression test, and some useful implications are deduced from these relations.

6. A procedure for measuring lateral strain for an unconfined test, by means of the Hveem stabilometer, is described and illustrated.

7. A method for the absolute determination of the physical constants μ and E and their ratio μ/E from the data taken in the ordinary routine triaxial compression test, without direct measurement of either vertical or lateral strain, is outlined and illustrated with numerical examples.