DETERMINATION AND USE OF THE DYNAMIC MODULUS OF ELASTICITY OF CONCRETE

EDMUND F. PREECE, *Principal Engineer, Washington District, Corps of Engineers, U. S. Army*

SYNOPSIS

The dynamic modulus of elasticity of portland cement concrete may be used to forecast the probable 14- or 28-day flexural modulus of the concrete when the specimen is but a few days of age. Extensive use of the procedure has led to certain practical developments which simplify the procedure somewhat and greatly reduce the labor required for the usual method. The purpose of this paper is to present these developments.

The use of the dynamic modulus of elasticity in forecasting the flexural strength of a concrete specimen has been reported in detail (*1*)[1] and only a brief description of the method will be presented here. Similarly, several excellent papers have been presented concerning the theory and the procedure for determining the dynamic modulus, a number of which are listed in the references which follow.

If the dynamic moduli of each of a group of similar specimens are determined as the specimens age and these values are plotted against age in days it will be found that a family of essentially similar curves are produced (Fig. 1). There will be more or less spread between those curves having the greatest and the least ordinate values but they will all have approximately the same shape. The amount of spread appears to be related more to the quality of the aggregate than to such factors as slump, mixing time, etc., assuming, of course, reasonable batching and mixing control. Shape, on the other hand, appears to depend on the overall conditions and a different aggregate, for example, generally produces a different family of curves.

Once a sufficient number of curves is available to determine an average shape it becomes practicable to project say a 5 day value forward to 14 or 28 days. The determination of this estimated future value is the first step of the procedure.

The relationship between the dynamic modulus and the corresponding flexural strength is not a linear one throughout the entire range of possible values. Nevertheless, within the range to be expected for a given set of conditions it may be considered to be linear without introducing an important error.

If the flexural strengths of a group of similar specimens are determined at a series of ages from say 7 days to 28 days in order to obtain a fairly wide spread between the lowest and highest values and these values are plotted against the respective dynamic modulus values at the breaking age it will be found that the points will form a definite grouping (Fig. 2). An average curve can then be fitted either visually or mathematically. Once this relationship is determined it becomes possible to use the estimated 28 day dynamic modulus to arrive at an approximation of the probable 28 day flexural strength.

The dynamic modulus value appears to be especially sensitive to variations in the quality of the aggregate and this has an influence on the magnitude of the probable error inherent in the procedure. As yet there has been no opportunity to determine the effect of variation in the amount of entrained air and this should be investigated. The aggregate represented by the plotted values in Figure 2 was both poor and variable. The departures of the individual points may not represent the worst condition but they certainly are much greater than should be expected with reasonably good material.

Computation of the dynamic modulus requires the determination of the dimensions, weight and resonant frequency of the specimen. The apparatus necessary for the determination of the resonant frequency includes an oscillator, support and driver to produce vibration of the specimen and a pick-up and a metering device to indicate the relative vibration amplitudes.

[1] Italicized figures in parentheses refer to the list of references at the end of the paper.

An oscillator, a cathode ray oscillograph and an amplifier with an output meter are shown in Figure 3. These units are similar to those used in most so-called sonic analyzers. This oscillator is continuously variable from 600 to 6000 cps. and has its own amplifier. The amplifier in the illustration is in the pick up circuit and the output may be indicated either by the output meter or the oscillograph.

Two types of driver are shown in Figure 4. One is an ordinary permanent magnet speaker in which a stem is cemented to the voice coil to bear against the specimen. This transmits the driving force with a minimum power loss. The other type is the speaker unit used for public address systems. It, too, has a cone to which a stem is cemented. The permanent magnet type is relatively light and must be attached to a support. This will be referred to as a "fixed" driver. The other driver is

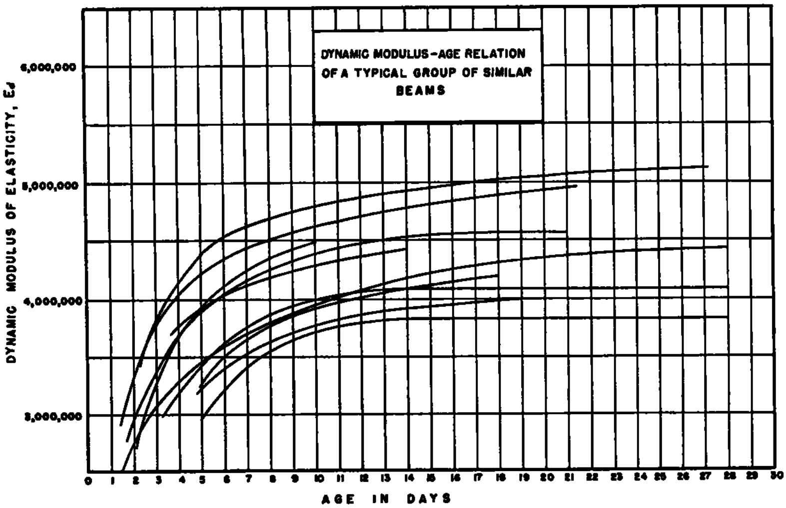

Figure 1. Dynamic Modulus—Age Relation

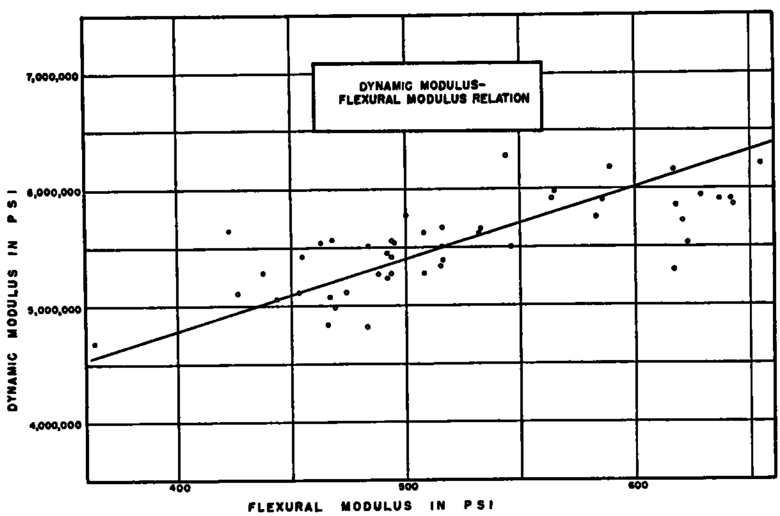

Figure 2. Dynamic Modulus—Flexural Modulus Relation

fairly heavy and needs no additional weight. It will be referred to as a "floating" driver. Matching transformers which are not visible in Figure 3 are inserted ahead of the drivers to improve their power output.

One type of support is shown in Figure 5. The housing is of oak so that its resonant frequency is lower than the lowest resonant frequencies encountered in any of the specimens tested. The permanent magnet driver

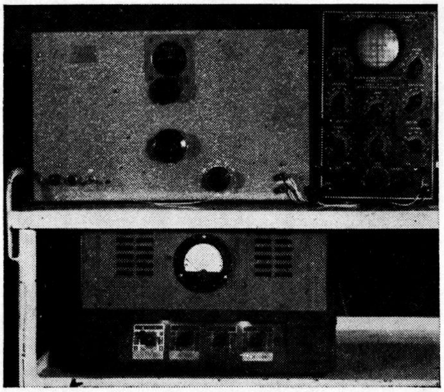

Figure 3. Oscillator, Amplifier, and Oscillograph

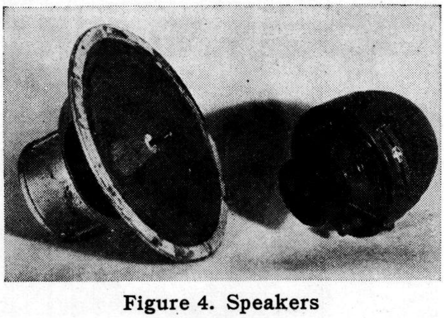

Figure 4. Speakers

is mounted within the box with the driving stem extending upward through the cover to rest on the beam. Two sharp edge bearings are provided the position of which may be adjusted to correspond to the nodal points of each specimen. The cover is hinged at one end and is supported on a hand operated screw at the other end so that the beam may be raised or lowered to bear properly on the driver stem.

The permanent magnet speaker has a paper cone and cannot be used in the humid room. Consequently, all beams must be carried to the sonic apparatus and this adds the cost of two helpers, a very important consideration where a large number of beams must be handled.

A beam supported on sponge rubber is illustrated in Figure 6. In this case the floating

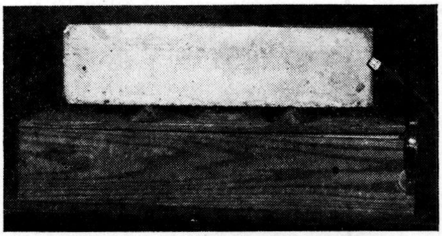

Figure 5. Nodal Support Driver

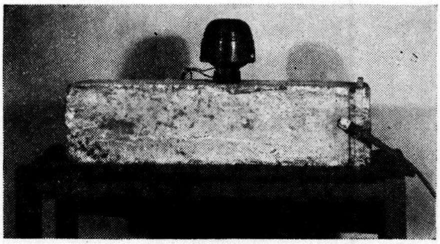

Figure 6. Rubber Support

TABLE 1
COMPARISON OF DRIVING METHOD

Beam No.	Dimensions	Resonant Frequency, *cps*: Supported At Nodal Points, Fixed Driver	Supported At Nodal Points, Floating Driver	Supported On Sponge Rubber Floating Driver
	in.			
B-2	6 x 6 x 36	685	685	685
D-2	6 x 6 x 36	635	635	635
S-2	6 x 6 x 24	1375	1375	1380
S-3	6 x 6 x 24	1405	1410	1410
S-4	3 x 4 x 18	1360, 1730	1360, 1730	1360, 1730
S-6	3 x 4 x 18	1350, 1700	1355, 1700	1350, 1700

driver is used resting on the top of the beam. The advantage of this type of support is quite apparent provided, of course, it permits the specimen to vibrate properly.

Data presented in Table 1 are typical of the results that have been obtained in a large number of tests. Two values are shown in each case for Beams S-4 and S-6 which were determined first with the 4-in. dimension vertical and the second with the 3-in. dimension vertical. The data show that the various methods are equally competent.

A tier of heavy wood shelves was erected in the humid room, each shelf being covered with a ½-in. thickness of sponge rubber. Specimens are weighed when received and again just before breaking. After the first weighing they are not moved until the time to break them. The resonant frequency is determined periodically in place on the shelves with the floating driver by a single operator in the same time as with the box support and there is a saving of the time of the two helpers formerly required.

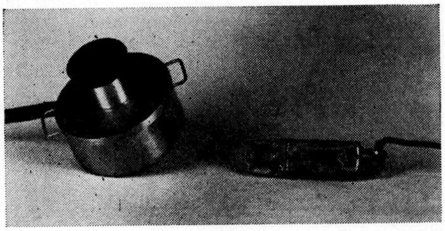

Figure 7. Pick-ups

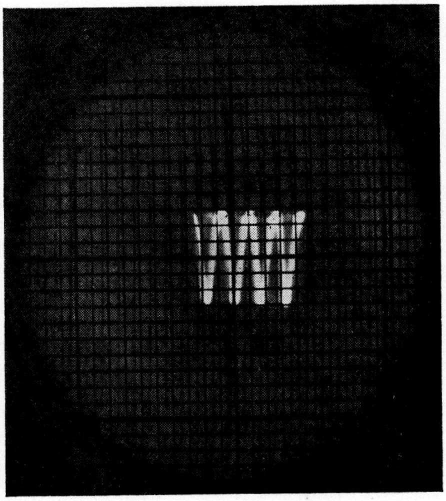

Figure 8. Sine Curve

Two types of pick-up are illustrated in Figure 7. The smaller of the two is a common phonograph pick-up; the larger is a Brush pick-up that was designed principally for use in cardiology. The Brush type is extremely sensitive and its use is presently being investigated for the purpose of decreasing the necessary driving energy.

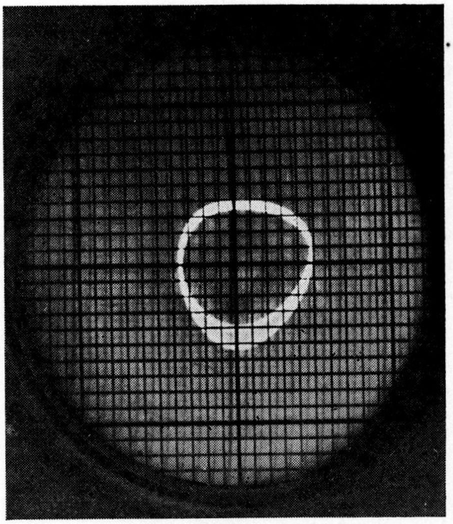

Figure 9. Full Circle

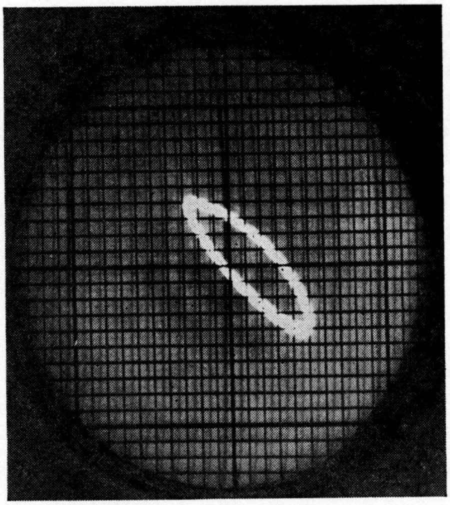

Figure 10. Skewed Circle

TABLE 2
SIGNIFICANCE OF BEAM DIMENSIONS

Beam No.	Dimensions	Dynamic Modulus $\times 10^{-6}$	
		Age 11 Days	Age 14 Days
	in.		
S-2	6 x 6 x 24	5.11	5.18
S-2	6 x 6 x 24	5.17	5.20
S-4	3 x 4 x 18	5.34	5.36
S-6	3 x 4 x 18	5.16	5.24

The pick-up output may be indicated by the oscillograph in two ways, (a) a sine wave pattern, Figure 8 and, (b) a closed or Lissajous figure, Figure 9. Since it is necessary only to determine that driving frequency which produces the greatest vertical amplitude, the sine wave pattern is usually narrowed to a single vertical line. The point of maximum amplitude is often difficult to distinguish, however, if extraneous vibrations are present.

The use of the Lissajous figure has been a practice of the Bureau of Standards and other investigators and has proven to be very superior to either the sine wave pattern or the output meter. When the beam is driven at its resonant frequency the trace is a circle or at least approximately so. A slight departure in either direction produces a skewed figure. Even severe external vibrations do not interfere with a satisfactory determination and this not only improves accuracy but, at the same time, speeds the work considerably if the building is subject to vibration.

In order to make additional savings in the cost and the time required both for testing and for preparing specimens consideration has been given to a reduction in the size of the specimens. For the present, at least, specifications must be based upon the flexural strength and it is, therefore, necessary to be able to express the results in this manner.

Assuming that the method of translating dynamic modulus into flexural strength as illustrated in Figure 2 is a valid one, it should be sufficient to determine only the dynamic modulus once the relationship is established for a given set of conditions. The accuracy of the assumption will, for the most part, vary directly with the uniformity of the conditions. The data of Table 2 are typical of the results with a poor grade of aggregate all four beams in this case being prepared from a single batch. Considerable work remains to be done before this modification of the procedure is either accepted or rejected. If it does prove to be usable, however, an important saving in the cost of making, transporting and storing specimens will result.

The Engineering Laboratory in which the work reported here was performed is part of the Office of the District Engineer, Washington District, Corps of Engineers, U. S. Army. Colonel Henry W. Wolfe, Jr., C. of E. is the District Engineer.

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