STRESSES AND DEFLECTIONS IN FLEXIBLE PIPE CULVERTS

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SYNOPSIS

Comparison of deflections of flexible pipe computed by the Iowa formula, as announced in 1941, with measured deflections secured at the University of North Carolina prior to 1929, and at Iowa State College indicates very close agreement between measured and computed values.

This recent study of the North Carolina data adds to greatly needed knowledge of the quantitative nature of the modulus of passive pressure of soil. Both the Iowa and North Carolina data support the theory that increase in lateral pressure at the sides of a pipe bears a constant relation to increase in horizontal deflection as height of fill increases, although further investigation is needed. There is also evidence, although not conclusive, that the modulus of passive pressure may vary with the inherent rigidity of a pipe, and that the modulus decreases as the pipe diameter increases.

The paper also discusses the design criterion, that deflection should be limited to some constant percentage of pipe diameter, in comparison with use of a constant value of fiber stress for the same purpose, and concludes that although the latter is more logical in some respects it is not a suitable criterion for design.

Research conducted by the Iowa Engineering Experiment Station in cooperation with the United States Public Roads Administration culminated in 1941 (*1*)[1] in the announcement of a formula by which the horizontal deflection of a flexible pipe under an earth embankment can be computed. Concurrently with the development of this formula, a number of experiments were conducted at Ames in which standard un-strutted corrugated pipe culverts were loaded with earth embankments. There were ten of these experimental pipes and they ranged in diameter from 36 in. to 60 in. The correlation between the measured deflections of the pipes and those computed by the Iowa formula was very good and the validity of the formula was believed to have been established at that time by these experiments.

The University of North Carolina (*2*) conducted a number of experiments prior to 1929 in which the loads on pipes under sand embankments were measured. In addition, the deflections of the pipes were observed and radial pressures on the pipes were measured at various points around the periphery by means of Goldbeck pressure cells. Five of the pipes used at Chapel Hill were of the flexible type and their diameters were 20 in. and 30 in. Fortunately, although this work was conducted for an entirely different purpose, the data obtained in the experiments were such that it is possible to make a comparison between the measured deflections and deflections computed by the Iowa formula for these five specimens. This comparison has recently been made and indicates a very close correlation between the measured and computed values. Confidence in the validity of the Iowa formula is materially strengthened by this fact. The purpose of this paper is to present the results of this comparison and to point out certain other indications from the North Carolina tests which are pertinent to the study of the supporting strength of flexible pipe culverts. The Iowa formula is

$$\Delta X = D_l \frac{K W_c r^3}{EI + .061\, e r^4} \qquad (1)$$

in which:

ΔX = horizontal deflection of the pipe (the vertical deflection is nearly the same at moderate values)
D_l = deflection lag factor
K = bedding constant
W_c = load per unit length of pipe
r = mean radius of pipe
E = modulus of elasticity of pipe metal
I = moment of inertia of the pipe wall, per unit length of pipe
e = modulus of passive resistance pressure of the soil sidefills, unit pressure per unit movement

[1] Italicized figures in parentheses refer to list of references at the end of the paper.

When the deflection lag factor equals unity, the formula represents the deflection of an elastic ring under the following load hypothesis:

1. The vertical load on a pipe may be determined by Marston's theory of loads on conduits and is distributed approximately uniformly over the breadth of the pipe.
2. The vertical reaction on the bottom of the pipe is equal to the vertical load and is distributed approximately uniformly over the width of bedding of the pipe.

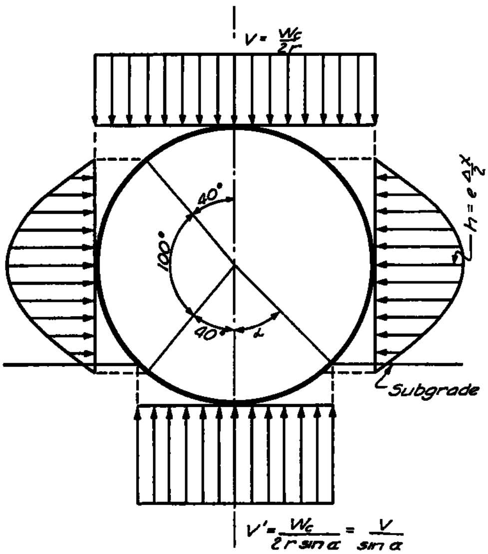

Figure 1. Assumed Distribution of Pressure on Flexible Pipe Culverts.

3. The passive horizontal pressures on the sides of the pipe are distributed parabolically over the middle 100 deg. of the pipe and the maximum unit pressure is equal to the modulus of passive pressure of the sidefill material multiplied by one-half of the horizontal deflection of the pipe.

The above load hypothesis is shown graphically in Figure 1.

In two of the Iowa experiments and all of the North Carolina experiments, the vertical loads on the pipes were measured by means of weighing levers and platform scales. In the other eight Iowa experiments the vertical loads were calculated by Marston's theory. The deflections of the pipes and the horizontal pressures at the sides were measured in all of the experiments. The values of the modulus of passive pressure of the sidefill materials were computed from these deflections and lateral pressures according to the formula

$$e = \frac{2h}{\Delta X} \tag{2}$$

in which:

h = lateral unit pressure on pipe at horizontal diameter

By substituting in the Iowa formula the measured loads, the computed values of the modulus of passive pressure, appropriate values of the bedding constant, and the physical properties of the pipes, computed values of deflections were obtained and compared with the actual measured deflections. The comparisons for the five North Carolina experiments are given in cols. 4 and 6 of Table 2. They are shown graphically for both the Iowa and North Carolina experiments in Figures 2 to 7. The close correlation between measured and computed deflections leaves little to be desired and the general correctness of the formula and the reasoning upon which it is based appear to be well established for the range of pipe sizes, gage thicknesses and construction conditions represented by these Iowa and North Carolina experiments. The pipe sizes and other pertinent information are summarized in Table 1.

Although the formula may now be accepted as valid, its application to design problems is hampered by a lack of information concerning the true quantitative nature of the modulus of passive pressure of soil. Extensive research is needed to discover the properties of soil and of the culvert pipes which influence the values of this modulus. This recent study of the North Carolina data adds somewhat to previously existing knowledge, though much remains to be learned in this field.

The Iowa experiments indicated that the modulus of passive resistance of soil is independent of the height of fill for a given culvert. That is to say, the increase in lateral pressure at the sides of a pipe bears a constant relationship to the increase in horizontal deflection as the height of fill is increased. This fact is illustrated graphically for two 42-in. pipes under two kinds of fill material (items 1 and 2, Table 1) in Figure 8. A similar study of the North Carolina data shows that this indication is upheld in three of the five cases, but in the other two the data are at variance with it, as shown in Table 2 and in Figure 9. The preponderance of available evidence, therefore, supports the constant relationship theory, but the evidence is not conclusive and the subject needs further study and experimentation. Attention is directed to the fact that in all the experiments referred to in this paper, the culvert pipes have been un-strutted; that is, they were not pre-deformed before loading and were allowed to deform freely during construction of the fill. It seems very probable that the above-mentioned constant relationship would not hold for the case of strutted pipe, although no factual evidence is available on this point.

Shafer (*3*) has suggested that the modulus of passive pressure may vary with the inherent rigidity of a pipe. There is evidence in the North Carolina experiments to support this contention, at least in the range of smaller pipe diameters. The fill material in all the North Carolina trials was a creek sand containing about 9 percent moisture. The same sand was used over and over for all the embankments and it was placed around and over the pipes in essentially the same manner each time. It is reasonable, therefore, to consider that the variations in values of the modulus of passive pressure in those experiments were independent of the sidefill material. Values of the modulus are plotted against the pipe wall stiffness in Figure 10. This plotting reveals that the modulus increased with the wall stiffness or inherent rigidity of the pipes and that the rate of increase was greater for the 20-in. diameter pipes than for the 30-in. group.

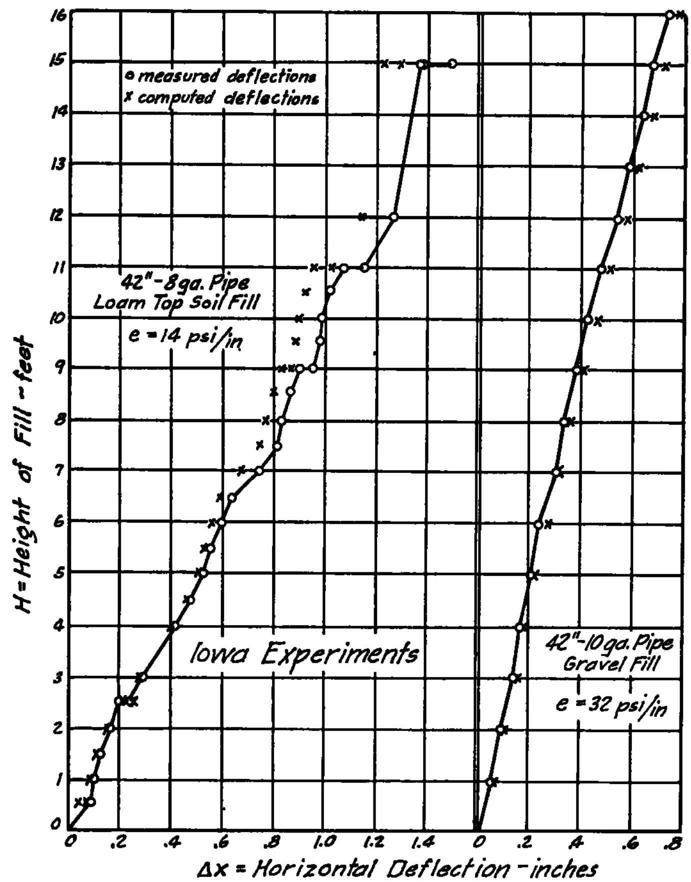

Figure 2

Also, it is indicated in Figure 10 that the modulus of passive pressure decreases as the pipe diameter increases, since the modulus was materially less in the case of the 30-in. pipes than for the 20-in. for pipes of similar wall stiffness. A rough extrapolation of the facts shown in Figure 10 relative to the effect of pipe diameters on the modulus leads to the hypothesis that the rate of decrease in the passive pressure modulus decreases rapidly as the pipe diameter increases. This indication is in harmony with the facts available from the Iowa experiments, where the modulus of passive resistance of the soil sidefills was essentially the same for the 36, 42, 48 and 60-in. pipes when the character of the sidefill materials was the same (see items 3, 5, 7, 9 and items 4, 6, 8, 10 in Table 1). However, the paucity of factual information on this point makes a definite conclusion impossible, and much further research is needed.

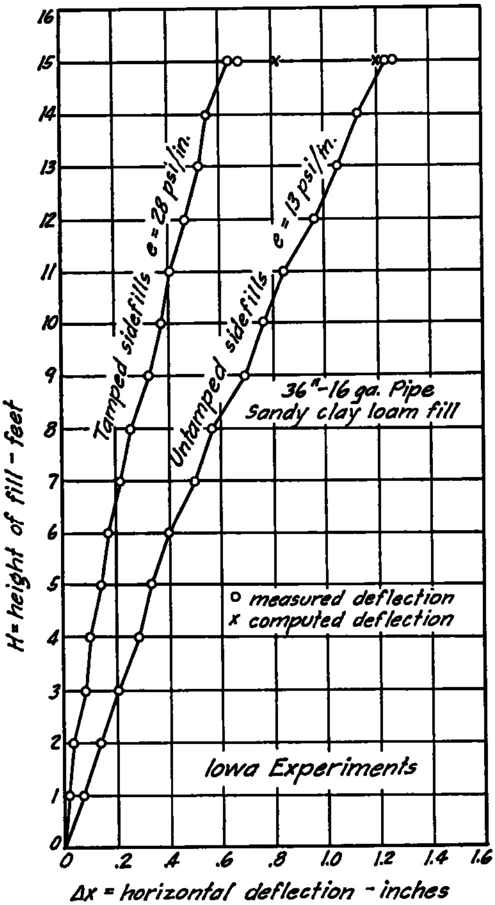

Figure 3

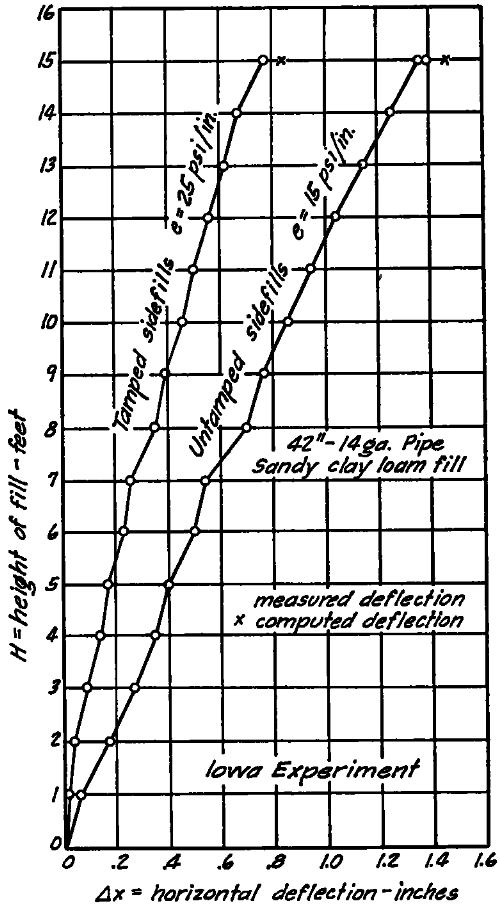

Figure 4

A question of considerable importance in the design of a flexible pipe culvert is the limiting deflection which should be allowed in design to obtain a structure which is both safe and economical. The corrugated pipe industry has for many years recommended a deflection limit of 5 percent of the nominal pipe diameter. According to Shafer (3) this recommendation is based upon observations which indicate that a pipe has reached incipient failure when it has deformed about 20 percent of its diameter. The 20 percent is divided by 4 to obtain the recommended 5 percent. However, no data are presented by the industry in support of

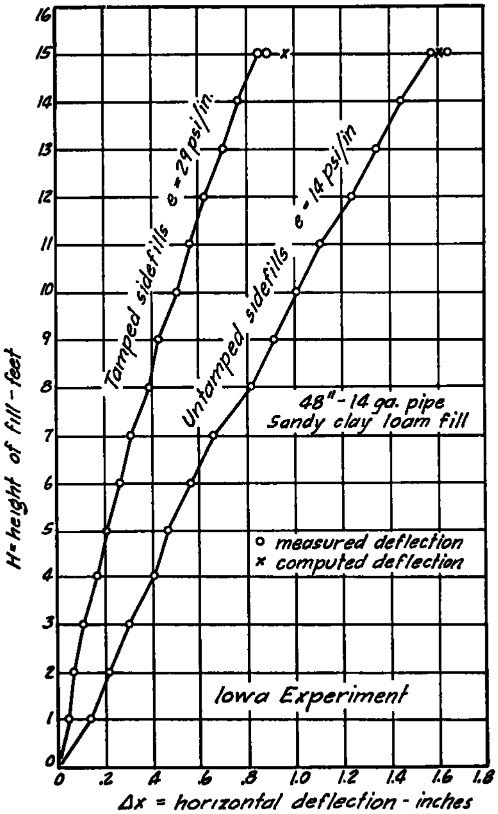

Figure 5

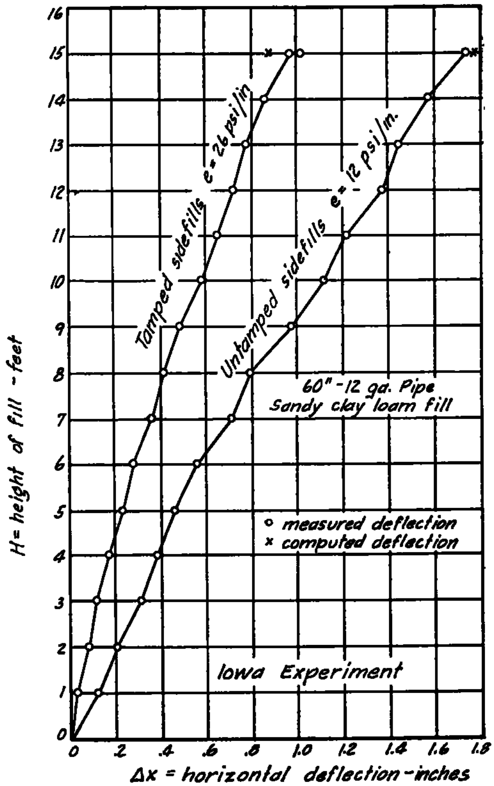

Figure 6

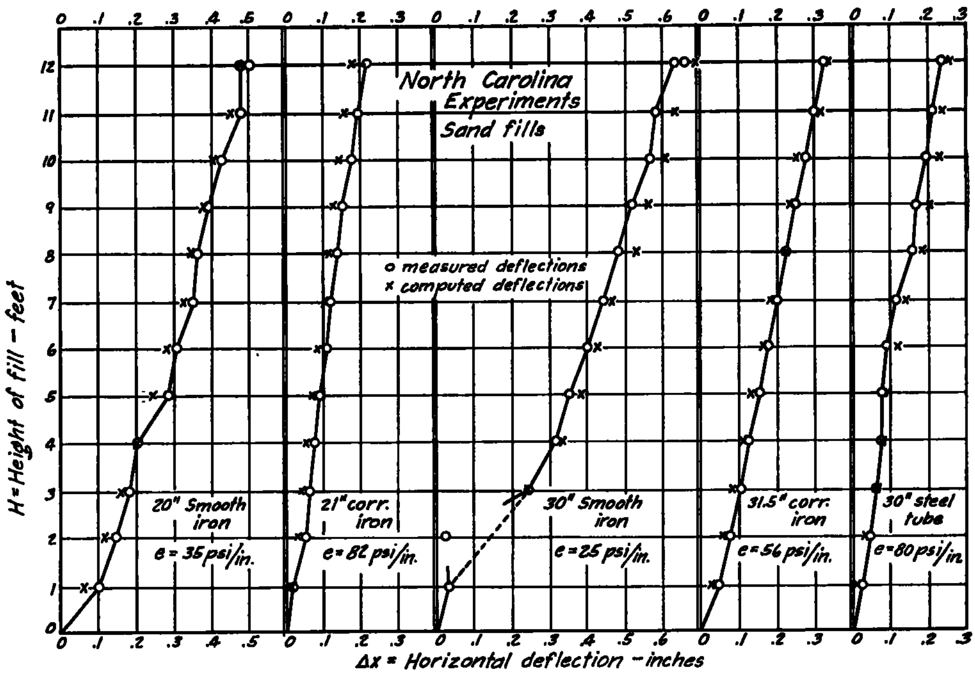

Figure 7

the claim that 20 percent deflection marks the beginning of failure and this author raises the question of whether or not pipes of all diameters have progressed the same relative distance along the path toward failure when they have deflected 20 percent. In other words, is a deflection of some constant percentage of nominal diameter a logical criterion for design of flexible culvert pipes? There is no immediate answer to this question, but some interesting facts pertinent to it may properly be discussed here.

TABLE 1
GENERAL DATA, FLEXIBLE PIPE LOAD AND DEFLECTION TESTS

Item no.	Experimental Pipes diam. & type	Wall Thickness, t, in.	Wall Stiffness, EI, lb.-in.	Sidefill soil	Ht. of fill, ft.	Av. Mod. of Passive pressure, e, psi. per in.
	Iowa Experiments					
1	42 in.—8 ga. corr. metal	0.172	165360	loam top soil	15	14
2	42 in —10 ga. corr. metal	0.141	131190	well graded gravel	16	32
3	36 in.—16 ga corr. metal	0.063	53100	sandy clay loam (tamped)	15	28
4	36 in.—16 ga corr metal	0.063	53100	sandy clay loam (untamped)	15	13
5	42 in.—14 ga. corr metal	0.078	73500	sandy clay loam (tamped)	15	25
6	42 in.—14 ga corr. metal	0.078	73500	sandy clay loam (untamped)	15	15
7	48 in.—14 ga corr. metal	0.078	73500	sandy clay loam (tamped)	15	29
8	48 in.—14 ga. corr. metal	0.078	73500	sandy clay loam (untamped)	15	14
9	60 in.—12 ga corr metal	0.109	101100	sandy clay loam (tamped)	15	26
10	60 in.—12 ga corr. metal	0.109	101100	sandy clay loam (untamped)	15	12
	North Carolina Experiments					
11	30 in. smooth iron	0.109	3240	sand	12	25
12	31.5 in. corr. metal	0.103	93330	sand	12	56
13	30 in. steel tube	0.349	106260	sand	12	80
14	20 in. smooth iron	0.076	1098	sand	12	35
15	21 in. corr. metal	0.076	67800	sand	12	82

Notes: All corrugated metal pipes were standard type having corrugations spaced $2\frac{2}{3}$ in and $\frac{1}{2}$ in. deep. For items 1, 2, 11, 12, 13, 14, 15 the modulus of elasticity of the pipe metal was assumed to be 30,000,000 psi. Values for the other items were determined by laboratory tests.

TABLE 2
MEASURED DATA AND COMPUTED DEFLECTIONS
North Carolina Experiments

1 Fill Height H ft.	2 Load W_e lb. per lin. in.	3 Horizontal Pressure h, psi.	4 Measured Hor. Defl. in.	5 Mod. of Passive Pressure e, psi. per in.	6 Computed Hor. Defl. Δ X in.
30-in. Smooth Iron Pipe					
1	19.4	1.4	.037		
2	40.0	2.4	.019		
3	56.1	3.2	.248	25.8	.241
4	70.0	3.8	.318	23.9	.323
5	81.5	4.0	.353	22.7	.394
6	93.3	4.9	.403	24.3	.418
7	105.1	5.5	.444	24.8	.468
8	117.2	5.8	.484	24.0	.538
9	129.0	6.5	.517	25.1	.568
10	137.3	7.1	.569	25.0	.607
11	151.0	7.7	.585	26.3	.635
12	162.4	8.3	.631	26.3	.711
12	163.0	9.3	.646	28.5	.636
12	161.8	8.5	.656	25.9	.691
Average				25.2	
31.5-in. Corrugated Pipe					
1	26.6	2.2	.047		
2	50.9	2.9	.078		
3	70.6	3.4	.104	65.4	.085
4	88.1	3.9	.128	61.0	.112
5	105.9	4.6	.155	59.4	.137
6	122.9	4.9	.177	55.4	.167
7	139.7	5.7	.200	57.0	.186
8	157.4	5.7	.224	50.9	.225
9	173.6	6.8	.243	56.0	.234
10	190.5	7.9	.274	57.7	.251
11	206.3	7.2	.300	48.0	.307
12	224.2	8.1	.322	50.3	.324
Average				56.1	
30-in. Steel Tube					
1	25.3	2.0	.025		
2	46.8	2.7	.044		
3	68.0	3.0	.066	91.0	.062
4	74.9	3.6	.080	90.0	.078
5	107.3	4.2	.080	105.0	.088
6	131.5	4.4	.093	94.6	.117
7	140.0	4.7	.111	84.7	.135
8	158.4	5.1	.156	65.4	.181
9	175.6	5.5	.170	64.7	.203
10	196.0	6.3	.194	65.0	.226
11	214.0	6.9	.211	65.4	.245
12	233.0	8.2	.234	70.0	.255
Average				79.6	
20-in. Smooth Iron					
1	13.4	2.0	.104		
2	26.3	2.9	.147		
3	38.1	3.6	.185	38.9	.162
4	45.6	4.2	.204	41.2	.160
5	52.0	5.1	.287	35.6	.240
6	58.9	5.2	.303	34.3	.282
7	66.0	5.8	.356	32.6	.331
8	71.6	6.1	.366	33.3	.352
9	77.0	6.4	.396	32.3	.390
10	84.1	7.1	.424	33.5	.411
11	91.5	8.0	.480	33.3	.450
12	98.0	8.0	.480	33.3	.479
12	98.0	8.3	.493	33.7	.475
Average				34.7	
21-in. Corrugated Pipe					
1	21.0	1.6	.018		
2	40.5	2.3	.045		
3	48.5	2.8	.057	98.3	.041
4	65.4	3.3	.070	94.3	.056
5	80.3	3.9	.088	88.7	.71
6	93.2	4.6	.103	89.3	.082
7	106.3	4.9	.119	82.4	.097
8	117.1	5.2	.134	77.6	.110
9	129.0	5.7	.150	76.0	.121
10	143.0	6.5	.176	74.0	.137
11	156.0	7.0	.196	71.5	.152
12	172.5	6.7	.212	63.2	.176
Average				81.5	

For a specific set of assumed conditions relative to the relationship between the height of fill over a pipe culvert and the load on the pipe produced by the fill, and for an assumed value of the modulus of passive pressure, it is possible to compute the height of fill which will cause

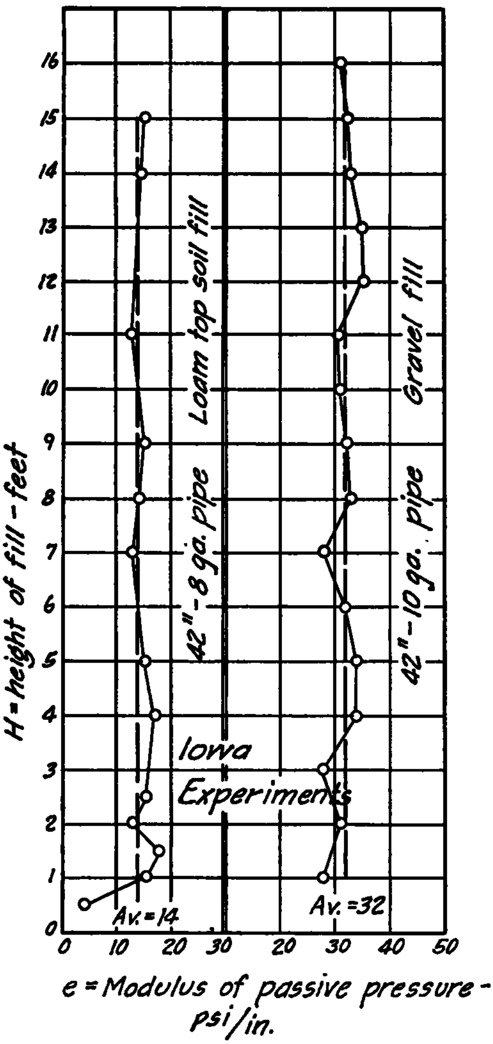

Figure 8

5 percent deflection in the case of a series of pipes having various diameters and gage thicknesses. Such computations have been made and the results are shown by the full lines in Figure 11. They show that the height of fill to produce 5 percent deflection is high for the smaller diameter pipes, decreases to a minimum value as the pipe size increases and then increases with further increase in diameter.[2] Intuition does not permit acceptance of this result since it is not reasonable to suppose that a 72-in. pipe can safely carry a higher fill than a 48-in. pipe of the same gage thickness.

In an attempt to throw further light on this matter, the author has turned to a study of the stresses in the pipe wall to see what effect variations in height of fill and pipe diameter may have on stress. Equation 25 on page 28 of reference (*1*) is an expression for the bending moment at the bottom of a pipe. It is as follows:

$$M_c = K'W_c r - .166\,hr^2 \qquad (3)$$

in which:

M_c = bending moment at the bottom of the pipe

K' = constant dependent upon the bedding angle α (See Fig. 1)

$$K' = .053 \sin^2 \alpha + .04 \frac{\alpha}{\sin \alpha} - .02 \frac{\sin 2\alpha}{\sin \alpha} - .08 \sin \alpha(\pi - \alpha) + .159 \cos \alpha + .135.$$

Values of the constant K' for various values of the bedding angle α are shown in Figure 12.

By means of the flexure formula and eq. 3, it is possible to compute the outer fiber stress due to bending moment at the bottom of a pipe for any condition of load and lateral pressure. There is also a horizontal thrust at the bottom of a pipe, but this thrust contributes only a very small percentage of the total fiber stress. Since the study being reported here is only qualitative in nature, the stresses due to thrust have been neglected as a simplifying measure.

The fiber stress in a 42-inch, 12-gage pipe at 5 percent deflection was arbitrarily chosen as a representative value of stress for the range of pipe sizes and gage thicknesses used in this study. The height of fill for these conditions is 21.3 feet and is shown at point A in Figure 11. The height of fill to produce this arbitrarily chosen fiber stress in all other pipes has been computed. These results are shown by the dashed lines in the figure. Also, the percentage deflection and the deflection in inches are shown in figures on the diagram. These lines show a marked reduction in the height of

[2] These facts were first brought to the author's attention by Mr. de Capiteau of the Toncan Culvert Manufacturers Association and later pointed out by Kelley (*4*).

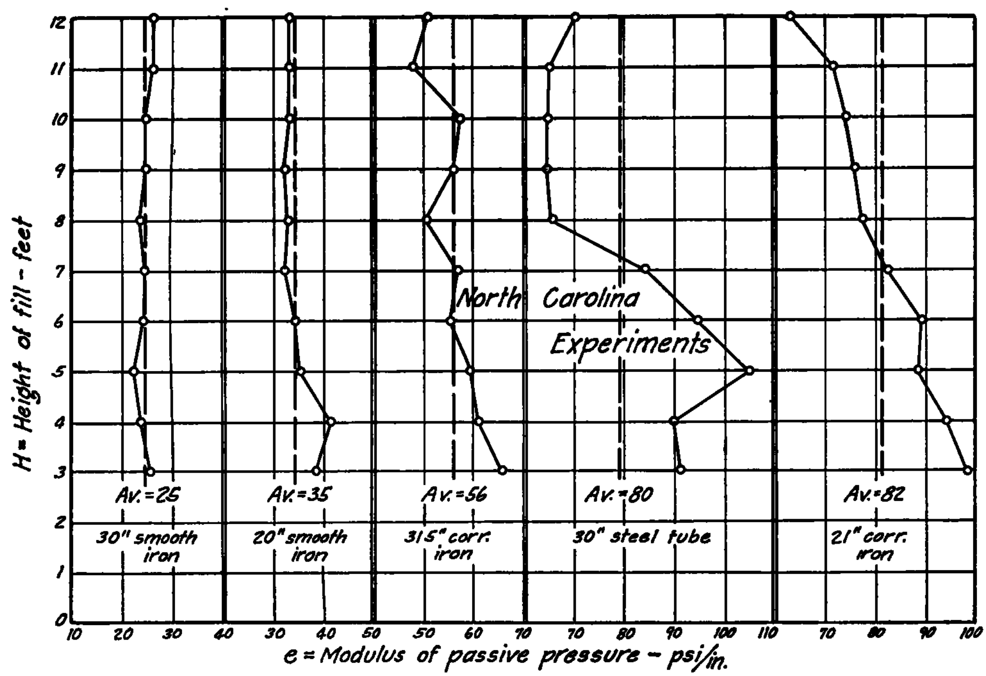

Figure 9

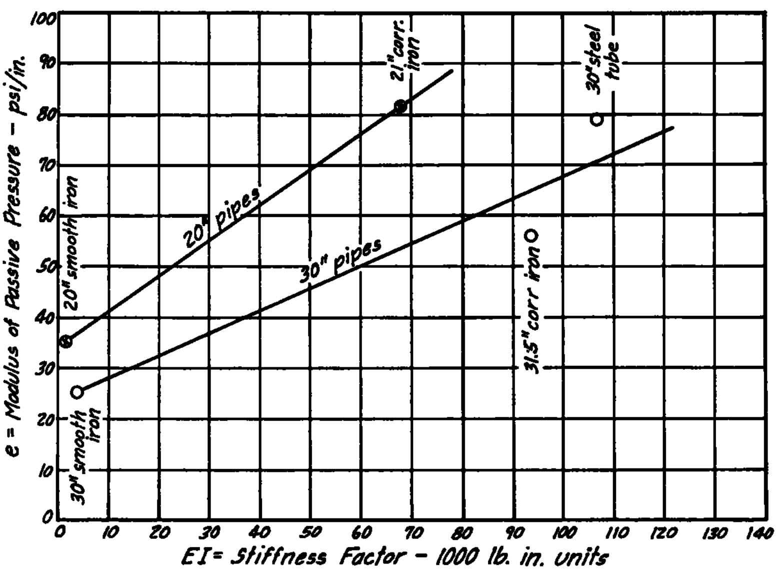

Figure 10

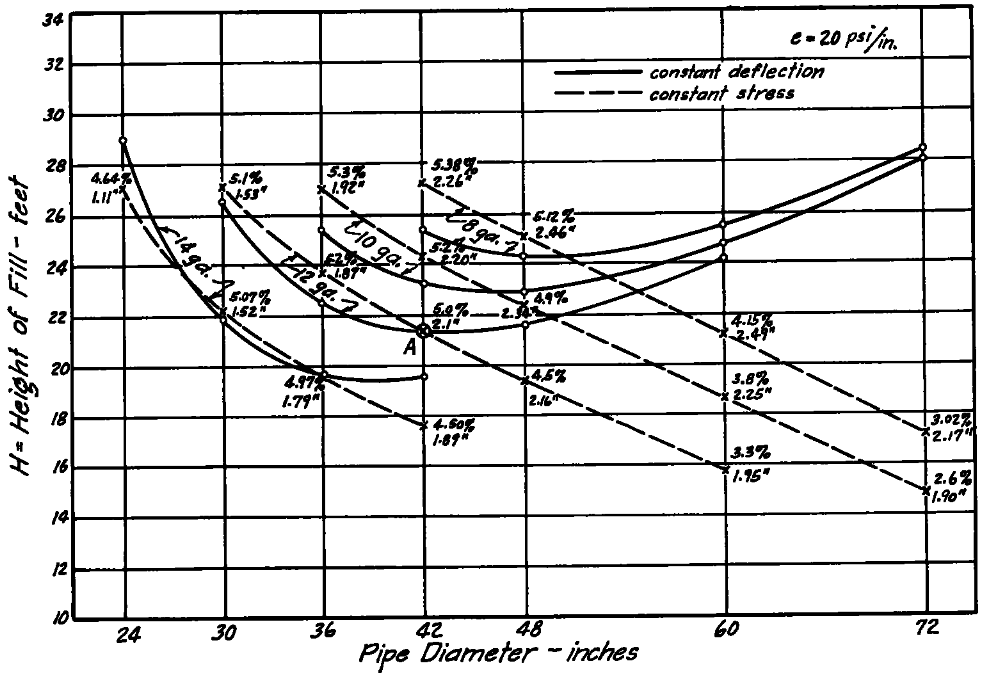

Figure 11

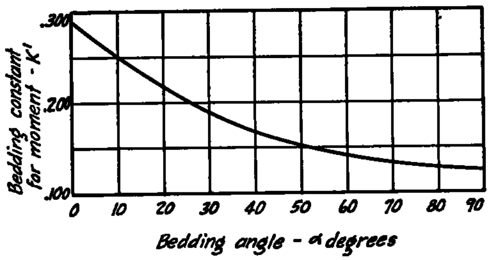

Figure 12

fill as the diameter of pipes increases, which seems at first glance to be quite reasonable. However, closer examination reveals that the deflection in inches tends to increase with pipe diameter until a maximum value is reached and then begins to decrease with further increase in diameter. This situation also violates intuitive judgment and it appears that constant value of fiber stress, though more logical in some respects than a constant percentage of diameter deflection, is not a suitable criterion for design purposes. Perhaps the true criterion lies somewhere between these two. At any rate it seems clear that much further study and experimental evidence will be required before a rational and acceptable criterion for an allowable deflection limit can be evolved.

REFERENCES

1. Spangler, M. G. "The Structural Design of Flexible Pipe Culverts." Iowa Engr. Exp. Sta., Bul. 153. 1941.
2. Braune, G. M., Wm. Cain and H. F. Janda. "Earth Pressure Experiments on Culvert Pipe." *Public Roads*, Vol. 10, No. 9, Nov., 1929.
3. Shafer, G. E. Discussion of "Underground Conduits—An Appraisal of Modern Research." *Proceedings* A.S.C.E., Vol. 74, No. 2, p. 267. Feb., 1948.
4. Kelley, E. F. Discussion of "Underground Conduits—An Appraisal of Modern Research." *Proceedings* A.S.C.E., Vol. 74, No. 3, Part 1, p. 381. March, 1948.

DISCUSSION

Mr. G. E. Shafer, *Armco Drainage and Metal Products, Inc.*—Because the writer discussed an A.S.C.E. paper of Mr. Spangler's on the design of flexible pipe,[1] it follows that this current paper, which is a continuation of the same subject, should also be discussed.

It seems Mr. Spangler is well satisfied that his equation is valid and does predict deflections of flexible pipe under certain conditions. He recognizes that its application to the actual design of flexible pipe is hampered by lack of information on the true quantitative nature of the passive resistance factor "e" and the working deflection of flexible pipe. He has made progress in recognizing that "e" in addition to being a "function of the properties of the soil, particularly the density"[2] may be a function of pipe rigidity or fill height or both. In the writer's opinion it would be worthwhile for the author to question the soundness of his "loading hypothesis" upon which his equation is based in addition to trying to find new limits of design based on deflection or stress. The author is aware that the equation and theory produce results that violate intuitive judgment, yet has not considered, presumably, a new equation based on a different loading hypothesis, which recognizes that the center of a pipe varies with deflection. This idea was discussed by the writer in the A.S.C.E. reference.[1] Such an equation might satisfy both the experimental data and a reasonable design criteria.

Mr. Spangler has introduced the idea that the modulus of passive resistance varies with pipe diameter, decreasing as the pipe diameter increases. This may be true but the writer believes that the "stiffness factor" (EI) is not the proper abscissa for Figure 10 in the author's paper. The product of EI may be part of the "stiffness factor" but the length of the section, or in the case of a pipe, the circumference or diameter should be taken into consideration.

If the elastic equation No. 14 from Spangler's Bulletin[2] for concentrated top and bottom load on a pipe is used to determine the inherent strength of the five North Carolina test pipes at 5 percent deflection, which is admittedly slightly beyond the elastic limit, and those loads or strengths are used instead of EI as abscissa in Figure 10, the results are quite different. Figure A shows "e" a function of pipe rigidity but not necessarily a function of diameter. The four Iowa test pipe the author refers to as producing data in harmony with his idea that "e" varies with diameter are of so near the same inherent strength that they cannot be used to predict values of "e" outside their range.

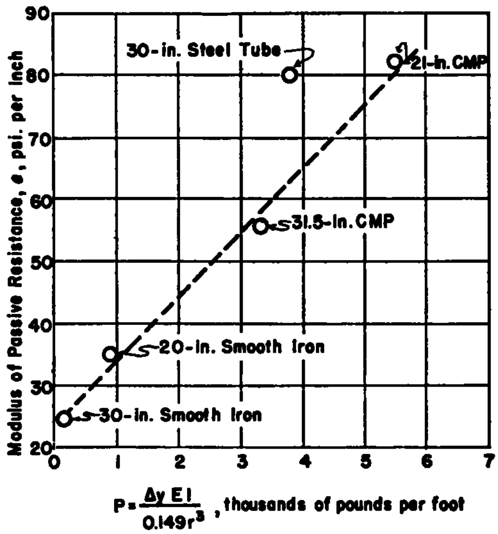

Figure A

Mr. Spangler rather chides the corrugated metal pipe industry for not having produced sufficient data to convince him that 20 percent deflection is the point of incipient failure and that 5 percent is a good design deflection for all diameters. The writer understands that when the A.R.E.A. (Farina) test was planned, the diameters and gages of corrugated pipe were so selected that it was thought the weakest pipe (48-in. 10-gage unstrutted) would be close to failure under the 35-ft. cover. The deflection was approximately 10 percent or about half the assumed ultimate. The fact that no corrugated pipe was tested to failure by the A.R.E.A., Iowa or North Carolina, as

[1] Underground Conduits—An Appraisal of Modern Research, *Proceedings* A.S.C.E. Vol. 73, No. 6, Part 1, page 855, June, 1947. Discussion by Shafer, Vol. 74, No. 2, page 267 February, 1948.

[2] Iowa Eng. Exp. Station Bulletin No. 153, "The Structural design of Flexible Pipe Culverts" by Spangler, Conclusion No. 5.

well as the few structural failures in the field, has materially hampered collecting the data the author desires. The excessive cost of the high fills necessary to produce failure has undoubtedly restrained research in this field.

The writer appreciates Mr. Spangler's calling attention to the fact that the factors affecting the design of strutted pipe may be different than for unstrutted pipe. The same might be said regarding the design of pipe under shallow cover where the principal load is due to dynamic loading.

MR. SPANGLER, *Closure:* Mr. Shafer suggests that the author should re-examine the load hypothesis upon which the Iowa formula for deflection of flexible culvert pipes is founded, and develop a new formula based upon a system of loading in which the horizontal pressures are assumed to be unsymmetrical about the horizontal axis. This, in the author's opinion, is asking for a degree of perfection which is neither necessary nor justified. The correlation between measured deflections and those computed by the Iowa formula in the case of fifteen individual experimental pipes, as reported in the paper, is sufficiently close to render further effort toward refinement practically valueless; unless and until competent evidence of a conflicting nature becomes available.

At least it can be said that the Iowa formula yields deflections which are very much nearer the actual measured values for the experimental pipes than does the only other formu'a of which the author is cognizant, i.e. the empirical formula recommended by Armco Drainage and Metal Products, Inc.[3] Evaluation of the empirical formula gives deflections which vary widely from those actually measured. This result is to be expected, since the empirical formula does not take into account any differences in magnitude of lateral pressures which develop at the sides of flexible pipe culverts due to differences in kind of soil, degree of soil compaction, amount of lateral movement, and other factors which influence the lateral pressures.

Mr. Shafer has misinterpreted the statement in the paper relative to the fact that certain computations give results which violate intuitive judgment. It is the assumption of a uniform limiting value of deflection equal to 5 percent of pipe diameter for pipes of all diameters which leads to unreasonable results, not the formula or the theory upon which it is based.

The balance of Mr. Shafer's discussion tends to confirm the author's stated viewpoint that much further research is needed to develop factual information concerning the modulus of passive resistance pressure of various soil sidefills and to provide the basis for a logical criterion for an allowable limit of deflection in design.

[3] Armco Drainage and Metal Products, Inc. "Handbook of Culvert and Drainage Practice," 1947, p. 65.